REF.TH.3415-CERN

FORWARD p-n REACTIONS AND NUCLEON-NUCLEON SCATTERING

*)

P.A.M. Guichon

CERN -- Geneva

ABSTRACT

Forward p-n reaction cross-sections are expressed in terms of nucleon-nucleon phase shifts. The ratio of the probabilities for triggering Gamow-Teller or Fermi transitions is found to be in quite good agreement with experiment. New n-p scattering experiments are also suggested in order to improve the interpretation of p-n reactions.

*) Permanent address: Service de Physique Théorique, I.P.N., Bâtiment 210, Université Claude Bernard, 69622 Villeurbanne Cedex, France.

1. - INTRODUCTION

The past few years have witnessed an intense activity in the field of p-n reactions. Thanks to the realization of long time of flight apparatus¹⁾, it has been possible to measure exclusive charge-exchange cross-sections at high incident energy (> 100 MeV). In this condition, the reaction is essentially direct so that its description is relatively simple, and useful information about nuclear structure can be extracted from the data. In particular, the forward cross-section can be expressed in terms of Fermi (F) and Gamow-Teller (GT) matrix elements between the target ground state and the final nuclear state. This possibility of measuring the GT strength without the kinematic limitation of β decay has opened up a new field of investigation of nuclear structure²⁾. The most exciting results concern the relevance of nucleon internal degrees of freedom in nuclei. Indeed, it has been known for a long time³⁾ that the renormalization of the GT operator in nuclear matter is a short-range effect due to the suppression of the s wave pion exchange contribution⁴⁾. Since the successful MIT bag model⁵⁾ and its descendants⁶⁾ all imply a large (~ 1 Fm) confinement domain, it is natural to invoke quark degrees of freedom whenever the truly long-range contribution is not dominant. Although this has been considered⁷⁾, the orthodox approach invokes the mixing of isobar and nucleon states by a zero-range particle-hole interaction⁸⁾.

Whatever the interpretation of the renormalization of the GT operator in nuclear matter, its experimental detection is of central importance. In that respect, the forward p-n reactions are, at present, the most powerful tool because they give access to sum rules which are, to a large extent, free from nuclear configuration mixing uncertainty. Despite many efforts⁹⁾, this uncertainty has always precluded the unambiguous detection of the renormalization of the GT operator from β decay experiments, because they are mostly single state transitions^{*)}.

The extraction of GT matrix elements from p-n reactions relies on the following expression for the forward cross-section¹⁰⁾ (irrelevant factors are omitted here):

*) Recent experimental advances at the CERN on-line isotope separator (ISOLDE), concerning the production of proton-rich nuclei, give some hope that (at least partial) sum rules will also be obtainable in β decay. This would be an ideal complement of p-n reactions (S. Damgaard et al., proposal submitted to the ISOLDE collaboration).

$$\frac{d\sigma}{d\Omega} \sim N_c |V_c|^2 B_F + N_{\sigma c} |V_{\sigma c}|^2 B_{GT} \quad (1)$$

Equation (1) results from the distorted wave Born approximation assuming a single step reaction. The factors N_T , $N_{\sigma T}$ describe the distortion, B_F and B_{GT} are the usual F and GT strengths from the target ground state to the final nuclear state, while $|V_T|^2$ and $|V_{\sigma T}|^2$ measure the probability of the reaction proceeding through F or GT transitions.

The ratio $N_{\sigma T}/N_T$ is close to unity¹⁰⁾ and its estimation from optical model calculations does not introduce serious errors. The knowledge of $R = |V_{\sigma T}/V_T|$ then allows the determination of the GT strength relative to the Fermi strength of the isobaric analogue state (IAS). Conversely, the knowledge of B_{GT} from β decay for some states allows an empirical determination of R [see Ref. 10) for more details].

In this paper we discuss the relation between R and the experimental nucleon-nucleon scattering matrix. Under reasonable assumptions, it can be calculated in terms of phase shifts. The phenomenological analysis of Ref. 10) shows that above $E_p \sim 70$ MeV (E_p = laboratory incident energy) and up to $E_p = 200$ MeV (the maximum energy presently available), R is approximately the same linear function of E_p for a number of nuclei. This universality suggests that, in this energy range, the reaction actually proceeds through direct charge exchange scattering on the nucleon which initiates the F or GT transition. Since the distortion factors N_T and $N_{\sigma T}$ are roughly equal, the ratio R corresponds, in this limit, to a Born approximation with isospin and spin-isospin densities controlled by B_F and B_{GT} . Then, apart from Fermi motion and binding, R will be identical to its value obtained from free nucleon-nucleon scattering. This is a stringent test of the whole procedure of extraction of GT strength from p-n reactions. At the same time, this provides a useful check of the nucleon-nucleon inputs which control the effective interaction used in distorted wave calculations.

In Section 2 we present the evaluation of R from **N-N** phase shifts and we show how the effect of the Fermi motion can be estimated. In Section 3 we present our choice of phase shifts and compare our results with experiments and with other calculations. In Section 4 we discuss the observables which should be measured in order to ascertain the ratio R as well as improve the effective interaction used in DWBA analysis of forward p-n reactions.

