REF.TH.3405-CERN

BARYONS IN QCD AND CHIRAL SYMMETRY BREAKING PARAMETERS

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ABSTRACT

We calculate all baryons in the 56 representation using QCD sum rules. All masses are well predicted and require stringent values of the chiral parameters $\langle 0 | \bar{u}u | 0 \rangle = -(230 \pm 15 \text{ MeV})^3$ and $\gamma = \langle \bar{s}s \rangle / \langle \bar{u}u \rangle - 1 = -0.19 \pm 0.02$. The determination of γ is the most precise to date, the strange quark mass and the quark condensate are also accurately fixed.

Recently there has been remarkable progress in calculating baryon masses in QCD. Ioffe [1] has pointed out that in the QCD sum rule scheme $M_B^3 \sim \langle 0 | \bar{q}q | 0 \rangle$ in the limit of massless quarks. In more recent papers his original results have been corrected [2] and expanded [3] to include several other states of the 56 representation.

In this letter we present a calculation of all states in the 56 representation. We confirm that all states can be obtained with only one new parameter $\gamma = \langle 0 | \bar{s}s | 0 \rangle / \langle 0 | \bar{u}u | 0 \rangle - 1$. It turns out that the octet baryons provide the best case for determining this chiral symmetry breaking parameter which was hitherto undetermined. Its value $\gamma = -0.19$ is interesting and unexpected.

We start with the choice for the nucleon current made by Ioffe [1]

$$\eta_N(x) = \epsilon_{abc} (u^a(x) C \gamma_\mu u^b(x)) \gamma_5 \gamma_\mu d^c(x) \quad (1)$$

Considering only currents without derivatives there is one more current with the same quantum numbers and with a non-relativistic limit

$$\eta'_N(x) = \epsilon_{abc} (u^a(x) C \sigma_{\mu\nu} u^b(x)) \sigma_{\mu\nu} \gamma_5 d^c(x) \quad (2)$$

Ioffe argues that the coupling of the nucleon to Eq. (2) should be small since the polarization operator induced by this current does not get any contribution from the lowest dimensional chiral symmetry breaking operators in the operator product expansion.

Starting from Eq. (1) it is straightforward to derive the currents which correspond to the other $J^P = \frac{1}{2}^+$ octet baryons

$$\eta_\Sigma(x) = \epsilon_{abc} (u^a(x) C \gamma_\mu u^b(x)) \gamma_5 \gamma_\mu s^c(x) \quad (3a)$$

$$\eta_\Lambda(x) = \sqrt{\frac{2}{3}} \epsilon_{abc} \left[(u^a(x) C \gamma_\mu s^b(x)) \gamma_5 \gamma_\mu d^c(x) - (d^a(x) C \gamma_\mu s^b(x)) \gamma_5 \gamma_\mu u^c(x) \right] \quad (3b)$$

$$\eta_\Xi(x) = -\epsilon_{abc} (s^a(x) C \gamma_\mu s^b(x)) \gamma_5 \gamma_\mu u^c(x) \quad (3c)$$

The lowest dimensional currents for the decuplet resonances are unique and can be written as follows

$$\eta_{\Delta}^{\mu} = \epsilon_{abc} (u^a(x) C \delta_{\mu} u^b(x)) u^c(x) \quad (4a)$$

$$\eta_{\Sigma^{*}}^{\mu} = \sqrt{\frac{1}{3}} \epsilon_{abc} \left[2(u^a(x) C \delta_{\mu} s^b(x)) u^c(x) + (u^a(x) C \delta_{\mu} u^b(x)) s^c(x) \right] \quad (4b)$$

$$\eta_{\Xi^{*}}^{\mu} = \sqrt{\frac{1}{3}} \epsilon_{abc} \left[2(s^a(x) C \delta_{\mu} u^b(x)) s^c(x) + (s^a(x) C \delta_{\mu} s^b(x)) u^c(x) \right] \quad (4c)$$

$$\eta_{\Omega}^{\mu} = \epsilon_{abc} (s^a(x) C \delta_{\mu} s^b(x)) s^c(x) \quad (4d)$$

To derive sum rules for these baryons we apply as usual [4] the operator product expansion to the two-point functions induced by the currents given above. It has been observed by Ioffe [1] and Chung et al. [5] that the dominant power corrections come from chiral symmetry breaking two- and four-quark operators. Let us first consider the nucleon case. The two-point function has two invariant amplitudes

$$i \int d^4x e^{ipx} \langle 0 | T(\eta_N(x) \bar{\eta}_N(0)) | 0 \rangle = \not{p} F_1(p^2) + F_2(p^2) \quad (5)$$

