B

08 NOV. 1982

GSI

GSI 82-33
7

GSI - 82 - 33
Preprint

CERN LIBRARIES, GENEVA

CM-P00069049

On the Role of Memory Effects for Dissipation
and Diffusion in Slow Collective Nuclear Motion

W. Cassing, W. Nörenberg

October 1982

Gesellschaft für Schwerionenforschung mbH
Planckstr. 1 · Postfach 110541 · D-6100 Darmstadt 11 · Germany

On the Role of Memory Effects for Dissipation and Diffusion in
Slow Collective Nuclear Motion

W. Cassing, W. Nörenberg

Gesellschaft für Schwerionenforschung (GSI)

D-6100 Darmstadt, F.R. Germany

Abstract:

The energy dissipation in slow collective nuclear motion is viewed as a combined effect of a diabatic production of particle-hole excitations, leading to a conservative storage of collective energy, and a subsequent equilibration due to residual two-body collisions. The effective equation of motion for the collective degree of freedom turns out to be nonlocal in time due to the large mean free path of the nucleons and allows for a simultaneous description of two different attitudes of nuclear matter. The elastic response of heavy nuclei for 'fast' collective motion switches over to pure friction for very slow collective motion. The time development of the fluctuations in the velocities may show oscillations for times comparable to the local equilibration time and hence, is qualitatively different from the classical limit. A first application of the diabatic dynamical approach is made for the quadrupole motion within a diabatic deformed harmonic oscillator basis.

- 2 -

1. Introduction

The observation of dissipative effects in slow collective nuclear motion of large amplitude, in particular in heavy-ion collisions somewhat above the Coulomb barrier, has initiated intensive theoretical studies on nuclear dissipation mechanisms^{1,2)}. Two main streams in the development of theoretical concepts can be distinguished which start from contradictory assumptions concerning the role of two-body collisions between nucleons.

The statistical transport theories start with the assumed separation of slow collective variables from the fast equilibrating intrinsic degrees of freedom. The fast equilibration within the intrinsic degrees of freedom for fixed values of the collective variables, so-called 'local' equilibration³⁾ is assumed to be a consequence of two-body collisions. In order to satisfy this assumption of local equilibrium throughout the process, the mean free path λ between subsequent two-body collisions has to be small enough, more precisely $\lambda \ll v_F \tau_{coll}$, where v_F and τ_{coll} denote the Fermi velocity and the characteristic time of the collective motion respectively.

In contrast to the statistical theories the time-dependent Hartree-Fock theory (TDHF)⁴⁾ requires a large mean free path, i.e. $\lambda \gg v_F \tau_{coll}$. Estimates³⁻⁷⁾ for the local equilibration time τ_{loc} suggest that TDHF is indeed a good approximation in the initial (approach) phase of a heavy-ion collision. However, at later stages the mean free path is strongly reduced due to the large excitation energies involved in the process. Thus the inclusion of residual two-body collisions in a self-consistent mean field theory (generalized or extended TDHF) is a natural step for a more realistic description of heavy-ion collisions⁸⁾. However, because of numerical difficulties, the applicability of such approaches to realistic collisions is doubtful.

The concept of dissipative diabatic dynamics (DDD) has been introduced in ref.^{3,9,10} in order to bridge the gap between current transport theories and TDHF in a simple and transparent way. The following study is devoted to a more general formulation of DDD including in particular the treatment of fluctuations and to an application to the quadrupole motion within an oscillator model.

In sect. 2 we introduce the concept of dissipation based on diabatic motion of nucleons and subsequent equilibration by two-body collisions and derive within a travelling single-particle basis a classical equation of motion for the collective degree of freedom. We further derive an equation for the time evolution of the fluctuations in the collective velocity which turn out to be qualitatively different from the usual diffusion behaviour. The theory is applied in section 3 to the quadrupole motion of a nucleus within a harmonic oscillator model. This model is highly transparent because almost all equations can be solved analytically and the model still contains the essential ingredients of the theory. A discussion of the physical phenomena related to the finite memory of the resulting friction kernel closes the paper.

2. Concept of dissipative diabatic dynamics

We want to consider large amplitude collective nuclear motion with collective velocities sufficiently small compared to the Fermi velocity ($v_{\text{coll}} \ll v_F$). In the adiabatic limit ($v_{\text{coll}} \rightarrow 0$) the individual nucleons can adjust completely to the collective motion in a time-reversible way while the residual interactions ensure that the collective motion proceeds through constrained correlated ground states of the system¹¹). For realistic collective velocities in fission and heavy-ion collisions, however, adiabaticity is not satisfied^{11,12}. Still

there exists an interesting range of collective kinetic energies per nucleon³⁾

$$\frac{1}{40} \text{ MeV} < E_{\text{coll}}/A \ll 40 \text{ MeV} \quad (2.1)$$

where the motion of the individual nucleons is almost adiabatic. Only near quasicrossings (Landau-Zener-crossing^{11,12}) of the adiabatic levels the single-particle motion becomes completely different from the adiabatic behaviour. Instead of staying in a given adiabatic state the nucleon jumps to the other adiabatic state when moving along the quasicrossing. The reason for this jump is that the nucleon tries to keep the nodal structure of its wave function and hence, stays on the diabatic (i.e. crossing) level¹¹.

The diabatic behaviour of the single-particle motion offers a simple picture for the dissipation of collective kinetic energy in fission and heavy-ion collisions: Via the diabatic shift of single-particle levels the collective kinetic energy is stored in a time-reversible way in the single-particle motion giving rise to a conservative diabatic potential⁹). This diabatic potential is destroyed by two-body collisions which randomly populate diabatic levels with slopes different from the initial ones. It is the latter process which leads to the dissipation of the collective kinetic energy in the usual sense of statistical mechanics.

In order to formulate these ideas quantitatively we first introduce in § 2.1 a suitable diabatic single-particle basis and then derive in §§ 2.2 and 2.3 equations for the mean values and the fluctuations of the collective variable.

2.1 Diabatic single-particle basis

For simplicity we restrict our considerations to a single collective parameter κ and define a stationary diabatic single-particle basis by

$$H_0(\kappa) \phi_\alpha(\kappa) = \epsilon_\alpha(\kappa) \phi_\alpha(\kappa) \quad (2.2)$$

where the diabatic Hamiltonian H_0 differs from the total single-particle Hamiltonian H by a non-diagonal part

$$H'(\kappa) = H(\kappa) - H_0(\kappa) \quad (2.3)$$

A diabatic basis (2.2) can be constructed from the adiabatic basis

$$H(\kappa)\psi_\mu(\kappa) = \bar{\epsilon}_\mu(\kappa) \psi_\mu(\kappa) \quad (2.4)$$

by employing the condition of maximum overlap. Another possibility is to define a diabatic Hamiltonian H_0 by retaining only terms with the symmetries of the collective motion. Both methods are studied in Ref. ¹³⁾. Thus we assume for our following considerations that a suitable single-particle basis (2.2) is known.

