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THE USE OF FINITE ENERGY SUM RULES FOR THE DESCRIPTION OF
THE HADRONIC PROPERTIES OF QCD

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ABSTRACT

We use finite energy sum rules, taking into account non-perturbative corrections, for the calculation of hadron masses within QCD. As a rule, we obtain reasonable agreement with experimental data. We also estimate the proton lifetime in the SU(5) model to be $\tau_p = 10^{28} (M_X/10^{14} \text{ GeV})^4$ years.

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1. - INTRODUCTION

As is well known, the calculation of the hadron mass spectrum is one of the most important tasks of QCD. Recently, the ITEP group¹⁾ suggested a method which allows one to calculate the meson resonance masses in QCD on the basis of the Wilson expansion and the Borel sum rules^{*)}. This method was used to estimate the baryon masses in Refs. 2) and 3). On the other hand, in Refs. 4) and 5), the FESR (finite energy sum rule) method was developed and used successfully for the description of the meson resonances in QCD.

In this paper we suggest a new method for the calculation of hadron masses in QCD using the Wilson expansion for the two-point correlation functions. In contrast to the approach of the ITEP group, we do not use Borel sum rules, but take the Källén-Lehmann representation for the two-point function and compare the theoretical and the "quasiexperimental" values of the correlators directly. It appears that our proposed method is closely related to the FESR method. In our opinion, our method for extracting information on hadronic properties is the more strict, and in several cases (scalar and tensor gluonium) it allows us to get more detailed information about the physical spectrum. The other advantage of our method is the possibility of obtaining analytic mass formulae for the hadrons.

The paper is organized as follows. In Section 2, using the ρ meson as an example, we explain the suggested method. In Section 3 the mass formulae for the vector octet is obtained. In Section 4 we consider the axial-vector mesons and in Sections 5 and 6 the cases of the scalar and pseudoscalar mesons. Sections 7, 8 and 9 are devoted to the description of the scalar, tensor and pseudoscalar gluonium. In Section 10 we calculate the proton and Δ^{++} masses. In Section 11 we calculate the proton lifetime in QCD. Finally, in Section 12, we make some concluding remarks.

2. - THE ρ MESON

The current corresponding to the ρ meson has the form

$$J_\mu^P = \frac{1}{2} (\bar{u} \gamma_\mu u - \bar{d} \gamma_\mu d) \quad (2.1)$$

*) The corresponding technique of the ITEP group is well known and we will not discuss it in detail.

To find the ρ meson mass it is convenient to consider the two-point function

$$\Pi_{\mu\nu}(q) = 8\pi^2 i \int_0^\infty \langle 0 | T \left(j_\mu^\rho(x) j_\nu^\rho(0) \right) | 0 \rangle d^4x \quad (2.2)$$

$$e^{iqx} d^4x = (q^\mu q^\nu - g^{\mu\nu} q^2) \Pi(Q^2), \quad Q^2 = -q^2$$

Using the Wilson expansion for the correlator (2.1) and the factorization hypothesis one can obtain¹⁾ that

$$\begin{aligned} \Pi(Q^2) = & - \ln \frac{Q^2}{\mu^2} + \frac{4\pi^2(m_u + m_d) \langle \bar{q}q \rangle}{Q^4} + \\ & + \frac{\pi^2}{3} \langle \frac{\alpha}{\pi} G^2 \rangle \frac{1}{Q^4} - \frac{896\pi^3}{81Q^6} (\langle \sqrt{\alpha} \bar{q}q \rangle)^2, \end{aligned} \quad (2.3)$$

where $\langle \bar{q}q \rangle$ is the quark vacuum condensate and $\langle \alpha/\pi G^2 \rangle$ is the gluonic one. It is convenient to introduce the renormalization group invariant quantity

$$\begin{aligned} D(Q^2) = & -Q^2 \frac{d\Pi}{dQ^2} = 1 + \frac{8\pi^2(m_u + m_d) \langle \bar{q}q \rangle}{Q^4} \\ & + \frac{2\pi^2}{3} \langle \frac{\alpha}{\pi} G^2 \rangle \frac{1}{Q^4} - \frac{896\pi^3}{27Q^6} (\langle \sqrt{\alpha} \bar{q}q \rangle)^2 + O\left(\frac{1}{Q^8}\right) \end{aligned} \quad (2.4)$$

The Källen-Lehmann representation for the function $D(Q^2)$ has the form

$$D(Q^2) = Q^2 \int_0^\infty \frac{\rho(s) ds}{(s + Q^2)^2} \quad (2.5)$$

Here $\rho(s)$ is the spectral density ($\rho(s) \geq 0$) and we chose for it the following ansatz

$$\rho^{\text{exp}}(s) = F \delta(s - m_\rho^2) + \theta(s - s_0) \quad (2.6)$$

which reflects rather well the characteristic features of the real spectrum. Substituting Eq. (2.6) into Eq. (2.5) and comparing the coefficients of equal powers of $1/Q^2$ we get the system of the equations

$$\begin{aligned} F - S_0 &= 0, \\ F m_\rho^2 - \frac{S_0^2}{2} &= -\frac{\pi^2}{3} \left\langle \frac{2}{\pi} G^2 \right\rangle + 4\pi^2 (m_u + m_d) \langle \bar{q}q \rangle, \\ F m_\rho^4 - \frac{S_0^3}{3} &= -\frac{896\pi^3}{81} (\langle \sqrt{2} \bar{q}q \rangle)^2 \end{aligned} \quad (2.7)$$

The system of Eqs. (2.7) allows us to find the parameters F , m_ρ^2 , S_0 . Note that the system of Eqs. (2.7) follows from the requirement of the equal asymptotic behaviour of the theoretically calculated quantity $D(Q^2)$ and the "quasiexperimental" one, obtained with the help of Källen-Lehmann representation from the ansatz (2.6), whereas in the ITEP group approach¹⁾ the values of the parameters are determined by the requirement of stability in relation to the variation of the Borel variable M^2 .

