Interstrip resistance measurement

Technical Note

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Introduction

In ATLAS SCT community two methods of interstrip resistance measurements are used: a) measuring the resistance between two strips and comparing it with a separately measured strip-to bias-rail resistance and b) applying DC voltage to one strip and measuring the current flowing to another strip. The method a) will further be referred to as Resistance Method and method b) as Induced Current Method. In the latter the current can be measured either directly or by a voltage drop on the bias resistor. All three techniques are analyzed in this Note and illustrated by measurements with the same sensor.

The measurements were performed at room temperature and ~7 % relative humidity with non-irradiated $n$-in-$p$ microstrip sensor w27-bz1-p7 produced by Hamamatsu within the ATLAS Tracker Upgrade R&D Program. The sensor has 104 strips with a pitch of 74.5 $\mu$m and a length of 8 mm. It was mounted on a test frame and put in the box flushed with dry Nitrogen. The sensor has no p-spray or p-stop interstrip isolation, which results in a relatively low interstrip resistance values even at quite high bias values. The full depletion voltage of the sensor is 153V. For better stability of the results the bias voltage was first raised to 300 V and the sensor was kept at this bias for ~5 hours. Then the measurements were performed at gradually decreasing bias. Three consecutive strips 38, 39 and 40 were used that allowed investigations with the nearest and next neighbour strips.
1. Basic relations

Consider a semi-infinite chain of bias, \( R_b \), and interstrip, \( R_{is} \), resistors as shown in Fig.1.1. Assuming that all \( R_b \) are equal and the same is true for \( R_{is} \), one can find an equivalent resistance \( R_{eq} \) of the chain presented in Fig.1.1.

![Fig.1.1. Circuit diagram for \( R_{eq} \)](image)

As shown in Appendix A, \( R_{eq} = bR_b \) where

\[
b = \frac{x + \sqrt{x^2 + 4x}}{2}
\]

and \( x = R_{is}/R_b \) is the parameter quantifying the interstrip isolation. Obviously \( b > x \). For \( x \to 0 \) \( b \to \sqrt{x} \) and \( R_{eq} \to \sqrt{R_b R_{is}} \). For \( x \to \infty \) \( b \to x \) and \( R_{eq} \to R_{is} \).

Also calculated in Appendix A is \( R_0 \) - the resistance between an individual strip and the bias rail.

\[
R_0 = R_b \frac{b}{b + 2} < R_b
\]

When \( x \to 0 \) \( R_0 \to \frac{\sqrt{R_b R_{is}}}{2} = \frac{R_{eq}}{2} \), while for \( x \to \infty \) \( b \to \infty \) \( R_0 \to R_b \).

2. Resistance Method

Typically the resistance is measured between two adjacent strips. However in some situations the access may be possible only to every second strip. Therefore this case is also considered for completeness.

2.1 Neighbour strips

The resistance \( R_1 \) between two adjacent strips can be expressed as (see Appendix B)

\[
R_1 = R_{is} \frac{2}{b + 2} < R_{is}
\]

For \( x \to 0 \) \( b \to 0 \) \( R_1 \to R_{is} \) while for \( x \to \infty \) \( b \to x \) \( R_1 \to 2R_{is}/x = 2R_b \). Note an interesting relation.
\[ \frac{R_0}{R_b} + \frac{R_1}{R_{is}} = 1 \]

The experimentally measured parameters \( R_0 \) and \( R_1 \) allow finding \( R_b \) and \( R_{is} \). It is useful to introduce parameter \( \rho_1 = \frac{2R_0}{R_1} \). Then as shown in Appendix B

\[
\rho_1 = \frac{b}{x} > 1; \ x = \frac{1}{\rho_1(\rho_1 - 1)}
\]

\[ R_b = R_b(2\rho_1 - 1) \]

\[ R_{is} = R_1 \frac{2\rho_1 - 1}{2(\rho_1 - 1)} \]

When \( x \to \infty \) \( (b \to x) \) \( \rho_1 \to 1 \), \( R_b \to R_0 \) and \( R_{is} \to \infty \). In this situation \( \rho_1 = \rho_1 - 1 \) is close to \( 1/x \).

It is the experimentally achievable accuracy in \( \rho_1 \) that limits the maximum reliably measurable \( R_{is} \). If e.g. the minimum reliably measurable \( \rho_1 \) is 0.05 then the maximum measurable \( x = R_{is}/R_b \) is \( \sim 20 \).

### 2.2 Next neighbour strips

As in the previous section introduce \( \rho_2 = \frac{2R_0}{R_2} \) where \( R_2 \) is the resistance between two next neighbour strips. As shown in Appendix B

\[
\rho_2 = \frac{(b + 1)^2}{b(b + 2)} > 1; \ x = \frac{\sqrt{\rho_2} - \sqrt{\rho_2 - 1}}{\sqrt{\rho_2}(\rho_2 - 1)}
\]

\[ R_b = R_0 \left( \sqrt{\rho_2} + \sqrt{(\rho_2 - 1)} \right)^2 \]

\[ R_{is} = R_2 \frac{\sqrt{\rho_2}}{2 \sqrt{\rho_2 - 1}} \]

When \( x \to \infty \) \( (b \to x) \) \( \rho_2 \to 1 \), \( R_b \to R_0 \) and \( R_{is} \to \infty \). In this situation \( \rho_2 = \rho_2 - 1 \) is close to \( 1/x^2 \). Again the accuracy in \( \rho_2 \) limits the maximum reliably measurable \( R_{is} \). For minimum reliably measurable \( \rho_2 = 0.05 \) the maximum measurable \( x = R_{is}/R_b \) is 2.8 i.e. \( \sim 7 \) times smaller than for the same accuracy in \( \rho_1 \). Thus the \( R_{is} \) reconstruction ability for measurements with next neighbours is significantly worse than that for the adjacent strips.

Fig. 2.1 shows the \( \rho_1 \) and \( \rho_2 \) as a function of \( x \). The lines are \( 1/x \) and \( 1/x^2 \) dependences.
Sometimes a more pragmatic approach is used when only “strip-strip” conductivity is measured as a function of bias and the voltage, above which this conductivity exceeds its plateau value by less than some fraction (e.g. $\delta<10\%$), is considered to be the “strip isolation” voltage. Note that without any additional information this approach doesn’t tell quantitatively how good the isolation is. If a usual assumption is made that the plateau value is equal to $1/(2R_0)$ then for the adjacent strips $\delta$ coincides with $\epsilon_1$ and the plot in Fig.2.1 (or corresponding formula) allow an estimate of the lower limit for $x$ (e.g. for $\delta<0.1$ $x>9$). The pragmatism here is in assuming the strip-strip conductivity plateau value to be one half of the conductivity between the strip and the bias rail without actually measuring the latter.

