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ON THE CALCULABILITY OF $W_L^+ - W_R^+$ GAUGE MIXING

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ABSTRACT

We investigate the conditions which must be satisfied in order for $W_L^+ - W_R^+$ gauge mixing to be a calculable quantity. In particular a model independent necessary constraint involving fermion mass matrices is derived.

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1. - INTRODUCTION

At present, there is impressive agreement between the Glashow-Salam-Weinberg model and all available experimental data. However, it may very well happen that the electroweak interactions actually have a much richer structure which will manifest itself at energies higher than M_{W_L} . An interesting departure from the standard model is to assume that the present left-handed character of the charged weak interactions is a low-energy phenomenon and that at higher energies right-handed currents will become apparent. One way of incorporating this idea within the framework of gauge theories is to introduce two charged vector bosons, one (W_L) coupled to a purely left-handed current and another (W_R) to a purely right-handed current. The physical charged gauge bosons (i.e., the mass eigenstates) are in general a mixture of W_L and W_R ¹⁾:

$$\begin{aligned} W_1 &= W_L \cos \xi - W_R \sin \xi \\ W_2 &= W_L \sin \xi + W_R \cos \xi \end{aligned} \quad (1.1)$$

Since the 2×2 charged gauge boson matrix is in principle arbitrary, M_{W_L} , M_{W_R} and ξ are independent parameters. It is clearly of paramount importance to put experimental limits on these quantities. Some years ago, Bég et al.¹⁾ used information from leptonic and semi-leptonic decays to derive an experimental limit on the mixing angle ξ :

$$|\xi| \leq .06 \quad (1.2)$$

This analysis has, however, to be reconsidered when very massive right-handed Majorana neutrinos are present^{2),3)}, since in that case the leptonic part of the right-handed charged current is completely suppressed at low energy. From the $\Delta I = 3/2$ contribution to non-leptonic decays, the authors of Ref. 2) obtained the bound

$$|\xi| \leq 1/120 \quad (1.3)$$

On the other hand, the $K_L - K_S$ mass difference, due to the appearance of an enhancement factor⁴⁾, again provides a bound⁵⁾

$$|\xi| \leq .06 \quad (1.4)$$

Recently⁶⁾ it has been emphasized that a negligible value for the mixing ξ is crucial in order to ensure a very small or even vanishing value for ε'/ε in the framework of $SU(2)_L \times SU(2)_R \times U(1)$ models and provides us, at the same time, with an upper bound for the right-handed charged gauge boson mass, namely $M_{WR} \lesssim 30$ TeV.

In this note, we investigate the conditions which must be satisfied in order for ξ to be naturally zero in the tree approximation. In this case, one is sure that ξ will only receive finite calculable contributions from radiative corrections⁷⁾.

2. - A NECESSARY CONDITION

Clearly a necessary condition for ξ to be a calculable quantity in the sense of the previous paragraph is that no infinite contribution to ξ should arise from higher orders of perturbation expansion. In particular, from the diagrams of Fig. 1, it will be seen that the following condition has to be satisfied:

$$\text{Tr} \{ M^{u\dagger} M^d + M^{v\dagger} M^l \} = 0 \quad (2.1)$$

where M^u, M^d are the up and down quarks tree level mass matrices, while M^v, M^l stand for the corresponding neutrinos^{*)} and charged leptons matrices. In order to derive (2.1), it is sufficient to notice that the infinite contribution I of the diagram of Fig. 1 is proportional to:

$$I \propto \{ \text{Tr} K_R^\dagger D^u K_L D^d + \dots \} \quad (2.2)$$

where $K_{L,R}$ are mixing matrices associated with left-handed and right-handed charged currents, while D^u, D^d are the diagonal mass matrices for up and down quarks. The dots in Eq. (2.2) stand for the corresponding contributions from leptons. Using the well-known relationship between $K_{L,R}$ and the unitary matrices which diagonalize $M^{u,d}$, Eq. (2.1) immediately follows. At this point we would like to emphasize that the condition of Eq. (2.1) is independent of the way the gauge symmetry breaking takes place. Indeed it holds both in the framework of dynamical symmetry breaking and scalar breaking.