The requirement of equal asymptotic behaviour of the theoretical and "experimental" D functions is closely connected with the validity of FESR^{4),5)}, namely the system of Eqs. (2.7) is a number of FESR. Really, the FESR have the form

$$\int_0^{S_1} \varphi(s) \rho^{\text{exp}}(s) ds = \int_0^{S_1} \rho^{\text{th}}(s) \varphi(s) ds \quad (2.8)$$

where $\varphi(s)$ is an arbitrary smooth function. Defining $\rho^{\text{th}}(s) = 1/\pi \operatorname{Im} \Gamma(-s - i\epsilon)$ and taking as $\varphi(s)$ the functions 1 , s , s^2 , we obtain the system (2.7) if we use the ansatz (2.6) for $\rho^{\text{exp}}(s)$. The sum rules used in Ref. 1) can be obtained from Eq. (2.8) if we take

$$\varphi(s) = \exp\left(-\frac{s}{M^2}\right), \quad S_1 = \infty$$

Let us now proceed with the analysis of Eqs. (2.7). Neglecting the masses of the light quarks and the contribution of the gluonic condensate - which is expected to be small¹⁾ - we find

$$\rho^{\text{exp}}(s) = 2m_\rho^2 \delta(s - m_\rho^2) + \theta(s - 2m_\rho^2),$$

$$m_\rho = 2.8 |(\langle \sqrt{2} \bar{q}q \rangle)|^{1/3}$$

Using the standard estimations

$$\langle \bar{q}q \rangle = - \frac{f_\pi^2 m_\pi^2}{m_u + m_d},$$

$$(m_u + m_d)(1 \text{ GeV}) \simeq 11 \text{ MeV}$$

we find $m_\rho = 700 \text{ MeV}$. The residue F is connected with the width of the ρ meson decay into e^+e^- . In our approach we have

$$\Gamma^{\text{th}}(\rho \rightarrow e^+e^-) = 4 \text{ KeV}$$

The experimental value for this quantity is $(5.8 \pm 0.8) \text{ keV}$ ⁶⁾. Taking into consideration the non-zero quark masses and the contribution of the gluonic condensate, we obtain

$$m_\rho = 630 \text{ MeV}, \quad \Gamma(\rho \rightarrow e^+e^-) = 3.9 \text{ KeV},$$

$$S_0 = 0.73 \text{ GeV}^2$$

Note that the numerical values of the condensates $\langle \bar{q}q \rangle$ and $\langle \alpha/\pi G^2 \rangle$ are not known with great accuracy. Thus we can reverse the procedure and, using the value of the ρ meson mass as an input parameter, we find

$$|\langle \sqrt{2} \bar{q}q \rangle|^{1/3} = 0.29 \text{ GeV}$$

which is in satisfactory agreement with the values obtained by other methods⁷⁾.

3. - VECTOR OCTET

A numerical analysis of the ITEP sum rules for φ and K^* mesons was performed in Ref. 1). Here, using the system of Eqs. (2.7), we obtain mass formulae in a linear approximation on the s quark mass m_s . These formulae have the form

$$m_\varphi^2 = m_\rho^2 - \frac{12\pi^2 m_s \langle \bar{S}S \rangle}{m_\rho^2}, \quad (3.1)$$

$$m_{K^*}^2 = m_\rho^2 - \frac{6\pi^2 m_s \langle \bar{s}s \rangle}{m_\rho^2} \quad (3.2)$$

The PCAC hypothesis gives

$$m_s \langle \bar{s}s \rangle = -f_K m_K^2 + \frac{1}{2} \pi^2 m_\pi^2 = -0.27 \cdot 10^{-2} \text{GeV}^4$$

where we use $f_K = 1.16 f_\pi$. The relations (3.1) and (3.2) are fulfilled with an accuracy of six per cent for the experimental values of the K^* and ϕ meson masses. For the lepton width of the ϕ meson we obtained

$$\Gamma(\phi \rightarrow e^+e^-) = 1.1 \text{keV}$$

while the experimental value is²³⁾

$$\Gamma^{\text{exp}}(\phi \rightarrow e^+e^-) = (1.3 \pm 0.2) \text{keV}$$

4. - AXIAL-VECTOR MESONS

For the correlator of the axial-vector currents

$$J_\mu^{A_1}(x) = \frac{1}{2} (\bar{u} \gamma_\mu \gamma_5 u - \bar{d} \gamma_\mu \gamma_5 d)$$

and

$$J_\mu^{Q_1} = \bar{s} \gamma_\mu \gamma_5 s$$

we have contributions not only from A_1 and Q_1 mesons, but also from π and K mesons. Due to the smallness of the u and d quark masses, the approximation $m_u = m_d = 0$ works rather well. The correlator of the isovector axial current was calculated in Ref. 1) and has the form

$$\begin{aligned} \Pi_{\mu\nu}^{A_1}(q) &= 8\pi^2 i \{ \langle 0 | T (J_\mu^{A_1}(x), J_\nu^{A_1}(0)) | 0 \rangle \\ e^{iqx} d^4x &= (g_\mu g_\nu - g_{\mu\nu} q^2) \left[-\ln \frac{q^2}{\mu^2} + \right. \\ &\frac{4\pi^2 (m_u + m_d) \langle \bar{q}q \rangle}{q^4} + \frac{\pi^2}{3} \langle \frac{2}{\pi} G^2 \rangle \frac{1}{q^4} + \\ &\left. \frac{1408}{8448} \pi^3 (\langle \sqrt{2} \bar{q}q \rangle)^2 \right] - g_{\mu\nu} \frac{(m_u + m_d) \langle \bar{q}q \rangle}{q^2} \end{aligned} \quad (4.1)$$

