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PRIMORDIAL SUPERSYMMETRIC INFLATION

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A B S T R A C T

We discuss the motivations for reconsidering cosmological inflation in supersymmetric theories as contrasted with conventional GUTs. Radiative corrections to the effective potential can be made arbitrarily small in supersymmetric GUTs, removing some of the obstacles to inflation. We analyze general renormalizable potentials at the tree level and show that the required fine-tuning of parameters becomes less acute if inflation takes place before the grand unified phase transition, a hypothesis we term primordial inflation. We show how the grand unified monopole problem can be solved in supersymmetric GUTs embodying primordial inflation.

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1. INTRODUCTION

One of the most interesting prospects offered to cosmologists by grand unified theories (GUTs) has been the possibility of exponential inflation during a GUT phase transition¹⁾. Inflation by a factor of $O(10^{28})$ or more in the cosmic scale $a(t)$ could explain many of the mysteries concerning the large-scale structure of the Universe. These include the puzzles of its homogeneity and isotropy on distance scales which would not have been in causal contact at earlier stages of a conventional Robertson-Walker-Friedman Universe expanding adiabatically, as well as its present flatness and longevity. In addition, inflation could also cure some of the cosmological difficulties that particle theorists have brought on themselves, such as the problematic abundances of grand unified monopoles²⁾ and of gravitinos³⁾.

At what stage in the evolution of the Universe should the inflationary epoch have occurred? Any time before Big Bang Nucleosynthesis would suffice for resolving the traditional cosmological problems. On the other hand, inflation should not have taken place after Big Bang Baryosynthesis, if one wants to avoid drowning the hard-won baryon asymmetry⁴⁾ in entropy. The grand unified monopole problem might be resolved if inflation took place during a first order grand unified phase transition, while inflation at an earlier epoch would leave the grand unified monopole problem untouched. Resolution⁵⁾ of the gravitino problem would require inflation after or perhaps during the Planck epoch. All these arguments are consistent with inflation during the GUT phase transition, which has been the focus of most recent research⁶⁻⁹⁾.

GUTs certainly provide a mechanism for driving inflation¹⁾, namely the vacuum energy to be expected before the spontaneous breakdown of a gauge symmetry if the cosmological constant is essentially zero afterwards. This vacuum energy could drive an exponential expansion while the Universe underwent a first order phase transition¹⁰⁾. Although earlier difficulties related to the termination of this de Sitter phase seem to be resolved⁶⁾, more detailed investigations of actual GUTs continue to reveal more difficulties¹¹⁻¹³⁾. The effective (mass)² parameter of the Higgs field ϕ making the transition must be carefully adjusted, and the effective

ϕ^4 coupling must be small if sufficient inflation is to be obtained. Radiative corrections in conventional GUTs are much larger than the fine tunings required^{11,12)}. Also, it seems likely that conventional GUTs, far from guaranteeing homogeneity on scales larger than the horizon size, actually freeze in perturbations $\delta\rho/\rho$ in the mass-energy density which are much too large when they cross the horizon¹³⁾.

Supersymmetric GUTs are very fashionable just now¹⁴⁾, and have some properties that are very desirable for realizing cosmological inflation¹²⁾. Radiative corrections to the effective Higgs potential vanish in an exactly supersymmetric theory¹⁵⁾, and can be kept arbitrarily small by a judicious choice of the supersymmetry breaking scale. Therefore they facilitate the fine-tuning needed for inflation, and may also relieve the difficulties with large-scale homogeneity. However, care should be taken to avoid having such a gradual inflation that Big Bang Baryosynthesis is not feasible⁵⁾.

In this paper we extend our previous work¹²⁾ by studying explicit supersymmetric GUT models. We work in the idealized limit of exact supersymmetry for two reasons. One is that supersymmetry frees us from the incubus of radiative corrections¹⁵⁾, and we wish to explore the new freedom this gives us. The other reason is that recent work¹⁶⁾ has strongly suggested that the primordial supersymmetry breaking scale M_s must be considerably smaller than the GUT scale $m_X = O(10^{16})$ GeV. If M_s is much larger than $O(10^{10})$ GeV, then supergravity corrections to scalar boson masses will be bigger than $O(M_s^2/m_P) > O(100)$ GeV, and hence unacceptably large¹⁶⁾.

Analysis of our supersymmetric toy model leads us naturally to the supposition that inflation took place before the GUT phase transition. This is the hypothesis that we call primordial inflation. Inflation before the GUT epoch seems more natural for two reasons. One is that the fine-tuning of parameters in the effective potential required to obtain sufficient inflation becomes less and less severe as the Planck scale is approached. The second reason is that the necessary tuning seems more reasonable if the Higgs field ϕ driving the inflation is a gauge singlet, and one might expect a gauge singlet field to have a vacuum expectation

value $O(m_p)$. Primordial inflation also has cosmological advantages, in that inflation using a vacuum energy density $\delta: \delta^{1/4} \ll m_p$ requires fine-tuning the primordial curvature¹⁷⁾ and rotation¹⁸⁾ to $O(\sqrt{\delta}/m_p^2)$. Taking δ close to m_p certainly makes these adjustments seem more natural. We are not able to discuss quantitatively what may happen when $\delta^{1/4} = O(m_p)$, since all manner of incomprehensible quantum gravitational effects must then become important. However, we do feel that inflation at or near this epoch is the most natural possibility.

A drawback of this primordial inflation is that it disregards the particle physics problem of excess monopole production²⁾. We investigate supersymmetric GUTs which offer two different classes of solution to the monopole problem. In one class the Universe cools to $O(10^{10})$ GeV before undergoing the GUT phase transition¹⁹⁾. This class again requires delicate adjustments of parameters. The second class of solution involves a second order GUT phase transition²⁰⁾. This may occur at $O(10^{15} \text{ to } 10^{16})$ GeV without generating too many monopoles. These may be the only classes of inflationary scenario which can accommodate an observable flux of grand unified monopoles.

The organization of this paper is as follows. In Section 2 we first review the aspirations and travails of conventional inflation during the GUT phase transition^{1,6-9)}, and the motivations for going supersymmetric^{11,12)}. Then we construct and investigate our toy supersymmetric model, and show that it can generate inflation in a way which becomes more and more natural as $\delta^{1/4}$ approaches m_p . We also discuss some of the qualitative questions arising when $\delta^{1/4} = O(m_p)$. In Section 3 we extend our toy model to complete supersymmetric GUTs embodying the two classes of solution to the monopole problem mentioned earlier. Section 4 summarizes our conclusions and prospects for further study.

