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SECOND ORDER TRANSITIONS IN $Z(2)$ GAUGE THEORY IN FOUR DIMENSIONS

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ABSTRACT

I study the four-dimensional $Z(2)$ gauge theory with asymmetric couplings (β_1 for the xy, xz, xt plaquettes and β_2 for the yz, yt, zt plaquettes) by Monte Carlo simulation. A rich phase structure is found in the β_1, β_2 plane. In particular, there are lines of second order transitions that meet in a tri-critical point at which a four-dimensional Lorentz invariant continuum limit might exist. Further, the limit $\beta_1 \rightarrow \infty, \beta_2 \rightarrow 0$ for which the Hamiltonian theory is obtained, is found to be the endpoint of a line of first order transitions. The prediction therefore is that the Hamiltonian formulation of this theory has a second order phase transition with infinite correlation length.

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Ever since the pioneering Monte-Carlo work of Creutz, Jacobs and Rebbi ¹⁾, it has been more or less accepted that the $Z(2)$ gauge theory in 4-dimensions has a first order phase transition and therefore no continuum limit. However, there have been some hints that this might be incorrect. In particular, strong coupling expansions ²⁾ suggest that the correlation length at the self-dual point seems to be quite large and may even be infinite. Numerically, the string tension seems to become quite small and there is some evidence that the theory is becoming almost scale invariant at the self-dual point ³⁾. In this paper, I study a $Z(2)$ gauge theory with asymmetric couplings to see whether any evidence for a possible continuum limit can be found. This study is motivated by the work of Wu and McCoy ⁴⁾ and the results of References 2), 3).

Let us consider the $Z(2)$ gauge theory in 4-dimensions with couplings β_1 for the xy, xz, xt plaquettes and β_2 for the yz, yt, zt plaquettes. The action of this theory is :

$$\mathcal{S} = \beta_1 \sum_x \sigma_{P,x} + \beta_2 \sum_{\bar{x}} \sigma_{P,\bar{x}} \quad (1)$$

where $\sigma_{P,x}$ is a plaquette with two x directed links and $\sigma_{P,\bar{x}}$ is one with no x directed links. σ_p is the product of four $Z(2)$ variables $\{\sigma\}$ which live on the links of the plaquette.

Define the order parameter :

$$\langle S \rangle = \frac{1}{N_p} \langle \beta_1 \sum_x \sigma_{P,x} + \beta_2 \sum_{\bar{x}} \sigma_{P,\bar{x}} \rangle \quad (2)$$

where N_p is the total number of plaquettes and the average of an operator $\hat{O}$ is defined by :

$$\langle \hat{O} \rangle = \frac{\sum_{\sigma} \hat{O} e^{\mathcal{S}}}{\sum_{\sigma} e^{\mathcal{S}}} \quad (3)$$

The coupling β_1 and β_2 are mutually dual. Under a duality transformation, a plaquette with x directed links goes into one with no x directed links. Hence, β_1 goes into β_2^* and β_2 into β_1^* . Thus,

$$\beta_1 \rightarrow -\frac{1}{2} \ln(\tanh \beta_2^*) \quad (4a)$$

$$\beta_2 \rightarrow -\frac{1}{2} \ln(\tanh \beta_1^*) \quad (4b)$$

The usual (symmetric) $Z(2)$ theory corresponds to $\beta_1 = \beta_2$. In the limit $\beta_1 = 0$ one obtains the $Z(2)$ gauge theory in three dimensions and for $\beta_2 = \infty$, its dual, the $Z(2)$ (Ising) spin theory in three dimensions. The limit $\beta_2 = 0$ corresponds to the self-dual two-dimensional gauge theory as does the limit $\beta_1 = \infty$.

The theory defined by Eqs. (1) and (2) was studied by standard Monte Carlo methods. The only unusual feature of the Monte Carlo procedure used was to exploit the fact that a $Z(2)$ variable is either 0 or 1 and therefore needs only one bit of storage in the computer. The word length of the CDC computers is 60 bits. Hence, potentially, 60 variables can be stored in every word. The procedure followed in the present simulation was to simultaneously do Monte Carlo simulations on 60 independent lattices by storing the corresponding variables of these systems in the same word. This results in an increase in the statistics by a factor of 60 at the cost of about a factor of 2 in computing time.

First, the two stable values of $\langle S \rangle$ on the $\beta_1 = \beta_2$ line at the self-dual point were measured by doing 200 iterations on the $60, 4^4$ lattices starting the lattice in ordered and disordered configurations respectively. This is useful to check the present calculations against the known behaviour of the symmetric theory¹⁾. Next, these two stable values of $\langle S \rangle$ were measured for different β_1, β_2 along the duality line around the region $\beta_1 = \beta_2$. It was found that at both ends, this first order line terminates. At $\beta_1 \approx 0.2$, $\beta_2 \approx 0.8$, the first order line branches into two second order lines which then go out to the 3-d gauge and Ising phase transitions at $(\beta_1 = 0, \beta_2 \approx 0.76)$ and $(\beta_2 = \infty, \beta_1 \approx 0.22)$ respectively. This feature was determined by doing hysteresis runs on $3^4 - 6^4$ lattices. At the other end, the first order line ends inside the $\beta_1\beta_2$ plane for all the finite lattices studied. However, for increasing lattice size, this endpoint moves out towards $\beta_1 = \infty, \beta_2 = 0$ along the duality line.

The phase diagram for the infinite volume system that emerges from this study is shown in Fig. 1. The dashed line is one of first order transitions separating a "disordered" phase R from a "completely ordered" phase O_1 . The phase O_2 is partially ordered (the yz, yt, zt plaquettes are "ordered") and partially "disordered" (the xy, xz, xt plaquettes are "disordered").

There are several interesting features of this phase diagram. First, contrary to established wisdom, the $Z(2)$ gauge theory in four dimensions does have several second order fixed points at which a correlation length diverges. The question of whether a four dimensional (as opposed to three dimensional)

Lorentz invariant continuum limit is obtained at any of these points is more subtle. The most likely candidate for where this might happen is the tri-critical point at the junction of the phases O_1 , O_2 and R. This is under study.

The Hamiltonian version of this theory corresponds to $\beta_2 = 0$, $\beta_1 = \infty$. This point is the endpoint of a first order line and so should be a second order fixed point. The prediction is that unlike the symmetric ($\beta_1 = \beta_2$) Euclidean formulation, which may or may not have infinite range correlations, the Hamiltonian formulation certainly must have a divergent correlation length at the critical point.

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FIGURE CAPTION

Figure 1 : Phase diagram in the $\beta_1\beta_2$ plane. Dashed lines are first order transitions, continuous lines are second order. The point $\beta_1 = \infty$, $\beta_2 = 0$ is a critical endpoint.