2.3 Experimental results
The resistance was measured by applying the voltage, $V$, between strip implant and the bias rail or between two implants and measuring the resulting current, $I$, using Keithley Source Measure Unit (SMU) K2410. The resistance was defined as the inverse slope of the linear fit to the measured $I/V$ data. To minimise the effect of measurements on the bias resistors the voltage scan range was only $\pm55$ mV.
The measurements were performed with strips: 38, 39, and 40. At each bias five voltage scans were performed. First, $R_0$, the resistivity between a strip and the bias rail, was measured for every strip. Then the resistivity $R_1$ between adjacent strips (38, 39) and $R_2$ between next-neighbour strips (38, 40) were measured. Fig.2.2. shows all five measured resistances. The strip-strip resistances $R_1$ and $R_2$ are divided by factor 2 to simplify the comparison.

Fig.2.2. Resistance $R_0$ for every strip, resistance $R_1$ between adjacent and $R_2$ between next neighbour strips. The two latter are divided by 2 to simplify the comparison.

For $U_{\text{bias}} > 40V$ both $\varepsilon_1$ and $\varepsilon_2$ are less than 1% and compatible with zero within measurement errors. Therefore the calculations of $R_{\text{is}}$ and $R_{\text{b}}$ were performed only for $U_{\text{bias}} \leq 40V$. The $R_0$ for these calculations was taken as the average of the two $R_0$ values for the strips included in corresponding “strip-strip” resistance measurement.

Fig.2.3 shows the interstrip resistance values found from the measurements with nearest ($iR_1$) and next neighbour ($iR_2$) strips. As expected the measurable limit for the latter is lower. Otherwise both values agree reasonably well.
Fig. 2.3. Interstrip resistance values calculated from the data for the nearest ($iR_1$) and next neighbour ($iR_2$) strips.

Fig. 2.4. Average value of $R_0$ measured for all 3 strips and the $R_b$ values reconstructed from the data for the nearest, $R_{b1}$, and next neighbour, $R_{b2}$, strips.
Fig. 2.4 shows the average of $R_0$ values measured for all 3 strips and the values of $R_b$ reconstructed from the data for the nearest, $R_{b1}$, and next neighbour, $R_{b2}$, strips. Below 45 V the $R_0$ decreases steadily with bias. The $R_b$ reconstruction compensates this decrease to some extent in a couple of points but fails to do so for lower voltages. This indicates that for low interstrip resistances the current distribution between the neighbouring strips differs from that assumed in the used simple model. Another manifestation of the same phenomenon is the variation of $R_0$ measured for different strips. As can be seen from Fig 2.2 at high bias all 3 $R_0$ values are very close but below 40V the variation between them increases significantly. Nevertheless a reasonable agreement between the two reconstructed $R_{in}$ values shows that basically the model used for the calculations is adequate.

3. Induced Current Method

When potential $V_0$ is applied between a strip and the bias rail it induces a current flowing via $R_{is}$ to the neighbouring strips. This current can be measured either directly or indirectly by connecting an ammeter or voltmeter in parallel to the bias resistor, $R_b$, of the investigated strip.

3.1 Voltage measurement

Typical approach is to connect between a strip and the bias rail a source-meter unit (SMU) providing potential $V_0$ varying by a few volts around zero. This potential will sometimes be called $V_{master}$. Measuring the current flowing out of the SMU provides $R_0$. Simultaneously a voltmeter connected between a nearest (or next) neighbour strip and the bias rail measures the potential $U$ (sometimes called $U_{slave}$) as a function of $V_0$. Ideally the induced potential $U$ should be simply proportional to $V_0$ and their ratio would characterise the inter-strip isolation. In practice however the slope $S=dU/dV_0$ is used instead.

3.1.1 Neighbour strips

The $U_{slave}$ measured at the nearest neighbour strip will be denoted as $V_1$ and the corresponding slope as $S_1 =dV_1/dV_0$. As shown in Appendix C the slope dependence on $x$ is very simply expressed via $b$:

$$S_1 = \frac{1}{1+b}$$
For $x \to 0 \ (b \to 0) \ S_1 \to 1$ while for $x \to \infty \ (b \to \infty) \ S_1 \to 1/x \to 0$. Experimentally measured parameters $R_0$ and $S_1$ allow reconstruction of the parameters in question: $R_b$ and $R_{is}$. As demonstrated in Appendix C

$$R_b = R_0 \frac{1 + S_1}{1 - S_1}$$

$$R_{is} = R_0 \left( \frac{1}{S_1} - S_1 \right).$$

In a typical situation when $x \to \infty$, $S_1 \to 0$ one gets $R_b \to R_0$ and $R_{is} \to R_0/S_1 = R_0 (dV_0/dV_1)$. A minimum detectable slope $S_1$ defines the maximum measurable $R_{is}$. For a proven to be detectable $S_1$ of $\sim 10^{-6}$ the limit for $R_{is}$ is $\sim 10^6 R_0$, which for a typical $R_0 \sim 1\,\text{M\Omega}$ corresponds to $R_{is} \sim 1000\,\text{G\Omega}$.

3.1.2 Next neighbour strips

As shown in Appendix C the slope of the voltage $V_2$ induced on the next neighbour strip $S_2 = dV_2/dV_0$ is related to $S_1$ in a very simple way: $S_2 = S_1^2$. Therefore the reconstruction formulae become

$$R_b = R_0 \frac{1 + \sqrt{S_2}}{1 - \sqrt{S_2}}$$

$$R_{is} = R_0 \left( \frac{1}{\sqrt{S_2}} - \sqrt{S_2} \right).$$

For $x \to \infty \ S_2 \to 0$ as $1/x^2$. Therefore for the same limit of measurable $S_2 \sim 10^{-6}$ the limit for the $R_{is}$ is $\sim 10^3 R_0$, which for a typical $R_0 \sim 1\,\text{M\Omega}$ corresponds to $R_{is} \sim 1\,\text{G\Omega}$.

Fig.3.1 shows $S_1$ and $S_2$ as a function of $x$ together with $1/x$ and $1/x^2$ lines. Similarly to the Resistance Method the measurements with neighbour strip have higher sensitivity compared to that for the next neighbour strip. But even for the latter the sensitivity is much higher than what can be achieved by the Resistance Method.
3.1.3 Effects of non-zero resistance to the ground

Another limit for the maximum measurable $R_{is}$ appears from a non-zero resistance to the ground $R_g$. As demonstrated in Appendix C in a typical situation $R_g << R_b << R_{is}$ the $R_{is}$ calculated from the measured $S_1$ represents the actual $R_{is}$ in parallel with the effective parasitic resistance $R_p = R_b^2 / R_g$. For a typical $R_0 ~ 1 \Omega$ even $R_g ~ 1 \Omega$ results in $R_p ~ 1000 \Omega$, which sets a practical limit for the sensitivity of the method.

As explained in Appendix C a non-zero $R_g$ produces an offset to the slope, which is the same for the neighbour and the next neighbour strips. Comparison of $S_1$ and $S_2$ measured under the same conditions allows decoupling of the effects related to $R_{is}$ and $R_g$.