2. SUPERSYMMETRIC INFLATION

2.1 Motivations

We assume that the early Universe can be described by a homogeneous Robertson-Walker-Friedmann metric of the form

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \right] \quad (2.1)$$

where $a(t)$ is the Robertson-Walker scale factor and $k = 0, \pm 1$ is a measure of curvature. The Einstein field equations for an isotropic Universe yield

$$H^2 \equiv \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi\rho}{3m_p^2} - \frac{k}{a^2} + \frac{\Lambda}{3} \quad (2.2)$$

where H is the Hubble parameter, ρ is the mass-energy density, Λ is the cosmological constant and $m_p \equiv G_N^{-1/2}$ is the Planck mass. We use units such that $\hbar = c = k_B = 1$. At the moment, the cosmological constant is negligible on any particle physics scale. Since the vacuum energy of the Universe decreases by an amount $\delta \propto \mu^4$ when the Universe passes through a phase transition in a gauge theory where some Higgs field ϕ acquires a vacuum expectation value (v.e.v.) $\langle 0|\phi|0 \rangle \equiv \mu$, it would have acted like a cosmological constant of magnitude

$$\Lambda = \frac{8\pi\delta}{m_p^2} \quad (2.3)$$

prior to the phase transition. We suppose that the Universe was radiation dominated at very early times so that

$$\rho = N(T) \frac{\pi^2}{30} T^4 \quad (2.4)$$

where $N(T)$ is the number of bosonic and fermionic degrees of freedom with masses $\ll T$: $N(T) = N_B(T) + \frac{7}{8}N_F(T)$. Radiation dominance (2.4) implies the evolution

$$a \propto T^{-1} \propto t^{1/2} \quad (2.5)$$

whereas if

$$\rho \lesssim \frac{\Lambda m_p^2}{8\pi} = \delta \quad (2.6)$$

and we neglect for the moment the curvature and any shear or rotation terms in Eq. (2.2), the dynamics of the expansion are governed by Λ and we find from Eq. (2.2) that the Universe approaches a De Sitter expansion¹⁰⁾

$$a \propto e^{Ht} \quad (2.7)$$

where

$$H = \left(\frac{\Lambda}{3}\right)^{1/2} = \left(\frac{8\pi\delta}{3m_p^2}\right)^{1/2} \quad (2.8)$$

characterizes the exponentially expanding Universe.

Recent interest in this inflationary Universe was triggered by Guth's observations¹⁾ that one could reasonably expect dominance by the vacuum energy density during a first-order GUT phase transition with supercooling, and that a prolonged period of inflationary expansion with

$$e^{H\tau} > O(10^{28}) \quad (2.9)$$

i.e.

$$\tau > 65/H$$

could neatly solve many of the cosmological problems mentioned in the Introduction. The near flatness of the present Universe [$k/a^2 \leq O(10) \times (8\pi/3)(\rho/m_p^2)$] despite its great age [$t_0 \approx O(10^{61}) \times t_p$] is explained by the early exponential growth (2.7) of the scale factor a . However, inflation during the GUT epoch is not a panacea. Large-scale homogeneity is not assured since density perturbations $\delta\rho/\rho$ may be $O(1)$ when they cross the horizon¹³⁾. It is not²¹⁾ clear that the cosmological abundance of grand unified monopoles would be sufficiently suppressed, since the exponential expansion takes place in a GUT-invariant phase, and in the original inflationary scenario there was no way of correlating the directions in which the GUT Higgs fields ϕ would eventually point after spontaneous symmetry breakdown in different parts of the Universe. Also, for the vacuum energy to dominate the expansion it is necessary that neither the curvature, nor any possible additional anisotropic shear or rotation terms in Eq. (2.1) be dominant when the mass-energy density ρ falls below δ (2.6). This requires that the dimensionless measure of curvature $\hat{k}$ defined by

$$\hat{k} \equiv \frac{k}{a^2 T^2} \quad (2.10)$$

which one might naively expect to be $O(1)$ at the Planck epoch should in fact be small even before inflation. Demanding

$$\frac{8\pi\rho}{3m_p^2} > \frac{k}{a^2} = \hat{k} T^2 \quad (2.11)$$

at the onset of the De Sitter phase, and using Eqs. (2.4) and (2.6) we find that we need

$$\hat{k} < \left(\frac{N\pi^2}{30}\right)^{1/2} \frac{8\pi}{3} \left(\frac{\mu^2}{m_p^2}\right) \quad (2.12)$$

which is redolent of some degree of primordial fine-tuning¹⁷⁾ if $\mu = O(10^{15})$ GeV as in conventional GUTs. There is no similar problem with including any shear term in Eq. (2.1) since it naturally falls as T^6 during a radiation-dominated epoch²²⁾, but there is an analogous puzzle concerning the rotation ω ¹⁸⁾. Introducing another dimensionless variable $\hat{\omega}$:

$$\hat{\omega} \equiv \frac{\omega}{T} \quad (2.13)$$

which also remains constant during a radiation-dominated epoch and could be anticipated to be $O(1)$ at the Planck time, we find that we avoid rotation dominance before the onset of De Sitter expansion only if

$$\hat{\omega}^2 < \left(\frac{N\pi^2}{30}\right)^{1/2} 4/\pi \left(\frac{\mu^2}{m_p^2}\right) \quad (2.14)$$

which poses another fine-tuning problem if $\mu = O(10^{15})$ GeV. Note that in the $k = +1$ case or if $\omega \neq 0$, the Universe may well recollapse before it has a chance to inflate.

However, the greatest snag of the original inflationary scenario was the apparent absence of a graceful exit from the De Sitter phase²³⁾. Bubbles of the new phase could never fill the entire Universe, which would as a result be unacceptably lumpy. This difficulty was finessed in the new inflationary scenario of Linde^{6,8)}, which postulated inflation after the Higgs field ϕ has started rolling over from its high temperature value $\langle 0|\phi|0\rangle = 0$ towards its ultimate v.e.v. $\langle 0|\phi|0\rangle = \mu$, as illustrated in Fig. 1. In this new inflationary scenario

the monopole problem disappears, though the other fine-tuning difficulties^{11,12,17,18)} mentioned above do persist. The most promising way of realizing this possibility in conventional GUTs such as SU(5) involved a Coleman-Weinberg²⁴⁾ potential where radiative corrections play a crucial role:

$$V(\phi) = A \phi^4 (\ln \phi^2/v^2 - 1/2) + D \phi^2 \quad (2.15)$$

with

$$A = \frac{1}{64\pi^2 v^4} \left[\sum_B g_B m_B^4 - \sum_F g_F m_F^4 \right] \quad (2.16)$$

where $g_{B(F)}$ is the number of boson (fermion) helicity states of mass $m_{B(F)}$ which can circulate in the loop of Fig. 2. Our notation will be that μ is the v.e.v. of an arbitrary Higgs field while the symbol v will denote the v.e.v. of an adjoint multiplet. For standard SU(5)²⁵⁾, the X and Y bosons dominate equation (2.16) so that

$$g_B = 36, \quad m_B^2 = (25/8) g^2 v^2$$

$$\text{and } A = \frac{5625}{1024 \pi^2} g^4 \quad (2.17)$$

where g is the SU(5) gauge coupling:

$$\frac{g^2}{4\pi} = \frac{1}{41} \Rightarrow g^2 = 0.3 \quad (2.18)$$

in minimal SU(5). The $D\phi^2$ in Eq. (2.15) serves as an effective mass term which can be subdivided as

$$D = \frac{1}{2} (m_0^2 + cT^2 + bR - 3\lambda \langle \phi^2 \rangle) \quad (2.19)$$

where m_0 is a bare mass term,

$$c = \frac{75}{8} g^2 \quad (2.20)$$

is the finite-temperature correction in an SU(5) gauge theory, b is an unknown parameter taking the value $1/6$ for a conformal coupling of the scalar fields ϕ , $R \equiv R_{\mu}^{\mu}$ is the scalar curvature, $-\lambda/4$ is the ϕ^4 self-coupling and $\langle \phi^2 \rangle$ the quantum expectation value of ϕ^2 in curved space^{26,11)}. At high temperatures D is large

and the field ϕ is confined near the origin, whereas at lower temperatures it may become possible for ϕ to fluctuate or tunnel across the potential barrier sketched in Fig. 1.

