A study of direct $\mathcal{CP}$-violation in charged $B$-meson decays with the LHCb experiment

Gareth James Rogers

of Trinity Hall
University of Cambridge

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Abstract

The LHCb experiment, operating at the Large Hadron Collider, is studying b-hadrons produced in proton-proton collisions at a centre-of-mass energy of 7 TeV. The LHCb detector consists of millions of detector elements and to manage these elements an industrial supervisory control and data acquisition system is used. The implementation of the Ring-Imaging Cherenkov sub-detector’s off-detector electronics configuration and monitoring within this framework is described.

The proton-proton collisions can last for periods of several hours and the real-time monitoring of the recorded data ensures that the detector performance does not degrade during this time. In this thesis the algorithms providing the real-time monitoring of the detector elements of the LHCb Ring-Imaging Cherenkov sub-detector are presented.

A measurement of the $C\!P$-asymmetry in $B^\pm$ decays is used to test the predictions of the Standard Model and to search for signs of new physics. A method to extract the $C\!P$-asymmetry in $B^\pm \to \phi K^\pm$ and $B^\pm \to \phi \pi^\pm$ decays using $B^\pm \to J/\psi K^\pm$ and $B^\pm \to J/\psi \pi^\pm$ decays respectively as control channels is presented. With a data set of 1 fb$^{-1}$ a measurement of the $C\!P$-asymmetry in the $B^\pm \to \phi K^\pm$ channel should provide the world’s best measurement. The $B^\pm \to \phi \pi^\pm$ channel is as yet undetected and it is expected that evidence for this channel could be found in a 2 fb$^{-1}$ data set, based on a prediction from the Standard Model. Finally, using a data set containing an integrated luminosity of 37 pb$^{-1}$ the ratio of branching fractions $B(B^\pm \to J/\psi \pi^\pm)/B(B^\pm \to J/\psi K^\pm)$ is measured as $0.051 \pm 0.005\,(\text{stat.}) \pm 0.03\,(\text{syst.})$ and an upper limit of $< 0.13$ at the 90 % confidence level has been set for the ratio of branching fractions $B(B^\pm \to \phi \pi^\pm)/B(B^\pm \to \phi K^\pm)$.
Declaration

This dissertation is the result of my own work, except where explicit reference is made to the work of others, and has not been submitted for another qualification to this or any other university. This dissertation does not exceed the word limit for the respective Degree Committee.

Gareth Rogers
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There is one person who deserves a special mention; they have done the most with one single act to preserve my sanity. Thank you to Susan Haines for taking the UKL1 phone off me.
Preface

This thesis describes the work I have performed while a member of the Large Hadron Collider beauty (LHCb) collaboration and is comprised of two major components. The first is the software written to control and monitor the off-detector readout electronics for the Ring-Imaging Cherenkov (RICH) sub-detector and the algorithms written to provide real-time monitoring of this sub-detector. The second is the analysis of four charged B-meson decays using the first 37 fb$^{-1}$ of data recorded by the LHCb experiment and simulated LHCb events.

Chapter 1 is a description of the theoretical concepts necessary to place the physics analysis in the context of the current theoretical understanding and latest experimental results. Chapter 2 begins with a description of the Large Hadron Collider and the experiments situated around the accelerator complex; the LHCb detector and support infrastructure is then described. Next the LHCb RICH sub-detector for the identification of charged particles is described in Chapter 3. This chapter contains details of the RICH off-detector readout electronics and its control software, and the software that provides real-time monitoring of the data recorded by the sub-detector. Chapters 4 to 7 describe the analysis of the charged B-meson decays: \(B^\pm \to J/\psi K^\pm\), \(B^\pm \to J/\psi \pi^\pm\), \(B^\pm \to \phi K^\pm\) and \(B^\pm \to \phi \pi^\pm\). Chapter 4 describes the optimisation of an event selection for \(B^\pm \to J/\psi h^\pm\) and \(B^\pm \to \phi h^\pm\) decays, where \(h\) represents either a kaon or a pion, using simulated events. The expected yields of the four decays are calculated using the performance of the event selection on the simulated events. The optimised event selections are applied to the 37 fb$^{-1}$ data set in Chapter 5 and the yields of the four signal channels are extracted using the RICH particle identification to separate the \(B^\pm \to J/\psi h^\pm\) and \(B^\pm \to \phi h^\pm\) data sets into kaon and pion samples. A method to measure the \(CP\)-asymmetry between the \(B^+\) and \(B^-\) decays is developed in Chapter 6 and the expected sensitivity of the measurements is predicted using the expected yields presented in Chapter 4. Finally, the method is used to extract the ratio of branching fractions of \(B(B^\pm \to J/\psi \pi^\pm)/B(B^\pm \to J/\psi K^\pm)\) and \(B(B^\pm \to \phi \pi^\pm)/B(B^\pm \to \phi K^\pm)\) in Chapter 7.
Contents