We now proceed in defining a travelling diabatic single-particle basis by

$$\{H_0(\kappa) - \lambda \dot{Q}\} \chi_\alpha(\kappa, \lambda) = \omega_\alpha(\kappa, \lambda) \chi_\alpha(\kappa, \lambda) \quad (2.5)$$

with

$$\dot{Q} \equiv i[H_0, Q] \quad (2.6)$$

The operator Q of the collective coordinate which is related to κ , depends on the particle coordinates only. In first-order perturbation theory with respect to $\lambda \dot{Q}$ the cranked wave functions are obtained by multiplying the stationary diabatic states (2.2) by a simple phase factor¹⁴⁾

$$\chi_\alpha(\kappa, \lambda) = \exp \{i\lambda Q\} \phi_\alpha(\kappa) \quad (2.7)$$

The parameters κ and λ are related to the expectation values of Q and $\dot{Q}$ by

$$q = \sum_\alpha \langle \phi_\alpha(\kappa) | Q | \phi_\alpha(\kappa) \rangle n_\alpha = q(\kappa) \quad (2.8)$$

$$\dot{q} = \lambda / B_q \quad (2.9)$$

with the mass parameter B_q defined by

$$B_q^{-1} \equiv i \sum_\alpha \langle \phi_\alpha(\kappa) | [\dot{Q}, Q] | \phi_\alpha(\kappa) \rangle n_\alpha \quad (2.10)$$

These expressions have been derived in Ref. ¹⁴⁾ for a single Slater determinant of the states χ_α .

Corrections to the relations (2.7 - 2.10) according to non-perturbative solutions of (2.5) have been investigated in Ref. ¹⁵⁾ for quadrupole motion.

The modifications of $\chi(\kappa, \lambda)$ and B_q turn out to be not significant for collective velocities v_{coll} smaller than half the Fermi velocity v_F ,

$$v_{coll} / v_F < 0.5 \quad (2.11)$$

In this respect we believe that (2.7) to (2.10) also hold for more general diabatic motions [cf. Ref. 11,16,17] if the condition (2.11) is satisfied.

2.2 Equation of motion for the collective variable

We consider a statistical mixture of Slater determinants $\phi_i(\kappa_i, t)$. Any mean value A of a quantity A is then defined by the sum

$$A = \sum_i |c_i|^2 A_i \quad (2.12)$$

where $|c_i|^2$ and A_i denote the weights of ϕ_i and the corresponding expectation values of A . For the fluctuations σ_A^2 of A we have

$$\sigma_A^2 = \sum_i |c_i|^2 (A_i - A)^2 \quad (2.13)$$

In the following we want to consider first the equation of motion for the mean value κ of the collective variable and then in the next subsection study the fluctuations in κ .

Since we are treating a closed system the total energy E is conserved.

Thus we can use

$$dE/dt = 0 \quad (2.14)$$

for determining the equation of motion for the collective variable. For the mean value κ this yields

$$B(\kappa)\ddot{\kappa} = -\frac{1}{2} \frac{\partial B}{\partial \kappa} \dot{\kappa}^2 + F(\kappa, t) \quad (2.15)$$

where the mass

$$B(\kappa) = B_q(q(\kappa)) \left(\frac{\partial q}{\partial \kappa}\right)^2 \quad (2.16)$$

is determined by (2.10). In writing down (2.15) and (2.16) we have neglected all fluctuations. Similar to the shell-correction method of Strutinski [cf. Ref. 18)] for the total energy we can renormalize the force

$$F = F_c(\kappa) + F'(\kappa, t) \quad (2.17)$$

to a smooth background $F_c(\kappa)$ which can be evaluated for example from the liquid-drop model¹⁹⁾ eventually at some temperature T_c . The shell-correction part

$$F'(\kappa, t) = - \sum_{\alpha} \frac{\partial \epsilon_{\alpha}}{\partial \kappa} [n_{\alpha}(t) - \bar{n}_{\alpha}(\kappa, \mu_c, T_c)] \quad (2.18)$$

depends explicitly on time via the occupation numbers $n_{\alpha}(t)$. The occupation probabilities $\bar{n}_{\alpha}(\kappa, \mu_c, T_c)$ account for the smooth background and can be taken as a Fermi distribution with chemical potential μ_c for a sufficiently large temperature T_c where shell effects on the force have vanished. Note that we have assumed energy conservation in the collision term²⁰⁾ such that

$\sum_{\alpha} n_{\alpha} \frac{dn_{\alpha}}{dt} = 0$. In order to simplify the notation in the following part we incorporate the smooth background terms by renormalizing the derivatives of the single-particle energies according to

$$\frac{\partial n_{\alpha}}{\partial \kappa} \equiv \frac{\partial \epsilon}{\partial \kappa} - \frac{1}{A} [F_c(\kappa) + \sum_{\beta} \frac{\partial \epsilon_{\beta}}{\partial \kappa} \bar{n}_{\beta}(\kappa, \mu_c, T_c)] \quad (2.19)$$

Thus the force F (cf. 2.17 and 2.18) is given by

$$F(\kappa, t) = - \sum_{\alpha} \frac{\partial n_{\alpha}}{\partial \kappa} n_{\alpha}(t) \quad (2.20)$$

where we have used $\sum_{\alpha} n_{\alpha}(t) = A$.

If the occupation probabilities do not depend on time, equation (2.15) is time-reversible and hence describes an elastic motion. However, due to μ' and particularly due to the residual two-body interactions the occupation probabilities tend to approach local equilibrium, i.e. equilibrium for a fixed value of the collective variable. For simplicity we approximate the collision term by a relaxation ansatz

$$\frac{d}{dt} n_{\alpha}(t) = -\tau_{loc}^{-1} [n_{\alpha}(t) - \bar{n}_{\alpha}(\kappa, \mu, T)] \quad (2.21)$$

with the local equilibration time $\tau_{loc}(t)$. The occupation probabilities at local equilibrium are denoted by $\bar{n}_{\alpha}$ and depend in addition to κ on the chemical potential μ and the temperature T which are determined by conservation of total mass and energy. Due to the time dependence of the single-particle levels $n_{\alpha}(t)$ the chemical potential and the temperature depend on time.

The local equilibration time has been estimated from the mean free path of nucleons to be of the order 10^{-21} s for heavy-ion collisions^{3,5)}. Calculations for the local equilibration time have been performed by Bertsch⁷⁾ with respect to the relaxation of a deformed Fermi sphere which is intimately related to diabatic single-particle motion (cf. for example Ref.³⁾). The result is

$$\begin{aligned} \tau_{loc} &= \left\{ \sigma v_F n \frac{3.2 \epsilon_T + 3.3 \epsilon_D}{\epsilon_F} \right\}^{-1} \quad (2.22) \\ &\approx \frac{2 \cdot 10^{-22} \text{ s} \cdot \text{MeV}}{\epsilon_T + \epsilon_D} \end{aligned}$$

where σ , n , ϵ_F and v_F denote the nucleon-nucleon cross-section, nuclear density, Fermi energy and velocity respectively. According to (2.22) the time scale of the relaxation process is determined by the sum of the deformation energy per nucleon ϵ_D of the deformed Fermi sphere and the thermal energy per nucleon ϵ_T . This energy is given by subtracting the adiabatic potential and the collective kinetic energy from the total energy. For peripheral collisions the relation (2.22) yields local equilibration times larger than 10^{-21} s which show that the system does not reach local equilibrium for this type of reactions.

