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MASS EFFECTS IN QCD CORRECTIONS OF WEAK DECAYS

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ABSTRACT

QCD corrections to the weak decay rate of a quark into three quarks are calculated taking full account of all the masses involved. The corrections are quite sensitive to the masses and change sign in most of the cases. Unexpectedly enough, in the whole range of the final fermion masses, the effects substantially enhance the rate compared to its zero-mass limit.

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We report new results of the first-order gluonic corrections to the total weak decay rates in which the masses of all the final fermions involved are taken into account. If some or all of the fermion masses are neglected, such corrections in both quantum electrodynamics ¹⁾ and quantum chromodynamics (QCD) ²⁾⁻⁴⁾ have already been computed. However, complete results for the case of entirely massive particles do not seem to be available. Yet mass effects in decay processes involving heavy quarks and/or heavy leptons can hardly be considered small compared, for example, with radiative corrections. At first sight, one might suspect that the mass effects tend to suppress the width, by the phase-space considerations. Our results indicate that actually they act in the opposite direction, thus substantially increasing the rate compared to its zero-mass limit.

In the following, we shall specifically consider the weak decay of a heavy fermion Q into three other less massive fermions $q_1, q_2, \bar{q}_3$. Our results will serve not only to give an accurate estimate of the rate for this type of decay, but also, when supplemented by appropriate energy distributions ⁵⁾, to determine all unknown masses. Moreover, the techniques of computation developed here can be extended with few modifications to study different processes: W, Z gauge bosons widths ⁶⁾, cross-sections of $e^+ + e^- \rightarrow$ hadrons at new thresholds, Drell-Yan production of heavy lepton pairs, quarkonia disintegrations into heavy fermion pairs, etc. . Our calculation generalizes the work of the authors of Ref. 3) who limit themselves to the zero-mass case. Here, as in that reference, a four-fermion weak interaction operator with non-renormalized coupling constant serves as the starting point of the calculation. Although one may argue that the renormalization effects due to short-range gluons are attenuated with an increasing mass scale and therefore that the present results can be also applicable to non-leptonic decays of heavy quarks, one must keep in mind that a more complete treatment of the problem must include contributions arising from anomalous dimension terms and resummation of leading logarithms in the effective weak Hamiltonian. Altarelli et al. ⁴⁾ have recently performed such a calculation for massless final quarks. They demonstrated in particular that the next-to-leading logarithm terms show the same enhancement-suppression pattern as the leading terms and are especially important in charm decay.

Let us then consider the decay process $Q(M) \rightarrow q_1(m_1) + q_2(m_2) + \bar{q}_3(m_3)$, where particle names and momenta are called by the same symbols and their respective masses shown in parentheses. The decay amplitude to the lowest-order in the Fermi coupling G , graphically shown in Fig. 1a, is given by a product of an "upper" (Qq_1) and a "lower" $(q_2\bar{q}_3)$ currents:

$$M_a = \frac{G}{\sqrt{2}} \bar{u}(q_1) \gamma_\mu (1-\gamma_5) u(Q) \bar{u}(q_2) \gamma_\mu (1-\gamma_5) v(q_3) \quad (1)$$

Introduction of gluons into the field, with the $\bar{q}qg$ -vertex specified by $g_s \gamma_\mu \lambda^a$ - g_s being the strong coupling, λ^a is the generators for $SU_c(3)$ - means that to order $\alpha_s = g_s^2/4\pi$, the above amplitude is to be supplemented by contributions from quark self-energies and gluon exchange between quarks (referred to together as vertex corrections, v) and emission of real gluons (bremsstrahlung, b) from both the upper (u) and lower (l) currents, shown in Fig. 1b-e and called M_{b-e} below. To this order α_s , contributions from gluon exchange between the two currents are absent and, furthermore, corrections for the upper and lower currents do not interfere, a consequence of the tracelessness of the matrices λ^a . Also absent from the computations are all terms proportional to γ_5 on account of the permutation symmetry between q_2 and q_3 when all the final states are summed over ³⁾.