Having taken an experimental density of the form

$$\rho^{\text{exp}}(s) = 8\pi^2 f_\pi^2 \delta(s) + F_{A_1} \delta(s - m_{A_1}^2) + \theta(s - s_0) \quad (4.2)$$

we get the system of the equations in the chiral limit $m_u = m_d = 0$

$$\begin{aligned} F_{A_1} - s_0 + 8\pi^2 f_\pi^2 &= 0, \\ F_{A_1} m_{A_1}^2 - \frac{s_0^2}{2} &= -\frac{\pi^2}{3} \langle \frac{2}{\pi} G^2 \rangle, \\ F_{A_1} m_{A_1}^4 - \frac{s_0^3}{3} &= \frac{1408}{81} \pi^3 (\langle \sqrt{\alpha} \bar{q}q \rangle)^2 \end{aligned} \quad (4.3)$$

The system (4.3) can be solved in the absence of non-perturbative corrections. That is, when the right-hand sides of Eqs. (4.3) are equal to zero we have

$$m_{A_1}^2 = \frac{2}{3} s_0, \quad F_{A_1} = \frac{3}{4} s_0, \quad s_0 = 32 \pi^2 f_\pi^2$$

The coupling constant of the current $j_\mu^{A_1}$ with the A_1 meson does not depend on s_0 if we define the matrix element as follows

$$\langle 0 | j_\mu^{A_1}(0) | A_1(\vec{k}, \lambda) \rangle = \frac{m_{A_1}^2}{f_{A_1}} \epsilon_\mu(\vec{k}, \lambda)$$

For f_{A_1} we have

$$\frac{4\pi}{f_{A_1}^2} = \frac{9}{16\pi} = 0.18$$

which agrees rather well with the experimental value⁸⁾. In our approach the A_1 meson mass is equal to 1.35 GeV. Taking into consideration the power corrections we obtain

$$\frac{4\pi}{f_{A_1}^2} = 0.16, \quad m_{A_1} = 1.27 \text{ GeV}$$

Note that the system of Eqs. (4.3) does not have solutions with $f_\pi = 0$. In other words, we have a direct indication that the spectrum in the axial channel differs from the vector one in a qualitative way.

For the current $j_\mu^{Q_1}(x)$ we took into account the non-zero s quark mass and obtained, in the linear approximation on m_s ,

$$m_{Q_1}^2 - m_{A_1}^2 = \frac{20}{3} \frac{4\pi^2 f_K^2 m_K^2}{m_{A_1}^2}, \quad (4.4)$$

$$F_{Q_1} = F_{A_1} + \frac{12}{m_{A_1}^2} \xi,$$

$$\xi = -4\pi^2 m_s \langle \bar{S} S \rangle$$

Numerically we have $m_{Q_1} = 1.22 \text{ GeV}$ ($m_{A_1} = 1.1 \text{ GeV}$), $m_{Q_1} = 1.40 \text{ GeV}$ ($m_{A_1} = 1.3 \text{ GeV}$). Analogous calculations for the $E(1420)$ meson which contains two strange quarks give

$$m_E = 1.32 \text{ GeV} \quad (m_{A_1} = 1.1 \text{ GeV})$$

$$m_E = 1.49 \text{ GeV} \quad (m_{A_1} = 1.3 \text{ GeV})$$

Once again, the agreement of the theoretical and experimental values is quite reasonable.

5. - SCALAR MESONS

The application of our method to the scalar mesons has some peculiarities which we shall discuss using the example of the $\delta(980)$ meson. This has quantum numbers $(I^G)_{J^{PC}} = (1^-)_{0^{++}}$ and the interpolating current has the form

$$j^\delta = \frac{1}{2} (\bar{u}u - \bar{d}d)$$

The Wilson expansion for the propagator of the j^δ current gives⁹⁾

$$\Pi(Q^2) \equiv \frac{16\pi^2}{3} i \int \langle 0 | T(j^\delta(x) j^\delta(0)) | 0 \rangle e^{iqx}$$

$$d^4x = Q^2 \ln \frac{Q^2}{\mu^2} + \frac{4\pi^2(m_u + m_d) \langle \bar{q}q \rangle}{Q^2} + \quad (5.1)$$

$$\frac{\pi^2}{3} \langle \frac{2}{\pi} G^2 \rangle \frac{1}{Q^2} - \frac{1408}{81Q^4} \pi^3 (\langle \sqrt{2} \bar{q}q \rangle)^2 -$$

$$\frac{32\pi^2/\rho_c^{-2}}{15Q^4} \langle \frac{2}{\pi} G^2 \rangle$$

where $\rho_c \sim 5 \text{ GeV}^{-1}$ is the cut-off used in the estimate of the three-gluon matrix element by saturating it with instantons¹⁾.