We return to the derivation of an equation of motion for the collective variable by considering the time derivative of the force $F(\kappa, t)$ minus its equilibrium value $\bar{F}(\kappa)$ which is defined from (2.20) by replacing $n_\alpha(t)$ by $\bar{n}_\alpha(\kappa, \mu, T)$. From (2.20) and (2.21) we obtain

$$\frac{d}{dt} [F(\kappa, t) - \bar{F}(\kappa)] = -W(\kappa, t) \dot{\kappa} - \tau_{loc}^{-1} [F(\kappa, t) - \bar{F}(\kappa)] \quad (2.23)$$

where $W(\kappa, t)$ is the sum of

$$W_0(\kappa) = - \sum_{\alpha} \frac{\partial^2 n_{\alpha}}{\partial \kappa^2} \frac{\partial \bar{n}_{\alpha}}{\partial \kappa} \quad (2.24)$$

and

$$W_1(\kappa, t) = \sum_{\alpha} \frac{\partial^2 n_{\alpha}}{\partial \kappa^2} [n_{\alpha}(t) - \bar{n}_{\alpha}(\kappa, \mu, T)]. \quad (2.25)$$

The first term W_0 accounts for the diabatic level crossings and is responsible for retardation effects as is shown below. Inserting the Fermi function for $\bar{n}_{\alpha}$ we obtain from (2.24)

$$W_0(\kappa) = T^{-1} \sum_{\alpha} \left[\frac{\partial^2 n_{\alpha}}{\partial \kappa^2} - \frac{\partial \mu}{\partial \kappa} \right] \frac{n_{\alpha}^{-1}}{T} \exp \left\{ \frac{n_{\alpha}^{-1}}{T} \right\} \quad (2.26)$$

showing that W_0 is positive (more precisely not negative). In deriving (2.26) we have used the conservation of the total occupation probability,

$\partial \left(\sum_{\alpha} \bar{n}_{\alpha} \right) / \partial \kappa = 0$. Furthermore, we have neglected the implicit time-dependence of $\bar{n}_{\alpha}$ resulting from the temperature $T(t)$. Because of random signs in the sum over α this contribution to W_0 is expected to be small. The quantity W_1 accounts for the difference in the mean second derivatives of the diabatic levels with respect to the actual occupation numbers $n_{\alpha}(t)$ as compared to their equilibrium values $\bar{n}_{\alpha}$. Using (2.21) for large times $t \gg \tau_{loc}$ (initial time $t_0 = 0$) and $\dot{\kappa} \partial \bar{n}_{\alpha} / \partial \kappa$ essentially constant over time intervals of order τ_{loc} the expression (2.25) for W_1 reads

$$W_1(\kappa, t) = - \sum_{\alpha} \frac{\partial^2 n_{\alpha}}{\partial \kappa^2} \dot{\kappa} \tau_{loc} \frac{\partial \bar{n}_{\alpha}}{\partial \kappa} \quad (2.27)$$

Initially W_1 vanishes because $n_{\alpha}(0) = \bar{n}_{\alpha}$. For large temperatures the terms in the sum over α have random signs such that W_1 will be negligible for large times and slow collective motion, too.

The equations (2.15), (2.22) and (2.23) determine the mean value κ of the collective variable. The quantities $B(\kappa, t)$, $W(\kappa, t)$ and $\bar{F}(\kappa)$ can be calculated microscopically from the diabatic levels and the relaxation ansatz (2.21) for $n_{\alpha}(t)$. Since the mass $B(\kappa, t)$ is given essentially by bulk properties we approximate $n_{\alpha}(t)$ by $n_{\alpha}(0)$ in (2.10) and thus neglect the explicit time dependence. The simple differential equations (2.15), (2.21) and (2.23) provide a unique description of the time evolution of the mean collective coordinate based on diabatic single-particle levels $\epsilon_{\alpha}(\kappa)$ and the effective cross-sections for residual two-body collisions which enter the determination of τ_{loc} and drive the system towards local equilibrium.

For simplicity we assume τ_{loc} to be constant ($\approx 10^{-21}$ s according to Ref. 3,9) in the following discussion. Integrating (2.23) and inserting the result into (2.17, 2.15) we find a closed form for the equation of motion,

$$B(\kappa)\ddot{\kappa} = -\frac{1}{2} \frac{\partial B(\kappa)}{\partial \kappa} \dot{\kappa}^2 + \overline{F}(\kappa) + [F(\kappa, t=0) - \overline{F}(\kappa)] \exp[-t/\tau_{10c}] - \int_0^t dt' W(\kappa, (t')) \dot{\kappa}(t') \exp[-(t-t')/\tau_{10c}]. \quad (2.28)$$

The last term represents a retarded force which depends on the history ($t' < t$) of the system. Specifically, $W_0(\kappa(t'))\dot{\kappa}(t')$ gives rise to a retarded friction term. This is a main source for memory effects in the time evolution of the collective variable resulting from the large, but finite mean free path of nucleons close to the Fermi energy. The formfactor W_0 in the friction kernel is positive semidefinite according to the previous discussion, cf. (2.26), and thus describes retarded dissipation of collective kinetic energy. The second term with $W_1(\kappa(t'), t')$ depends on κ in a non-trivial manner describing an additional retarded force which in many cases will be negligible according to the discussion below.