3.1.4 Experimental results

The bias voltage was changing downwards from 300V after the sensor was kept at this bias for ~5 hours. Three consecutive strips 38, 39 and 40 were used. At each bias value two separate $V_{master}$ scans were performed with the SMU Keithley K2410 connected either to strip 39 or 38 (with the connection to the other of these two strips floating) and the potential $U_{slave}$ induced at the strip 40 was measured. In this way
both slopes $S_1$ and $S_2$ were measured for each bias point. Depending on the strength of the induced signal different ranges were used in the master voltage scans. For a strong signal (low bias, low $R_{is}$) the range was ±55 mV, as in the Resistance Method tests described above. For weakest signals (high bias, high $R_{is}$) the range was ±4V. Sometimes an intermediate range ±0.9V was used.

Fig.3.2a shows $S_1$ and $S_2$ as a function of bias voltage. As expected $S_2$ is usually lower than $S_1$ but at $U_{bias} \geq 80$V the slopes are close and do not change with bias. This indicates that in this bias range both slopes are dominated by the $R_g$ contribution, which sets a limit on measurable $R_{is}$.

The reconstructed $R_{is}$ are presented in Fig.3.2b. In measurements with the nearest neighbour the $R_{is}$ limit is ~1000GΩ while with the next neighbour it is ~1GΩ. Outside the plateau area the $R_{is}$ reconstructed from $S_1$ and $S_2$ agree quite well. Better sensitivity of the measurement with the nearest neighbour strip allows a wider range of measurable $R_{is}$. 

![](image.png)

**Fig.3.2a. Bias dependence of $S_1$ and $S_2$**
Fig. 3.2b. Interstrip resistances $iR_1$ and $iR_2$ calculated from $S_1$ and $S_2$ respectively.

Measured resistances $R_0$ and reconstructed values of $R_b$ are shown in Fig. 3.3 for measurement with the nearest (1) and next (2) neighbours.

Fig. 3.3. Measured, $R_0$, and reconstructed, $R_b$, resistances.
At $U_{\text{bias}} < 50\text{V}$ the measured resistance $R_0$ is significantly lower than its plateau level corresponding to the bias resistor value $R_b$. Down to $\sim30\text{V}$ the $R_b$ reconstruction compensates this to some extent, but fails to do so at lower bias. This indicates more complicated distribution of the master current than assumed in the used simple model. This effect was already discussed in the end of section 2.3.

Reasonable consistency of the results obtained from the measurements with nearest and next neighbour strips validates in general the model used in the calculations.

### 3.2 Current measurement

Direct measurement of the induced current is quite straightforward but has its own difficulties. First, an ordinary DMM (Digital Multimeter), adequate for the voltage measurements, can’t measure the induced current of pA level and a specialised device, picoammeter, is required. Second point is that connection of the ammeter with very low input resistance parallel to the bias resistor changes the electrical circuitry on the sensor which may lead to distortions of the results. Third problem is related to the voltage burden (also called burden voltage) - a DC voltage present at the input terminals of the device. Depending on the instrument the voltage burden (VB) can be either constant or proportional to the current through the ammeter or a mixture of both. It may also depend on the measurement range. For a typical picoammeter the VB is $\sim200\mu\text{V}$. It induces parasitic current, $I_p$, in the bias resistor parallel to the ammeter that in turn results in the same current but with the opposite sign flowing through the ammeter. Typical in our situation $I_p$ of $200\mu\text{V}/1\text{M}\Omega = 200\text{pA}$ should be compared with a current produced by a potential of 1V applied to interstrip resistance of $\sim100\text{G}\Omega$: $1\text{V}/100\text{G}\Omega = 10\text{pA}$, which is 20 times lower. Ideally the constant $I_p$ should simply shift the measured dependence of the ammeter current on master voltage and have no effect on the slope of this characteristic. However when the measured current is much lower than $I_p$ small variations in the latter may distort the results for $R_{\text{is}}$ significantly.

The interstrip resistance measurements by this method were performed in the same way as the voltage measurements described in section 3.1.4. The Kethley
picoammeter K6485 with the specified resolution of 10 fA and voltage burden of <200 µV was used. In the data analysis the ammeter input resistance was supposed to be zero and the VB independent of current. The latter assumption means that the VB was expected to have no influence on the slope of measured dependence of the current \( I_{\text{slave}} \) on the master voltage \( V_0 \).

3.2.1. Nearest neighbour measurements
The ammeter was connected between the implant at strip 40 and the ground, while the SMU between the implant of strip 39 and the ground. The bias rail was grounded. Two currents were measured as a function of the SMU voltage \( V_0 \): the one flowing from the SMU, \( I_0 \), and the one detected by the picoammeter, \( I_1 \). The interstrip resistance \( R_{\text{is}} \) and the bias resistor \( R_b \) were calculated from the measured currents as described in Appendix D. With the notation \( dV_0/dI_0 = R_{0a} \) the results look like follows.

\[
R_{\text{is}} = \frac{dV_0}{dI_1}
\]

\[
R_b = R_{0a} \frac{R_{0a}^2}{(R_{\text{is}} - R_{0a})^2}
\]

The interstrip resistance is simply an inverse measured slope \( S_1 = dI_1/dV_0 \) and doesn’t depend on the \( R_{0a} \). This directness and simplicity is one of the major advantages of the current measurement method. In a typical situation when \( R_{\text{is}} >> R_{0a} \) the value of \( R_b \) given by the second equation is close to \( R_{0a} \).

3.2.2. Next neighbour measurements
In this case the SMU was connected to strip 38. Denote the current measured by the picoammeter (again connected at strip 40) as \( I_2 \), and introduce the corresponding effective resistance \( R_2 = dV_0/dI_2 \) (in addition to the previously defined \( R_{0a} \)). The results can then be expressed like follows (see Appendix D).

\[
R_{\text{is}} = R_2 \sqrt{\frac{R_2 R_{0a}}{R_2 + R_{0a}}}
\]

\[
R_b = R_{0a} \frac{R_{0a}^2}{(R_2 + R_{0a})(\sqrt{R_2} - \sqrt{R_{0a}})^2}
\]

In this case the interstrip resistance depends on both measured currents \( I_2 \) and \( I_0 \). For a typical situation: \( R_2 >> R_{0a} \), the above equations revert to:
\[ R_{i_h} = \sqrt{R_z R_{i_a}}; \]
\[ R_{i_b} = R_{0a} \]

3.2.3. Effects of a non-zero resistance to the ground

For the current measurements a non-zero resistance to the ground \( R_g \) also sets a limit for the maximum measurable \( R_{i_s} \). It is shown in Appendix D that in a typical situation \( R_g \ll R_b \ll R_{i_s} \) the \( R_{i_s} \) calculated from the measured \( S_1 \) represents the actual \( R_{i_s} \) in parallel with the effective parasitic resistance \( R_p = R_b^2 / R_g \). The latter value is exactly the same as in the voltage measurement method and for a typical \( R_b \sim 1\,\text{M}\Omega \) the \( R_g \) of \( \sim 1\,\Omega \) gives \( R_p \) of \( \sim 1000\,\text{G}\Omega \).