Using the ansatz

$$\rho^{\text{exp}}(s) = F_{\delta} \delta(s - m_{\delta}^2) + s \theta(s - s_0) \quad (5.2)$$

it is easy to obtain a system of equations which guarantees the equality of the asymptotic behaviour $\Pi^{\text{th}}(Q^2)$ and $\Pi^{\text{exp}}(Q^2)$ when $Q^2 \rightarrow \infty$. Namely, for the ansatz (5.2) we have

$$\begin{aligned} F_{\delta} m_{\delta}^2 - \frac{s_0^2}{2} &= A, \\ F_{\delta} m_{\delta}^4 - \frac{s_0^3}{3} &= B, \end{aligned} \quad (5.3)$$

where $A = 0.04 \text{ GeV}^2$, $B = 0.13 \text{ GeV}^6$ [we took the standard estimates¹⁾]. Unlike the previous examples, the system of Eqs. (5.3) does not allow us to find the parameters of the model without additional assumptions. We shall consider two methods of closing the system (5.3). The first method is to identify the beginning of the continuum and the position of the δ resonance ($s_0 = m_{\delta}^2$). Then, from Eqs. (5.3), we find $m_{\delta} = 0.92 \text{ GeV}$, which is in good agreement with the experimental value $m_{\delta} = 0.98 \text{ GeV}$. Note that the authors of Ref. 9) choose the beginning of the continuum to be 1.5 GeV^2 . If we take this value of s_0 we obtain $m_{\delta} = 1.04 \text{ GeV}$. It is seen that the mass of the δ meson depends slightly on the exact position of the continuum threshold if it belongs to some allowed region. It is on this remark that the second method is based. This method is closely related to the ideas developed in Ref. 10). It requires the independence of the mass of the physical particles from the positions of the continuum beginning. So we demand the minimal dependence of the δ meson mass on the continuum threshold s_0 .

Having neglected the contribution of the gluonic condensate in (5.1) ($A=0$), we obtain

$$m_{\delta}^2(s_0) = \frac{2B}{s_0^2} + \frac{2}{3} s_0$$

We find the value s_0 using the principle of the minimal dependence $m_{\delta}^2(s_0)$ on s_0 , i.e., we chose s_0 such that

$$\frac{dm_{\delta}^2(s_0)}{ds_0} = 0 \quad (5.4)$$

Using Eq. (5.4) we find $m_0^2 = S_0 = (6B)^{1/3} \approx 0.9 \text{ GeV}$. For the $\epsilon(1400)$ scalar meson, which contains two strange quarks, in the approximation $S_0 = m_0^2$ we obtain

$$m_\epsilon^2 - m_\delta^2 = - \frac{16\pi^2 m_s \langle \bar{s}s \rangle}{m_\delta^2} = 0.51 \text{ GeV}^2$$

(the experimental value is equal to 0.73 GeV^2). We conclude that the proposed picture is reproduced rather well in the scalar channel.

6. - PSEUDOSCALAR MESONS

The current corresponding to the Π meson is chosen to have the usual form

$$j^\Pi = \frac{1}{2} (\bar{u} \gamma_5 u - \bar{d} \gamma_5 d) \quad (6.1)$$

The corresponding correlator can be represented in the form⁹⁾

$$\begin{aligned} \Pi(q^2) &= q^2 \ln \frac{q^2}{\mu^2} + \frac{4\pi^2 (m_u + m_d) \langle \bar{q}q \rangle}{q^2} + \\ &\frac{\pi^2}{3} \langle \frac{2}{\pi} G^2 \rangle \frac{1}{q^2} - \frac{1408\pi^3}{81 q^4} (\langle \sqrt{2} \bar{q}q \rangle)^2 \end{aligned} \quad (6.2)$$

As a physical density for $\rho(s)$ we take the simple ansatz including the contributions of the π meson, its first radial excitation $[m_{\pi'} = 1.2 \text{ GeV}^{11})]$ and continuum, which is described by perturbation theory

$$\rho^{\text{exp}}(s) = \frac{16\pi^2}{3} \left(\frac{\sqrt{2} + \pi m_\pi^2}{m_u + m_d} \right)^2 \delta(s) + \quad (6.3)$$

$$F_{\pi'}^2 \delta(s - m_{\pi'}^2) + s \theta(s - s_0)$$

The requirement of the same asymptotic behaviour leads to a system of equations for the determination of the parameters of the excited state π' (we consider the contribution of the π meson as known and do not calculate it) and the beginning of the continuum threshold

$$\begin{aligned} F_{\pi'} m_{\pi'}^2 - \frac{s_0^2}{2} &= -1.4 \text{ GeV}^4, \\ F_{\pi'} m_{\pi'}^4 - \frac{s_0^3}{3} &= 0.13 \text{ GeV}^6 \end{aligned} \quad (6.4)$$

In this case, the situation is analogous to the previous one and two different methods of closing the system (6.4) lead to the result $m_{\pi'} = 1.7 \text{ GeV}$. The fact that two different methods of closing the system (6.4) give the same result, is an indication in favour of these methods, while the numerical disagreement we explain as the effect of the uncertainties in the values of the parameters. For example, if we take another estimate for the quark masses, $m_u + m_d \sim 20 \text{ MeV}$ which was advocated by a number of authors¹²⁾, we obtain $m_{\pi'} = 1.26 \text{ GeV}$ in agreement with the experimental value.

7. - SCALAR GLUONIUM

As is well known, QCD predicts the existence¹³⁾ of new hadrons connected with the gluon degrees of freedom. The operator $\alpha G_{\mu\nu}^2$ can describe a particle with quantum numbers $J_{PC} = 0^{++}$, which can be thought of as a bound state of two gluons in the same way as the δ meson is a bound state of two quarks. To estimate the mass of this new state, the Wilson expansion and the Borel summation technique were used in Ref. 14).