It is interesting to consider the short-time and long-time behaviour of equation (2.28). For $t \ll \tau_{10c}$ the equation of motion reduces to

$$B(\kappa)\ddot{\kappa} = -\frac{1}{2} \frac{\partial B(\kappa)}{\partial \kappa} \dot{\kappa}^2 + F_d(\kappa) \quad (2.29)$$

with the diabatic force

$$F_d(\kappa) = F(\kappa, t=0) - W(\kappa, t=0) \quad (\kappa = \kappa(t=0)) \quad (2.30)$$

simply describing conservative collective motion. For $t \gg \tau_{10c}$ and $\dot{\kappa}$ slowly varying we obtain

$$B(\kappa)\ddot{\kappa} = -\frac{1}{2} \frac{\partial B(\kappa)}{\partial \kappa} \dot{\kappa}^2 - \gamma(\kappa)\dot{\kappa} + \overline{F}(\kappa) + F_{con}(\kappa, \dot{\kappa}) \quad (2.31)$$

with the conservative force, cf. (2.27)

$$F_{con}(\kappa, \dot{\kappa}) = \tau_{10c} \sum_{\alpha} \frac{\partial^2 \eta_{\alpha}}{\partial \kappa^2} \dot{\kappa}^2 \quad (2.32)$$

and the friction coefficient, cf. (2.24),

$$\gamma(\kappa) = \tau_{10c} \sum_{\alpha} \left\{ -\frac{\partial \eta_{\alpha}}{\partial \kappa} \frac{\partial \eta_{\alpha}}{\partial \kappa} \right\} > 0 \quad (2.33)$$

which is proportional to τ_{10c} and according to (2.22) proportional to $\epsilon_T^{-1}(\epsilon_D \ll \epsilon_T$ for $t \gg \tau_{10c}$ and small $\dot{\kappa}$) or τ^{-2} if

$$\epsilon_T = \frac{1}{g} \tau^2. \quad (2.34)$$

is used. Thus for short times ($t \ll \tau_{10c}$) the nuclear system behaves like an elastic solid while it shows the properties of a viscous fluid for long times ($t \gg \tau_{10c}$) where the collective energy is almost dissipated such that the collective velocity has become very small.

2.3 Time evolution of fluctuations

In the preceding § 2.2 we have considered only the mean values of the occupation probabilities and the collective variable. We assume that the stochastic properties of the process is determined by residual two-body collisions which induce stochastic transitions between Slater determinants.

Thus the system is described at any time by a statistical mixture (ensemble) of Slater determinants $\phi_i(t)$ which stochastically change the single-particle states $\phi_{\alpha}^{(i)}$. In other words, the single-particle occupation numbers $n_{\alpha}^{(i)}$ of an ensemble member i change stochastically between 0 and 1. We can

describe this behaviour by generalizing the relaxation ansatz (2.21) for the mean values $n_{\alpha}(t) = \langle n_{\alpha}^{(i)}(t) \rangle$ over the ensemble by a Langevin-type equation

$$\frac{dn_{\alpha}^{(i)}(t)}{dt} = -\frac{1}{\tau_{10c}} [n_{\alpha}^{(i)}(t) - \overline{n}_{\alpha}(\kappa, \mu, T)] + g_{\alpha}^{(i)}(t; \kappa_i, \mu, T). \quad (2.35)$$

The quantity $g_{\alpha}^{(i)}(t)$ is a sequence of δ -functions in the time and thus is responsible for the fluctuations in $n_{\alpha}^{(i)}(t)$. This term is analogous to the fluctuating force in the Langevin equation of Brownian motion²¹⁾.

According to the stochastic changes of the occupation probabilities (2.35) the fluctuations in the collective variable originate from the corresponding

stochastic variations of the force (cf. 2.20)

$$F_i(k_i, t) = - \sum_{\alpha} \frac{\partial n_{\alpha}^{(i)}(t)}{\partial k_i} \quad (2.36)$$

where $k_i(t)$ denotes the value of the collective variable for the ensemble member i . This is substantially different from Brownian motion where the fluctuations originate from stochastic variations in the velocities. In the following we consider the fluctuations in the velocities $\dot{k}$ which result from the fluctuations of the force (2.36) and discuss the difference to normal Brownian motion.

For simplicity we neglect the k -dependence and the fluctuations of the mass parameter B such that

$$B \ddot{k}_i(t) = F_i(k_i, t) \quad (2.37)$$

is the relation between the fluctuating force F_i and the corresponding collective acceleration $\ddot{k}_i$. In order to evaluate the time evolution of the fluctuations in $\dot{k}$ we consider

$$\frac{d^2}{dt^2} \langle \dot{k}_i^2 \rangle = 2 \langle \ddot{k}_i^2 \rangle + 2 \langle \ddot{k}_i \dot{k}_i \rangle. \quad (2.38)$$

Using (2.35) to (2.38) we find

$$\begin{aligned} \frac{d^2}{dt^2} \langle \dot{k}_i^2 \rangle &= \frac{2}{B^2} F_i^2(k, t) - \frac{2}{B} \langle w_i(k, t) \dot{k}_i^2 \rangle \\ &- \frac{1}{T \text{loc}} \frac{d}{dt} \langle \dot{k}_i^2 \rangle + \frac{2}{B} \bar{F}(k) \langle \dot{k}_i \rangle \\ &- \sum_{\alpha} \frac{\partial n_{\alpha}^{(i)}(t)}{\partial k_i} g_{\alpha}^{(i)}(t) \end{aligned} \quad (2.39)$$

with the notation

$$w_i(k, t) = + \sum_{\alpha} \frac{\partial^2 n_{\alpha}^{(i)}(t)}{\partial k^2} \quad (2.40)$$

In writing down (2.39) we have replaced k_i by k , thus restricting our considerations to small fluctuations in k . The last term on the r.h.s. of (2.39) vanishes because $g_{\alpha}^{(i)}(t)$ is not correlated with $\dot{k}_i$. This is like in Brownian motion where the fluctuating force is not correlated with the position of the Brownian particle²¹⁾.

Subtracting from (2.39) the expression for $d^2 \langle \dot{k}_i \rangle^2 / dt^2$ we find the differential equation of second order

$$\begin{aligned} \frac{d^2}{dt^2} \sigma_k^2 &= 2B^{-2} [\langle F_i^2(k, t) \rangle - \langle F_i(k, t) \rangle^2] \\ &- 2B^{-1} \bar{w}(k, t) \sigma_k^2 - \frac{1}{T \text{loc}} \frac{d}{dt} \sigma_k^2 \end{aligned} \quad (2.41)$$

for the variance $\sigma_k^2 \equiv \langle \dot{k}^2 \rangle - \langle \dot{k} \rangle^2$ where we have approximated $\langle w_i k_i^2 \rangle \approx \langle w_i \dot{k}_i \rangle \langle \dot{k}_i \rangle$ by $\langle w_i \rangle \{ \langle \dot{k}_i^2 \rangle - \langle \dot{k}_i \rangle^2 \} \equiv \bar{w} \sigma_k^2$ because we do not expect correlations between w_i and $\dot{k}_i$. As the source for fluctuations in $\dot{k}$ the variance in the force enters in (2.41). This variance can be related to the mean occupation numbers n_{α} as

$$\begin{aligned} \langle F_i^2(k, t) \rangle &= \langle F_i(k, t) \rangle^2 \\ &= \sum_{\alpha, \beta} \frac{\partial n_{\alpha}}{\partial k} \frac{\partial n_{\beta}}{\partial k} \{ \langle n_{\alpha}^{(i)}(t) n_{\beta}^{(i)}(t) \rangle - n_{\alpha} n_{\beta} \} \\ &\approx \sum_{\alpha} \left(\frac{\partial n_{\alpha}}{\partial k} \right)^2 \{ \langle n_{\alpha}^{(i)}(t) n_{\alpha}^{(i)}(t) \rangle - n_{\alpha}^2 \} \\ &= \sum_{\alpha} \left(\frac{\partial n_{\alpha}}{\partial k} \right)^2 n_{\alpha} (1 - n_{\alpha}). \end{aligned} \quad (2.42)$$

We have dropped the double summation in the second line because the slopes $\partial n_{\alpha} / \partial k$ of different levels are randomly distributed around a small mean value. The contribution from this mean values multiplied with the two-body correlations ($\alpha \neq \beta$) is expected to be negligible. Thus the variance of the force is directly related to the variance of the occupation probabilities which is determined by the mean values alone (since $\langle n_{\alpha}^{(i)}(t) n_{\alpha}^{(i)}(t) \rangle = \langle n_{\alpha}^{(i)} \rangle = n_{\alpha}$ for a statistical mixture of Slater determinants).