2. - EVALUATION OF R

2.1 Relating p-n Reactions to p-n Scattering

We consider the reaction:

with the neutron emitted in the forward direction. N and N' are the initial and final nuclear states. We assume that the energy loss can be neglected before the incident energy, as is the case in most applications. Corrections for finite energy transfer can be evaluated in the DWBA.

We take the proton laboratory energy E_p in the range $100 \div 300$ MeV. The upper bound allows a non-relativistic treatment of the problem as well as the neglect of inelasticity. (This is for simplification. Extension to higher energy does not cause difficulties.) The lower bound is what one usually requires for the reaction (2) to be approximated by a one-step process. The idea is that compound nucleus formation involves the overlap between the nuclear and the proton or neutron wave functions. This overlap is small when the proton energy is much greater than the Fermi energy.

We replace reaction (2) by off-shell elastic charge exchange scattering as shown in Fig. 1. Multiple scattering is not excluded but, due to Pauli blocking, its dominant contribution comes from scattering on the nucleus without charge exchange. This is accounted for by the distortion of the in or outgoing waves. (Note that multiple scattering of the same pair must not be considered. It is already included in the p-n scattering kernel of Fig. 1. The latter also contains the antisymmetrization.)

In order to simplify the calculation of the reaction amplitude, let us remark that the isospin-flip character of the nuclear transition prevents the isospin saturated part of the nucleus from contributing. This means that only the external shells are involved in the reaction. Moreover, in the energy range considered, the mean free path of a nucleon inside nuclear matter is small (~ 1 Fm) so that transitions which take place inside the nuclear volume practically do not contribute to the cross-section. Thus the relevant transitions occur at the surface where the density is low. In first approximation we can neglect the effect of distortion except for the loss of flux due to the shadow of the whole nucleus.

We do not expect this approximation to be sufficient for an absolute evaluation of the reaction amplitude, but we need in fact only a ratio of amplitudes to estimate R . For this purpose, our approach should make sense. It is strongly supported by the near equality of the distortion factors N_T and $N_{\sigma T}$ ¹⁰⁾.

We can now remark that for a zero range n - p interaction, the absorption would be the same irrespective of the transition induced by the scattering. (The imaginary part of the optical potential is essentially central and charge independent. It does not change the spin-isospin state of the in or outgoing nucleon.) In the forward direction, direct pion exchange does not contribute so that the interaction at work is rather short-ranged. We take this as our justification for assuming that absorption is the same for every piece of the reaction amplitude. Since ultimately we are interested in a ratio, we forget about it.

This leads us to compute the forward reaction T matrix according to

$$\langle \vec{k}_n, N' | T | \vec{k}_p, N \rangle = \sum_{\alpha\beta} \langle \vec{k}_n, \vec{k}_\alpha | t | \vec{k}_p, \vec{k}_\beta \rangle \langle N' | a_\alpha^\dagger a_\beta | N \rangle \quad (3)$$

We work in the laboratory frame. $\vec{k}_p, \vec{k}_n$ are the proton and neutron momenta. Neglecting the energy loss, we have, in the forward direction, $\vec{k}_p = \vec{k}_n$ and there is no recoil. $a_\alpha^\dagger, a_\beta$ are creation and annihilation operators of a nucleon in a plane wave state $\vec{k}_\alpha, \vec{k}_\beta$ ($\vec{k}_\alpha = \vec{k}_\beta$ due to momentum conservation, but we keep a different notation for clarity's sake. Spin isospin co-ordinates are omitted until explicitly needed.) The first factor in Eq. (3) is the antisymmetrized t matrix for nucleon-nucleon scattering which we take to be the same as in free space. This is justified by the low density at which the transition takes place. Since we neglect the energy loss, only the energy diagonal t matrix elements are needed and we can take them from experiment. In the non-relativistic limit, the t matrix is invariant under a boost from the laboratory to the CM frame so that we can write:

$$\langle \vec{k}_n, \vec{k}_\alpha | t | \vec{k}_p, \vec{k}_\beta \rangle = \langle \vec{k}_p | t^{CM} | \vec{k}_i \rangle \quad (4)$$

with t^{CM} the scattering matrix in the CM of the pair. $\vec{k}_i = (\vec{k}_p - \vec{k}_\beta)/2$, $\vec{k}_f = (\vec{k}_n - \vec{k}_\alpha)/2$ ($\vec{k}_i = \vec{k}_f$) are the initial and final momenta in the CM, respectively. We recall that the states $|\vec{k}_p, \vec{k}_\beta\rangle$ or $|\vec{k}_n, \vec{k}_\alpha\rangle$ are simple products of plane wave states. If we restore spin-isospin co-ordinates we have the connection

$$|\vec{k}_p \sigma_p \tau_p, \vec{k}_\beta \sigma_\beta \tau_\beta\rangle \rightarrow |\vec{k}_i \sigma_p \tau_p, -\vec{k}_i \sigma_\beta \tau_\beta\rangle = |\vec{k}_i \sigma_p \tau_p, \sigma_\beta \tau_\beta\rangle \quad (5)$$