*) We note that Majorana masses for neutrinos do not spoil this relation since they only give rise to effective terms of at least dimension six (see Fig. 2).

In principle, Eq. (2.1) can be implemented in a variety of ways, namely:

- i) a natural cancellation between quark and lepton contributions to Eq. (2.1).
One can envisage such a cancellation to occur in grand unified theories such as $SO(10)$;
- ii) a separate cancellation of each one of the terms occurring in Eq. (2.1).
This can occur, for example, if the Yukawa couplings have some extra symmetries.

In the following sections we will analyze in detail how scenario (ii) can be achieved within the framework of a $SU(2)_L \times SU(2)_R \times U(1)$ gauge theory.

3. - THE CASE OF $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

We assume that the left-handed and right-handed fundamental fermions transform under $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ in the following way:

$$\begin{aligned}\Psi_L &\sim (1/2, 0, B-L) \\ \Psi_R &\sim (0, 1/2, B-L)\end{aligned}\tag{3.1}$$

the electric charge being defined as:

$$Q = T_{3L} + T_{3R} + \frac{B-L}{2}\tag{3.2}$$

In order to give fermions a Dirac mass, one needs to introduce $(1/2, 1/2^*)$ representations for the scalar fields. For simplicity, let us consider first a single ϕ , with its four complex fields written as:

$$\phi = \begin{pmatrix} \varphi_1^0 & \varphi_1^+ \\ \varphi_2^- & \varphi_2^0 \end{pmatrix}\tag{3.3}$$

It is well known that one can simultaneously construct a second $(1/2, 1/2^*, 0)$ representation, namely:

$$\tilde{\phi} \equiv \tau_2 \phi^* \tau_2 = \begin{pmatrix} \varphi_2^{0*} & -\varphi_2^+ \\ -\varphi_1^- & \varphi_1^{0*} \end{pmatrix}\tag{3.4}$$

Therefore, the following Yukawa couplings are in general allowed:

$$\mathcal{L}_Y = \bar{\Psi}_{iL} (g_{ij} \phi + \tilde{g}_{ij} \tilde{\phi}) \Psi_{jR} \quad (3.5)$$

giving rise to arbitrary mass matrices for quarks and leptons, once the neutral scalar fields acquire vacuum expectation values:

$$\langle \varphi_1^0 \rangle = k \quad ; \quad \langle \varphi_2^0 \rangle = k' \quad (3.6)$$

However, this also implies that in principle there will be a square mass term mixing the left-handed and right-handed charged gauge bosons

$$\mu_{W_L^+ W_R^-}^2 = g^2 \langle T_L \tau_+ \phi T_L \phi^\dagger \rangle = g^2 k k' \quad (3.7)$$

This already stresses the important connection between fermion mass spectrum and the $W_L - W_R$ mixing ξ which can be expressed as:

$$\tan 2\xi = 2 \frac{\mu_{W_L^+ W_R^-}^2}{\mu_{W_R}^2 - \mu_{W_L}^2} \quad (3.8)$$

Since we are interested in having a scheme where the mixing angle ξ is calculable, we require that:

$$\langle \varphi_1^0 \rangle \neq 0 \quad ; \quad \langle \varphi_2^0 \rangle_{\text{tree}} = 0 \quad (3.9)$$