The two-point correlator for the current $\alpha G^2(x)$ has the form¹⁴⁾

$$\begin{aligned} \Pi(Q^2) &= \frac{\pi^2}{A} i \int \langle 0 | T(\alpha G^2(x), \alpha G^2(0)) | 0 \rangle \\ &e^{iqx} d^4x = \alpha^2 Q^4 \left[-\ln \frac{Q^2}{\mu^2} + \frac{2\pi^3}{\alpha Q^4} \langle \frac{\alpha}{\pi} G^2 \rangle \right. \\ &+ \frac{48\pi^3}{5\alpha Q^6} \rho_c^{-2} \langle \frac{\alpha}{\pi} G^2 \rangle + \frac{9\pi^5}{4\alpha Q^8} \\ &\left. \langle \frac{\alpha}{\pi} G^2 \rangle^2 \right] \end{aligned} \quad (7.1)$$

For the spectral density we chose the form

$$\rho^{\text{exp}}(s) = F \delta(s - m^2) + \alpha s^2 \theta(s - S_0) \quad (7.2)$$

The corresponding system of equations based on the requirement of the equality of the asymptotic behaviour, and the validity of FESR with the weight function $1/S$, has the form

$$\begin{aligned} F m^2 - \frac{S_0^2}{2} + \frac{2\pi^3}{\alpha} \left\langle \frac{\alpha}{\pi} G^2 \right\rangle &= 0, \\ F m^4 - \frac{S_0^3}{3} - \frac{48\pi^3}{5\alpha} \rho_c^{-2} \left\langle \frac{\alpha}{\pi} G^2 \right\rangle &= 0, \\ F m^6 - \frac{S_0^4}{4} + \frac{9\pi^5}{4\alpha} \left(\left\langle \frac{\alpha}{\pi} G^2 \right\rangle \right)^2 &= 0 \end{aligned} \quad (7.3)$$

Neglecting the contribution of power corrections in $1/Q^6$ and $1/Q^8$ (this approximation is expected to be reasonable because of their smallness) we obtain

$$\begin{aligned} m^2 &= \frac{3}{4} S_0 = 5 \text{ GeV}^2, \quad F = \frac{16}{27} S_0 = 4 \text{ GeV}^2, \\ S_0 &= 6\pi \sqrt{\frac{\pi}{\alpha} \left\langle \frac{\alpha}{\pi} G^2 \right\rangle} = 6.7 \text{ GeV}^2 \end{aligned} \quad (7.4)$$

Here we give the numerical values for $\langle \alpha/\pi G^2 \rangle = 0.012 \text{ GeV}^4$ and $\alpha = 0.3$. Note that the duality interval S_0 is approximately equal to the value obtained in Ref. 13), and is really much larger than the corresponding interval in the case of the ρ meson ($S_{0\rho} = 2m_\rho^2$). We recall that the latter was determined by a comparison of the effective scale parameters Λ_{eff} for the scalar gluonium and vector quark channels¹⁵⁾. Taking into account the $1/Q^6$ and $1/Q^8$ corrections in (7.1) leads to

$$F = 4.2 \text{ GeV}^2, \quad m^2 = 5.2 \text{ GeV}^2, \quad S_0 = 6.9 \text{ GeV}^2$$

If we further demand the validity of sum rules with the weight function $1/S^2$ in Eq. (2.8), one can try to extract more detailed information about the spectrum. In this case we will have four equations and the simplest ansatz (7.2) does not work. However, it is possible to make it work if we use two resonances and

choose the continuum threshold to coincide with the position of the second resonance. In this case, the solution has the form

$$S_0 = m_2^2 = 9 \text{ GeV}^2, \quad m_1^2 = 3 \text{ GeV}^2, \quad F_2 = 2.2 \text{ GeV}^2, \quad F_1 = 6.8 \text{ GeV}^2.$$

It is seen that the picture did not change on the qualitative level. However, it seems that at the present time we not only have difficulty in fixing the quantitative features of the gluonium spectrum, but even its qualitative behaviour.

8. - PSEUDOSCALAR GLUONIUM

Unlike the previous cases, the Wilson expansion for the two-point function of the pseudoscalar gluonic current $\alpha \tilde{G} \tilde{G}$ ¹⁶⁾ does not lead to a system of equations which have a solution under any simple ansatz for the form of the physical spectrum. Thus, the only conclusion which we can draw is that either the spectrum in the pseudoscalar channel differs qualitatively from the scalar one, or the sum rules with the weight functions $1/S$ and $1/S^2$ are not valid.

9. - TENSOR GLUONIUM

The current with the quantum numbers $J^{PC} = 2^{++}$ can be chosen in the form

$$J_{\mu\nu} = G_{\mu\alpha}^a G_{\alpha\nu}^a - \frac{1}{4} g_{\mu\nu} G^2$$

The corresponding correlator was calculated in Ref. 9)

$$\begin{aligned} \Pi_{\mu\nu, \mu'\nu'}(q) &= 20\pi^2 i \int \langle 0 | T (J_{\mu\nu}(x), \\ J_{\mu'\nu'}(0)) | 0 \rangle e^{iqx} d^4x &= \\ \eta_{\mu\nu, \mu'\nu'}(q) \Pi(q^2), & \\ q^2 = -q^2 & \end{aligned} \tag{9.1}$$

where $\eta_{\mu\nu, \mu'\nu'}$ is the density matrix for the particle with spin 2.