The new inflationary scenario only works for a certain range of values of D . To get a lower bound on the tunnelling rate, we consider the proposal of Hawking and Moss⁹⁾ that a region of space larger than the De Sitter horizon volume may jump simultaneously to a local maximum V_1 of the potential. Hawking and Moss⁹⁾ estimated the resulting tunnelling probability to be

$$P \sim D^2 e^{-B} \quad (2.21)$$

where*)

$$B = \frac{3m_p^4}{8} \left[\frac{1}{V_0} - \frac{1}{V_1} \right] \approx \frac{3m_p^4}{8} \left[\frac{V_1 - V_0}{V_0^2} \right] \quad (2.22)$$

in the limit where $V_1 - V_0 \ll V_0$. For the Coleman-Weinberg potential (2.15)

$$V_0 = \frac{1}{2} A v^4, \quad V_1 = V_0 + \frac{D^2}{8A} \quad (2.23)$$

and the Hawking-Moss action is therefore

$$B \approx \frac{3m_p^4 D^2}{16 A^3 v^8} \quad (2.24)$$

It is only sensible²⁷⁾ to use the picture that a De Sitter horizon volume jumps to a local maximum of V_1 if $B > 0(1)$ so that other values of ϕ are disfavoured.

However, even this may not be sufficient, either because the Universe finds some other way of tunnelling or fluctuating, or else because the amount of inflation after the jump is less than that (2.9) required. A plausible estimate of τ is

$$\tau \approx \frac{3H}{2D} \quad (2.25)$$

Applying the twin conditions that B (2.24) be > 1 and that τ (2.25) be sufficiently large we obtain the twin constraints

$$\frac{4A^{3/2}v^4}{\sqrt{3}m_p^2} < D < \frac{4\pi A v^4}{130 m_p^2} \quad (2.26)$$

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*) This formula differs by a factor of 3 from that quoted by Hawking and Moss⁹⁾. We thank S. Hawking and N. Straumann for clarifying this point.

where we have substituted Eq. (2.23) into the expression (2.8) for the Hubble parameter H .

Equation (2.26) is one of the main reasons leading us¹²⁾ to propose that cosmological inflation would benefit from supersymmetry. The two inequalities are only consistent if

$$A < 3\left(\frac{\pi}{130}\right)^2 = O(10^{-3}) \quad (2.27)$$

which is certainly not the case in minimal SU(5) (2.17) for which $A = O(1/10)$.

The general equation (2.16) immediately suggests a way out: introduce fermions so as to cancel out the X and Y bosons in the offending expression (2.17). This is most naturally done in the context of supersymmetry¹⁴⁾, where if

$$m_B^2 - m_F^2 = m_S^2 \ll m_B^2 \quad (2.28)$$

in Eq. (2.16) we get

$$A = \frac{\sum g_B (= g_F) m_S^2 m_B^2}{32 \pi^2 \nu^4} \quad (2.29)$$

The effective supersymmetric mass splittings m_S in Eqs. (2.28) and (2.29) should not be confused with a primordial supersymmetry breaking scale $\sqrt{d}$. They may be much smaller than $\sqrt{d}$ by powers of coupling constants and other numerical factors, and need not be universal for different supermultiplets²⁸⁾. Also, the vacuum energy V_1 ($\approx V_0$) that we are using to drive inflation should not be confused with the vacuum energy $O(d^2)$ of a theory with broken supersymmetry. It is the broken supersymmetry contribution to the potential $V(\phi)$ when $\langle 0|\phi|0\rangle = v$ which is $O(d^2)$, and must be mysteriously cancelled (quantum supergravity?)²⁹⁾ to get the required negligible cosmological constant at low temperatures. The vacuum energy V_0 is the difference between this global minimum and the energy of a local extremum at $\phi = 0$ which may be much less supersymmetric. Hence the expression

$$A = \frac{75}{32 \pi^2 \nu^2} g^2 m_S^2 \quad (2.30)$$

obtained in minimal supersymmetric SU(5) [cf. Eq. (2.17)] should not astound the reader. Inserting the expression (2.30) into the bound (2.27) we find¹²⁾ that

$$gm_s < 5 \times 10^{-2} v \quad (2.31)$$

which is not a disastrous restriction on the supersymmetry breaking mass splitting m_s .

Supersymmetry can aid inflation in other ways. Returning to the effective mass parameter D of Eq. (2.19), it enables us to tune the bare mass parameter m_0 to whatever value we require, and fret less about the possible destabilizing effects of radiative corrections¹⁵⁾. Supersymmetry also enables us to make c arbitrarily small in a theory where the operational Higgs field ϕ is a gauge singlet, in which case c has the magnitude of an adjustable Yukawa coupling. This point is welcome since we have³⁰⁾

$$T = \frac{H}{2\pi} = \left(\frac{A v^4}{3\pi m_p^2} \right)^{1/2} \quad (2.32)$$

during the De Sitter phase, making the contribution of the $SU(5)$ gauge multiplet

$$\frac{1}{2} c T^2 = \frac{25 g^2 A v^4}{16\pi m_p^2} \quad (2.33)$$

to D somewhat larger than the upper limit (2.26), thus necessitating a cancellation. As for the third term equation (2.19) in the effective mass term D , it vanishes in a conventionally Weyl-rescaled model of chiral supermultiplets coupled to simple $N = 1$ supergravity²⁹⁾. This is again good news, since

$$\frac{1}{2} b R = 6 b H^2 = \frac{8\pi b A v^4}{m_p^2} \quad (2.34)$$

is again uncomfortably larger than the upper bound in Eq. (2.26) for values of $b = O(1)$ such as the conformal coupling $b = 1/6$. Finally, Linde¹¹⁾ has pointed out that the quantum correction term $-3\lambda/2\langle\phi^2\rangle$ in Eq. (2.19) must be quite small for the inflationary epoch (2.25) to be sufficiently long. He deduces¹¹⁾

$$\lambda \lesssim \frac{\pi^2}{1800} \sim 5 \times 10^{-3} \quad (2.35)$$

which is difficult to realize in conventional GUTs [the A coupling (2.17) is $O(1/10)$] but can easily be arranged in supersymmetric GUTs¹²⁾.

By now we have accumulated five reasons for believing that cosmological inflation would be easier to realize in GUTs with at least an approximate supersymmetry. There are actually good reasons to think that supersymmetry may be very nearly exact at the GUT scales of interest to us. One is that one generally expects

$$m_s \lesssim \sqrt{d} \quad (2.36)$$

and in most scenarios $\sqrt{d} \lesssim m_X$. In particular, for the gauge multiplet

$$m_s^2 \lesssim O(1) T_{UV}^2 \quad (2.37)$$

and it may even be much smaller for the X and Y gauge bosons if $\sqrt{d} < m_X$. Furthermore, in some models there are cancellations which enforce

$$\sum_B g_B m_B^4 - \sum_F g_F m_F^4 = O(m_s^4) \quad (2.38)$$

Finally, there are arguments^{16,31)} based on the coupling to supergravity and the mass of the gravitino which suggest that

$$\sqrt{d} = O(\sqrt{m_g m_p}) \lesssim O(\sqrt{m_w m_p}) = O(10^{10}) \text{ GeV} \quad (2.39)$$

in which case supersymmetry breaking effects are negligible at or before the GUT epoch.

We are therefore led to study a toy model entirely at the tree level, and discard all radiative corrections. In so doing, we will see that arguments of naturalness lead us to entertain the possibility that $\mu > O(m_X)$, thus incidentally alleviating the cosmological fine-tuning problems^{17,18)} (2.12, 2.14) which have not been emphasized up to now.