According to (2.41) the equilibrium value of σ_K^2 is determined by

$$\overline{\sigma_K^2} = \overline{\langle F - \langle F \rangle \rangle^2} / \overline{B\omega} \quad (2.43)$$

with

$$\begin{aligned} \overline{\langle F - \langle F \rangle \rangle^2} &= \sum_{\alpha} \left(\frac{\partial \bar{n}_{\alpha}}{\partial \kappa} \right)^2 \bar{n}_{\alpha} (1 - \bar{n}_{\alpha}) \\ &= -T \sum_{\alpha} \left(\frac{\partial \bar{n}_{\alpha}}{\partial \kappa} \right)^2 \frac{\partial \bar{n}_{\alpha}}{\partial \bar{n}_{\alpha}}, \end{aligned} \quad (2.44)$$

$$\bar{w} = \sum_{\alpha} \frac{\partial^2 \bar{n}_{\alpha}}{\partial \kappa^2} \bar{n}_{\alpha}, \quad (2.45)$$

$$\bar{n}_{\alpha} = \left\{ 1 + \exp \left[\frac{\bar{n}_{\alpha} - 1}{T} \right] \right\}^{-1}. \quad (2.46)$$

Since terms with $\partial \bar{F} / \partial \kappa$ have been neglected in (2.41) we should also consider (2.43) for

$$\frac{\partial \bar{F}}{\partial \kappa} = 0 = - \sum_{\alpha} \frac{\partial^2 \bar{n}_{\alpha}}{\partial \kappa^2} \bar{n}_{\alpha} - \sum_{\alpha} \left(\frac{\partial \bar{n}_{\alpha}}{\partial \kappa} \right)^2 \frac{\partial \bar{n}_{\alpha}}{\partial \bar{n}_{\alpha}} \quad (2.47)$$

which results in the relation

$$T \bar{w} = \overline{\langle F - \langle F \rangle \rangle^2} \quad (2.48)$$

and hence from (2.43)

$$\frac{1}{2} B \overline{\sigma_K^2} = \frac{1}{2} T \quad (2.49)$$

the equipartition theorem for the fluctuations in the kinetic energy.

The time evolution of the fluctuations in $\dot{\kappa}$ is qualitatively different from the classical behaviour (Brownian motion). We may consider the limit $\tau_{loc} \rightarrow 0$ as the classical limit. For times $t \gg \tau_{loc}$ the second derivative of σ_K^2 in (2.38) can be neglected and hence, the fluctuations develop monotonous towards the equilibrium value (2.49). For times $t \approx \tau_{loc}$, however, the presence of $d^2 \sigma_K^2 / dt^2$ allows for some oscillations around an increasing average value, because (2.41) is analogous to the equation of motion for the mean value of a damped harmonic oscillator.

To summarize this section we want to point out that the concept of diabaticity and subsequent randomization due to two-body collisions leads to effective equations of motions which include rather large memory effects and which show a different time evolution of the mean value and the variance of the collective variable. It is the aim of the next section to follow more precisely these new dynamics for the quadrupole motion within a specific model.

3. Application to the quadrupole motion

In this section we study the dynamics of quadrupole deformation of the mean field of a 3-dimensional harmonic oscillator. This model allows to perform analytical calculations and thus exhibits a high degree of transparency. In this model eq. (2.2) can easily be solved for the diabatic Hamiltonian $H_0(\delta)$ which coincides with the adiabatic Hamiltonian $H(\delta)$

$$H_0(\delta) = -\frac{\hbar^2}{2m} \Delta + \frac{m}{2} \omega_0^2(\delta) \cdot \left\{ \left(1 + \frac{2}{3}\delta\right) (x^2 + y^2) + (1 - 4/3\delta) z^2 \right\} \quad (3.1)$$

$$\text{with } \omega_0(\delta) = \omega^0 \left(1 + \frac{2}{9}\delta^2 + \dots\right)$$

ensuring volume conservation. The parameter δ denotes the actual value of the quadrupole deformation. The diabatic single-particle states $\phi_{n_x, n_y, n_z}(\delta)$ and diabatic levels

$$\epsilon_{n_x, n_y, n_z}(\delta) = \hbar \omega_0(\delta) \left[\sqrt{1 + 2/3\delta} (n_x + n_y + 1) + \sqrt{1 - 4/3\delta} (n_z + 1/2) \right] \quad (3.2)$$

are easily obtained analytically as well as the matrix elements of the quadrupole operator

$$Q = \sum_{k=1}^A (2Z_k^2 - X_k^2 - Y_k^2), \quad (3.3)$$

$$Q = -\frac{2i\hbar}{m} \sum_{k=1}^A \left(2Z_k \frac{\partial}{\partial Z_k} - Y_k \frac{\partial}{\partial Y_k} \right), \quad (3.4)$$

$$B_q^{-1} = \frac{4}{m} \sum_{k=1}^A (4Z_k^2 + X_k^2 + Y_k^2) \quad (3.5)$$

and $B(\delta)$ according to eq. (2.16). Neglecting the renormalization of the single-particle states along the line of eq. (2.19) in the following we use $\eta_\alpha(\delta) = \epsilon_\alpha(\delta)$. We do not take into account the explicit time dependence of the mass via the occupation numbers $\eta_x, \eta_y, \eta_z(t)$ because $B(\delta)$ essentially is given by the bulk properties. Thus we evaluate $B(\delta)$ with respect to the ground state where (for $\delta=0$) all states with $\eta_x + \eta_y + \eta_z \leq 5$ are occupied (see fig. 1). The nucleus described in this way corresponds to a spherical closed-shell nucleus ($A=224$). The oscillator frequency ω^0 is given as usual by

$$\hbar\omega^0 = 41 A^{-1/3} \text{ [MeV]}. \quad (3.6)$$

By taking first and second derivatives of the diabatic levels (3.2) with respect to δ and summing up over all occupied states ($\eta_x + \eta_y + \eta_z \leq 5$) we obtain $F(\delta, t=0)$ and the mean second derivative of the diabatic s.p. levels

$$\hat{W}(\delta, t=0) = \sum_{\alpha} \frac{\partial^2 \eta_{\alpha}}{\partial \delta^2} \eta_{\alpha}(t=0). \quad (3.7)$$