The $O(\alpha_s)$ QCD corrections to the decay rate then arise either from interference terms, $\Gamma_\ell^v \sim \text{Re}(M_a^* M_b)$ and $\Gamma_u^v \sim \text{Re}(M_a^* M_d)$, or from squared amplitudes, $\Gamma_\ell^b \sim |M_c|^2$ and $\Gamma_u^b \sim |M_e|^2$, summed over colours, spins and integrated over all final momenta. Taken separately, each partial rate Γ_k^α ($k = u$ or ℓ , $\alpha = v$ or b) diverges because of the presence of massless gluons (infra-red divergences). This difficulty can be elegantly resolved by applying the now familiar dimensional regularization technique ^{7),8)}, which extends the underlying four-dimensional space-time to a hyperspace of arbitrary dimension $d \neq 4$. It can then be shown that each partial rate Γ_k^α can be written in the limit $d \sim 4$ as the sum of a singular term (S_k^α) with a simple pole in $\epsilon = d-4$ and a regular term (R_k^α):

$$\Gamma_k^\alpha = \alpha_s (G^2 M^5 / 24 \pi^4) \int_{\sigma'}^{\delta_1'} d\xi \left[\frac{S_k^\alpha(\xi)}{\epsilon} + R_k^\alpha(\xi) \right] \frac{w w_1}{\xi} \quad (2)$$

where $\xi = (Q - q_1)^2 / M^2$, $\sigma' = (\sqrt{\rho_2} + \sqrt{\rho_3})^2$, $\delta_1' = (1 - \sqrt{\rho_1})^2$, $\rho_j = (m_j / M)^2$, $j = 1, 2, 3$. Table A below defines w and by extension w_1 , as well as many other symbols that we will need. Table B gives the definitions of three sets of basic integrals to be used; we have computed them analytically but do not explicate their forms here; because they are rather cumbersome, involving many logarithm and dilogarithm functions ⁹⁾ as well as their products. However, the combinations j_2 and j_3 of J_n , as defined below, are compact. Finally, Table C gives the functions $S_k^\alpha(\xi)$ and $R_k^\alpha(\xi)$ needed to calculate Γ_k^α .

Table A. General symbols

$$\sigma = \rho_2 + \rho_3, \quad \delta = \rho_2 - \rho_3, \quad \sigma' = (\sqrt{\rho_2} + \sqrt{\rho_3})^2, \quad \delta' = (\sqrt{\rho_2} - \sqrt{\rho_3})^2$$

$$w = w(\xi) = (\xi^2 - 2\xi\sigma + \delta^2)^{1/2}$$

$$\lambda^\pm = (\xi - \delta \pm w)/2\xi$$

$$\Sigma = (\xi - \sigma + w)/(\xi - \sigma - w)$$

$$\Delta = (\xi\sigma - \delta^2 + \delta w)/(\xi\sigma - \delta^2 - \delta w)$$

$$f = 1 + \sigma/\xi - 2\delta^2/\xi^2$$

$$g = 1 - \sigma/2\xi - \delta^2/2\xi^2$$

$$f' = -0.5 (\sigma/\xi - \delta^2/\xi^2)$$

$$\alpha = (1 - \rho_1)^2 - \xi(1 + \rho_1)$$

$$\beta = 2\xi(1 + \rho_1 - \xi)$$

$$F = \alpha f + \beta g$$

$$F' = \alpha f' + 0.5 \beta (f' + g)$$

$$a = -(\ln \Sigma)/w$$

$$b = -\{2\xi + (2\xi - \delta) \ln(\rho_2/\rho_3) + a[2\xi^2 - \xi(3\rho_2 - \rho_3) + \delta^2]\}/2\xi^2$$

$$d = [2\xi + (\xi - \delta) \ln(\rho_2/\rho_3) + a(\xi^2 - 2\xi\rho_2 + \delta^2)]/2\xi^2$$