As in the voltage measurements a non-zero \( R_g \) results in the slope offset the same for the nearest neighbour and next neighbour strips. Comparison of \( S_1 \) and \( S_2 \) allows decoupling of the effects related to \( R_{i_s} \) and \( R_g \). As shown in Appendix D the effective parasitic resistance for the next neighbour measurement is again equal to that for the voltage measurements: \( R_p = R_b \sqrt{\frac{R_b}{R_g}} \). For typical values of \( R_b = 1\,\text{M}\Omega \) and \( R_g = 1\,\Omega \) the \( R_p = 1\,\text{G}\Omega \).

3.2.4. Experimental results

Fig. 3.4a shows the slopes \( S_1 \) and \( S_2 \) measured with the nearest and next neighbours respectively as a function of bias voltage. As expected both slopes have about the same value at high bias voltages due to the effect of non-zero resistance to the ground. Outside the plateau region the slope \( S_2 \) is lower than \( S_1 \). The interstrip resistance values reconstructed from these slopes are shown in Fig. 3.4b.

As expected the range of measurable interstrip resistance values is smaller for the next neighbour measurements. Otherwise the results are close.
Fig. 3.4a. Bias dependence of the slope $\frac{dI_{\text{slave}}}{dV_{\text{master}}}$ for the nearest (1) and next (2) neighbour strips.

Fig. 3.4b. Interstrip resistances $iR_1$ and $iR_2$ calculated from the current measurements with nearest and next neighbour strips respectively.
The values of measured $R_{0a}$ and reconstructed bias resistor, $R_b$, are presented in Fig.3.5 for the nearest and next neighbour measurements. They are similar to those obtained in voltage measurements and shown in Fig.3.3.

In general the results are quite consistent that validates the assumptions made in the used theoretical model. No effects of the voltage burden were noticed.

![Graph](image)

Fig.3.5. Measured values of $R_{0a}$ and the reconstructed bias resistor $R_b$ for the measurements with nearest (1) and the next (2) neighbour strips.

3.3 Discussion

The measurements performed with the same sensor in the same conditions gave very similar results for the voltage and current measurement options, which therefore can be regarded as equivalent. In both cases the measurable value of the interstrip resistance is limited by ~1000 GΩ resulting from a non-zero resistance to the ground. For the measured bias resistor of 1.33 MΩ the limit $R_p=1000$ GΩ corresponds to the resistance to the ground of $R_b^2/R_p=1.8$ Ω. This quite low value can well be due to the
resistivity of the bias rail itself. Due to the same reasons the $R_{is}$ measurable with the next neighbours is limited by the value of $\sim 1 \ G\Omega$ in agreement with expectations. Outside the plateau area the $R_{is}$ measured with nearest and next neighbour strips agree quite well that validates the theoretical models. Note also that the interstrip resistance is very sensitive to the charge on the sensor surface, which in turn is a function of the sensor biasing history, especially in dry conditions when the surface charging processes are extremely slow. Therefore the voltage and current measurement results obtained on different days have slightly different bias voltage dependence. This is not important however for comparison of the nearest and next neighbour results, which were always obtained one after another within a few minutes time.

Though the voltage and current measurement options are equivalent, the first one has several advantages. First is that an ordinary DMM (Digital Multimeter) can achieve a reasonable accuracy ($\sim 10\%$) in measuring the induced voltage with a slope of $\sim 1\mu V/V$ corresponding to the maximum measurable $R_{is}$ of $\sim 1000G\Omega$. The current measurement in this situation requires an ability to measure the slope of $\sim 1pA/V$ i.e. a specialised device - picoammeter. The accuracy achieved in our current measurements using Keithley K6485 was also $\sim 10\%$. To get a similar accuracy in voltage measurements much cheaper DMM Keithley K2000 was sufficient. Second reason is that the voltmeter connection in parallel to $R_b$ has no effect on electrical circuit of the sensor because the input resistance of modern DMM’s in DC voltage mode is typically of $\sim 10G\Omega$ that is by 4 orders of magnitude higher than the bias resistor, $R_b$. Ammeter connection is more intrusive. Its internal resistance practically shortens the bias resistor changing the electrical circuit of the sensor. This may lead to a distortion of the results especially in a case of high current through bias resistor. Finally, for the voltage measurements there is no problem of voltage burden, discussed in the beginning of section 3.2, potentially quite dangerous for the current measurements. The corresponding problem in the voltage measurements, the offset stability, is far less severe. For example, for K2000 used in our measurements, the offset drift is specified to be $<3 \ \mu V$ during 24 hours and nowhere it is mentioned to be dependent on the measured voltage, at least within the same measurement range.
Sometimes to increase the measurement accuracy the master voltage is applied to two strips surrounding the one where the induced current is measured. Comparison of such configuration with a simple measurement at the nearest neighbour strip is shown in Fig.3.6. The slope for the “two-master strip” option is divided by factor 2 to simplify the comparison. The errors are derived from the points spread around the straight line fits of the $I_{\text{slave}}$ vs. $V_{\text{master}}$ data. Also shown are the linear fits to the slope points with the errors taken into account.

![Graph showing $I_{\text{slave}}$ vs. $V_{\text{master}}$ for $n_{\text{master}} = 1$ and $n_{\text{master}} = 2$.](image)

Fig.3.6. Slope of the $I_{\text{slave}}$ vs. $V_{\text{master}}$ dependence measured at the nearest neighbour with one or two master strips (divided by factor 2 for the latter to simplify the comparison).

As expected, the errors in the two master data are smaller. The voltage dependence gradient is also slightly different but the average values are very close. Since, as discussed above, what is measured here is not an actual interstrip resistance but the effects of a finite resistance to the ground, the subtle differences between the two sets of results hardly justify the use of an extra probe contact needed for a second master strip. If three consecutive strips are probed it is more useful to measure the signal both at the nearest and next neighbour strips to decouple the effect of the interstrip resistance from that of a non-zero resistance to the ground.
4. Summary

The Resistance Method (section 2) is considerably less powerful than the Induced Current Method (section 3). The former can measure $R_{in}$ up to ~100 MΩ and is limited by the accuracy and systematic effects in the $I$-$V$ measurements. With the latter an interstrip resistance up to ~1000 GΩ can be measured using either voltmeter or ammeter connected to the nearest neighbour strip. The limit is due not to the measurement accuracy but to practically unavoidable few Ohm resistance to the ground. The latter includes in particular the resistance of the bias rail between the points where the investigated strips are connected to it and the bias rail contact to the outside world. If contacting is possible only to the next neighbour strip the interstrip resistance can be measured up to ~1 GΩ level. However even this limit is much higher than a typical bias resistor of ~1MΩ.