with obvious notations. Note that we take the forward n-p scattering as

$$p + n \longrightarrow n + p \quad (6)$$

so as to select the isovector piece of the t matrix. It is related to the usual scattering

$$p + n \longrightarrow p + n \quad (7)$$

by the exchange of the final particle states. This amounts to multiplying the t matrix by $P_{\sigma P_T} = (1 + \vec{\sigma}_1 \cdot \vec{\sigma}_2)(1 + \vec{\tau}_1 \cdot \vec{\tau}_2)/4$ and changing $\vec{k}_f$ into $-\vec{k}_f$.

The t matrix is related to the scattering amplitude by (units are such that $\hbar = c = 1$):

$$\langle k_f | t^{CM} | k_i \rangle = - (2\pi^2 M)^{-1} f(E, \theta) \quad (8)$$

with M the nucleon mass, θ the CM angle and E the total CM energy $E = k_1^2/M = k_f^2/M$. f is scalar in isospin space and we write:

$$f = A \vec{\sigma}_1 \cdot \vec{\sigma}_2 + B \quad (9)$$

B does not contribute to the scattering (6), so from now on we consider only A . We make explicit the spin dependence of the latter through the usual expansion¹¹⁾:

$$A = a + c (\vec{\sigma}_1 \cdot \hat{n} + \vec{\sigma}_2 \cdot \hat{n}) + m \vec{\sigma}_1 \cdot \hat{n} \vec{\sigma}_2 \cdot \hat{n} \\ + (g+h) \vec{\sigma}_1 \cdot \hat{p} \vec{\sigma}_2 \cdot \hat{p} + (g-h) \vec{\sigma}_1 \cdot \hat{q} \vec{\sigma}_2 \cdot \hat{q} \quad (10)$$

where $a, \dots, h$ are scalar functions of E and θ , $\hat{p}, \hat{q}, \hat{n}$ are the unit vectors $\hat{p} = \widehat{k_i + k_f}$, $\hat{q} = \widehat{k_f - k_i}$, $\hat{n} = \hat{p} \times \hat{q}$. In our case, $\vec{k}_i = \vec{k}_f = \vec{k}$ so only $\hat{p}$ can appear in the expansion of A . One easily verifies that Eq. (10) reduces to:

$$A(\vec{k}_i = \vec{k}_f) = a(E) + m(E) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + 2h(E) \vec{\sigma}_1 \cdot \hat{p} \vec{\sigma}_2 \cdot \hat{p} \quad (11)$$

We do not specify that the angular argument of a, m, h is zero. The matrix elements of f between spin-isospin coupled states $|SM_S, TM_T\rangle$ have the partial wave expansion:

$$\begin{aligned} \langle S'M'_S T'M'_T | f | SM_S TM_T \rangle &= \delta(S,S') \delta(T,T') \delta(M_T, M'_T) \times \\ &\sum_{\ell, \ell', J, M_J} 4\pi Y_{m'}^{\ell'}(\hat{k}_f) Y_m^{\ell}(\hat{k}_i) \langle \ell' m' S' M'_S | J M_J \rangle \langle \ell m S M_S | J M_J \rangle \\ &\quad [1 - (-)^{\ell+S+T}] \langle \ell' S J T | (S-1)/2 i k | \ell S J T \rangle \end{aligned} \quad (12)$$

where the spherical harmonics and the Clebsch-Gordan coefficients follow the conventions of Ref. 12). The factor $[1 - (-)^{\ell+S+T}]$ comes from the antisymmetrization. In terms of nuclear bar phase shifts¹¹⁾, the partial wave matrix elements of (S-1) are:

$$\begin{aligned} \langle \ell' S J | S-1 | \ell S J \rangle &= \cos(2 \epsilon_{\ell S J}) (e^{2i\delta_{\ell S J}} - 1) \quad \text{if } \ell = \ell' \\ &= -i \sin(2 \epsilon_{\ell S J}) e^{i(\delta_{\ell S J} + \delta_{\ell' S J})} \quad \text{if } \ell \neq \ell' \end{aligned} \quad (13)$$

with $\delta_{\ell S J}$ and $\epsilon_{\ell S J}$ the usual phase shifts and mixing parameters. We have used the convention $\epsilon_{\ell S J} = \epsilon_{\ell' S J} = 0$ when $\ell = J$. Otherwise the mixing parameter is simply noted ϵ_J . Note that the superfluous isospin dependence has been suppressed. Using Eqs. (9), (11) and (12) one gets the following expressions for a, m, h:

$$a = (x - y)/8k, \quad m = (-x - y - z)/8k, \quad h = (x - 3z/2)/8k \quad (14)$$

with:

$$\begin{aligned} x &= \sum_J (-)^J (2J+1) \langle J 0 J | (S-1)/2 i | J 0 J \rangle, \\ y &= \sum_{\ell, J} (-)^{\ell} (2J+1) \langle \ell 1 J | (S-1)/2 i | \ell 1 J \rangle, \\ z &= \sum_{\ell, \ell', J} (-)^{\ell} C(\ell, J) C(\ell', J) \langle \ell' 1 J | (S-1)/2 i | \ell 1 J \rangle \end{aligned}$$

$$\text{and: } C(\ell, J) = \sqrt{J}, \sqrt{J+1}, \sqrt{2J+1}, \quad (15)$$

according to $\ell = J+1, J-1, J$. In the practical calculation we cut the expansion at $\ell = 5$ as it is usually done in phase shifts analyses and for $\ell > 5$ we take the partial wave matrix element of (S-1) from the one-pion exchange potential¹³⁾.