$$B = d(\rho_2 \leftrightarrow \rho_3)$$

$$C = b(\rho_2 \leftrightarrow \rho_3)$$

$$A' = 2 + (\xi - \sigma) a$$

$$A = -2 + 0.5 \ln(\rho_2\rho_3) + (\delta/4\xi) \ln(\rho_2/\rho_3)$$

$$- a[3\xi^2 - 4\xi\sigma + \delta^2 + 2\xi(\sigma - \xi) \ln \xi]/4\xi + \gamma(\lambda^+, \lambda^-)(\xi - \sigma)/2\xi$$

$$L = \mathcal{L}_2(2w/(w - \xi - \delta)) + \mathcal{L}_2(2w/(w - \xi + \delta)) - \mathcal{L}_2(2w/(w + \xi + \delta)) - \mathcal{L}_2(2w/(w + \xi - \delta))$$

$\mathcal{L}_2(x)$ is the dilogarithm function as defined in 9). The above list is to be supplemented by symbols obtained from the corresponding ones given above by the substitution $\rho_2 \rightarrow \rho_1$ and $\rho_3 \rightarrow 1$. These will carry the subscript 1, e.g. $\sigma_1 = 1 + \rho_1$, $\delta_1 = \rho_1 - 1$, $w_1 = (\xi^2 - 2\xi\sigma_1 + \delta_1^2)^{1/2}$, $\lambda_1^\pm = (\xi - \delta_1 \pm w_1)/2\xi$, $\Sigma_1 = (\xi - \sigma_1 + w_1)/(\xi - \sigma_1 - w_1)$, etc., L_1 .

Table B. Integrals

$$Y(\lambda_i, \lambda_j) = \text{real} \int_0^1 dy [(y - \lambda_i)(y - \lambda_j)]^{-1} \ln [(y - \lambda_i)(y - \lambda_j)]$$

In fact, the integral corresponding to $Y(\lambda^+, \lambda^-)$ is complex while that corresponding to $Y(\lambda_1^+, \lambda_1^-)$ is already real.

$$I_n(\xi) = \int_{\sigma'}^{\xi} dz z^n w(z) \quad , n = -3, -2, \dots, 1$$

$$J_n(\xi) = \int_{\sigma'}^{\xi} dz \ln(\xi - z)/[z^n w(z)] \quad , n = 0, 1, \dots, 4$$

As an explicit example, we give :

$$J_0(\xi) = \ln(\sqrt{\Sigma}) \ln\left[\frac{w + \xi - \delta'}{2}\right] + 2 \mathcal{L}_2\left(\frac{1}{\sqrt{\Sigma}}\right) + \mathcal{L}_2\left(\frac{w + \xi - \sigma'}{2w}\right) - \mathcal{L}_2\left(\frac{w - \xi + \sigma'}{2w}\right) - \frac{\pi^2}{3}$$

The combinations given later have a rather compact form :

$$j_2 = \frac{3\xi}{w} [(\sigma^2 + \delta^2)J_2 - 4\sigma\delta^2J_3 + 2\delta^4J_4]$$

$$j_3 = \frac{3\xi}{w} [\sigma J_1 - \sigma^2J_2 - \sigma\delta^2J_3 + \delta^4J_4]$$

We find :

$$j_2 = \frac{3\delta^2 - 2\xi\sigma}{\xi^2} - \frac{\xi}{2w} \ln \Sigma + \frac{(3\xi\sigma - 2\delta^2)\delta}{2\xi^2w} \ln \Delta + \frac{\xi^2 + \xi\sigma - 2\delta^2}{2\xi^2} \ln\left(\frac{w^4}{\rho_2\rho_3}\right)$$

$$j_3 = \frac{3\delta^2 + \xi\sigma}{2\xi^2} - \frac{\xi}{w} \ln \Sigma + \frac{[3\xi^2(\sigma^2 + \delta^2) - 2\delta^4]}{4\xi^2w\delta} \ln \Delta + \frac{2\xi^2 - \xi\sigma - \delta^2}{2\xi^2} \ln\left(\frac{w^4}{\rho_2\rho_3}\right)$$