Measuring the induced voltage and induced current are equivalent in terms of their capabilities. However the former can be done with less expensive equipment. Enhancing a weak signal by connecting an additional strip as a source of induced current is not an effective way of using an extra probe contact to the sensor. More efficient would be to use three contacts for measuring the signal induced both at the nearest and next neighbour strips to identify the contribution from a non-zero resistance to the ground.
Appendix A. Basic relations details

Looking at the circuit presented in Fig.1.1 one may notice that $R_{eq}$ can be presented as $R_{xx}$ plus $R_{b}$ parallel to $R_{eq}$ that results in the following equation:

$$R_{eq} = R_{xx} + \frac{R_{b}R_{eq}}{R_{b} + R_{eq}}$$  \hspace{1cm} (A.1)

Using parameters $x=R_{xx}/R_{b}$ and $b=R_{eq}/R_{b}$ the eq. (A.1) can be re-written as

$$b = x + \frac{b}{1+b}$$  \hspace{1cm} (A.2)

or

$$b^2 - xb - x = 0$$  \hspace{1cm} (A.3)

from which it follows:

$$x = \frac{b^2}{1+b}$$  \hspace{1cm} (A.4)

and

$$b = \frac{x + \sqrt{x^2 + 4x}}{2}.$$  \hspace{1cm} (A.5)

The last is the result of solving eq. (A.3) vs. $b$ and keeping only the positive solution.

The resistance $R_0$ between a strip and the bias rail consists of three resistors in parallel: bias resistor $R_{b}$ and two $R_{eq}$ i.e.

$$\frac{1}{R_0} = \frac{1}{R_{b}} + \frac{2}{R_{eq}}$$  \hspace{1cm} (A.6)

Using $R_{eq}=bR_{b}$ one gets from eq. (A.6)

$$R_0 = R_{b} \frac{b}{b+2}$$  \hspace{1cm} (A.7)

Appendix B. Resistance method calculations

a) Adjacent strips

An equivalent circuit diagram for measuring resistance $R_1$ between two adjacent strips is shown in Fig.B1. $R_1$ is the resistance between the points A and B and can be expressed as follows

$$\frac{1}{R_1} = \frac{1}{R_{xx}} + \frac{R_{b} + R_{eq}}{2R_{b}R_{eq}}$$  \hspace{1cm} (B.1)

* I am indebted to Nobu Unno for the idea of this calculation. – A.C.
Fig. B1. Equivalent circuit diagram for measuring $R_1$

Using parameters $x = R_{\text{is}}/R_b$ and $b = R_{\text{eq}}/R_b$ the eq. (B.1) can be re-written as

$$\frac{R_{\text{is}}}{R_1} = 1 + \frac{x(b + 1)}{2b}$$

(B.2)

Expressing $x$ via $b$ using eq. (A.4) one gets from (B.2)

$$R_1 = R_{\text{is}} \frac{2}{b + 2}$$

(B.3)

Introduce parameter $\rho_1 = 2R_0/R_1$. Using eqs. (A.7) and (B.3) one obtains

$$\rho_1 = \frac{bR_b}{R_{\text{is}}} = \frac{b}{x}$$

(B.4)

Expressing $b$ via $x$ from eq. (A.5) and finding $x$ from the resulting equation one gets

$$x = \frac{1}{\rho_1(\rho_1 - 1)}$$

(B.5)

Combining eqs. (B.4) and (B.5) one can express $b$ vs. $\rho_1$:

$$b = \frac{1}{\rho_1 - 1}$$

(B.6)

Using eq. (B.6) one obtains from (A.7)

$$R_b = R_0(2\rho_1 - 1)$$

(B.7)

and from (B.3)

$$R_{\text{is}} = R_1 \frac{2\rho_1 - 1}{2(\rho_1 - 1)}$$

(B.8)

Experimentally both $R_0$ and $R_1$ are found from independent voltage scans with similar accuracy. Therefore the errors for $R_b$ and $R_{\text{is}}$ depend on the errors for both $R_0$ and $R_1$ denoted as $\sigma_0$ and $\sigma_1$ respectively. Substituting $\rho_1$ by $2R_0/R_1$ in eqs. (B.7, B.8) and using standard error propagation formulae one obtains:

$$\sigma^2(R_{\text{is}}) = \frac{\sigma_0^2}{(\rho_1 - 1)^4} + \frac{\sigma_1^2}{4} \left(1 + \frac{\rho_1^2}{(\rho_1 - 1)^2}\right)^2$$

(B.9)

$$\sigma^2(R_b) = \sigma_0^2 (4\rho_1 - 1)^2 + \sigma_1^2 \rho_1^4$$

(B.10)
b) Next-neighbour strips

An equivalent circuit diagram for measuring resistance $R_2$ between two next-neighbour strips is shown in Fig.B2. Due to the symmetry there is no potential difference between the ends of the central bias resistor. Therefore it can be either removed or replaced by a short.

![Equivalent circuit diagram for measuring $R_2$](image)

In both cases the resistance $R_2$ between the points A and B can be expressed as:

$$\frac{1}{R_2} = \frac{1}{2} \left( \frac{1}{R_{is}} + \frac{1}{R_b} + \frac{1}{R_{eq}} \right)$$  \hspace{1cm} (B.9)

From (B.9) the parameter $\rho_2 = 2R_0/R_2$ can be expressed as (using also eq. (A.7)):

$$\rho_2 = \frac{R_0}{R_b} \left( 1 + \frac{R_b}{R_{is}} + \frac{R_b}{R_{eq}} \right) = \frac{b}{b + 2} \left( 1 + \frac{1}{x} + \frac{1}{b} \right)$$  \hspace{1cm} (B.10)

Using for $x$ its form of eq. (A.4) one finally obtains:

$$\rho_2 = \frac{(b + 1)^2}{b(b + 2)}$$  \hspace{1cm} (B.11)

As follows from (B.11)

$$\rho_2 - 1 = \frac{1}{b(b + 2)}; \quad \rho_2 = (b + 1)^2; \quad b + 1 = \frac{\sqrt{\rho_2}}{\sqrt{\rho_2 - 1}}$$  \hspace{1cm} (B.12)

The last part can be transformed into

$$b^2 = \left( \frac{\sqrt{\rho_2} - \sqrt{\rho_2 - 1}}{\rho_2 - 1} \right)^2$$  \hspace{1cm} (B.13)

Substituting in eq. (A.4) $(b + 1)$ from (B.12) and $b^2$ from (B.13) one obtains

$$x = \frac{\left( \sqrt{\rho_2} - \sqrt{\rho_2 - 1} \right)^2}{\sqrt{\rho_2} \left( \rho_2 - 1 \right)}$$  \hspace{1cm} (B.14)

To express $R_b$ via $R_0$ and $\rho_2$ one can re-write eq. (A.7)
\[ R_b = R_0 \frac{b+2}{b} = R_0 \frac{b(b+2)}{b^2} \quad \text{(B.15)} \]

Substituting \( b(b+2) \) by \( 1/(\rho_2-1) \) from eq. (B.12) and using \( b^2 \) from (B.13) one gets
\[
R_b = \frac{R_0}{\left(\sqrt{\rho_2} - \sqrt{\rho_2 - 1}\right)^2} = R_0 \left(\sqrt{\rho_2} + \sqrt{\rho_2 - 1}\right)^2 \quad \text{(B.16)}
\]