The renormalization group invariant function $D(Q^2)$ has the form

$$D(Q^2) = -Q^2 \frac{d\pi}{dQ^2} = 1 - \frac{25\pi^5}{2} \left(\langle \frac{\alpha}{\pi} G^2 \rangle \right)^2$$

$$\frac{1}{Q^8} = Q^2 \int_0^\infty \frac{\rho(s) ds}{(s+Q^2)^2} \quad (9.2)$$

We chose an ansatz for $\rho(s)$ of the form

$$\rho^{\text{exp}}(s) = F_1 \delta(s - m_1^2) + F_2 \delta(s - m_2^2) + \theta(s - m_2^2) \quad (9.3)$$

The comparison of the asymptotic behaviour of the theoretical $D(Q^2)$ and $D^{\text{exp}}(Q^2)$ gives

$$m_1^2 = \frac{1}{3} m_2^2, \quad F_1 = \frac{3}{4} m_2^2, \quad F_2 = \frac{1}{4} m_2^2,$$

$$m_2^2 = \sqrt{\frac{15}{2\sqrt{2}} \langle \frac{\alpha}{\pi} G^2 \rangle} \quad (9.4)$$

The numerical results for $\langle \alpha/\pi G^2 \rangle = 0.012 \text{ GeV}^4$ and $\alpha = 0.3$ are as follows

$$m_1^2 = 0.8 \text{ GeV}^2, \quad m_2^2 = 2.4 \text{ GeV}^2,$$

$$F_1 = 1.8 \text{ GeV}^2, \quad F_2 = 0.6 \text{ GeV}^2$$

Note that the same results can be obtained by closing the system of equations with the requirement $m_2^2 = 3m_1^2$, which follows from the absence of the term $1/Q^4$ and the suggestion that the FESR should be valid for each resonance separately⁵⁾.

10. - THE PROTON AND Δ ISOBAR MASS CALCULATIONS

To calculate the baryon masses by the FESR method, we will use the results of the two-point function calculations of the baryonic currents^{2),3)}.

The interpolating currents of the proton and Δ isobar have the form

$$\begin{aligned} J^p &= u^a c \not{f}_\mu u^b \not{f}_5 \not{f}_\nu d^c \varepsilon^{abc}, \\ J_\mu^\Delta &= u^a c \not{f}_\mu u^b u^c \varepsilon^{abc}, \end{aligned} \quad (10.1)$$

where c is the charge conjugation matrix. Note that the form of the interpolating currents (10.1) is not a unique one^{2),3)}. So there is a problem of the choice of the interpolating current. For definiteness we consider the proton case in detail, and for the Δ isobar case we give only the main results.

The two-point function of the currents with the proton quantum numbers has the form

$$\begin{aligned} \Pi(q) &\equiv i \int d^4x e^{iqx} \langle 0 | T (J^p(x), \bar{J}^p(0)) | 0 \rangle = \\ &\hat{q} \left[\frac{-(q^2)^2 \ln \frac{-q^2}{\mu^2}}{8\pi (2\pi)^3} - \frac{2}{3} (\langle 0 | \bar{q} q | 0 \rangle)^2 \frac{1}{q^2} \right] + \\ &\frac{1}{(2\pi)^2} \langle 0 | \bar{q} q | 0 \rangle q^2 \ln \frac{-q^2}{\mu^2} + \\ &\frac{1}{2(2\pi)^2} g_s \langle 0 | \bar{q} G_{\mu\nu} G_{\mu\nu}^a \frac{\lambda^a}{2} q | 0 \rangle \ln \frac{-q^2}{\mu^2} \end{aligned} \quad (10.2)$$

This expression was obtained in the chiral limit $m_u = m_d = m_s = 0$ and it includes several leading terms in the expansion on $1/q^2$ only.

For the quantity $\Pi(q)$ the Källen-Lehmann representation has the form

$$\Pi(q^2) = \int_0^\infty \frac{\hat{q} \rho_1(s) + \rho_2(s)}{s + q^2} ds - \text{subtractions}, \quad (10.3)$$

$q^2 = -q^2$

In the approximation (10.2) this leads to the results

$$\rho_1^{\text{th}}(s) = \frac{s^2}{8\pi (2\pi)^3} + \frac{2}{3} (\langle 0 | \bar{q} q | 0 \rangle)^2 \delta(s), \quad (10.4)$$

$$\rho_2^{th}(s) = - \frac{s}{(2\pi)^2} \langle 0 | \bar{q} q | 0 \rangle -$$

$$\frac{1}{2(2\pi)^2} g_s \langle 0 | \bar{q} G_{\mu\nu} \frac{\lambda^a}{2} G_{\mu\nu}^a q | 0 \rangle$$

(10.4)
cont.

The representation (10.3) and the expressions (10.4) for $\rho_{th}^{1,2}(s)$ allow us to obtain the FESR

$$\int_0^{s_0} \rho_1^{th}(s) ds = \int_0^{s_0} \rho_1^{exp}(s) ds,$$

$$\int_0^{s_0} s \rho_1^{th}(s) ds = \int_0^{s_0} s \rho_1^{exp}(s) ds,$$

$$\int_0^{s_0} \rho_2^{th}(s) ds = \int_0^{s_0} \rho_2^{exp}(s) ds$$

(10.5)

Having saturated the right-hand side of Eqs. (10.5) by the proton contribution only, we get a system of equations for the determination of the parameters

S_0 , M_p and λ_p

$$\lambda_p^2 = \frac{s_0^3}{24\pi(2\pi)^3} + \frac{2}{3} (\langle 0 | \bar{q} q | 0 \rangle)^2,$$

$$\lambda_p^2 M_p^2 = \frac{s_0^4}{32(2\pi)^3},$$

$$\lambda_p^2 M_p = - \frac{s_0^2}{2(2\pi)^3} \langle 0 | \bar{q} q | 0 \rangle - \frac{s_0}{2(2\pi)^2} g_s \langle 0 | \bar{q} G_{\mu\nu} G_{\mu\nu}^a \frac{\lambda_a}{2} q | 0 \rangle$$