If the relation (3.9) is implemented naturally⁷⁾ (we use here the technical meaning of the word), then it ensures that radiative corrections are finite. It is trivial to check that (3.9) does indeed correspond to a minimum of the classical scalar potential, for a finite range of its parameters, provided that terms like $\text{Tr} \phi^\dagger \tilde{\phi}$ are not included. If these couplings were present, they would give rise to linear terms in k' , thus making it impossible to implement (3.9) naturally. In order to avoid these couplings, it is sufficient to impose a discrete phase transformation symmetry:

$$R : \phi \rightarrow e^{i\alpha} \phi \quad \alpha \neq n\pi \quad (3.10)$$

It should be stressed that if only terms linear in k' are forbidden and no other terms, the scalar potential automatically acquires the symmetry R , with $\alpha = \pi/2$. At the same time, this allows for terms like $[\text{Tr} \phi^\dagger \tilde{\phi}]^2$, thus preventing an unwanted continuous global symmetry. We will show that the necessity of such a discrete symmetry R has a dramatic effect as far as radiative corrections to the gauge boson mixing ξ are concerned. To see how this comes about, let us consider the following subgroup of $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times R$:

$$R' : e^{i\eta^L T_3^L} \otimes e^{i\eta^R T_3^R} \otimes e^{i\beta(\frac{B-L}{2})} \otimes R \quad (3.11)$$

under which:

$$\begin{aligned} W_L^+ &\longrightarrow e^{i\eta^L} W_L^+ \\ W_R^+ &\longrightarrow e^{i\eta^R} W_R^+ \\ \phi &\longrightarrow e^{i\eta^L \sigma_3} \phi e^{-i\eta^R \sigma_3} e^{i\alpha} \end{aligned} \quad (3.12)$$

In particular, the neutral components of ϕ transform under R' like

$$\begin{aligned} \varphi_1^0 &\longrightarrow e^{i/2 [\eta^L - \eta^R + 2\alpha]} \varphi_1^0 \\ \varphi_2^0 &\longrightarrow e^{-i/2 [\eta^L - \eta^R - 2\alpha]} \varphi_2^0 \end{aligned} \quad (3.13)$$

It is clear from (3.13) that if we choose

$$\eta^R - \eta^L = 2\alpha \quad (3.14)$$

the symmetry R is unbroken by the tree approximation vacuum (3.9), since then

$$\begin{aligned} \langle e_1^0 \rangle &\longrightarrow \langle e_1^0 \rangle \\ \langle e_2^0 \rangle &\longrightarrow e^{2i\alpha} \langle e_2^0 \rangle \end{aligned} \quad (3.15)$$

Using the Georgi-Pais theorem⁸⁾, one can immediately conclude that $k' = 0$ to all orders of perturbation expansion and the $W_L - W_R$ mixing will be exactly zero. It is apparent that the symmetry R plays a crucial rôle in the derivation of the previous result. Without it (i.e., $\alpha = 0$), the symmetry which remains unbroken after spontaneous symmetry breaking is obtained from (3.12) by choosing $\eta_L = \eta_R$ and simply corresponds to conservation of electric charge. We also note that the unbroken R' symmetry comes from a combination of a gauge symmetry with a global symmetry, a situation very similar to the "accidental" unbroken B-L symmetry arising in a minimal $SU(5)$ grand unification model.

So far we have considered the case of only one ϕ multiplet. We will next show that the generalization to an arbitrary number of ϕ multiplets is straightforward. Consider n $(1/2, 1/2^*, 0)$ representations for the scalars such that:

$$\langle \phi_i \rangle = \begin{pmatrix} k_i & 0 \\ 0 & k'_i \end{pmatrix} \quad i=1 \dots m \quad (3.16)$$

We can always define a linear combination of these multiplets:

$$F_m = \frac{1}{\sqrt{\sum k_i^2}} \sum_{j=1}^m k_j \phi_j \quad (3.17)$$

with

$$\langle F_m \rangle = \frac{1}{\sqrt{\sum k_i^2}} \begin{pmatrix} \sum_j k_j^2 & 0 \\ 0 & \sum_j k_j k'_j \end{pmatrix} \quad (3.18)$$

while the $(n-1)$ orthogonal combinations F_i ($i = 1, \dots, n-1$) are such that

$$\langle F_i \rangle = \begin{pmatrix} 0 & 0 \\ 0 & 1_i \end{pmatrix} \quad (3.19)$$

and therefore do not contribute to the mixing ξ . The way to guarantee a vanishing ξ at tree level is to forbid again terms like $\text{Tr } F_n^\dagger \tilde{F}_n$, through the introduction of a symmetry under which