To find \( R_{is} \) one can transform (B.16) as follows
\[
R_{is} = xR_b = \frac{xR_0}{\left(\sqrt{\rho_2} - \sqrt{\rho_2 - 1}\right)^2} \quad \text{(B.17)}
\]

Using the relation \( R_0 = (\rho_2 R_2)/2 \) following from the \( \rho_2 \) definition and \( x \) from (B.14) one obtains from (B.17)
\[
R_{is} = \frac{R_2}{2} \sqrt{\frac{\rho_2}{\rho_2 - 1}} \quad \text{(B.18)}
\]

Errors for \( R_{is} \) and \( R_b \) can be found in the same way as for the adjacent strips (end of section a). Denoting the errors in \( R_0 \) and \( R_2 \) as \( \sigma_0 \) and \( \sigma_2 \) respectively one can write:
\[
\sigma^2(R_{is}) = \frac{\sigma_0^2}{4 \rho_2 (\rho_2 - 1)^3} + \frac{\sigma_2^2 \rho_2^3}{4 (\rho_2 - 1)^3} \quad \text{(B.19)}
\]
\[
\sigma^2(R_b) = \sigma_0^2 \left(\sqrt{\rho_2} + \sqrt{\rho_2 - 1}\right)^6 + \sigma_2^2 \rho_2^4 \left(1 + \sqrt{\frac{\rho_2 - 1}{\rho_2}} + \frac{1}{2 \sqrt{\rho_2 (\rho_2 - 1)}}\right)^2 \quad \text{(B.20)}
\]

Appendix C. Induced voltage calculations

An equivalent circuit diagram for the \( R_{is} \) measurement using the voltage induced at the neighbour strip is presented in Fig.C1. The SMU is connected between the point marked \( V_0 \) and the ground while the induced voltage is measured between the point marked \( V_1 \) and the ground.

![Equivalent circuit diagram for measurement with the neighbour strip](image)

Fig.C1. Equivalent circuit diagram for measurement with the neighbour strip

First let us consider a situation when the resistance to the ground, \( R_g \), is zero.
a) Zero $R_g$.

As follows from the diagram in Fig.C1 the induced voltage $V_1$ can be expressed via applied voltage $V_0$ as

$$V_1 = V_0 \frac{R_g R_{eq}}{R_b + R_{eq}} = V_0 \frac{R_g R_{eq}}{R_{is}(R_b + R_{eq}) + R_b R_{eq}} \quad (C.1)$$

Using the relations $R_{is} = x R_b$, $R_{eq} = b R_b$ and the eq. (A.4) one finds from (C.1)

$$S_1 = \frac{V_1}{V_0} = \frac{1}{1 + b} \quad (C.2)$$

Expressing from (C.2) $b$ via $S_1$ one gets

$$b = \frac{1 - S_1}{S_1} \quad (C.3)$$

Substituting $b$ in eq. (A.7) by its expression from (C.3) one obtains

$$R_b = R_0 \frac{1 + S_1}{1 - S_1} \quad (C.4)$$

Substituting $b$ in eq. (A.4) by its expression from (C.3) one obtains

$$x = \frac{(1 - S_1)^2}{S_1} \quad (C.5)$$

Using the relation $R_{is} = x R_b$ and eqs. (C.4), (C.5) one gets

$$R_{is} = R_0 \left( \frac{1}{S_1} S_1 \right) \quad (C.6)$$

Typically the parameter $R_0$ is measured with a good accuracy while $S_1$ (especially when it is very small) has a significant relative error $\sigma(S_1)/S_1$. Using (C.6) one can calculate the uncertainty in $R_{is}$ due to the error in $S_1$

$$\sigma(R_{is}) = R_0 \frac{\sigma(S_1)}{S_1} \left( \frac{1}{S_1} + S_1 \right) \quad (C.7)$$

For measurement with the next neighbour strip the diagram shown in Fig.C1 can also be used but with potential $V_1$ instead of $V_0$ and $V_2$ instead of $V_1$. Obviously one gets in this case

$$\frac{V_2}{V_1} = \frac{V_1}{V_0} \rightarrow S_2 = \frac{V_2}{V_0} = \left( \frac{V_1}{V_0} \right)^2 = S_1^2 \rightarrow S_1 = \sqrt{S_2} \quad (C.8)$$
Therefore for the measurements with next neighbour strips the eqs. (C.4) and (C.6) can be written as

\[ R_b = R_0 \frac{1 + \sqrt{S_2}}{1 - \sqrt{S_2}} \]  \hspace{1cm} (C.9)

\[ R_{is} = R_0 \left( \frac{1}{\sqrt{S_2}} - \sqrt{S_2} \right) \]  \hspace{1cm} (C.10)

while the eq. (C.7) is transformed to

\[ \sigma(R_{is}) = \frac{R_0}{2} \sigma(S_2) \left( \frac{1}{\sqrt{S_2}} + \sqrt{S_2} \right). \]  \hspace{1cm} (C.11)

b) Effects of non-zero \( R_g \).

As can be seen from the circuit diagram presented in Fig.C1 a non-zero resistor \( R_g \) results in a voltage drop on it

\[ V_g = V_0 \frac{R_g}{R_0 + R_g}, \]  \hspace{1cm} (C.12)

which adds up to \( V_1 \) or \( V_2 \) that would be measured with \( R_g=0 \). In other words the measured slopes \( S_1, S_2 \) will include an additional component

\[ S_g = \frac{V_g}{V_0} = \frac{R_g}{R_0 + R_g}, \]  \hspace{1cm} (C.13)

which is the same for neighbour and the next neighbour strips. The \( S_g \) may be measured e.g. as the slope in the situation \( S_1=S_2 \). It then can be subtracted from the slopes measured under other conditions thus suppressing the effects from non-zero \( R_g \).

To verify this model a special measurement was made with 1 k\( \Omega \) resistor inserted between the bias rail and the ground. The measurement was performed at \( U_{bias}=200V \) with the same sensor as in the study described in section 3.1.4. The signal was measured at the nearest neighbour strip. Fig.C2 summarises the results.

In a standard measurement with grounded bias rail shown in Fig.C2a the slope \( \frac{dV_1}{dV_0} \) of the induced voltage was found to be \( 1.65\pm0.11 \mu V/V \). When 1.003 k\( \Omega \) resistor was inserted between the bias rail and the ground, the slope increased to \( 753.3\pm0.1 \mu V/V \) as shown in Fig.C2b.
In the second case the $R_0$ was measured to be 1334 kΩ (which includes already the 1kΩ resistor). Using eq. (C.13) the slope resulting from 1 kΩ resistor can be calculated as: $1.003\text{kΩ}/1334\text{kΩ} = 751.9\mu\text{V/V}$. Adding to it the slope of 1.6 $\mu\text{V/V}$ due
to the resistor existed already before the insertion of 1 kΩ one obtains 753.5 μV/V in a perfect agreement with the results presented in Fig.C2b.