2.2 A supersymmetric model

We consider first the most general renormalizable tree potential constructed out of a single gauge singlet Higgs field ϕ :

$$V(\phi; T) = \alpha \phi^4 - \beta \phi^3 + (\gamma + c T^2) \phi^2 + \delta \quad (2.40)$$

We call the v.e.v. of the Higgs field μ and use it to scale the dimensional parameters in the potential (2.40):

$$\hat{\beta} \equiv \beta/\mu, \quad \hat{\gamma} \equiv \gamma/\mu^2, \quad \hat{\delta} \equiv \delta/\mu^4 \quad (2.41)$$

The condition that the vacuum energy vanish when $\langle 0|\phi|0\rangle = \mu$ at zero temperature implies

$$V(\mu; 0) = 0 : \quad \alpha - \hat{\beta} + \hat{\gamma} + \hat{\delta} = 0 \quad (2.42)$$

while the condition that $\langle 0|\phi|0\rangle = \mu$ be a stationary point is

$$\frac{\partial V}{\partial \phi}(\mu; 0) = 0 : \quad 4\alpha - 3\hat{\beta} + 2\hat{\gamma} = 0 \quad (2.43)$$

and it is a minimum if

$$\frac{\partial^2 V}{\partial \phi^2}(\mu; 0) > 0 : \quad 12\alpha - 6\hat{\beta} + 2\hat{\gamma} > 0 \quad (2.44)$$

We can use the vacuum conditions (2.42, 2.43) to relate the parameters $\hat{\gamma}$ and $\hat{\delta}$ to α and $\hat{\beta}$:

$$\left. \begin{aligned} \hat{\gamma} &= \frac{3}{2}\hat{\beta} - 2\alpha \\ \hat{\delta} &= \alpha - \frac{\hat{\beta}}{2} \end{aligned} \right\} \quad (2.45)$$

Using these, the minimality condition (2.44) becomes

$$8\alpha - 3\hat{\beta} > 0 \quad (2.46)$$

The potential (2.40) has a local extremum at $\phi = 0$, where it takes the value

$$V_0 = V(0; 0) = \delta \quad (2.47)$$

at zero temperature. At finite temperature the other two extrema of the potential (2.40) are at

$$\phi = \pm \frac{3\beta}{8\alpha} \pm \frac{\sqrt{9\beta^2 - 32\alpha(\gamma + cT^2)}}{8\alpha} \quad (2.48)$$

If the vacuum energy δ dominates the expansion rate, then the associated Hawking temperature is $T_H = H/2\pi$ which can be calculated using Eq. (2.8). We therefore have

$$c T_H^2 = \frac{2c}{3\pi} \frac{\mu^4}{m_P^2} \hat{\delta} \quad (2.49)$$

which we will also write in the scaled form

$$c T_H^2 / \mu^2 \equiv \chi \hat{\delta} \quad ; \quad \chi = \frac{2c}{3\pi} \frac{\mu^2}{m_p^2} \quad (2.50)$$

It follows from Eqs. (2.8), (2.9), (2.25) and (2.47) that we obtain sufficient inflation if

$$\frac{3H^2}{2D} = \frac{4\pi(\mu^2/m_p^2)\hat{\delta}}{\hat{\gamma} + \chi\hat{\delta}} > 65 \quad (2.51)$$

which we can write in the form

$$\hat{\gamma} + \chi\hat{\delta} = \gamma\hat{\delta} \quad (2.52a)$$

where

$$\gamma = \frac{4\pi}{65} \frac{\mu^2}{m_p^2} \hat{\gamma} \quad ; \quad 0 < \hat{\gamma} \leq 1 \quad (2.52b)$$

We can then use Eq. (2.48) to find that in the limit $\gamma \ll 1$ the local maximum of the potential is at

$$\phi_1 \approx \frac{\gamma}{3\rho} \mu \quad (2.53a)$$

where

$$\rho \equiv 2 + \gamma - \chi \quad (2.53b)$$

The three parameters (2.41) may also be expressed in terms of α and ρ :

$$\hat{\beta} = \frac{2\alpha\rho}{\rho+1} \quad , \quad \hat{\gamma} = \frac{\alpha(\rho-2)}{\rho+1} \quad , \quad \hat{\delta} = \frac{\alpha}{\rho+1} \quad (2.54)$$

The difference between the value V_1 of the potential at the local maximum ϕ_1 and the value V_0 at the local minimum is then clearly much smaller than V_0 itself, so we are justified in using the approximation (2.22) to the Hawking-Moss action⁹), which yields in our case

$$B \approx \frac{m_p^4 (\rho+1) \gamma^3}{72 \mu^4 \alpha \rho^2} \gg 1 \quad ? \quad (2.55)$$

As we mentioned before requiring $B \gg 1$ is certainly a necessary²⁷⁾ condition for the new inflationary scenario to be valid, but it may well not be sufficient.

Before analysing the constraints (2.51), (2.55) numerically, we first recall that for the reasons discussed in the previous subsection we are working in the

context of exact supersymmetry, and for reasons which will become clearer later on [but remember also Eq. (2.33)] we want ϕ to be a gauge singlet. Then the potential (2.40) can be easily obtained from the superpotential³²⁾

$$W(\phi, X, Y) = aX\phi(\phi - \mu) + bY(\phi^2 - \mu^2) \quad (2.56)$$

which yields at zero temperature

$$\begin{aligned} V(\phi, X, Y; 0) &= |W_X|^2 + |W_Y|^2 + |W_\phi|^2 \\ &= |a\phi(\phi - \mu)|^2 + |b(\phi^2 - \mu^2)|^2 + |aX(2\phi - \mu) + 2bY\phi|^2 \end{aligned} \quad (2.57)$$

The relevant finite temperature corrections³³⁾ to $V(\phi, X, Y)$ take the form

$$\begin{aligned} \delta V(\phi, X, Y; T) &= (2|W_{X\phi}|^2 + 2|W_{Y\phi}|^2 + |W_{\phi\phi}|^2) \frac{T^2}{8} \\ &= \frac{T^2}{8} [2|a(2\phi - \mu)|^2 + 2|2b\phi|^2 + |2aX + 2bY|^2] \end{aligned} \quad (2.58)$$

We deduce from Eq. (2.58) that finite temperature effects prefer $2aX + 2bY = 0$, in which case Eq. (2.57) tells us that the $|W_\phi|^2$ term prefers $X = 0$ and hence $Y = 0$ also. It is also obvious that the potential does not want a non-zero v.e.v. for the imaginary part of ϕ . We therefore have the following finite temperature effective potential for the real part of ϕ :

$$\begin{aligned} V(\phi; T) &= (a^2 + b^2)\phi^4 - 2a^2\mu\phi^3 + (a^2 - 2b^2)\mu^2\phi^2 \\ &\quad + b^2\mu^4 + (a^2 + b^2)T^2\phi^2 - a^2\mu T^2\phi + a^2T^2\mu^2 \end{aligned} \quad (2.59)$$