The total number of dimensions on the left-hand side is odd. This implies that $F_1(p^2)$ has an even number of dimensions and $F_2(p^2)$ an odd number. Therefore, contributions to $F_2(p^2)$ from even dimensional operators in the OPE will necessarily be multiplied by a small quark mass while the odd dimensional quark condensate operator $\bar{q}q$ will get no small quark mass factor and will dominate. This constitutes an important difference with the meson case (see Refs. [4] and [6]). The invariant function $F_1(p^2)$ gets contributions from all operators. The contribution from $G_{\mu\nu}^a G_{\mu\nu}^a$ has explicitly been calculated by Smilga in Ref. [2]. It is responsible for about 10% of the mass of the nucleon and should therefore be taken into account. At present we neglect first order α_s perturbative corrections, which we hope to calculate later. However, they can be absorbed in the continuum contribution and reliable mass estimates can be obtained without them.

Our procedure is identical to the one used before for light quark mesons [4,7]. Since there are two independent tensor structures, we get two independent sum rules. We have calculated the various Wilson coefficients, saturated the left-hand side of Eq. (5) by the nucleon with a coupling λ_N to the current and taken the Borel transform, which gives [2]

$$M^6 - bM^2 + \frac{4}{3}a^2 = 2(2\pi)^4 \lambda_N^2 e^{-M_N^2/M^2} \quad (6a)$$

$$2aM^4 = 2(2\pi)^4 M_N \lambda_N^2 e^{-M_N^2/M^2} \quad (6b)$$

where $a = -(2\pi)^2 \langle 0 | \bar{q}q | 0 \rangle$ and $b = \pi^2 \langle 0 | \alpha_s / \pi G_{\mu\nu}^a G_{\mu\nu}^a | 0 \rangle$. The coupling λ_N can be eliminated by taking the ratio of Eqs. (6b) and (6a) which results in an equation for M_N as a function of M^2 . To analyse the sum rules (6) we introduce a continuum contribution with a threshold s_0 in the form of a θ -function to account for higher resonances in the sum rule. We only include a continuum contribution in Eq. (6a). The reasons for this are that a) in sum rule (6b) opposite parity states tend to cancel [1,5] and b) the continuum contribution can be absorbed in a slight change in a . In the sum rule (6a) we put the explicit Borel transform of the perturbative quark cross-section which a) gives again a small contribution and b) mimics the perturbative contribution from gluon exchange. Therefore we get

$$M_N(M^2) = \frac{2aM^4}{M^6 \left[1 - e^{-s_0/M^2} \left(\frac{s_0^2}{2M^4} + \frac{s_0}{M^2} + 1 \right) \right] - bM^2 + \frac{4}{3}a^2} \quad (7)$$

In Fig. 1 we have plotted M_N as a function of M^2 for a few values of s_0 and for $a = 0.5$ [i.e. $\langle 0 | \bar{q}q | 0 \rangle = -(230 \text{ MeV})^3$] and $b \approx 0.13$ as determined from charmonium [4,6] or lattice calculations [8]. We see from Fig. 1 that a plateau develops for $s_0 \approx 5 \text{ GeV}^2$ at a mass value around 0.9 GeV and that inclusion of the continuum changes the prediction for the mass by about 5%. This determines the accuracy of our calculations, and in the following we will not take any continuum contributions into account in the sum rules, and give the maximum of the curve as our prediction for the mass.

Taking also corrections to first order in the strange quark mass m into account, we can derive sum rules like (6) for the other members of the baryon octet which results in the equations:

$$M_N(M^2) = \frac{2aM^4}{M^6 - bM^2 + \frac{4}{3}a^2} \quad (8a)$$

$$M_\Lambda(M^2) = \frac{-2mM^6 + 2a(3-\gamma)M^4 + \frac{8}{3}a^2m(2-\gamma)}{3M^6 + 2am(1-3\gamma)M^2 - 3bM^2 + \frac{4}{3}a^2(3+4\gamma)} \quad (8b)$$

$$M_\Sigma(M^2) = \frac{2mM^6 + 2a(1+\gamma)M^4 + \frac{8}{3}a^2m}{M^6 - 2am(1+\gamma)M^2 - bM^2 + \frac{4}{3}a^2} \quad (8c)$$

$$M_\Xi(M^2) = \frac{2aM^4 + 4a^2m(1+\gamma)}{M^6 - bM^2 + \frac{4}{3}a^2(1+\gamma)^2} \quad (8d)$$

where $\gamma = [\langle 0|\bar{s}s|0\rangle - \langle 0|\bar{u}u|0\rangle]/\langle 0|\bar{u}u|0\rangle$. A check of these formulae is given by keeping the octet pieces of the symmetry breaking which gives the Gell-Mann-Okubo mass formula. Investigating these formulae one finds that the mass splittings are very sensitive to the parameter γ . We will come back to this later. We have kept γ -corrections in the first order m terms for completeness although they are of higher order.