$$F_m \longrightarrow e^{i\alpha} F_m \quad (3.20)$$

or equivalently:

$$\phi_i \longrightarrow e^{i\alpha} \phi_i \quad (i = 1 \dots m) \quad (3.21)$$

Using (3.21), the previous arguments go through, thus proving that a vanishing tree level mixing angle ξ is stable against radiative corrections, when an arbitrary number of $(1/2, 1/2^*, 0)$ scalars are considered. However, other scalar representations clearly have to be introduced since so far the charged gauge bosons are degenerated in mass and moreover $U(1)_{T_{3L}+T_{3R}} \times U(1)_{B-L}$ remains unbroken. A minimal approach consists of introducing only two new multiplets:

$$\begin{aligned} \chi_L &\sim \left(\frac{m}{2}, 0, B-L \right) \\ \chi_R &\sim \left(0, \frac{m}{2}, B-L \right) \end{aligned} \quad (3.22)$$

The scalar potential now contains a new sector

$$V = V_1(\chi_L^\dagger \chi_L, \chi_R^\dagger \chi_R, \phi^\dagger \phi) + V_2(\chi_L, \chi_R, \phi) \quad (3.23)$$

where V_1 contains the trivial couplings which are always present, independently of the transformation properties of χ_L, χ_R under R , while V_2 contains non-trivial couplings. In general, under R the new scalars transform like:

$$\begin{aligned} \chi_L &\longrightarrow e^{i\alpha^L} \chi_L \\ \chi_R &\longrightarrow e^{i\alpha^R} \chi_R \end{aligned} \quad (3.24)$$

where $\alpha_L + \alpha_R = 0$ *). At the neutral components level, this gives the following transformation properties under R' :

$$\begin{aligned} \chi_L^0 &\longrightarrow e^{i[\eta^L T_3 + \beta(\frac{B-L}{2}) + \alpha^L]} \chi_L^0 \\ \chi_R^0 &\longrightarrow e^{i[\eta^R T_3 + \beta(\frac{B-L}{2}) + \alpha^R]} \chi_R^0 \end{aligned} \quad (3.25)$$

From (3.25) and (3.14), one concludes that R' will be broken by $\langle \chi_L^0 \rangle, \langle \chi_R^0 \rangle$ if

$$\alpha^R - \alpha^L \neq (B-L)\alpha \quad (3.26)$$

One could be tempted to consider the case of $\alpha_L = \alpha_R = 0$. However, this implies $V_2 = 0$ and the potential in that case would have a continuous accidental symmetry

$$\begin{aligned} \chi_L &\longrightarrow e^{i\theta} \chi_L \\ \chi_R &\longrightarrow e^{i\theta} \chi_R \end{aligned} \quad (3.27)$$

in addition to $U(1)_{B-L}$. An unwanted Goldstone boson is avoided only if $\langle \chi_L^0 \rangle = 0$, but in this case a suitable choice of the parameter β in (3.25) ensures again an unbroken R' . One, therefore, needs non-trivial couplings between $\chi_{L,R}$ and the ϕ multiplets, which will define the transformation properties of $\chi_{L,R}$ under R . There are only two possible choices for χ_L and χ_R which allow for such non-trivial couplings and at the same time have neutral components. Let us consider first:

$$\begin{aligned} \chi_L &: (\frac{1}{2}, 0, 1) \quad ; \quad \langle \chi_L \rangle = \begin{pmatrix} 0 \\ v_L \end{pmatrix} \\ \chi_R &: (0, \frac{1}{2}, 1) \quad ; \quad \langle \chi_R \rangle = \begin{pmatrix} 0 \\ v_R \end{pmatrix} \end{aligned} \quad (3.28)$$

*) We already know that the complete Lagrangian is invariant under the local $U(1)_{B-L}$.