(10.6)

where the coupling constant λ_p of the proton with the proton interpolating current is determined as

$$\langle 0 | J^p(0) | P(\vec{k}, \lambda) \rangle = \lambda_p u_p(\vec{k}, \lambda),$$

$$\bar{u}_p u_p = 2M_p$$

(10.7)

The numerical value of the matrix element $g_s \langle 0 | \bar{q} \gamma_{\mu\nu} \lambda^a / 2 q G^{a\mu\nu} | 0 \rangle$ was estimated in Ref. 17)

$$g_s \langle 0 | \bar{q} \gamma_{\mu\nu} \frac{\lambda^a}{2} G_{\mu\nu}^a q | 0 \rangle = m_0^2 \langle \bar{q} q \rangle$$

where $m_0^2 \sim (0.5 \div 1) \text{ GeV}^2$. With this numerical value the contribution of this operator to the system of Eqs. (10.6) happens to be small, and we will neglect it. The solution of Eqs. (10.6) has the form

$$\begin{aligned} S_0 &= 1.6 M_P^2, \\ \lambda_P^2 &= 0.26 \cdot 10^{-3} M_P^6, \\ M_P &= 4.98 | \langle 0 | \bar{q} q | 0 \rangle |^{1/3} \end{aligned} \quad (10.8)$$

The standard value $\langle \bar{q} q \rangle = -(0.24 \text{ GeV})^3$ (for $m_u + m_d = 11 \text{ MeV}$) leads to the proton mass estimate $M_P = 1.245 \text{ GeV}$. As we remarked earlier, the numerical value of the matrix element $\langle 0 | \bar{q} q | 0 \rangle$ is rather poorly known. Reversing the problem and calculating $\langle 0 | \bar{q} q | 0 \rangle$ from the proton mass we obtain $| \langle 0 | \bar{q} q | 0 \rangle |^{1/3} = 0.188 \text{ GeV}$. Note that the light quark mass calculations¹²⁾ on the base of FESR give $m_u + m_d \approx 20 \text{ MeV}$ which corresponds to the above estimate.

The analogous calculations for the Δ isobar using the spectral densities for the J_μ^Δ propagator²⁾ lead to

$$\begin{aligned} \rho_1^\Delta(s) &= -\frac{s^2}{20\pi(2\pi)^3} - \frac{4}{3} (\langle 0 | \bar{q} q | 0 \rangle)^2 \delta(s), \\ \rho_2^\Delta(s) &= \frac{1}{3\pi^2} \langle 0 | \bar{q} q | 0 \rangle s - \\ &\quad \frac{1}{3\pi^2} \frac{1}{8} g_s \langle 0 | \bar{q} \gamma_{\mu\nu} \frac{\lambda^a}{2} G_{\mu\nu}^a q | 0 \rangle \end{aligned} \quad (10.9)$$

The use of the FESR (10.5) gives

$$S_0 = 1.44 M_\Delta^2, \quad (10.10)$$

$$\begin{aligned}\lambda_{\Delta}^2 &= 0.69 \cdot 10^{-4} M_{\Delta}^6, \\ M_{\Delta} &= 7.97 |\langle 0 | \bar{q} q | 0 \rangle|^{1/3}\end{aligned}\quad (10.10) \text{ cont.}$$

Using (10.8) and (10.10), we obtain $M_{\Delta}/M_p = 1.6$ while the experimental value is 1.3.

In the linear approximation on the s quark mass, the corresponding spectral densities of the interpolating currents of Σ and Σ^* baryons

$$\begin{aligned}J^{\Sigma} &= u^a c_{\mu} u^b j_5 j^{\mu} s^c \varepsilon^{abc}, \\ J^{\Sigma^*} &= \sqrt{\frac{1}{3}} [2 (u^a c_{\mu} s^b) u^c + \\ &\quad (u^d c_{\mu} u^b) s^c] \varepsilon^{abc}\end{aligned}\quad (10.11)$$

have the form

$$\begin{aligned}\rho_1^{\Sigma^*} &= \rho_1^{\Delta} + \frac{m_s}{3\pi^2} \langle 0 | \bar{q} q | 0 \rangle, \\ \rho_2^{\Sigma^*} &= \rho_2^{\Delta} - m_s \frac{s}{8\pi(2\pi)^3} - \frac{2m_s}{3} (\langle 0 | \bar{q} q | 0 \rangle)^2 \delta(s)\end{aligned}\quad (10.12)$$

and

$$\begin{aligned}\rho_1^{\Sigma} &= \rho_1^p, \\ \rho_2^{\Sigma} &= \rho_2^p + \frac{m_s s^2}{4\pi(2\pi)^3} + \frac{4}{3} m_s (\langle 0 | \bar{q} q | 0 \rangle)^2 \delta(s)\end{aligned}\quad (10.13)$$

Using the approximation (10.13), we obtained the following formula for the mass splitting

$$m_{\Sigma}^2 - m_p^2 = 30\pi^2 \frac{f_K^2 m_K^2}{M_p^2} = 0.71 \text{ GeV}^2 \quad (10.14)$$

(the experimental value is 0.54 GeV^2). The analogous formula for the Δ isobar has the form

$$M_{\Sigma^*}^2 - M_{\Delta}^2 = 33 \pi^2 \frac{f_K^2 M_K^2}{M_{\Delta}^2} = 0.46 \text{ GeV}^2 \quad (10.15)$$

(the experimental value is 0.4 GeV^2).