Comparing with our prototype tree potential (2.40) we notice that we can eliminate the errant linear term in Eq. (2.59) by a translation

$$\hat{\phi} = \phi - \phi_0 \quad : \quad \phi_0 = \frac{a^2 T^2}{2(a^2 - 2b^2)\mu} \quad (2.60)$$

in the limit of small T . [Recall that T_H is expected (2.49) to be much smaller than μ since $\hat{\delta} \ll 1$]. After making the translation (2.60) we can identify the parameters of our toy potential (2.40) as functions of the parameters of our supersymmetric model (2.56):

$$\alpha = a^2 + b^2 \quad (2.61a)$$

$$\beta = 2a^2\mu \quad (2.61b)$$

$$\gamma = (a^2 - 2b^2)\mu^2 \quad (2.61c)$$

$$c = - \left(\frac{2a^4 + a^2b^2 + 2b^4}{a^2 - 2b^2} \right) \quad (2.61d)$$

$$\delta = b^2\mu^4 \quad (2.61e)$$

$$\rho = a^2/b^2 \quad (2.61f)$$

where we have neglected terms of higher order in T/v . Also we find that the quantity x defined in Eq. (2.50) takes the value

$$x \approx \frac{-2}{3\pi} \frac{\mu^2}{m_p^2} \left[\frac{2a^4 + a^2b^2 + 2b^4}{a^2 - 2b^2} \right] \approx \frac{-4\mu^2}{\pi m_p^2} \left[\frac{a^2}{\rho - 2} \right] \quad (2.62)$$

for $\rho \approx 2$

and we will see later that the quantity in square brackets is expected to be $O(10^{-5})$ (see Fig. 3 for a sneak preview). Having established the correspondence of our supersymmetric model to the original toy model (2.40), we are now in a position to analyse the inflationary capability of the superpotential (2.56).

We examine the constraints (2.52), (2.55) for a range of values of $\langle 0|\phi|0\rangle = \mu$ between $O(10^{16})$ GeV and $O(10^{19})$ GeV. Figure 3 shows the allowed values of a^2 together with the corresponding values of the combination $\rho - 2 = (a^2/b^2 - 2)$ indicating the degree of fine-tuning between a and b necessary in the superpotential (2.56). We see that both a and b must be rather small if $\mu = O(10^{16})$ GeV:

$$a, b \sim O(10^{-6}) \quad (2.63)$$

and that their ratio must be finely tuned:

$$\rho - 2 = a^2/b^2 - 2 = O(10^{-7}) \quad (2.64)$$

However, the picture looks rather different if we push μ up to $O(10^{19})$ GeV:

$$a, b \sim O(10^{-3}) \quad (2.65)$$

and

$$\rho - 2 = a^2/b^2 - 2 \sim O(10^{-1}) \quad (2.66)$$

Thus while inflation is attainable in a supersymmetric model, it looks rather unnatural (2.63), (2.64) if $\mu = O(10^{16})$ GeV, but rather less unnatural if $\mu = O(10^{19})$ GeV.

This is the observation which leads us to propose primordial inflation, i.e. inflation at an epoch prior to the grand unified phase transition and very possibly during the Planck era^{*)}. We also remind the reader that inflation at an energy scale $\gg m_X$ also alleviates the cosmological fine-tuning problems associated with the curvature¹⁷⁾ (2.12) and rotation¹⁸⁾ (2.14) of the Universe. This all sounds wonderful: so why have previous authors not discussed primordial inflation, and whether they have or not, what are the hidden snags?

2.3 Remarks on primordial inflation

Well, of course previous authors^{34,35)} have discussed the possibility of inflation before the GUT era, especially at the Planck epoch. And it is probably the plethora of attendant technical complications which has deterred prospective adherents of primordial inflation.

Planck-era inflation has been proposed as a way to avoid any initial singularity in the Universe³⁴⁾, and it has been suggested as a natural consequence of quantum gravitational effects³⁵⁾. Herein lies the rub: once one gets close to the Planck mass or temperature it is no longer adequate to treat gravitational effects just as a classical background field. First order gravitational (FOG) quantum fluctuations can be expected to be $O(1)$ in importance. Also, one expects conventional renormalizable particle interactions to be supplemented by additional non-renormalizable interactions scaled by inverse powers of the Planck mass, which should also be $O(1)$ in importance at this scale. Furthermore, there could well be many as yet unseen new particle degrees of freedom at the Planck scale. For all these reasons it is not yet possible to make any quantitative statements

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*) We now see *a posteriori* the reason for choosing a gauge singlet field ϕ : the conventional GUT scenario with $m_X \ll m_P$ could not tolerate $\mu \gg m_X$.

about Planck-era inflation, but one can use model theories as qualitative guides to some of the possible new effects to be expected.

Since we work in a supersymmetric framework, the natural class of theories for us to study are those with local supersymmetries, i.e. supergravity theories. It is plausible that interactions at energy scales $\lesssim m_p$ can be described by an effective $N = 1$ supergravity theory. The general form of gauge and chiral supermultiplet couplings in $N = 1$ supergravity is known²⁹). As already remarked, when suitably Weyl-rescaled there is no bR curvature term in the scalar potential. One does indeed find extra non-renormalizable interactions proportional to

$$\left(\phi^2/m_p^2\right)^n \quad : \quad n \geq 1 \quad (2.67)$$

which means that our simple form (2.40), (2.59) of the tree-level potential is no longer a good approximation when $\mu = O(m_p)$. This means that the constraints of Fig. 3 on the effective potential couplings must in general be modified by FOG corrections $O(\mu^2/m_p^2)$ at the very least, and the lines in Fig. 3 should only be interpreted qualitatively for $\mu \gtrsim O(10^{18})$ GeV. However, we find it plausible that the main qualitative feature of Fig. 3, namely that inflation becomes less unnatural as μ approaches m_p , may be of more general validity despite the onset of FOG effects.

As a weak argument in favour of this point of view, we point out that it is in principle possible to obtain an effective potential in a supergravity theory which is arbitrarily close to the toy model (2.40) for a single real scalar field. We recall that the general form for the effective potential for a complex field Z in an $N = 1$ supergravity theory²⁹) is

$$V = \exp \left[-G(Z, Z^*) \right] \left[2 \left| \frac{\partial G}{\partial Z} \right|^2 - 3 \right] \quad (2.68)$$

where

$$G = +J - \ln \frac{|W|^2}{4} \quad (2.69)$$

with J a function of Z and Z^* which has a series expansion starting

$$J = -\frac{1}{2} Z Z^* + \dots \quad (2.70)$$

and $W(Z)$ is a conventional superpotential as in Eq. (2.51). However, W need no longer be truncated at the cubic order as was the expression (2.51). Restricting ourselves to the real part X of Z we get

$$V(X) = \exp[-G(X)] \left[2 \left(\frac{\partial G}{\partial X} \right)^2 - 3 \right] \quad (2.71)$$

Our remark is that it is possible to mock up any desired polynomial potential $P(X)$ by an appropriate choice of $G(X)$ obeying the equation:

$$\frac{\partial G}{\partial X} = \sqrt{\frac{3 + P(X) \exp G(X)}{2}} \quad (2.72)$$

This equation clearly has a well-defined series expansion solution and can clearly be extended to complex Z in the canonical form (2.69) in an infinity of different ways. We can only speculate that at least one of these extensions to complex Z may not have any complex minima lower than the real one $Z = X = \mu$ that we explored in subsection 2.2. Even if it does, the Universe may be stable^{32,36)} against tunnelling into it.

This observation is clearly not to be taken very seriously, but it does serve as an existence proof that there are supergravity models where inflation becomes more and more natural as the scale of the Higgs v.e.v. approaches the Planck mass. We therefore feel that the idea of primordial inflation deserves more serious consideration, though there are an enormous number of technical issues to be resolved before it can be judged acceptable. There is however one phenomenological issue that can be addressed immediately. It is the problem of mating primordial inflation with a cosmologically and phenomenologically acceptable GUT. In particular one must deal with the grand unified monopole problem, which was previously solved by the new GUT inflationary scenario, and now requires a different solution.