The time derivative of the total force $F(\delta, t)$ according to (2.23) is given by

$$\frac{d}{dt} F(\delta, t) = - \hat{W}(\delta, t) \cdot \dot{\delta} - \tau_{loc}^{-1} [F(\delta, t) - \bar{F}(\delta, \mu, T)] \quad (3.8)$$

which in addition to $F(\delta, 0)$ and $\hat{W}(\delta, 0)$ is determined by the equilibrium force $\bar{F}(\delta, \mu, T)$, which finally should drive the model nucleus to spherical shape again. Due to the simplicity of the model we do not calculate $\bar{F}$ microscopically according to eq. (2.20) but insert a reasonable estimate given by

$$\bar{F}(\delta, \mu, T) = \bar{F}(\delta) = -B(\delta) \omega_S^2 \cdot \delta \quad (3.9)$$

with

$$\hbar\omega_S \approx 2 \text{ MeV}.$$

This choice corresponds to a low lying surface vibration which is determined by the balance between Coulomb and surface energies. With (3.9) the solution of eq. (3.8) is entirely determined once the initial conditions $\delta(t=0)$ and $\dot{\delta}(t=0)$ are given.

Before going over to the numerical solution of eqs. (2.15, 3.8) which describe the collective dynamics, it is instructive to look at the approximate analytical results valid for $\delta \ll 1$. In this limit we obtain

$$B(\delta) = B(\delta=0) = B_0, \quad (3.10)$$

$$\tilde{W}(\delta, t) = \tilde{W}(\delta=0, t=0) = B_0 \Omega^2 \quad (3.11)$$

with

$$\hbar\Omega = 2\hbar\omega^0 = 82 \cdot A^{-1/3} \text{ [MeV]}, \quad (3.12)$$

which is identified with the energy of the isoscalar giant quadrupole vibration (experimental energy: $\hbar\Omega_{ex} \approx 63 \cdot A^{-1/3} \text{ [MeV]}$).

The effective equation of motion for $\delta(t)$ then reads (according to eq.

$$(2.28) \text{ for } \tau_{loc} = \text{const}) \quad (3.13)$$

$$\ddot{\delta} + \int_0^t e^{-(t-t')/\tau_{loc}} [\Omega^2 \dot{\delta}(t') + \frac{1}{\tau_{loc}} \omega_S^2 \delta(t')] dt' = 0$$

which for $\omega_S = 0$ is identical to eq. (14) of Ref. 3). In eq. (3.13) we have employed the initial condition $F(\delta, t=0) = 0$. For $t \ll \tau_{loc}$ eq. (3.13) reduces to

$$\ddot{\delta} + \Omega^2 \cdot (\delta - \delta(t=0)) = 0 \quad (3.14)$$

showing an elastic vibration with the frequency of the giant quadrupole vibration (3.12), while for $t \gg \tau_{loc}$ we obtain

$$\ddot{\delta} + \tau_{loc} \Omega^2 \dot{\delta} + \omega_S^2 \delta = 0, \quad (3.15)$$

i.e. a damped surface oscillation with an effective friction coefficient

$$\gamma_f = \tau_{loc} \Omega^2/2. \quad (3.16)$$

It is worth to reestablish the relations between the formfactors in eq. (3.13) with those given in the general treatment in § 2.2 by W_0 (2.24) and W_1 (2.25).

From

$$\begin{aligned} \frac{d\bar{F}}{d\delta} &= -B_0 \omega_s^2 = -\sum_{\alpha} \frac{\partial^2 \eta_{\alpha}}{\partial \delta^2} \bar{n}_{\alpha}(\delta) - \sum_{\alpha} \frac{\partial \eta_{\alpha}}{\partial \delta} \frac{\partial \bar{n}_{\alpha}}{\partial \delta} \\ &= -B_0 \Omega^2 + W_0 \end{aligned} \quad (3.17)$$

we obtain

$$W_0 = B_0(\Omega^2 - \omega_s^2) \approx B_0 \Omega^2 \quad (3.18)$$

and

$$W_1 = 0.$$

This implies that the formfactor for retarded friction W_0 is approximately given by the sum over the second derivatives of the diabatic levels while the retarded dynamical potential W_1 is zero in the limit (3.11).

In order to study the effect of the nonlocal friction kernel with respect to $\delta(t)$ we make comparison to a damped oscillator

$$\ddot{\delta} + \frac{1}{\tau} \dot{\delta} + \Omega^2 \delta = 0 \quad (3.20)$$

and assume $\omega_s=0$ for a while. Whereas the solution of eq. (3.20) for the initial condition $\delta(0)=0$, $\dot{\delta}(0) = \dot{\delta}_0$ is given by

$$\delta(t) = e^{-t/2\tau} \cdot \sin(\omega_{\delta} t) \dot{\delta}_0 / \omega_{\delta} \quad (3.21)$$

with $\omega_{\delta}^2 = \Omega^2 - \frac{1}{4\tau^2}$

the solution of eq. (3.13) reads

$$\delta(t) = \delta(\infty) + e^{-t/2\tau} \tau_{loc} \cdot [-\dot{\delta}(\infty) \cos(\omega_{\delta} t) + b \sin(\omega_{\delta} t)] \quad (3.2)$$

with

$$\delta(\infty) = \frac{4\tau_{loc} \dot{\delta}_0}{1+4\tau_{loc}^2 \omega_{\delta}^2}, \quad \omega_{\delta}^2 = \Omega^2 - \frac{1}{4\tau_{loc}^2}$$

and

$$b = -\frac{\dot{\delta}_0(1-4\tau_{loc}^2 \omega_{\delta}^2)}{\omega_{\delta}(1+4\tau_{loc}^2 \omega_{\delta}^2)}$$

and approaches a finite value for $t \rightarrow \infty$. The energy dissipation instead for both cases evolves according to the same formula if we identify $\tau = \tau_{loc}$

$$\frac{d}{dt} E^* = \frac{B_0}{\tau_{loc}} \dot{\delta}^2 \quad (3.32)$$

with only slightly different $\delta(t)$ however.