2.2 p-n Cross-Section

We return to Eq. (3) for the expression of the reaction T matrix which, according to the above discussion, becomes (spin-isospin co-ordinates are restored):

$$\begin{aligned} \langle \vec{k}_p \sigma_n \tau_n, N' | T | \vec{k}_p \sigma_p \tau_p, N \rangle = -[2\pi^2 M]^{-1} \times \\ \sum_{\vec{k}_\alpha \sigma_\alpha \tau_\alpha} \langle \sigma_n \tau_n, \sigma_\alpha \tau_\alpha | [a(E) + m(E) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + 2h(E) \vec{\sigma}_1 \cdot \hat{p} \vec{\sigma}_2 \cdot \hat{p}] \vec{\tau}_1 \cdot \vec{\tau}_2 | \sigma_p \tau_p, \sigma_\beta \tau_\beta \rangle \times \\ \langle N' | a_{\vec{k}_\alpha \sigma_\alpha \tau_\alpha}^\dagger a_{\vec{k}_\alpha \sigma_\beta \tau_\beta} | N \rangle \end{aligned} \quad (16)$$

The summation cannot be performed since both E and $\hat{p}$ depend on $\vec{k}_\alpha$. However, this is a mild problem insofar as the incident energy is much larger than the Fermi energy because the nuclear matrix element on the right-hand side of Eq. (16) selects values of k_α which are small before the incident momentum. We can then try an expansion of Eq. (16) in powers of k_F/k_p , k_F being the Fermi momentum, with the explicit assumption that the term of order zero is a good starting point and that corrections can be evaluated in an approximate manner. This is ultimately justified by the final result which shows that the corrections are actually small.

To zeroth order in k_F/k_p , Eq. (16) simplifies:

$$\begin{aligned} \langle T \rangle = -[\pi^2 M]^{-1} [a(E_0) \delta(\sigma_n \sigma_p) \langle N' | \sum_i \tau_i^+ | N' \rangle + \\ m(E_0) \langle \sigma_n | \vec{\sigma} | \sigma_p \rangle \cdot \langle N' | \sum_i \tau_i^+ \vec{\sigma}_i | N \rangle + \\ 2h(E_0) \hat{k}_p \cdot \langle \sigma_n | \vec{\sigma} | \sigma_p \rangle \hat{k}_p \cdot \langle N' | \sum_i \tau_i^+ \vec{\sigma}_i | N \rangle] \end{aligned} \quad (17)$$

where we have used $\langle n | \vec{\tau}_1 | p \rangle \cdot \vec{\tau}_2 = 2\tau_2^+ (= \tau_2^X + i\tau_2^Y)$ and $E_0 = k_p^2/4M$. We recognize the F and GT operators for the nuclear transition from N to N'. Let us write $|N\rangle = |J_I M_I\rangle$, $|N'\rangle = |J_F M_F\rangle$. The unpolarized cross-section is:

$$\begin{aligned} \frac{d\sigma(I \rightarrow F)}{d\Omega} = (2\pi)^4 M^2 [2Q_{I \rightarrow F}]^2 \sum_{M_I, M_F} T_n |T|^2 \\ = 16 [|V_c|^2 B_F(I \rightarrow F) + |V_{\sigma\tau}|^2 B_{GT}(I \rightarrow F)] \end{aligned} \quad (18)$$

where:

$$\begin{aligned} B_F(I \rightarrow F) &= (2J_I + 1)^{-1} |\langle J_I || \sum_i \tau_i^+ || J_I \rangle|^2, \\ B_{GT}(I \rightarrow F) &= (2J_I + 1)^{-1} |\langle J_F || \sum_i \tau_i^+ \vec{\sigma}_i || J_I \rangle|^2 \end{aligned} \quad (19)$$

are the F and GT strengths [the reduced matrix element is the same as in Ref. 12]. In terms of these strengths, the inverse β decay rate from F to I is $6200/\text{ft} \approx B_F(I \rightarrow F) + B_{GT}(I \rightarrow F) \times g_A^2 \times (2J_I + 1)/(2J_F + 1)$, $g_A \sim 1.25$. The expressions for $|V_T|$ and $|V_{GT}|$ are:

$$\begin{aligned} |V_T|^2 &= |a(E_0)|^2, \\ |V_{GT}|^2 &= [2|m(E_0)|^2 + |m(E_0) + 2h(E_0)|^2]/3. \end{aligned} \quad (20)$$

We recall that absorption factors have been omitted so that Eq. (18) has only a relative significance. Equation (20) allows us to compute the ratio $R = |V_{GT}/V_T|$ in terms of nucleon-nucleon scattering data. We now indicate how corrections due to the Fermi motion can be estimated.