3. GUTs AFTER PRIMORDIAL INFLATION

We have argued in the previous sections that it is most natural to attribute the inflationary period to a phase transition that precedes the grand unified phase transition. Having divorced inflation from GUTs, we must now re-examine in

the context of supersymmetric GUTs the problems of the abundance of grand unified monopoles²⁾ and of cosmological baryosynthesis⁴⁾. In this section we propose two possible scenarios for incorporating supersymmetric GUTs into a model with primordial inflation.

3.1 Supercosmology revisited

Starting with the minimal supersymmetric SU(5), the most general superspace potential for the adjoint Higgs superfield is

$$W = \frac{m}{2} \text{Tr}(\Sigma^2) + \frac{\lambda}{3} \text{Tr}(\Sigma^3) \quad (3.1)$$

and its associated Higgs potential is

$$V = \text{Tr} \left| m \Sigma + \lambda \Sigma^2 - \frac{\lambda}{5} \text{Tr}(\Sigma^2) \mathbf{1} \right|^2 + \frac{g^2}{2} \text{Tr}([\Sigma, \Sigma^\dagger]^2) \quad (3.2)$$

Finite temperature corrections³³⁾ modify (3.2) by

$$\delta V(\Sigma; T) = c_1 \text{Tr}|\Sigma|^2 T^2 + c_2 T^4 \quad (3.3a)$$

where

$$c_1 \simeq \frac{75}{8} g^2 \quad \text{and} \quad c_2 \simeq -\pi^2 \left(\frac{93}{24} \right) \quad (3.3b)$$

The degenerate minima of (3.2) corresponding to zero vacuum energy are shifted by (3.3), and the SU(5) extremum is the lowest lying because c_2 is most negative for this case. Therefore an SU(5) symmetric Universe would remain symmetric down to low energies as long as (3.3) remains valid, as seen in Fig. 4a.

However, the gauge interactions of an unbroken SU(5) theory increase as we approach lower temperatures (energies) (as long as the theory is asymptotically free) and at $T \sim \Lambda_{\text{SU}(5)}$ they become strong invalidating the naive effective potential¹⁹⁾. The number of degrees of freedom of a strongly coupled SU(5) is drastically reduced and as a consequence the depth of the SU(5) minimum determined by c_2 (3.3b) is reduced as in Fig. 4b. Thus, at temperatures of $O(\Lambda_{\text{SU}(5)})$ the phase transition to a broken phase is in principle possible, and the $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$ minimum is energetically preferred if there are sufficiently many Higgs supermultiplets, as we will see below. Transition to a broken minimum is

however possible in practice only when the barrier separating the two phases is small^{32,19)}. For example, in the case where $m \sim O(10^{16})$ GeV $\Lambda_{SU(5)}$ comes out to be roughly 10^9 GeV. Then the height $O(m^4/\lambda^2)$ of the barrier is a huge $(10^{16} \text{ GeV})^4$ while the typical range of thermal fluctuations is only 10^{10} GeV. Such a transition would therefore require an unacceptably long time to be completed. It is however possible to construct models with a low barrier so that the transition can be completed rapidly. Consider the previous superpotential (3.1) but take $m = O(m_W)$ and $\lambda = O(m_W/m_X)$ -- we recall that adjustment of λ to an arbitrarily small value is technically natural in a supersymmetric theory¹⁵⁾. As long as we are in the symmetric phase the adjoint Higgs superfield remains essentially massless. Since we will be dealing with the problem of the baryon asymmetry⁴⁾ let us also assume that the model is augmented by four "light" pentaplets of mass $O(10^{10})$ GeV³⁷⁾, as dictated³⁸⁾ by the lower mass limit for colour-triplets from proton decay. Thus

$$\delta W = h \bar{H} \Sigma H + f \bar{H}' \Sigma H' + \mu (\bar{H} \cdot H + \bar{H}' \cdot H') \quad (3.4)$$

where $h, f = O(10^{-5})$ and $\mu = O(10^{10} \text{ GeV})$. Assuming that $m_X = 3.6 \times 10^{15} \text{ GeV}$ and $1/\alpha_{GUT} \approx 5.8$ (these numbers will be justified in a moment), we obtain $\Lambda_{SU(5)} \approx 10^9 \text{ GeV}$. Thus in such a model the GUT phase transition can begin at temperatures $O(10^9 \text{ to } 10^{10}) \text{ GeV}$. The expectation value of the Higgs field Σ in the $SU(3) \times SU(2) \times U(1)$ direction will be

$$\langle \Sigma \rangle = v \text{diag} (2, 2, 2, -3, -3) \quad (3.5)$$

with

$$v = O\left(\frac{m}{\lambda}\right) \sim m_W / (m_W/m_X) = O(m_X)$$

The adjoint superfield receiving a tiny mass $O(m_W)$ from W will receive a contribution from the D-term after the breaking. Nevertheless a colour octet and a weak isotriplet component of Σ will remain light [i.e. with masses $O(m_W)$]. Looking at this theory now on the scale of energy we can compute³⁹⁾ m_X , $\sin^2 \theta_W$, α_{GUT} and m_b/m_τ taking into account the contributions of four isodoublets, an isotriplet and a colour octet with masses $O(m_W)$, as well as the contributions of four colour

triplets with masses $O(10^{10})$ GeV. It is possible to get reasonable values of $\sin^2 \theta_W$ and m_X if one includes more Higgs superfields. For example, introducing three pairs of Higgses in the $\underline{10}$ and $\overline{10}$ representations in such a way that only their charged singlet components remain light $O(m_W)$, while the rest are super-heavy⁴⁰⁾, leads us to

$$\sin^2 \theta_W = 0.202, \quad m_X = 3.6 \times 10^{15} \text{ GeV}, \quad \alpha_{GUT}^{-1} = 5.8, \quad (3.6)$$

$$m_b/m_\tau = 0.9 (m_b/m_\tau)_{\text{minimal SU(5)}}.$$

in the one-loop approximation to the renormalization group equations. This can be achieved with the addition of

$$\delta W = h_{ij} \overline{10}^i \Sigma \underline{10}^j + v_{ij} \overline{10}^i \underline{10}^j \quad (3.7)$$

It is clear that such an enlargement does not spoil our finite temperature analysis since all the $\underline{10}$'s are superheavy ($v = O(m_X)$) as long as $\langle \Sigma \rangle = 0$.

The model described above will start exhibiting strong coupling phenomena in the supersymmetric phase at $T \sim 10^9$ GeV. At such a temperature the barrier is typically of a size

$$O(m_W^2 m_X^2) \sim 10^{35} \text{ GeV}^4 \quad (3.8)$$

and so will be negligible and easily traversed. It should also be stressed that the supergravity effects recently discussed by Weinberg³⁶⁾ will be negligible since the shift in the vacuum energies due to gravitational effects

$$\delta V \sim \frac{1}{m_p^2} (m_W m_X^2)^2 \sim m_W^2 m_X^2 \left(\frac{m_X}{m_p}\right)^2 \sim 10^{35} \text{ GeV}^4 \times O(10^{-7}) \quad (3.9)$$

is much smaller than the typical size (3.8) of the barrier.