With respect to fluctuations in δ we essentially obtain different equations. The fluctuations for the damped harmonic oscillator are determined by the Fokker-Planck equation

$$\frac{\partial}{\partial t} f(\delta, \dot{\delta}; t) = \left\{ -\dot{\delta} \frac{\partial}{\partial \delta} + \Omega^2 \delta \frac{\partial}{\partial \dot{\delta}} + \tau_{loc}^{-1} \frac{\partial^2}{\partial \dot{\delta}^2} \right\} f(\delta, \dot{\delta}; t) \quad (3.24)$$

which is nothing but the extension of eq. (3.20) including diffusion in phase-space according to the Einstein-relation

$$D(t) = T(t)/(\tau_{loc} \cdot B_0). \quad (3.25)$$

In eq. (3.25) $T(t)$ denotes the nuclear temperature of the intrinsic single-particle motion. The fluctuations in δ then are determined by the following set of coupled differential equations

$$\frac{d}{dt} \sigma_{\delta}^2 = 2\sigma_{\dot{\delta}}^2 \quad (3.26)$$

$$\frac{d}{dt} \sigma_{\dot{\delta}}^2 = \sigma_{\delta}^2 - \tau_{loc}^{-1} \sigma_{\delta\dot{\delta}}^2 - \Omega^2 \sigma_{\delta}^2 \quad (3.27)$$

$$\frac{d}{dt} \sigma_{\delta\dot{\delta}}^2 = 2\tau_{loc}^{-1} \sigma_{\delta}^2 - 2\Omega^2 \sigma_{\delta\dot{\delta}}^2 + 2D(t) \quad (3.28)$$

which have to be solved with the initial condition $\sigma_{\delta}^2(t=0) = \sigma_{\dot{\delta}}^2(t=0) = \sigma_{\delta\dot{\delta}}^2(t=0) = 0$.

Diabatic s.p. motion instead leads to the following equation for σ_δ^2
(comp. eq. (2.41))

$$\frac{d^2}{dt^2} \sigma_\delta^2(t) = -\tau_{1oc}^{-1} \frac{d}{dt} \sigma_\delta^2(t) - 2\Omega^2 \sigma_\delta^2(t) + 2B_0^{-1} \Omega^2 \cdot T(t), \quad (3.29)$$

where we have adopted the relation (2.48) in order to determine the fluctuations with respect to the force.

The solutions for the standard deviations in δ according to eq. (3.29)

(full line) and eqs. (3.26-3.28) (dashed line) are displayed in fig. 2 for $\tau_{1oc} = 100bs = 10^{-21} sec$ and $\tau_{1oc} = 10bs$ together with the asymptotic value of $\sigma_\delta^2(\omega)$ indicated by the open circle. The intrinsic temperature was calculated via

$$T(t) = \{8 E^*(t)/A\}^{1/2} \quad (3.30)$$

with $E^*(t)$ given by (3.23). While for $t \rightarrow \infty$ the solutions for σ_δ^2 in both cases are the same according to the equipartition theorem (given by (2.49) and reflected in (3.25)) the time evolutions for $t < \tau_{1oc}$ are significantly different. For $\Omega^2 \gg \tau_{1oc}^{-2}$ large oscillations show up according to eq. (3.29) while for $\Omega^2 \approx \tau_{1oc}^{-2}$ (i.e. $\tau_{1oc} \approx 10bs$) the spreading rises less steeply than the corresponding solution of eqs. (3.26 - 3.28). The initial slope of $\sigma_\delta^2(t)$ thus sensitively depends on the ratio of Ω and τ_{1oc}^{-1} for fixed asymptotic $\sigma_\delta^2(t \rightarrow \infty)$ and may differ significantly from the slope determined by the Einstein-relation (3.25).

In order to study the general solution of eq. (3.13) with $\omega_s^2 \neq 0$ we look at its Laplace transform

$$\begin{aligned} \{S^3 + \frac{S^2}{\tau_{1oc}} + \Omega^2 S + \frac{1}{\tau_{1oc}} \omega_s^2\} \delta(s) \\ = (S^2 + \frac{S}{\tau_{1oc}}) \delta(0) + (S + \frac{1}{\tau_{1oc}}) \dot{\delta}(0) + \Omega^2 \delta(0) + \frac{F(0)}{B_0}, \end{aligned} \quad (3.31)$$

which has three different basic solutions because the polynomial on the l.h.s. of eq. (3.31) is of third order. There is a real solution for

$$S_1 = (-q + \sqrt{q^2 + p^3})^{1/3} + (-q - \sqrt{q^2 + p^3})^{1/3} \quad (3.32a)$$

and two complex solutions for

$$S_2 = (-\frac{1}{2} + i\frac{\sqrt{3}}{2}) (-q + \sqrt{q^2 + p^3})^{1/3} + (-\frac{1}{2} - i\frac{\sqrt{3}}{2}) (-q - \sqrt{q^2 + p^3})^{1/3} \quad (3.32b)$$

and

$$S_3 = (-\frac{1}{2} - i\frac{\sqrt{3}}{2}) (-q + \sqrt{q^2 + p^3})^{1/3} + (-\frac{1}{2} + i\frac{\sqrt{3}}{2}) (-q - \sqrt{q^2 + p^3})^{1/3} \quad (3.32c)$$

with

$$p = \frac{\Omega^2}{3} - \frac{1}{9\tau_{1oc}^2}, \quad q = \frac{\Omega^2}{27\tau_{1oc}^3} - \frac{\Omega^2}{6\tau_{1oc}} + \frac{\omega_s^2}{2\tau_{1oc}}$$

For $\Omega^2 \gg \tau_{1oc}^{-2}$, ω_s^2 , a condition we actually meet experimentally, the solutions reduce to the pocket-size version with

$$S_1 \approx -\frac{\omega_s^2}{\Omega^2 \tau_{1oc}}, \quad (3.33a)$$

$$S_2 \approx -\frac{1}{2\tau_{1oc}} + i\sqrt{\frac{\Omega^2}{4\tau_{1oc}^2} - \frac{1}{4\tau_{1oc}^2}}, \quad (3.33b)$$

$$S_3 \approx -\frac{1}{2\tau_{1oc}} - i\sqrt{\frac{\Omega^2}{4\tau_{1oc}^2} - \frac{1}{4\tau_{1oc}^2}}, \quad (3.33c)$$

which represent a mode very slowly decaying in time according to S_1 and a fast oscillatory motion damped within the time-scale of τ_{1oc} .

The dissipation mechanism thus is twofold: for times $t \ll 2\tau_{1oc}$ we observe a rather fast relaxation of the isoscalar giant quadrupole motion by which the intrinsic system is heated up, while for $t \gg \tau_{1oc}$ a slow "adiabatic" relaxation takes place with a different effective friction constant. Comparing the effective friction constants for $t < \tau_{1oc}$ (i.e. $\gamma_i = \frac{1}{2\tau_{1oc}}$) and $t \gg \tau_{1oc}$ (cp. eq. (3.16)) we find

$$\gamma_f \gg \gamma_i, \quad (3.34)$$

stating that the final stage is fairly overdamped ($\omega_S^2 \ll \Omega^2$).

The numerical solution of the effective equations of motion (2.15, 3.8) for the case of nuclear quadrupole motion within the deformed harmonic oscillator basis (3.2) (the equilibrium force $\bar{F}$ given by eq. (3.9)) is displayed in fig. 3 for the model nucleus ($A=224$) described above. This calculation does not include the restrictive approximations (3.10, 3.11) made for the analytical investigation and essentially reproduces the results of the analytical study (eqs. (3.33, 3.34), $\tau_{loc} = 100$ bs).