In further discussion let’s restrict ourselves to a typical in practice situation $R_g \ll R_0$. Then the usual gradient $dV_0/dI_0$ still correctly measures $R_0$ and the additional slope $S_g = R_g/R_0 \ll 1$. Let us now consider only the situation when the real slopes $S_1$, $S_2$ are comparable with $S_g$ and therefore the effects of $R_g$ are essential. The measured slopes $S_1^{\text{meas}}$, $S_2^{\text{meas}}$ are then also $\ll 1$. For the measurements with the neighbour strip we then obtain from eq. (C.6) for the measured interstrip resistance

$$\frac{1}{R_{\text{Is}}^{\text{meas}}} = \frac{1}{R_0} S_1^{\text{meas}} = \frac{1}{R_0} \left( S_1 + S_g \right) = \frac{1}{R_0} \frac{R_g}{R_0} + \frac{R_g}{R_0}.$$  \hspace{1cm} (C.14)

As follows from (C.14) the measured $R_{\text{Is}}$ is equal to the actual $R_{\text{Is}}$ with an effective parasitic resistance $R_p = R_0^2/R_g$ connected in parallel and restricting the measurable $R_{\text{Is}}$. For typical values of $R_0 = 1\, \text{MΩ}$ and $R_g = 1\, \Omega$ $R_p = 1000\, \text{GΩ}$.

For the measurements with next neighbour strip one can obtain from eq. (C.10)

$$\frac{1}{(R_{\text{Is}}^{\text{meas}})^2} = \frac{1}{R_0} S_2^{\text{meas}} = \frac{1}{R_0^2} \left( S_2 + S_g \right) = \frac{1}{R_0^2} \frac{R_g}{R_0} + \frac{R_g}{R_0^3}.$$  \hspace{1cm} (C.15)

Thus for the next neighbour the measured $R_{\text{Is}}$ is limited by an effective parasitic resistance $R_p = R_0 \sqrt{\frac{R_0}{R_g}}$. For typical values of $R_0 = 1\, \text{MΩ}$ and $R_g = 1\, \Omega$ the $R_p = 1\, \text{GΩ}$.

Appendix D. Current measurement calculations

In contrast to the voltage measurement the ammeter connection changes the electrical circuit at the sensor. Therefore the resistance $R_0$ measured between the master strip and the bias rail becomes dependent on the ammeter position and the formulae calculated for $R_0$ in Appendix A are no longer valid. To remind about this difference the slope $dV_0/dI_{\text{master}}$ will in this section be denoted as $R_{0a}$ – the $R_0$ with connected ammeter. In Fig.D1 the ammeter is shown connected in point 1 at the nearest neighbour to the master strip at which the master potential $V_0$ is applied. For measurements with the next neighbour strip it should be moved to point 2.
As mentioned in the beginning of section 3.2 the exact effect of an ammeter depends on the voltage burden (VB) of the instrument. In the calculations below the simplest VB behaviour is assumed: VB=const. In this case the current, $I$, measured by the ammeter is shifted relative to the actual current by a constant value but the normally used differential resistance $dV_0/dI$ isn’t affected. Note that there exist many other offsets, both in $V_0$ and $I$, which are all eliminated by using $dV_0/dI$ instead of $V_0/I$. The internal resistance of the ammeter is supposed to be negligible i.e. the point of its connection is considered grounded.

a) Zero $R_g$

First let us consider a situation when the resistance to the ground, $R_g$, is zero. Start with a situation when the test voltage, $V_0$, is applied to the immediate neighbour of the strip to which the picoammeter is connected. Sometimes to double the signal in the picoammeter the test voltage is applied to both immediate neighbours of the tested strip. The extension of the formulae given below to this case is quite straightforward.

The interstrip resistance can be found directly from the slope $S_1$ of the measured $I$-$V_0$ characteristic:

$$R_{is} = \frac{dV_0}{dl} = \frac{1}{S_1}. \quad (D.1)$$

If only the $R_{is}$ value is required the eq. (D.1) alone is sufficient. This directness and simplicity is a major advantage of the current measurement approach. The error in $R_{is}$ is easily calculated from $\sigma(S_1)$ - the error in $S_1$ found from the fit to experimental data.

Fig.D1. Equivalent circuit diagram for current measurement
\[ \sigma(R_{ii}) = R_{ii} \frac{\sigma(S_i)}{S_i} \]

Simultaneously measured is \( R_{0a} \) - the resistance between the point where the test voltage is applied and the bias rail:

\[ R_{0a} = \frac{dV_0}{dI_0}, \]  

(D.2)

where \( I_0 \) is the current from the source-meter unit. This resistance together with the directly measured \( R_{is} \) allows finding the bias resistance \( R_b \). In the situation considered here the \( R_{0a} \) consists of three resistors connected in parallel: \( R_{is}, R_b \) and \( R_{eq} \) (defined in Fig.1.1). (An assumption \( V_0 \gg V_B \) is also made here indirectly.) Thus

\[ \frac{1}{R_{0a}} = \frac{1}{R_{is}} + \frac{1}{R_b} + \frac{1}{R_{eq}}. \]  

(D.3)

Using the relations \( R_{is}=xR_b \) and \( R_{eq}=bR_b \) the eq. (D.3) can be rewritten as

\[ \frac{1}{R_{0a}} = \frac{1}{R_{is}} \left( 1 + \frac{x}{b} \right). \]  

(D.4)

From the eqs. (A.4) and (A.2) it follows

\[ \frac{x}{b} = \frac{b}{1+b} = b - x. \]  

(D.5)

Using (D.5) the eq. (D.4) can be rewritten as

\[ \frac{R_{is}}{R_{0a}} = 1 + b. \]  

(D.6)

Obviously \( R_{0a}<R_{is} \). From (D.6) it follows that for \( x \to 0, \ (b \to \sqrt{x} \to 0) \ R_{0a} \to R_{is} \) and for \( x \to \infty \ (b \to x \to \infty) \ R_{0a} \to R_{is}/x = R_b \). The eq. (D.6) can also be written as

\[ b = \frac{R_{is}}{R_{0a}} - 1 = \frac{R_{is} - R_{0a}}{R_{0a}}. \]  

(D.7)

Using eqs. (A.4), (D.6), (D.7) and the definition \( x = R_{is}/R_b \) one finds

\[ x = \frac{b^2}{1+b} = \left( \frac{R_{is} - R_{0a}}{R_{is}} \right)^2 = \frac{R_{is}}{R_b}. \]  

(D.8)

This in turn gives

\[ R_b = R_{0a} \frac{R_{is}^2}{(R_{is} - R_{0a})^2}. \]  

(D.9)

When \( x \to \infty \) the \( R_{is} \) is much larger than \( R_{0a} \) and eq. (D.9) becomes simply \( R_b = R_{0a} \). In other words \( R_{is} \) and \( R_b \) in this case are defined by two separate measurements.
$x \to 0$ then $R_{0a} \to R_{is}$ and the successful $R_b$ reconstruction relies on the accuracy of a small difference between two independently measured values of $R_{is}$ and $R_{0a}$.