Since the model we have just described is augmented by four "light" colour triplets which could in principle have different couplings to quarks, it is straightforward to obtain a non-vanishing baryon asymmetry from the decays of the "light" ($\sim 10^{10}$ GeV) Higgses³⁷⁾. Strong coupling phenomena might play an essential role here in ensuring sufficient deviation from equilibrium¹⁹⁾.

What about the monopole problem in this scenario? It is not difficult to see that since the temperature is delayed to low temperatures $O(10^9)$ GeV, the number of monopoles per photon could¹⁹⁾ be as low as is suggested by the size of causal domains at $T \sim 10^9$ GeV, namely

$$\frac{n_m}{n_\gamma} \sim 10^3 \left(\frac{T}{m_p} \right)^3 \sim 10^3 \left(\frac{10^9}{10^{19}} \right)^3 \sim 10^{-27} \quad (3.10)$$

which is below the present observational limit. Thus, monopoles are suppressed but the amount of their suppression allows for a cosmologically interesting low abundance, in contrast with their suppression in conventional inflationary scenarios^{1,6)} which would necessarily lead to an unobservable monopole flux far below this limit.

3.2 A second order phase transition

However attractive the above scenario might appear at first sight, it requires an extremely accurate adjustment of the self-coupling in the superspace potential (3.1) to $O(m_W/m_X)$, and this lacks an explanation for the time being. Also the model requires a proliferation of extra Higgs superfields. Other appealing alternatives are models in which the SU(5) minimum exists only at high temperatures and ceases to be a minimum at low temperatures.

A model of this type is readily constructed. Let us consider the superspace potential

$$W = \lambda Z (Tr \Sigma^2 - m^2) \quad (3.11)$$

where Z is a singlet superfield. The associated potential is

$$V = \lambda^2 |Tr \Sigma^2 - m^2|^2 + 4\lambda^2 |Z|^2 Tr |\Sigma|^2 + c_1 Tr |\Sigma|^2 T^2 + c_2 T^4 + c_3 |Z|^2 T^2 \quad (3.12)$$

Clearly finite temperature effects push $|Z|$ to 0, and we neglect Z henceforth.

At $T = 0$ only a broken phase corresponds to a supersymmetric minimum. For simplicity we restrict ourselves to the $SU(3) \times SU(2) \times U(1)$ direction

$$\Sigma = \frac{\sigma}{\sqrt{30}} \text{diag} (2, 2, 2, -3, -3) \quad (3.13)$$

and the potential (3.12) then becomes

$$V = \lambda^2 |\sigma^2 - m^2|^2 + c_1 |\sigma|^2 T^2 + c_2 T^4 \quad (3.14)$$

For temperatures $T^2 > 2\lambda^2 m^2 / c_1$, the origin is a minimum, while for lower temperatures it turns into a local maximum with the simultaneous appearance of a minimum at

$$\sigma^2 = m^2 - c_1 T^2 / 2\lambda^2 \quad (3.15)$$

as depicted in Fig. 5. If $T_c \equiv \lambda\sqrt{2}m/\sqrt{c_1}$ is of order m_X , then since the transition is (3.15) of second order, monopoles can be suppressed by the thermal fluctuation arguments of Bais and Rudaz²⁰⁾. It should be pointed out however that if we are willing to fine-tune λ to $O(10^{-6}$ to $10^{-7})$ so that $T_c = O(10^9)$ GeV, then the same suppression mechanism described in the previous subsection also applies since $(T_c/m_p)^3 = O(10^{-30})$. Once again, we see that this second order phase transition scenario also allows for an observable flux of grand unified monopoles.

The generation of the baryon asymmetry⁴⁾ poses no great problem. It can easily be achieved as in standard non-supersymmetric GUTs by means of colour triplet Higgses of mass 10^{10} to 10^{16} GeV, depending on the critical temperature T_c which is chosen.

4. CONCLUSIONS AND PROSPECTS

We have argued previously¹²⁾ that achieving cosmological inflation is likely to be easier in a supersymmetric theory, and this paper has expanded and extended our previous analysis. In addition, we have found that various particle physics and cosmological fine-tuning arguments tend to favour a scenario with primordial inflation before the GUT phase transition. We have also shown that it is in principle possible to solve the grand unified monopole problem²⁾ and achieve sufficient Big Bang baryosynthesis⁴⁾ in supersymmetric GUTs with primordial inflation.

However, there are still a number of open questions about our scenario of primordial supersymmetric inflation. When the vacuum energy driving inflation is $O(m_p^4)$ there are many new technical complications which come into play because

higher order gravitational (HOG) effects [quantum corrections, new non-renormalizable interactions, new particles with masses $O(m_p)$] are no longer negligible. For this reason, our analysis may appear rather negative, as it suggests that cosmological inflation is shrouded in fog. However, it may be that some reliable approximation scheme for FOG and HOG effects can be set up which enables one to analyse primordial inflation quantitatively. This is certainly a vital area for future study.

A related question is the fate of the gravitino problem. Weinberg³⁾ has emphasized that certain gravitino mass ranges raise severe problems in the context of conventional cosmology. These difficulties can be avoided⁵⁾ if there has been some inflation subsequent to the Planck epoch, but it is not clear whether sufficient suppression of gravitinos can be achieved if inflation takes place at the Planck epoch itself. However, we are cautiously optimistic since solving the gravitino problem does not require a very large inflationary suppression of their number density⁵⁾, and the gravitino problem does not exist at all for sufficiently light gravitinos³⁾. Nevertheless, this is another important area for future study.

A problem with conventional inflationary scenarios is the spectrum of perturbations leading to large-scale inhomogeneities. It seems difficult to keep $\delta\rho/\rho$ below acceptable levels in non-supersymmetric GUTs¹³⁾. As shown in the last paper of Ref. 13, we show that because supersymmetric theories offer much more freedom to adjust parameters, it is possible to solve this fluctuation problem in a realistic supersymmetric inflationary model.

Finally, although we believe that either of our proposed scenarios for solving the grand unified monopole problem subsequent to primordial inflation may work satisfactorily, neither of them is as clear-cut as the conventional inflationary solution¹⁾, and both scenarios merit further clarification. In the revisited supercosmology scenario¹⁹⁾ one may wonder whether the number density of monopoles is really as low as $O(1)$ per causal domain of the Universe at a temperature $O(10^9)$ GeV, or whether there are more monopoles because of a shorter Higgs correlation length. The number of grand unified monopoles after a second order

phase transition is also rather uncertain²⁰⁾. We believe that both of these scenarios should be investigated more thoroughly as possible alternatives to conventional inflationary suppression, particularly in view of the technical problems accompanying inflation during the GUT phase transition.

Cosmological inflation¹⁾ is still a seductive idea, despite all its frustrations. Therefore we still hope that a complete inflationary cosmology can eventually be constructed.