Before matching the equations for the octet baryons to the experimental masses by varying m , a , and γ , we also include the decuplet resonances given by the currents (4). Since we deal with $J = 3/2$ states the tensor structure is more complicated. We saturate the phenomenological side of the sum rule by a single resonance and use the Rarita-Schwinger formalism for the propagator. On the theoretical side there is a large number of invariant functions but the two proportional to $g_{\mu\nu}$ and $\not{p}g_{\mu\nu}$ receive contributions from $J = 3/2$ states only (at least for massless quarks). We will therefore write sum rules by equating the $g_{\mu\nu}$ and $\not{p}g_{\mu\nu}$

structures on both sides. For more details see Ref. [1]. The sum rules for the Δ read

$$M^6 - \frac{25}{18} b M^2 + \frac{20}{3} a^2 = 5 (2\pi)^4 \lambda_\Delta^2 e^{-M_\Delta^2/M^2} \quad (9a)$$

$$\frac{20}{3} a M^4 (1 - \frac{1}{2} \frac{m_0^2}{M^2}) = 5 (2\pi)^4 M_\Delta \lambda_\Delta^2 e^{-M_\Delta^2/M^2} \quad (9b)$$

where the parameter m_0 is defined as

$$\langle 0 | g_s \bar{q} \sigma_{\mu\nu} \frac{\lambda^a}{2} q G_{\mu\nu} | 0 \rangle \equiv m_0^2 \langle 0 | \bar{q} q | 0 \rangle \quad (10)$$

For the octet baryons the coefficient of this operator vanishes [2]. In Ref. [9] m_0^2 has been estimated to be 0.5-1 GeV² from analysing open charm states, so its contribution in Eq. (9b) cannot be neglected. We will treat it as a free parameter restricted between 0.5 and 1 GeV². In Eqs. (9) we have again included the gluon condensate contribution calculated by Smilga [2].

Taking the ratio of Eqs. (9b) and (9a) we find the following equation for M_Δ as a function of M^2

$$M_\Delta(M^2) = \frac{\frac{20}{3} a M^2 (1 - \frac{1}{2} \frac{m_0^2}{M^2})}{M^6 - \frac{25}{18} b M^2 + \frac{20}{3} a^2} \quad (11a)$$

Similarly we find for the other decuplet resonances

$$M_{\Sigma^*}(M^2) = \frac{\frac{20}{3} a M^4 (1 + \frac{1}{3} \gamma) (1 - \frac{1}{2} \frac{m_0^2}{M^2}) + \frac{5}{2} m M^6 + \frac{10}{3} a^2 m}{M^6 + 5 a m (1 - \frac{1}{3} \gamma) M^2 - \frac{25}{18} b M^2 + \frac{20}{3} a^2 (1 + \frac{2}{3} \gamma)} \quad (11b)$$

$$M_{\Xi^*}(M^2) = \frac{\frac{20}{3} a M^4 (1 + \frac{2}{3} \gamma) (1 - \frac{1}{2} \frac{m_0^2}{M^2}) + 5 m M^6 + \frac{20}{3} a^2 m (1 + \gamma)}{M^6 + 10 a m (1 + \frac{1}{3} \gamma) M^2 - \frac{25}{18} b M^2 + \frac{20}{3} a^2 (1 + \gamma) (1 + \frac{1}{3} \gamma)} \quad (11c)$$

$$M_{\Omega}(M^2) = \frac{\frac{20}{3} a M^4 (1 + \gamma) (1 - \frac{1}{2} \frac{m_0^2}{M^2}) + \frac{15}{2} m M^6 + 10 a^2 m (1 + \gamma)^2}{M^6 + 15 a m (1 + \gamma) M^2 - \frac{25}{18} b M^2 + \frac{20}{3} a^2 (1 + \gamma)^2} \quad (11d)$$

The simplicity of the structure of these formulae reflects the fact that for the decuplet all quark pairs are in a spin one state.

At this stage we have expressions for eight masses as a function of M^2 and the parameters a , b , m , m_0 and γ . These parameters are not completely free; a is restricted by current algebra to lie between 0.3 and 0.7 GeV^3 [$\langle 0|\bar{u}u|0\rangle^{1/3} \approx -(200-260) \text{ MeV}$], m_s should be about 150 MeV, b has been determined before from charmonium [4,6] and should be about 0.13 GeV^4 , m_0^2 has been estimated to be 0.5-1 GeV^2 , and the absolute value of γ is expected to be not larger than 0.3, since it measures SU(3) symmetry breaking. As we neglect all continuum contributions to the sum rules we cannot expect our mass values to agree better than within five per cent with the experimental values.