For the next-neighbour measurements the ammeter in Fig.D1 should be moved to the point 2. Introduce the experimentally measured resistances $R_{0a}$ still defined by the equation (D.2) and $R_2 = dV_0/dI$. The latter is not equal to $R_{is}$ anymore but as follows from the circuit diagram can be presented as follows

$$R_2 = R_{is} \left( \frac{R_{is}}{R_b} + 2 \right) \quad \text{(D.10)}$$

For $R_{0a}$ the eq. (D.3) transforms into

$$\frac{1}{R_{0a}} = \frac{1}{R_{is}} + \frac{1}{R_b} + \frac{1}{R_{eq}} \quad \text{(D.11)}$$

Using the relations $R_{is} = xR_b$ and $R_{eq} = bR_b$, the eqs. (D.10) and (D.11) can be rewritten as

$$R_2 = R_b x(x + 2) \quad \text{(D.12)}$$

$$\frac{1}{R_{0a}} = \frac{1}{R_b} \left( 1 + \frac{1}{b} + \frac{1+x}{x(x+2)} \right) \quad \text{(D.13)}$$

Introduce parameter $A$ - the ratio of measured resistances

$$A = \frac{R_2}{R_{0a}} \quad \text{(D.14)}$$

Multiplying eqs. (D.12) and (D.13) one finds

$$A = x(x+2) \frac{1+b}{b} + x + 1 \quad \text{(D.15)}$$

Using eqs. (D.5) and (A.4) the eq. (D.15) can be re-written as

$$A = b(x + 2) + x + 1 = 1 + 2b + x(b + 1) = 1 + 2b + b^2 = (1 + b)^2 \quad \text{(D.16)}$$

Thus parameter $b$ can be simply expressed via experimentally measured parameter $A$

$$b = \sqrt{A} - 1 \quad \text{(D.17)}$$

Now using eqs. (A.4) and (D.17) one express $x$ and $x(x+2)$ via $A$

$$x = \frac{\left(\sqrt{A} - 1\right)^2}{\sqrt{A}} \quad \text{(D.18)}$$

$$x(x+2) = \left(\sqrt{A} - 1\right)^2 \frac{A + 1}{A} \quad \text{(D.19)}$$

Using eq. (D.14) the last two expressions can be re-written as
The last expression together with eq. (D.12) allows finding $R_b$

$$R_b = R_{oa} \frac{R_2^2}{\left(\sqrt{R_2} - \sqrt{R_{oa}}\right)^2 (R_2 + R_{oa})}$$ (D.22)

Multiplying the last expression by $x$ from eq. (D.20) one obtains $R_{is}$

$$R_{is} = R_2 \frac{\sqrt{R_2 R_{oa}}}{R_2 + R_{oa}}$$ (D.23)

For $x >> 1$ ($R_2 >> R_{oa}$) these equations are simplified to

$$R_b = R_{oa}$$ (D.22’)

$$R_{is} = \sqrt{R_2 R_{oa}}$$ (D.23’)

If $x\rightarrow 0$ than $b\rightarrow 0$ and as follows from eq. (D.17) $A\rightarrow 1$ or $R_2\rightarrow R_{oa}$. Then eq. (D.23) becomes simply $R_{is}=R_2/2$. Successful reconstruction of $R_b$ by eq. (D.22) in this situation relies on the accuracy of a difference between square roots of two close and separately measured resistances $R_2$ and $R_{oa}$.

Typically the accuracy of the $R_{is}$ is dominated by the accuracy of $R_2$, which in turn is determined by the accuracy $\sigma(S_2)$ of the experimentally measured slope $S_2=\frac{dI}{dV_0}$. Obviously

$$\sigma(R_2) = R_2 \frac{\sigma(S_2)}{S_2}$$ (D.24)

Differentiating eq. (D.23) by $R_2$ one finds for the error in $R_{is}$

$$\sigma(R_{is}) = R_{is} \frac{R_2 + 3R_{oa}}{2(R_2 + R_{oa})} \frac{\sigma(R_2)}{R_2}$$ (D.25)

For $x >> 1$ ($R_2 >> R_{oa}$) this equation reverts to

$$\sigma(R_{is}) = R_{is} \frac{\sigma(R_2)}{2R_2}$$ (D.26)

obviously following also from the eq. (D.23’).
b) Effects of non-zero $R_g$

As follows from the diagram in Fig. D1 a non-zero $R_g$ results in a potential at the bias rail $V_g = I_0 R_g$. This in turn produces an additional current flowing from the bias rail to the ammeter. For simplicity let us consider only the following typical and practically important situation: $R_g << R_b << R_{is}$. In this case $V_g = V_0 R_g / R_b$ that produces in the ammeter the current $I_g = V_g / R_b = V_0 R_g / R_b^2$. This current adds up to that via interstrip resistance and as a result the measured $R_{is}$ will be a real $R_{is}$ in parallel with a parasitic resistance, which for the measurements at a nearest neighbour strip can be calculated from eq. (D.1)

$$R_p = \frac{dV_0}{dI_g} = \frac{R_g^2}{R_{is}} \quad \text{(D.27)}$$

Clearly the same current will flow also through the ammeter connected at the next neighbour strip resulting in the observed $R_2$ with the same value as in eq. (D.27). The limit on the observable $R_{is}$ in this case can be deduced from the eqs. (D.22',D.23')

$$R_{is} = \sqrt{\frac{R_b^2}{R_g}} R_b = R_b \sqrt{\frac{R_b}{R_g}} \quad \text{(D.28)}$$

To verify this model a special measurement was made with 1 kΩ resistor inserted between the bias rail and the ground. The measurement was performed at $U_{bias}=200$V with the same sensor as in the study described in section 3.2.4. The induced signal was measured at the nearest neighbour strip. Fig.D2 summarises the results.

In a standard measurement with grounded bias rail shown in Fig.D2a the slope $dI_1/dV_0$ was found to be $1.04 \pm 0.15$ pA/V. When 1.003 kΩ resistor was inserted between the bias rail and the ground, the slope increased to $565.2 \pm 0.1$ pA/V as shown in Fig.D2b.

In the second case the $R_0$ was measured to be 1333 kΩ (which includes already the additional 1 kΩ resistor). The resistance from the bias rail to the ground consisted of 1.003 kΩ in parallel with separately measured $R_b=1334.5$ kΩ between the bias rail and the ammeter, which gives 1.002 kΩ. The voltage produced by $V_0=1$ V at this resistance can be calculated as: $1V \times 1.002kΩ/1333kΩ = 751.7 \mu$V, which results in the current via ammeter of $751.7\mu$V/1334.5kΩ = 563.3 pA/V. Adding to it the slope of 1.0 pA/V due to the resistor existed already before the insertion of 1 kΩ one obtains 564.3 pA/V in an excellent agreement with the results in Fig.D2b.
Fig. D2a. $I_1$ vs. $V_0$ for grounded bias rail

Fig. D2b. $I_1$ vs. $V_0$ for 1 kΩ resistor between the bias rail and the ground