Table C. Functions $S_k^\alpha(\xi)$ and $R_k^\alpha(\xi)$

$$(1) \quad S_\ell^V = A'F$$

$$R_\ell^V = \alpha [Af + A'f' + 0.5 (B\rho_3 + d\rho_2) f - C\rho_3 (w^2 + 3\xi\rho_2)/\xi^2 - b\rho_2 (w^2 + 3\xi\rho_3)/\xi^2]$$

$$+ \beta \{Ag + 0.5 A' (g + f') + 3a\rho_2\rho_3/\xi - 0.25 (w/\xi)^2 [(B+C)\rho_3 + (b+d)\rho_2]\}$$

$$(2) S_{\ell}^b = - S_{\ell}^v$$

$$R_{\ell}^b = - (\alpha \mathcal{A} + \beta \mathcal{B})$$

$$\mathcal{A} = - 11/4 - (15/4) \sigma/\xi - 1.5 \sigma^2/\xi^2 + 9\delta^2/\xi^2 + (1+0.5\sigma/\xi-\delta^2/\xi^2) \delta \ln \Delta / w \\ + Zf + 0.5 a \{ 1.5 \xi (1-\delta^2/2\xi^2) + \sigma [1-0.75 \sigma/\xi + 1.5 (\sigma^2-\delta^2)/\xi^2] \}$$

$$\mathcal{B} = - 7/4 + (9/8) \sigma/\xi - (3/8) \sigma^2/\xi^2 + 2\delta^2/\xi^2 + (1-\sigma/4\xi-\delta^2/4\xi^2) \delta \ln \Delta / w \\ + Zg + (a/16) \{ 20 \xi (1-0.1 \delta^2/\xi^2) + \sigma[-18 + 6\sigma/\xi + (3\sigma^2-\delta^2)/\xi^2] \}$$

$$Z = \ln (w^4/\rho_2\rho_3) - 0.5 A' \ln \xi - (\xi-\sigma)(4J_0+L)/2w$$

$$(3) S_u^v = A_1' F$$

$$R_u^v = A_1 F + A_1' F' + 2 a_1 \rho_1 \xi (f-4g) + 0.5 (B_1+d_1 \rho_1) [(\alpha+0.5\beta) f - \beta g] \\ + 0.5 c_1 [(\delta_1+\xi)^2 f - 4\rho_1 \xi g] + 0.5 b_1 \rho_1 [(\delta_1-\xi)^2 f - 4\xi g]$$

$$(4) S_u^b = - S_u^v$$

$$R_u^b = - \{ \alpha [Xf + A_1' (f'+j_2)] + \beta [Xg + 0.5 A_1' (f'+g+j_3)] \} + \sum_{i=-3}^1 \gamma_i I_i / w$$

$$\theta = - 0.5 \xi a_1$$

$$X = - \ln \xi - \theta [1 + (\sigma_1-\xi)(\ln \xi)/\xi] + 0.5 [\delta_1 \ln \Delta_1 + (\sigma_1-\xi) L_1] / w_1$$

$$j_2 = (3\xi/w) [(\sigma^2+\delta^2) J_2 - 4\sigma \delta^2 J_3 + 2 \delta^4 J_4]$$

$$j_3 = (3\xi/w) [\sigma J_1 - \sigma^2 J_2 - \sigma \delta^2 J_3 + \delta^4 J_4]$$

$$\gamma_1 = 2 (1-\theta)$$

$$\gamma_0 = \xi \gamma_1 + 3\theta \sigma_1 - 0.5 \sigma \gamma_1$$

$$\gamma_{-1} = 2\xi^2 \gamma_1 + \xi [-2\sigma_1 - \sigma + \theta (7\sigma_1+\sigma)] - \delta^2 - \theta [\sigma_1(2\sigma_1+3\sigma) - \delta^2]$$

$$\gamma_{-2} = -\xi^2 \sigma \gamma_1 + \xi [\sigma \sigma_1 (4-5\theta) - 0.5\delta^2 \gamma_1] + 4\theta \sigma \sigma_1^2$$

$$\gamma_{-3} = - 2\delta^2 [0.5\xi^2 \gamma_1 + \xi \sigma_1 (1+\theta) + \theta \sigma_1^2]$$