11. - THE PROTON LIFETIME IN THE SU(5) MODEL

As is well known, the models of grand unification predict the instability of the proton¹⁸⁾. The proton lifetime estimates in the models of grand unification are very important, because the search for proton decay is in fact the only direct experimental test of the grand unification philosophy. Note that the existing experiment constraint on the proton lifetime is $\tau_p \geq (1 \div 2) \cdot 10^{30} \text{ years}$ ¹⁹⁾.

The Lagrangian describing proton decay in the SU(5) model has the form¹⁸⁾

$$\mathcal{L} = \frac{G_u}{\sqrt{2}} \xi \bar{e}_\alpha^c (1 + 3\gamma_5) O_{1\alpha}, \quad (11.1)$$

$$O_{1\alpha} = (u^k C \gamma_\mu (1 - \gamma_5) u^j) (\gamma^\mu d^i)_\alpha \varepsilon^{ijk},$$

(we neglect the Cabibbo mixing due to the smallness of the Cabibbo angle $\sin^2 \theta_c \sim 1/25$) where $G_u/\sqrt{2} = g^2/8M^2$, g is the gauge coupling constant, and M is the X and Y boson mass. The factor $\xi = 3-4$ reflects the renormalization effects due to the strong and electroweak interactions. The proton lifetime estimate in the SU(5) model (as in any other GUT model) is divided into two parts. The first part is the calculation of the parameter $G_u/\sqrt{2}$ in the effective Lagrangian (11.1). The second part is the estimate of the matrix elements for the Lagrangian (11.1). Here we estimate the matrix elements for proton decay using the pole model [see, for example, Ref. 20)] with the help of FESR. We should, of course, point out that earlier estimates of the matrix elements were made using the non-relativistic SU(6) model, the bag model²¹⁾ and the technique of the ITEP group²⁰⁾. Since the proton lifetime estimate is a very important one, we think that it is useful to get the numerical values for the matrix elements with the help of our new technique. From the relation (10.8) and the Lagrangian (11.1) describing the proton decay in terms of the quark fields, we obtain the effective Lagrangian

$$\mathcal{L}_{eff} = \frac{3\lambda_p}{\sqrt{2}} G_u \xi \bar{e}_2^c \psi_2^p, \quad (11.2)$$

where ψ_p is the wave function of the proton. The constant λ_p was calculated by us in Section 10 and it is equal to

$$\lambda_p^2 = 1.8 \cdot 10^{-4} \text{ GeV}^6 \quad (11.3)$$

Our result differs rather strongly from the one ($\lambda_p^2 = 1.2 \cdot 10^{-3} \text{ GeV}^6$) obtained in Ref. 2) and used in Ref. 20) to estimate the proton lifetime in the SU(5) model. It seems reasonable enough by analogy with the case of mesons⁵⁾ that the averaging interval S_0 in the formulae (10.5) must lie between the mass square of the proton and the mass square of the first proton excitation $N(1470)$. Because of this, the natural constraint on S_0 is $M_p^2 \leq S_0 \leq M_{N(1470)}^2$. Using the formulae (10.6) we therefore find

$$0.33 \cdot 10^{-4} \text{ GeV}^6 \leq \lambda_p^2 \leq 7 \cdot 10^{-4} \text{ GeV}^6 \quad (11.4)$$

The quantity $g^2/8M_X^2$ was estimated in the SU(5) model by many authors; see, for example, Ref. 18). The main uncertainty in the estimate of $g^2/8M_X^2$ is connected with the uncertainty in the choice of the strong interaction scale parameter Λ . The analysis of experimental data leads to the value $\Lambda_{\overline{MS}} = (0.1 \div 0.2) \text{ GeV}$ ¹⁸⁾. This value of $\Lambda_{\overline{MS}}$, after taking into consideration all other uncertainties, gives [J. Ellis, Ref. 18)]

$$M_X = (1 \div 4) \cdot 10^{14} \text{ GeV} \quad (11.5)$$

The quantity $\alpha_s(M_X)$ is rather well known and is equal to¹⁸⁾

$$\alpha_s(M_X) = 1/41$$

In the pole model we have²⁰⁾

$$\Gamma(p \rightarrow e^+ \pi^0) = \frac{5}{16\pi} G_u^2 \xi^2 \lambda_p^2 \frac{g_\pi^2}{m_p^2} \approx 0.5 \Gamma_{tot}$$

where g_π is the pion-nucleon coupling constant $g_\pi^2/4\pi = 14$. Numerically for $\xi^2 = 10$ we have

$$\tau_p = 10^{28} \left(\frac{M_x}{10^{14} \text{ GeV}} \right)^4 \text{ years} \quad (11.6)$$

To estimate the above limit of the proton lifetime, we take $\lambda_p^2 = 0.33 \cdot 10^{-4} \text{ GeV}^6$ - from the inequality (11.4) - and $M_x = 5 \cdot 10^{14} \text{ GeV}$; we then have

$$\tau_p \lesssim 4 \cdot 10^{31} \text{ years} \quad (11.7)$$

Our estimate of the proton lifetime is close to the estimates based on the non-relativistic SU(6) model, approximately seven times larger than that of Ref. 20) and three to 14 times less than the bag model estimate.

12. - CONCLUSION

In this paper we applied the FESR method for the calculation of a number of characteristics of the hadrons (masses, coupling constants). Our results are in reasonable agreement with experimental data.

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NOTE ADDED

We recently heard about a paper by J.J. Sakurai [Phys. Lett. 46B (1973) 207] where FESR were used for the description of the e^+e^- cross-section. We are indebted to Professor Sakurai for pointing this reference out to us.

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