In this case,

$$V_2 = \lambda_1 \text{Tr } \chi_L^\dagger \phi \chi_R + \lambda_2 \text{Tr } \chi_L^\dagger \tilde{\phi} \chi_R + \text{h.c.} \quad (3.29)$$

However, $\alpha_{L,R}$ have to be chosen in such a way that the λ_1 coupling is naturally forbidden, since it is linear in k' . This implies:

$$\alpha^R - \alpha^L = \alpha \quad (3.30)$$

From (3.30) and (3.26) we find that the condition to break R' cannot be satisfied, since in our case $(B-L) = 1$. The other possibility consists of taking:

$$\begin{aligned} \chi_L &: (1, 0, 2) \quad ; \quad \langle \chi_L \rangle = \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix} \\ \chi_R &: (0, 1, 2) \quad ; \quad \langle \chi_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \end{aligned} \quad (3.31)$$

The non-trivial couplings in this case are:

$$V_2 = \lambda_1 \text{Tr } \chi_L^\dagger \phi \chi_R \tilde{\phi}^\dagger + \lambda_2 \text{Tr } \chi_L^\dagger \tilde{\phi} \chi_R \phi^\dagger + \text{h.c.} \quad (3.32)$$

Note that after spontaneous symmetry breaking these terms give rise to $v_L v_R (\lambda_1 k'^2 + \lambda_2 k^2)$, allowing therefore a minimum of the potential with vanishing K' but non-vanishing v_L, v_R . In order to avoid a continuous accidental symmetry, one has to take $\alpha = \pi/2$, which implies $\alpha_L - \alpha_R = \pm\pi$. Once again one finds that condition (3.26) cannot be satisfied and the R' symmetry remains unbroken. This completes our proof that once $W_L - W_R$ mixing is required to be naturally vanishing in the tree approximation, then an unbroken R' symmetry keeps it zero to all orders of perturbation, independently of the Higgs system chosen.

4. - IMPLICATIONS FOR FERMION MASSES AND MIXING ANGLES

In the previous section we have analyzed the situation where $W_L - W_R$ mixing is naturally zero in the tree approximation. Since the theory is renormalizable, no infinite contribution to ξ can arise in higher orders and in particular Eq. (2.1) should be automatically satisfied in the scheme described in Section 3. In order to show that this is indeed the case, we notice that when the symmetry R is extended to fermions, the elements of the fermion mass matrices necessarily satisfy the following conditions:

$$\begin{aligned} M_{ij}^u \neq 0 &\implies M_{ij}^d = 0 \\ M_{ij}^d \neq 0 &\implies M_{ij}^u = 0 \end{aligned} \quad (4.1)$$

and similarly for the leptonic sector^{*)}. In other words, the non-zero elements of M^u, M^d form disjoint sets. It should be emphasized that this result is independent of the transformation properties of fermions under R . This is seen by noticing that if both M_{ij}^u and M_{ij}^d were non-vanishing for some (i,j) , then both the couplings $\bar{\psi}_{Li} \psi_{Rj} \phi^\alpha$ and $\bar{\psi}_{Li} \psi_{Rj} \tilde{\phi}^\beta$ would exist. This would in turn imply that $\phi^\alpha, \tilde{\phi}^\beta$ have the same transformation properties under R , which, as we have seen in the previous section [see Eq. (3.21)], cannot be the case. From property (4.1) it follows that $\text{Tr}(M^{u\dagger} M^d)$ as well as $\text{Tr}(M^{v\dagger} M^l)$ automatically vanish. In fact, from the results of the previous section, one expects that the fermion mass matrices will be further restricted, since it has already been shown that an unbroken R' symmetry prevents the appearance of any $W_L - W_R$ mixing, even in higher orders. In particular, even the finite part of the diagram of Fig. 1 has to vanish. In order to illustrate how these constraints on the Yukawa couplings arise, we will consider for simplicity the case of only two generations. Consider the following two mass matrices:

$$M^u = \begin{pmatrix} 0 & a \\ c & b \end{pmatrix} ; \quad M^d = \begin{pmatrix} d & 0 \\ 0 & 0 \end{pmatrix} \quad (4.2)$$

which obviously satisfy Eq. (2.1).