If we change the equilibrium force in such a way that the corresponding potential has a saddle for e.g. $\delta \approx 0.25$ the solution again shows a fast oscillatory motion at the beginning ($t < 2\tau_{loc}$) and a slow motion from the saddle to scission for large times ($t \gg \tau_{loc}$). It is important to note again that the concept of retarded friction based on the mechanism of relaxed diabatic motion is able to describe dissipative phenomena with significantly different time scales and effective friction constants on the basis of the same friction kernel. It thus allows for a simultaneous description of the elastic behaviour of nuclear matter for short times ($t \ll \tau_{loc}$) and of the viscous attitudes of excited nuclei for long times ($t \gg \tau_{loc}$).

Conclusion

In this paper we studied large amplitude collective nuclear motion in which the dissipation mechanism is viewed as a combined effect of diabatic particle-hole excitations and subsequent equilibration by two-body collisions. An important result of this approach is the large memory showing up in the effective equation of motion for the collective degree of freedom which also leads to a different equation for the time evolution of the spreading in the collective velocity. An application

of the diabatic dynamical approach to the quadrupole motion within a 3-dim. deformed harmonic oscillator basis showed that the effective equation of motion (2.28) describes a gradual transition from the elastic properties of nuclei at the early stages to the viscous behaviour at times large compared to the local equilibration time. In this way different effective friction constants for $t < \tau_{loc}$ and $t \gg \tau_{loc}$ are simultaneously described by the same friction kernel. With respect to the fluctuations in the collective velocity we found out that the time evolution of this quantity sensitively depends on the value of the local equilibration time and differs significantly from the time evolution determined through a conventional transport equation where drift and diffusion coefficients are related via an Einstein relation.

Nevertheless we have to convey that it is rather difficult to determine approximate values of τ_{loc} experimentally via mean values and variances of collective variables, but other diabatic phenomena like the direct emission of light particles in heavy-ion collisions¹³⁾ should yield a sensitive test of diabatic single-particle motion in large amplitude collective nuclear motion.

The authors acknowledge numerous discussions with their colleagues at GSI and TH Darmstadt.

References

1. H.A. Weidemüller, in: Progress in Particle and Nuclear Physics, vol. 3, ed. by D.H. Wilkinson (Pergamon Press, Oxford, 1980)
A. Gobbi and W. Nörenberg, in: Heavy-Ion Collisions, vol. 2, ed. by R. Bock (North-Holland, Amsterdam 1980)
W. Nörenberg and H.A. Weidemüller, Introduction to the Theory of Heavy-Ion Collisions, Lecture Notes in Physics, vol. 51, 2nd edition (Springer-Verlag, Heidelberg, 1980)
2. W.J. Swiatecki, in: Progress in Particle and Nuclear Physics, vol. 4, ed. by D. Wilkinson (Pergamon Press, Oxford, 1980)
J. Randrup, Nukleonika 22 (1977) 1029
J. Blocki, Y. Boneh, J.R. Nix, J. Randrup, M. Robel, A. Sierk, W.J. Swiatecki, Ann. of Phys. 113 (1978) 330
3. W. Nörenberg, Phys. Lett. 104B (1981) 107
4. J.W. Negele, NATO Advanced Study Institute of Theoretical Methods in Medium-Energy and Heavy-Ion Physics, ed. by K.W. McVoy and W.A. Friedman (Plenum Press, New York, 1978)
P. Bonche, B. Grammaticos and S.E. Koonin, Phys. Rev. C17 (1978) 1700
5. P. Morel and P. Nozières, Phys. Rev. 126 (1962) 1909
6. W. Nörenberg, in "Nuclear Physics", ed. by Dasso, Proceedings of the Nuclear Physics Workshop, I.C.T.P., Trieste, Miramare, Italy, 5-30. Oct. 1981, (North-Holland, Amsterdam, 1982) p. 543-563
7. G.F. Bertsch, Z. Phys. A 289 (1978) 103
8. C.Y. Wong and H.F. Tang, Phys. Rev. Lett. 40 (1978) 1070
H. Orland and R. Schaeffer, Z. Phys. A290 (1979) 191
S. Ayik, Z. Phys. A298 (1980) 83
S. Ayik, Nucl. Phys. A370 (1981) 317
H.S. Köhler, Nucl. Phys. A343 (1980) 315
P. Grangé, J. Richert, G. Woloschin and H.A. Weidemüller, Nucl. Phys. 356 (1981) 260

- P. Grangé, H.A. Weidemüller and G. Woloschin, Ann. Phys. (NY) 136 (1981) 190
R. Balian and M. Vénéroni, Ann. Phys. (N.Y.) 135 (1981) 270
9. W. Nörenberg and C. Riedel, Z. Phys. A290 (1979) 385
H.L. Yadav and W. Nörenberg, Phys. Lett. 115B (1982) 179
10. W. Cassing and W. Nörenberg, Preprint GSI-81-41
11. D.L. Hill and J.A. Wheeler, Phys. Rev. 89 (1953) 1102
12. G. Schütte, Phys. Rep. 80 (1981) 113
13. A. Lukasiak, W. Cassing and W. Nörenberg, Proceedings of the International Workshop on Gross Properties of Nuclei and Nuclear Excitations X, Hirschegg, Jan. 82, p. 80-84 and to be published.
14. H.C. Pauli and L. Wilets, Z. Phys. A 277 (1976) 83
15. S.J. Wang, W. Cassing and W. Nörenberg, Communications of the International Conference on Selected Aspects of Heavy-Ion Reactions, Saclay, May 82, p. 57 and Preprint GSI-82-13
16. P. Möller and J.R. Nix, Nucl. Phys. A296 (1978) 289
17. J.J. Griffin and K.K. Kan, in 'Physics and Chemistry of Fission 1973', Vol. 1, p. 533 (International Atomic Energy Agency, Vienna, 1974)
18. A. Bohr and B.R. Mottelson, Nuclear Structure Vol. II, Benjamin, London - Amsterdam, 1975
19. W.D. Myers, Dropted Model of Atomic Nuclei, (IFI, Plenum, 1977)
20. S. Ayik and W. Nörenberg, Preprint GSI-82-25, Z. Phys. A (in press)
21. R. Becker, Theorie der Wärme, p. 272

Figure Captions

Fig. 1: The effective mass for the quadrupole motion as a function of the deformation δ .

Fig. 2: Comparison of the time evolution for the standard deviation in the velocity $\dot{\delta}$ within the damped oscillator model (dashed line) and the relaxed diabatic approach (full line) for different local equilibration times τ_{loc} .

Fig. 3: Solution of the effective equation of motion (2.28) for $\delta(t)$ in the relaxed diabatic approach for $\tau_{loc} = 10^{-21}$ s and the model nucleus ($A=224$) described in the text.

Fig. 1

Fig. 2

Fig. 3