2.3 Fermi motion

In our approach, the Fermi motion effect cannot be computed exactly, but our goal is essentially to show that it is small. For this we can use rather rough approximations. The summation over $\vec{k}_\alpha$ in Eq. (16) amounts to averaging the scattering t matrix with a weight factor proportional to the transition density $\langle N' | a_{\vec{k}_\alpha}^+ a_{\vec{k}_\alpha} | N \rangle$. In the Fermi gas model (FGM) this would be $\delta(|\vec{k}_\alpha| - k_F)$ up to a factor, due to Pauli blocking. In more realistic models, it would be essentially the product of two single particle wave functions written in momentum space, but the tendency to peak at a given momentum (whose order of magnitude is k_F) is respected. In fact, it does not matter very much whether $|\vec{k}_\alpha|$ is completely defined or runs over a range of values because the transition density multiplies amplitudes which turn out to be smooth functions of E . So we do not make a large error if we replace the first factor in Eq. (16) by the ansatz suggested by the FGM:

$$(4\pi k_F^2)^{-1} \int d\vec{k}_\alpha \delta(|\vec{k}_\alpha| - k_F) [a(E) + m(E) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + 2h(E) \vec{\sigma}_1 \cdot \hat{p} \vec{\sigma}_2 \cdot \hat{p}] \quad (21)$$

The limit $k_F = 0$ in Eq. (21) gives back the zeroth order approximation and thus provides the means to investigate the corrections due to the Fermi motion. The exact value of k_F is not crucial insofar as the corrections are small, and it turns out to be the case in the energy range we are considering.

If we insert Eq. (21) into Eq. (16) we can perform all the summations and we get the same results as in zeroth order provided we replace $a(E_0)$, $m(E_0)$, $h(E_0)$ by:

$$\begin{aligned} a(E_0, k_F) &= \frac{1}{2} \int_{-1}^{+1} du \, a(E) \quad , \\ m(E_0, k_F) &= \frac{1}{2} \int_{-1}^{+1} du \, m(E) + h(E)(1-u^2) \left(\frac{k_F}{p} \right)^2 \quad , \\ h(E_0, k_F) &= \frac{1}{2} \int_{-1}^{+1} du \, h(E) \left[1 - 2yu + \frac{3u^2-1}{2} y^2 \right] \left(\frac{k_F}{p} \right)^2 \quad , \end{aligned}$$

with: $p^2 = k_p^2 + k_F^2 - 2k_p k_F u$, $E = \frac{p^2}{4M}$, $y = \frac{k_F}{k_p}$ (22)

If we choose $k_F = 1 \text{ Fm}^{-1}$, the upper bound of our energy range $E_p = 300 \text{ MeV}$ requires the knowledge of the amplitudes up to $E_p \approx 500 \text{ MeV}$. We have checked that relativistic and inelasticity effects do not change the result by more than $1 \div 2\%$. The lower bound $E_p = 100 \text{ MeV}$ requires the amplitudes down to $E_p = 30 \text{ MeV}$. This is still large enough to prevent the calculation from being sensitive to the very rapid variations of the amplitudes at threshold. We can, of course, compute the amplitude only at those energies where the phase shifts analysis has been performed, but it is a simple technical problem to fit a curve through these points in order to make the numerical integration.

Before going on to the results, we would like to point out that the ansatz (22) is not unique due to the term $\vec{\sigma}_1 \cdot \hat{p} \vec{\sigma}_2 \cdot \hat{p}$ in Eq. (16). The ratio $h(E)/E$ is also smoothly varying and we could have chosen the ansatz:

$$\frac{\vec{\sigma}_1 \cdot \vec{p} \vec{\sigma}_2 \cdot \vec{p}}{4M} (4\pi k_F^2)^{-1} \int d\vec{k}_\alpha \delta(|\vec{k}_\alpha| - k_F) \frac{h(E)}{E} \quad (23)$$

Then, since $\vec{p} = (\vec{k}_p - \vec{k}_\alpha)/2$, we easily realize that new nucleon operators will come into play in the cross-section: $\sum_i [\vec{\sigma}_i \times \vec{p}_i]^k$, $\sum_i \vec{p}_i \vec{\sigma}_i \cdot \vec{p}_i$, with $\vec{p}_i$ the momentum of the i th nucleon. The contribution of the last operator is presumably negligible because of the order of the Fermi energy, but the first one, of order k_F , may be interesting. In particular, when $k = 0$, it is the impulse approximation to the axial charge density of the nucleus. This quantity is known to be highly sensible to non-nucleonic degrees of freedom¹⁴⁾, but very difficult to measure. It is not clear whether p - n reactions can actually provide a new means of measuring this quantity, but it may be worthwhile studying this problem.