The Yukawa couplings necessary to achieve (4.2) would, however, also imply the existence of the quartic coupling

$$T_\Lambda (\phi_a^\dagger \phi_b) T_\Lambda (\phi_c^\dagger \tilde{\phi}_d) \quad (4.3)$$

where, for example, ϕ_a denotes the Higgs field responsible for M_{12}^u and similarly for the others. Since the coupling (4.3) is forbidden by the symmetry R , the structure (4.2) for the fermion mass matrices is also forbidden. Indeed, one expects this result, since (4.2) would imply a finite but non-vanishing gauge mixing through the graph of Fig. 3. This illustrates once more the important

^{*)} In this sector, the simplest solution consists of ensuring $M^v = 0$ through an appropriate choice of the transformation properties of the chiral lepton fields under R .

connection between calculability of $W_L - W_R$ mixing and the structure of the fermion mass matrix. It is easy to convince oneself that in the case of two generations the only allowed structure for M^u, M^d is:

$$M^u = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} ; \quad M^d = \begin{pmatrix} 0 & c \\ d & 0 \end{pmatrix} \quad (4.4)$$

since a tree level vanishing mass matrix element M_{ij} receives no radiative corrections owing to the unbroken R' symmetry. The corresponding "tadpole", as well as the self-energy type of contributions, indeed vanish ($\delta k' = 0$ and $\mu_{WLWR}^2 = 0$, respectively) in such a way that tree level massless quarks are excluded. The generalization to more flavours is straightforward, and the only allowed structure is

$$M^u = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix} ; \quad M^d = \begin{pmatrix} 0 & C \\ D & 0 \end{pmatrix} \quad (4.5)$$

where A, B, C, D are square matrices. This leads to an "orthogonal" structure for the left- and right-handed charged currents. This was found previously in a more restrictive context where neutrinos were required to be massless in all orders of perturbation expansion⁹⁾. If we assume that there are six "light quarks" and taking into account the bottom quark decay modes through W_L exchange the structure of Eq. (4.5) requires the existence of six generations of fermions.

5. - COMMENTS AND CONCLUSIONS

Previously⁶⁾ it has been emphasized that having a negligible $W_L - W_R$ mixing is crucial for obtaining a very small ϵ'/ϵ ratio in the context of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge theories, implying at the same time a suppressed one-loop contribution to the neutron dipole moment¹⁰⁾. Motivated by these results, we have looked for the constraints which have to be satisfied in order to obtain a naturally vanishing $W_L - W_R$ mixing in the tree approximation, since this would assure us that higher order contributions would be calculable and small. In fact, it turned out that in the context $SU(2)_L \times SU(2)_R \times U(1)$ gauge models, a calculable gauge mixing leads to dramatic effects on the fermion

masses, as we have explained in the previous section. However, as we mentioned in Section 2, calculability of $W_L - W_R$ mixing can also be implemented¹¹⁾ in Grand Unified Theories (GUTs) such as $SO(10)$ ¹²⁾, where quark and lepton masses are related. At this stage, it is worth recalling that calculability of the electroweak gauge mixing angle θ_W at the $SU(2)_L \times U(1)$ level leads straightforwardly to the famous relation $\sin^2 \theta_W = 3/8$ by simply requiring that the coefficient of the divergence in Fig. 4 vanish. As is well known, this can be naturally implemented in minimal GUTs where quark and lepton charges are related.

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FIGURE CAPTIONS

- Fig. 1 : One-loop fermionic contribution to the mixing square mass $\mu_{W_L W_R}^2$.
- Fig. 2 : A Majorana masses contribution to $\mu_{W_L W_R}^2$.
- Fig. 3 : One-loop graph giving rise to an effective $W_L - W_R$ mixing.
- Fig. 4 : One-loop fermionic contribution to the electroweak mixing θ_W in $SU(2)_L \times U(1)$.

FIG.1

FIG.2

FIG3

FIG.4