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REFERENCES

- 1) A.H. Guth, Phys. Rev. D 23, 347 (1981).
- 2) J.P. Preskill, Phys. Rev. Lett. 43, 1365 (1979).
Ya.B. Zel'dovich and M.Yu. Khlopov, Phys. Lett. 79B, 239 (1979).
- 3) S. Weinberg, Phys. Rev. Lett. 48, 1303 (1982).
- 4) A.D. Sakharov, Zh. Eksp. Teor. Fiz. Pis'ma Red. 5, 32 (1967).
- 5) J. Ellis, A.D. Linde and D.V. Nanopoulos, CERN preprint TH.3356 (1982).
- 6) A.D. Linde, Phys. Lett. 108B, 389 (1982).
- 7) L.F. Abbott, Nucl. Phys. B185, 233 (1981).
M. Sher, Phys. Rev. D 24, 1699 (1981).
P. Hut and F. Klinkhamer, Phys. Lett. 104B, 439 (1981).
A. Billoire and K. Tamvakis, Nucl. Phys. B200, [FS4] 329 (1982).
K. Tamvakis and C.E. Vayonakis, Phys. Lett. 109B, 283 (1982).
G.P. Cook and K.T. Mahanthappa, Phys. Rev. D 23, 1321 (1981) and D 25, 1154 (1982).
- 8) A. Albrecht and P.J. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982).
- 9) S.W. Hawking and I.G. Moss, Phys. Lett. 110B, 35 (1982).
- 10) D. Kazanas, Ap. J. 241, L59 (1980).
- 11) A.D. Linde, Lebedev Physical Institute Preprint "Scalar Field Fluctuations in Expanding Universe and the New Inflationary Universe Scenario" (1982).
- 12) J. Ellis, D.V. Nanopoulos, K.A. Olive and K. Tamvakis, CERN preprint TH.3364 (1982).
- 13) S.W. Hawking, Phys. Lett. 115B, 295 (1982).
A.H. Guth and S.-Y. Pi, MIT preprint CTP-1014 (1982).
J. Bardeen, P. Steinhardt, M.S. Turner and A. Starobinskii, private communications (1982).

For a discussion of how to resolve this difficulty using a supersymmetric model, see J. Ellis, D.V. Nanopoulos, K.A. Olive and K. Tamvakis, CERN preprint TH.3437 (1982).

- 14) S. Dimopoulos and H. Georgi, Nucl. Phys. B193, 150 (1981).
N. Sakai, Zeit. für Phys. C11, 153 (1982).
- 15) J. Wess and B. Zumino, Phys. Lett. 49B, 52 (1974).
J. Iliopoulos and B. Zumino, Nucl. Phys. B76, 310 (1974).
S. Ferrara, J. Iliopoulos and B. Zumino, Nucl. Phys. B77, 413 (1974).
- 16) J. Ellis and D.V. Nanopoulos, Phys. Lett. 116B, 133 (1982).
- 17) G. Steigman, "Unification of the Fundamental Particle Interactions II",
ed. J. Ellis and S. Ferrara (Plenum Publ. Corp., N.Y., 1982), p. 263.
- 18) J. Ellis and K.A. Olive, CERN preprint TH.3399 (1982).
- 19) D.V. Nanopoulos and K. Tamvakis, Phys. Lett. 110B, 449 (1982).
M. Srednicki, Nucl. Phys. B202, 327 (1982).
D.V. Nanopoulos, K.A. Olive and K. Tamvakis, Phys. Lett. 115B, 15 (1982).
M. Srednicki, Princeton University preprint (1982).
- 20) F.A. Bais and S. Rudaz, Nucl. Phys. B170 [FS1], 507 (1980).
- 21) J. Ellis, M.K. Gaillard, D.V. Nanopoulos and S. Rudaz, Nature 293, 41 (1981).
- 22) G.F.R. Ellis, Cargèse Lectures in Physics, Vol. 6, ed. E. Schatzman
(Gordon and Breach, New York, 1973), p. 1.
- 23) A.H. Guth and E. Weinberg, Phys. Rev. D 23, 826 (1981).
- 24) S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).
- 25) H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32, 438 (1974).
A.J. Buras, J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Nucl. Phys. B135,
66 (1978).
- 26) G.M. Shore, Ann. Phys. 128, 376 (1980).
A. Vilenkin, Tufts University preprint TUTP-82-5 (1982).
A. Vilenkin and L.H. Ford, Tufts University preprint TUTP-82-7 (1982).
A.A. Starobinskii, private communication (1982).
- 27) P.J. Steinhardt, Univ. of Pennsylvania preprint, "A New Cosmology" (1982).
- 28) R. Barbieri, S. Ferrara and D.V. Nanopoulos, Zeit. für Physik C13, 276 (1982)
and CERN preprint TH.3305 (1982).
J. Ellis, L.E. Ibáñez and G.G. Ross, Phys. Lett. 113B, 283 (1982) and CERN
preprint TH.3382 (1982).

- 29) E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello and
P. van Nieuwenhuizen, Phys. Lett. 79B, 231 (1978) and Nucl. Phys. B147,
105 (1979).
E. Cremmer, S. Ferrara, L. Girardello and A. van Proeyen, CERN preprints
TH.3312 and TH.3348 (1982).
- 30) G. Gibbons and S.W. Hawking, Phys. Rev. D 15, 2738 (1977).
- 31) R. Barbieri, S. Ferrara, D.V. Nanopoulos and K.S. Stelle, Phys. Lett. 113B,
219 (1982).
M.K. Gaillard, Lawrence Berkeley Laboratory preprint LBL-14647 (1982).
- 32) J. Ellis, C.H. Llewellyn Smith and G.G. Ross, Phys. Lett. 114B, 227 (1982).
- 33) L. Girardello, M.T. Grisaru and P. Salomonson, Nucl. Phys. B178, 331 (1981).
- 34) R. Brout, F. Englert and E. Gunzig, Ann. Phys. (N.Y.) 115, 78 (1978).
A.A. Starobinskii, Phys. Lett. 91B, 99 (1980).
- 35) M.V. Fischetti, J.B. Hartle and B.-L. Hu, Phys. Rev. D 20, 1757 (1979).
J.B. Hartle and B.-L. Hu, Phys. Rev. D 20, 1772 (1979) and D 21, 2756 (1980).
J.B. Hartle, Phys. Rev. D 22, 2091 (1980) and D 23, 2121 (1981).
A. Vilenkin, private communication (1982).
- 36) S. Weinberg, Phys. Rev. Lett. 48, 1776 (1982).
- 37) D.V. Nanopoulos and K. Tamvakis, Phys. Lett. 113B, 151 (1982) and 114B,
235 (1982).
- 38) J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Phys. Lett. 80B, 360 (1979).
- 39) C. Kounnas, A.B. Lahanas, D.V. Nanopoulos and K. Tamvakis, CERN preprint
TH.3359 (1982).
- 40) A. Masiero, D.V. Nanopoulos, K. Tamvakis and T. Yanagida, Phys. Lett. 115B,
298 (1982) and CERN preprint TH.3324 (1982).

Figure captions

- Fig. 1 : Sketch of the effective potential for a generic field ϕ , including the local minimum of height $\delta = V_0$ at the origin, the barrier provided by the local maximum V_1 at ϕ_1 , and the global minimum at $\phi = \mu$ which has zero vacuum energy.
- Fig. 2 : Examples of one-loop contributions to the effective potential (2.15) which cancel in an exactly supersymmetric theory.
- Fig. 3 : Illustration of the fine-tuning of couplings necessary for supersymmetric inflation at an energy scale $\mu \lesssim O(m_P)$. The lines shown are upper bounds on the quantities indicated; i.e. $\hat{y} = 1$.
- Fig. 4 : Sketches of the effective potential in the SU(5) theory (3.1), with the SU(5), SU(4) $\times$ U(1) and SU(3) $\times$ SU(2) $\times$ U(1) minima exhibited.
a) In the absence of strong coupling effects the Universe falls naturally into the SU(5) invariant phase as indicated. b) The SU(5) minimum may be disfavoured by strong coupling corrections.
- Fig. 5 : Sketch of the effective potential in the SU(5) theory (3.11), showing how the SU(5) minimum becomes a local maximum at low temperatures.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5