As far as we are concerned, this alternative treatment of the Fermi motion makes little difference because the term $h(E) \vec{\sigma}_1 \hat{p} \vec{\sigma}_2 \hat{p}$ is always a numerically small correction to the term $m(E) \vec{\sigma}_1 \cdot \vec{\sigma}_2$. So this difference could only manifest itself when the GT operator is completely suppressed by parity. This refers then to another problem of the measure of the GT strength in nuclei.

3. - RESULTS

3.1 Phase Shifts

We have chosen to work with the set of phase shifts of Arndt et al.¹⁵⁾ (set I) although a more recent analysis¹⁶⁾ (set II) has added the last TRIUMF results¹⁷⁾ to the data base and thus should have been preferred, a priori. The reasons for our choice are the following.

- i) Set I results from an energy-dependent analysis in the range (0,400÷500 MeV). This is at variance with set II for which the fit has been performed in a narrow band around some energies, which makes the correlation between the data of the base much weaker.
- ii) The energy increment in Set I is only 25 MeV, so that between two successive energies, the amplitudes vary little. We can reasonably fit a continuous curve through the values obtained in order to make possible the estimation of the Fermi motion effect. This is impossible with set II: first the separation in energy between successive experimental points is too large (~ 100 MeV). Second, due to the independent systematic errors, there are fluctuations in the amplitudes from one energy to the following one.
- iii) The new data used for set II are either zero in the forward direction or insensitive to R (see Section 4). Thus, as far as this particular quantity is concerned, they do not give really strong constraints. Moreover, they have not been performed in the strict forward direction (except for the differential cross-section for which sets I and II give the same result). In Section 4 we discuss n - p scattering experiments which could, at least in principle, determine directly R .

3.2 Results

We first discuss $|V_\tau|$ and $|V_{\sigma\tau}|$ separately. The results we obtain with set I are shown in Fig. 2 with $k_F = 0$ or $k_F = 1 \text{ fm}^{-1}$ (for illustration, the energy range is extended down to 50 MeV). The quite moderate difference between the two cases provides support to our approximate evaluation of the Fermi motion effect. Above 100 MeV, the static approximation is already adequate and, as expected, gets better and better as the energy increases. The two curves become practically indistinguishable above 200 MeV, both for V_τ and $V_{\sigma\tau}$. In the 100 ÷ 300 MeV case, the magnitude of the difference is probably comparable to the uncertainties associated with the phase shifts, so we shall keep $k_F = 0$ in the following.

The energy dependence of $|V_{\sigma\tau}|$ and $|V_\tau|$ respects the well-known general trend. $|V_\tau|$ decreases rapidly until 200 MeV while $|V_{\sigma\tau}|$ is nearly constant above 100 MeV. A partial explanation is that $|V_{\sigma\tau}|$ receives a strong contribution ($\sim 60\%$ above 100 MeV) from the single pion exchange amplitude which has a very weak energy dependence [only the Pauli exchange term contributes and goes like $(1+m_\pi^2/2ME)^{-1}$]. This does not work for V_τ because the pion contribution, although strong, is nearly cancelled by the real part of the remaining amplitude. The detailed understanding of the energy dependence of the nucleon-nucleon amplitudes is a difficult problem and is not within the scope of this work. We take it from phenomenology and use it as an input for p-n reactions.

One cannot measure separately $|V_\tau|$ and $|V_{\sigma\tau}|$ in p-n reactions due to absorption but, as pointed out in Ref. 10), the ratio $R = |V_{\sigma\tau}/V_\tau|$ can be extracted from the forward cross-sections with some confidence. In fact, what can be measured is $\sqrt{N_{\sigma\tau}}|V_{\sigma\tau}|/\sqrt{N_\tau}|V_\tau|$ but, within our approximations, this makes no difference. Since we adopt the limit $k_F = 0$, consistency requires that we also take the limit $N_{\sigma\tau}/N_\tau \sim 1$, valid in the zero density limit. In Fig. 3 we have plotted our result for R (curve I) and the phenomenological straight line fitted in Ref. 10). The experimental points are, of course, distributed around it, but above 70 MeV the linear dependence is striking. [For illustrative purposes, we have again extended the energy range down to 50 MeV and we have plotted some experimental points taken from Ref. 10). They refer to ^{13}C .] Clearly, the agreement is quite good above 100 MeV, i.e., in the range where our approximation begins to make sense. The departure from the linear dependence starts at 180 MeV, but at 200 MeV (the last experimental point) the difference is still tiny. Above 200 MeV, R decreases slightly but remains larger than 3. It would be interesting to have an experimental confirmation of this trend.

In Fig. 3 we have also plotted R as computed from the widely-used effective interaction of Love and Franey¹⁸⁾ (LF), [curve II taken from Ref. 10)]. In this case, the agreement with experiment is rather poor above 140 MeV. It is not easy to understand the origin of the discrepancy because two effects may be at work: first the interaction is a local parametric one and this may introduce some errors due to the neglect of velocity-dependent terms, for instance. Second, it has been fitted to the phase shifts of set II. If we compute R with this set we find a result close to that of LF at 210 MeV, but very different at 315 MeV (40% below).