As expected on the basis of the Kinoshita theorem¹⁰⁾, the one-gluon radiative corrections to the total rate are free from infra-red divergences; we find out explicitly that the singular terms cancel out, $S_\ell^v + S_\ell^b = 0$ and $S_u^v + S_u^b = 0$, leaving non-vanishing divergence-free terms. These finite terms R_k^α , taken separately, depend on the regularization procedure used to derive them. However the sum $R_k = R_k^v + R_k^b$ ($k = u$ or ℓ) is unique and independent of the regularization procedure, and so is the corresponding width $\Gamma_k = \Gamma_k^v + \Gamma_k^b$ ($k = u$ or ℓ). Let $\Gamma^{(0)} = \alpha_s G^2 M^5 / 64 \pi^4$, then in the zero-mass limit $m_i \rightarrow 0$ ($i = 1, 2, 3$) we have¹⁾⁻³⁾:

$$\Gamma_\ell / \Gamma^{(0)} \rightarrow 1, \quad (3)$$

$$\Gamma_u / \Gamma^{(0)} \rightarrow -\frac{2}{3} \left(\pi^2 - \frac{25}{4} \right) \approx -2.41 \quad (4)$$

The width correction Γ_u has also been tabulated for $\rho_2 = \rho_3 = 0$ and different values of ρ_1 ²⁾. From our general formula given above (Tables A, B, C), we derive analytically the limits (3) and (4) and also reproduce numerically results of Ref. 2). We remind the reader that both Γ_ℓ and Γ_u are symmetrical by the exchange $\rho_2 \leftrightarrow \rho_3$. Some representative numerical results are presented in Table D and Figs 2, 3. The following points are worth noting :

1. For every fixed pair of variables (m_i, m_j) and considered as a function of growing m_k ($i \neq j \neq k$), Γ_u is always a smoothly increasing function (remember that it has a negative sign) as shown in Fig. 2.
2. At every fixed value of m_1 , and as function of m_2 (or m_3), Γ_ℓ first increases and passes through a maximum near m_2/M (or m_3/M) $\sim 0.15 - 0.2$, then decreases inevitably to zero. In many cases, we observe that even if $\sum_{i=1}^3 m_i/M \sim 0.7 - 0.8$, although very suppressed by phase-space, Γ_ℓ remains bigger than its zero-mass limit which is $\Gamma^{(0)}$, as shown in Fig. 3. This indeed provides an unexpected non-trivial result. Finally, Γ_ℓ decreases smoothly with increasing m_1 , as expected.
3. The total correction $\Gamma_\ell + \Gamma_u$ changes sign, becoming positive quite early (even for small values of m_i). See Table D. In the whole range of final fermion masses, the total correction exceeds its zero-mass limit $2/3 (31/4 - \pi^2) \Gamma^{(0)}$, consequently, the mass effects tend always to increase the radiative correction rate, contrary to the non-radiative case (zero order in α_s) for which the mass effects act in the opposite direction. A straightforward application of our calculations can be used in heavy lepton τ , charmed and "beauty" $B(b\bar{q})$ meson decays.