As pointed out above the octet mass splittings are extremely sensitive to the value of γ . In Fig. 2 we have plotted the octet masses as a function of γ . It can be seen that the correct range of levels can only be obtained for $0.17 < -\gamma < 0.21$. This is essentially independent of reasonable changes in the other parameters. The decuplet masses are not sensitive to γ .

In Table 1 we have listed the results for the masses for a few values of a . For each value of a , m_0 has been adjusted to give the best possible values for the decuplet masses. The mass values in Table 1 correspond to the maxima (as a function of M^2) of the sum rules (8) and (11). It can be seen from the table that it is not difficult to get predictions for the masses within 5% of the experimental values. The results are not sensitive to the mass of the strange quark and the value of b ; γ has been kept fixed at -0.19. We arrive at the following set of parameter values

$$\begin{aligned}
 a &= 0.4-0.6 \text{ GeV}^3, \text{ i.e., } \langle 0|\bar{u}u|0\rangle = -(230 \pm 15 \text{ MeV})^3 \\
 b &= 0.15 \text{ GeV}^4, \text{ i.e., } \langle 0|\frac{\alpha_s}{\pi} G_{\mu\nu}^a G_{\mu\nu}^a|0\rangle = (350 \text{ MeV})^4 \\
 m &= 140 \pm 20 \text{ MeV} \\
 m_0^2 &= 0.5-1.0 \text{ GeV}^2 \\
 \gamma &= -0.19 \pm 0.02, \text{ i.e., } \langle 0|\bar{s}s|0\rangle = 0.8 \langle 0|\bar{u}u|0\rangle
 \end{aligned} \tag{12}$$

The parameter γ has been argued to be positive [10]. The argument relies on positivity and the first order mass effect in chiral perturbation theory. However, our definition of the condensate in which all perturbative contributions are subtracted is not the same*). It is amusing that the heavy quark mass expansion of $\langle 0 | \bar{s}s | 0 \rangle$ also gives $\gamma < 0$ at the value of m_s . Also, using other sum rules Mallik [11] has obtained a rough value of γ in agreement with our result in sign and magnitude.

We conclude that QCD sum rules naturally explain the levels in baryon systems of light quarks to a remarkable accuracy (5%) in the lowest order approximation in the operator expansion. Our main result here, however, is the observation that the appearance of baryon masses and mass splittings is very sensitive to some parameters that characterize spontaneous chiral symmetry breaking (a) and SU(3) symmetry breaking (γ). The theory fixes $\langle 0 | \bar{u}u | 0 \rangle = -(230 \pm 15 \text{ MeV})^3$ and $\gamma = -0.19 \pm 0.02$. These quantities can be reinterpreted in the f and d couplings which are the basis of octet dominance for symmetry breaking. In fact, the negative sign of γ is due to the fact that the splitting between Ξ and N (f -type) is much larger than the one between Σ and Λ (d -type). QCD is therefore able to predict dynamically the strength of the two octet components, as well as the absolute baryon masses, the distance between the octet and decuplet, m_s and γ . Confirmation of the values of these parameters elsewhere is important.

*) One of us (HRR) thanks H. Leutwyler and P. Minkowski for enlightening discussions on this point.

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Table 1

Results for the octet and decuplet masses for a few values of a ;
 m_0^2 has been adjusted to give the best possible decuplet masses;
 m is fixed at 140 MeV, b at 0.15 GeV^4 and γ at -0.19 .

	$a = 0.4$ $m_0^2 = 0.5$	$a = 0.5$ $m_0^2 = 0.7$	$a = 0.6$ $m_0^2 = 0.9$	exp.
M_N	0.85	0.88	0.90	0.94 GeV
M_Λ	1.03	1.03	1.06	1.12
M_Σ	1.24	1.19	1.18	1.19
M_Ξ	1.39	1.37	1.35	1.32
M_Δ	1.22	1.23	1.24	1.23 GeV
M_{Σ^*}	1.34	1.36	1.38	1.38
M_{Ξ^*}	1.48	1.52	1.54	1.53
M_Ω	1.67	1.70	1.74	1.67

Figure captions

- Fig. 1 : Plot of the nucleon mass M_N as a function of M^2 [Eq. (7)] for a few values of the continuum threshold s_0 (x: $s_0 = 4.5 \text{ GeV}^2$, o: $s_0 = 5.25 \text{ GeV}^2$, +: $s_0 = \infty$); $a = 0.5 \text{ GeV}^3$ and $b = 0.13 \text{ GeV}^4$.
- Fig. 2 : Plot of the octet masses as a function of γ ($a = 0.6 \text{ GeV}^3$, $m = 140 \text{ MeV}$, and $b = 0.15 \text{ GeV}^4$).

Fig. 1

Fig. 2