In any case, the ratio R computed with set II is very different from that obtained with set I (about 60% at 315 MeV) and we do not conceal the fact that this is somewhat disturbing. Indeed, if set II was really to be chosen, the whole interpretation of p - n reactions would lose its simplicity and possibly its credibility concerning the determination of the GT strength in nuclei. We do not think that the good agreement with set I is purely fortuitous. On the other hand, it is highly desirable to clarify the situation concerning the nucleon-nucleon scattering. In the next section, we discuss how it could be done.

4. - OBSERVABLES IN n - p SCATTERING

We shall see below that a direct measure of R in n - p scattering implies rather difficult polarization experiments in the forward proton direction so that the phenomenological determination of this quantity will probably always use the phase shifts. However, it is useful to express R in terms of physical observables at zero angle because this suggests which experiments should be included in the data base of the phase shift analysis for a correct determination of this ratio. In the forward direction, a number of observables vanish and among the remaining ones we retain those with no more than three polarizations. The reason is that the experiment must be reasonably feasible. This excludes any detection of the final neutron since it has zero energy in the laboratory. According to Section II we still consider the proton as the scattered particle. Usually, it is referred to as the recoil particle. This changes the definition of some observables, but the correspondence is obvious ($\theta^{\text{cm}} = 0^\circ$ for the proton means $\theta^{\text{cm}} = 180^\circ$ for the neutron; polarization parameters are exchanged with transfer parameters and so on.)

The set of observables we have chosen is not unique since a number of them have the same expression in the forward direction. Also, some of them differ only by terms of order $|h/m|^2$ which is small in the interesting energy range and in this case we have kept only one of them. Consequently, what follows describes

only one possibility from which many others can be inferred. With the same notations as in Ref 11), we retain:

$$I_0 = |V_z|^2 + 3 |V_{\sigma z}|^2 \quad (24)$$

$$I_0 D = |a|^2 - |m+2h|^2 \quad (25)$$

$$I_0 A_{zz} = -2|m|^2 + 2 \operatorname{Re}[a(m+2h)^*] \quad (26)$$

$$I_0 C_{np}^q = -2 \operatorname{Im}[a(m+2h)^*] \quad (27)$$

The laboratory cross-section is $16 I_0$. This is Eq. (18) with B_F and B_{GT} computed for the nucleon. To get D one must measure the transverse polarization of the proton, assuming a neutron beam polarized in the same direction. For A_{zz} (resp. C_{np}^q) one must use a longitudinally polarized neutron beam and a longitudinally (resp. transversally) polarized target. A_{zz} can be obtained by measuring an asymmetry in the forward direction with respect to a reversal of the target polarization. Through time-reversal, one gets C_{np}^q by measuring the transverse polarization of the proton along a direction perpendicular to the target polarization [see Ref. 11) for details].

It is obvious from Eq. (24) that the cross-section determines essentially $V_{\sigma T}$ since in the energy range considered $1/3 |V_T/V_{\sigma T}|^2 \sim 0.1 \div 0.03$. We have already pointed out that h is small before m so we have the rough approximation:

$$I_0 D \simeq |a|^2 - |m|^2 \simeq |V_z|^2 - |V_{\sigma z}|^2 \quad (28)$$

which, together with Eq. (24), leads to $D \simeq -1/3 + 4/3 |V_T/V_{\sigma T}|^2$. The dependence of D (this corresponds to D_t or R_t) on $|V_T/V_{\sigma T}|$ is thus weak since this quantity is small. This explains why this observable does not put a strong constraint on the phase shift analysis in what concerns $|V_{\sigma T}/V_T|$. Clearly, one must look for observables where "a" enters linearly and A_{zz} is the simplest one. However, it turns out that between 100 MeV and 300 MeV, $(m+2h)$ is practically real so that A_{zz} tests only the real part of "a". Since the imaginary part of the latter is not at all negligible, we have been forced to add Eq. (27). The

system is then complete and can be solved for $|V_{\sigma\tau}/V_{\tau}|$. After some algebra, we get :

$$\left| \frac{V_{\sigma\tau}}{V_{\tau}} \right|^2 = \frac{4}{3} \frac{1 + A_{zz}}{(1 + D + A_{zz})^2 + C_{np}^q} - \frac{1}{3} \quad (29)$$

In our energy range $1 + D + A_{zz} \approx 0$ and $C_{pn}^q \approx 0$ since we know that the right-hand side of Eq. (29) is large. In the limit $|V_{\tau}| = 0$, $D = -1/3$, $A_{zz} = -2/3$. To get a more precise idea of how things are matched, we give the values of D , A_{zz} and C_{np}^q at 200 MeV when computed with set I: $D = -1/3 + 0.0417$, $A_{zz} = -2/3 - 0.00767$, $C_{np}^q = -0.1961$. Thus, C_{np}^q is likely to control the accuracy. If we assume that D and C_{np}^q can be measured up to 10% and A_{zz} up to 5% (this is optimistic but not unreasonable in view of the TRIUMF data) R would then be determined up to $15 \div 20\%$ and this can be improved by fitting the right-hand side of Eq. (29) over a range of angles.