If one assumes that $\tau \rightarrow \nu_\tau + \text{hadrons}$ is given by $\tau \rightarrow \nu_\tau \bar{u}d$ (at the Cabibbo favoured level), then the semi-leptonic branching ratio $B_{SL} = \Gamma(\tau \rightarrow \nu_\tau e \bar{\nu}) / \Gamma(\tau \rightarrow \text{All})$ will be:

$$(B_{SL})^{-1} = 2 + 3 \left[f(x) + \frac{\alpha_s}{\pi} g(x) \right]$$

where $x = m_{u,d}/m_\tau$ and $m_{\nu_\tau} = 0$

$$f(x) = (1-4x^2)^{1/2} [1-14x^2-2x^4-12x^6] + 24x^4(1-x^4) \ln \frac{1+\sqrt{1-4x^2}}{1-\sqrt{1-4x^2}}$$

$g(x) = \Gamma_\ell(0, x, x) / \Gamma^{(0)}$ as calculated in this work.

Taking α_s to be between 0.2 and 0.4, the experimental value $B_{SL} = (17.5 \pm 1.1) \%$ can only be satisfied for $m_{u,d} \leq 180$ Mev. Within this range of u, d quark mass, the semi-leptonic branching ratio of charmed D^+ mesons is predicted to be $\leq 14\%$, assuming the spectator picture. A detailed account of our calculations will be given elsewhere.

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Table D. Dependence of radiative corrections of weak decay rates on final masses (given in units of decaying quark mass). $m_i \equiv m_l$ for cols. 2 and 3, $m_i \equiv m_2$ for cols. 4 and 5. Each entry gives, in order, Γ_u , Γ_ℓ and $\Gamma_u + \Gamma_\ell$ in units of $\Gamma^{(0)} = \alpha_s G^2 M^5 / 64\pi^4$.

m_i	$m_2 \sim 0.0$ $m_3 \sim 0.0$	$m_2 = 0.2$ $m_3 = 0.1$	$m_l \sim 0.0$ $m_3 \sim 0.0$	$m_l = 0.2$ $m_3 = 0.1$
0.0	- 2.41 1.00 - 1.412	- 1.58 2.27 0.68	- 2.41 1.00 - 1.41	- 1.27 1.14 - 0.12
0.1	- 2.00 0.92 - 1.076	- 1.27 2.12 0.85	- 2.22 1.48 - 0.73	- 1.13 1.50 0.37
0.2	- 1.40 0.74 - 0.661	- 0.82 1.73 0.91	- 1.75 1.92 0.17	- 0.82 1.73 0.91
0.3	- 0.86 0.52 - 0.345	- 0.45 1.23 0.78	- 1.19 1.89 0.70	- 0.48 1.52 1.04
0.4	- 0.47 0.31 - 0.152	- 0.20 0.74 0.54	- 0.69 1.48 0.79	- 0.22 1.02 0.80
0.5	- 0.21 0.16 - 0.054	- 0.06 0.34 0.28	- 0.33 0.93 0.60	- 0.06 0.48 0.42
0.6	- 0.079 0.065 - 0.014	- 0.01 0.09 0.08	- 0.12 0.45 0.33	- 0.01 0.13 0.12
0.7	- 0.021 0.019 - 0.002		- 0.03 0.15 0.12	

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FIGURE CAPTIONS

- Figure 1 : a) Zero order diagram
b) Lower vertex correction
c) Bremsstrahlung at lower vertex
d) Upper vertex correction
e) Bremsstrahlung at upper vertex
f) Vertex correction
g) Bremsstrahlung (solid lines indicate fermions, dashed lines gluons and wavy lines W bosons).
- Figure 2 : $\Gamma_u/\Gamma^{(0)}$ as function of m_1/M for different values of m_2/M and m_3/M .
Solid lines ($m_3/M = 0.3$), dashed lines ($m_3/M = 0.10$).
- Figure 3 : $\Gamma_\ell/\Gamma^{(0)}$ as function of m_2/M for different values of m_1/M and m_3/M .
Solid lines ($m_1/M = 0.15$), dashed lines ($m_1/M = 0.25$).

- Figure 1 -

- Figure 2 -

- Figure 3 -