Although Eq. (29) provides a means of determining R directly from n-p scattering experiments, it is not sure that the required measurements will ever be performed in the forward direction with enough accuracy. However, we expect that some measurements at large neutron angles of A_{zz} and/or C_{np}^q (or any observable with a similar content) would help to reduce the ambiguity evoked at the end of Section 3 concerning R .

5. - CONCLUSION

Under reasonable assumptions, we have shown how the relative probability of inducing GT or F nuclear transitions with a forward p-n reaction can be related to nucleon-nucleon scattering. In the energy range where most experiments have been performed, the p-n reaction at zero degree looks like elastic charge exchange scattering on a fictitious nucleon at rest with spin and isospin described by nuclear F and GT operators. The validity of this simple picture seems to be confirmed by the good agreement between the ratio $R = |V_{\sigma\tau}/V_{\tau}|$ computed from nucleon-nucleon phase shifts and the ratio deduced from the p-n reaction.

However, we cannot rule out the possibility of a fortuitous agreement due to the use of inadequate phase shifts. This would destroy all the simplicity of the reaction and cast doubts on its use as a clean probe of spin-isospin densities in nuclei. This is why we have drawn attention to n-p scattering experiments which would be decisive to settle this issue. These experiments are difficult

due to the smallness of $|V_T|$ which forces us to look for observables where (loosely speaking) it enters linearly. Such experiments require the simultaneous use of a polarized neutron beam and proton target. We hope that in the not too distant future some progress will be realized on this side. Alternatively, and awaiting such experiments, one may consider using the observed ratio R to restrict the nucleon-nucleon amplitudes.

ACKNOWLEDGEMENTS

It is a pleasure to acknowledge stimulating discussions with T.E.O. Ericson and M. Ericson.

REFERENCES

- 1) C.D. Goodman et al., IEEE Transactions on Nuclear Science, NS. 26 (1979) 2248.
- 2) C. Gaarde, J.S. Larsen and J. Rapaport, Int. Conf. on Spin Excitations in Nuclei, Telluride, Colorado (1982).
- 3) J. Delorme, M. Ericson, A. Figureau and C. Thévenet, Ann. Phys. 102 (1976) 273.
- 4) K. Kubodera, J. Delorme and M. Rho, Phys. Rev. Lett. 40B (1978) 755.
- 5) A. Chodos, R.L. Jaffe, C.B. Thorn and V. Weisskopf, Phys. Rev. D9 (1974) 3471.
- 6) R.L. Jaffe and F. Low, Phys. Rev. D19 (1979) 2105;
S. Théberge, A.W. Thomas and G.A. Miller, Phys. Rev. D22 (1980) 2838.
- 7) J. Delorme, M. Ericson and P. Guichon, Phys. Lett. 115B (1982) 86.
- 8) For references, see the review of A. Richter, Nucl. Phys. A374 (1982) 177C.
- 9) D.H. Wilkinson, Nucl. Phys. A225 (1974) 365.
- 10) T.N. Waddeucci et al., Phys. Rev. 25C (1981) 1094.
- 11) N. Hoshizaki, Sup. Prog. Th. Phys. 42 (1968) 107.
- 12) A.R. Edmonds, Angular Momentum in Quantum Mechanics, Princeton University Press (1960).
- 13) P. Cziffra, M.H. MacGregor, M.J. Moravcsik and H.P. Stapp, Phys. Rev. 114 (1959) 880.
- 14) P.A.M. Guichon and C. Samour, Nucl. Phys. A382 (1982) 461.
- 15) R.A. Arndt, R.H. Hackman and L.D. Roper, Phys. Rev. C15 (1977) 1002.
- 16) D.V. Bugg et al., Phys. Rev. C21 (1980) 1004.
- 17) A.S. Clough et al., Phys. Rev. C21 (1980) 988;
D. Axen et al., Phys. Rev. C21 (1980) 998.
- 18) W.G. Love and M.A. Franey, Phys. Rev. C24 (1981) 1073.

FIGURE CAPTIONS

Figure 1 : One-step approximation for p-n reactions.

Figure 2 : Plot of V_{τ} (a) and $V_{\sigma\tau}$ (b) versus incident proton energy. In both cases, the continuous curve is obtained by assuming the target neutron at rest. The dashed curve takes into account the Fermi motion effect with $k_F = 1 \text{ F}_m^{-1}$.

Figure 3 : Plot of $R = |V_{\sigma\tau}/V_{\tau}|$ versus incident proton energy. The straight line is the experimental fit of Ref. 10). The points correspond to $^{13}\text{C}_0$. The continuous curve (I) is our result. The dashed curve (II) is obtained with the Love and Franey interaction.

$\sim$

Fig. 1

Fig. 2

Fig. 3