1 Charge Parity Violation in the B-meson System  1
  1.1 Introduction . ................................. 1
  1.2 The $C$, $P$ and $T$ Operators .............. 4
  1.3 Model Independent $CP$-violation .......... 5
      1.3.1 Neutral B-meson Mixing ........................................... 5
      1.3.2 Neutral B-meson Decay ......................................... 7
      1.3.3 The Types of $CP$-violation ...................................... 8
  1.4 $CP$-violation in the Standard Model ......... 11
  1.5 The CKM Matrix ...................................... 13
  1.6 The Unitarity Triangle ......................... 15
  1.7 Constraining the CKM Parameters ............. 18
      1.7.1 Indirect Measurements ...................................... 19
      1.7.2 Direct Measurements .................................... 20
      1.7.3 Global Constraints from Direct and Indirect Measurements .... 25
  1.8 Direct $CP$-violation in Charged B-meson Decays 26
      1.8.1 Direct $CP$-violation in $B^\pm \to J/\psi K^\pm$ Decays ........... 26
      1.8.2 Direct $CP$-violation in $B^\pm \to J/\psi \pi^\pm$ Decays .......... 28
      1.8.3 Direct $CP$-violation in $B^\pm \to \phi K^\pm$ Decays ............ 29
      1.8.4 Direct $CP$-violation in $B^\pm \to \phi \pi^\pm$ Decays ............ 31
  1.9 Summary ........................................... 31

2 The LHC and the LHCb Experiment  33
  2.1 The LHC ........................................... 33
  2.2 The LHC Experiments ............................ 34
  2.3 B-production at the LHC ....................... 36
  2.4 The LHCb Experiment ............................ 37
      2.4.1 Beam-Pipe ..................................................... 41
      2.4.2 The LHCb Magnet ............................................ 42
## Contents

2.4.3 The Tracking System ........................................... 44
2.4.4 Particle Identification ........................................ 54
2.4.5 Data Acquisition and Experiment Control .................. 62
2.4.6 The Trigger ................................................... 66
2.4.7 The Stripping ................................................. 70
2.5 The LHCb Software ................................................ 71
  2.5.1 Gaudi ......................................................... 71
  2.5.2 Event Simulation ............................................. 71
  2.5.3 The Software Trigger ....................................... 71
  2.5.4 Reconstruction and Analysis ................................ 72
  2.5.5 Data Processing ............................................. 72
2.6 The 2010 Data Taking Period .................................... 73
2.7 Summary ......................................................... 75

3 The LHCb RICH Detectors ............................................ 77
  3.1 Charged Particle Identification at LHCb ....................... 77
    3.1.1 The Cherenkov Effect .................................... 77
    3.1.2 The RICH System ........................................ 78
    3.1.3 Particle Identification .................................... 83
    3.1.4 The RICH PID Performance ............................... 83
  3.2 The UKL1 Board ................................................ 86
    3.2.1 The Experiment Control System Architecture ............ 88
    3.2.2 The UKL1 Control System ................................ 91
  3.3 The RICH Monitoring ........................................... 97
    3.3.1 Panoptes .................................................. 99
  3.4 Summary ....................................................... 104

4 Event Reconstruction and Selection ................................. 107
  4.1 The Simulated Events ......................................... 107
  4.2 Event Reconstruction ......................................... 108
  4.3 Event Selection Optimisation .................................. 111
    4.3.1 Optimisation Method ..................................... 111
    4.3.2 Optimisation Results ..................................... 116
  4.4 Selection Performance ......................................... 122
  4.5 Expected Yields ............................................... 129
  4.6 Summary ....................................................... 131
Chapter 1

Charge Parity Violation in the B-meson System

This chapter gives an introduction to the theory of $C\!P$-violation and the current experimental status of $C\!P$-violation in the Standard Model of particles physics. It also provides the theoretical motivation for the analysis that is the subject of this thesis. The chapter begins with a brief introduction to the Standard Model, Section 1.1, to place the later discussion of $C\!P$-violation in the Standard Model into context. The $C$, $P$ and $T$ operators are described in Section 1.2 and Section 1.3 describes the mechanisms for $C\!P$-violation and the three types of $C\!P$-violation that exist. Section 1.4 introduces $C\!P$-violation in the Standard Model and the origin of the quark mixing matrix. The quark mixing matrix is discussed in detail in Section 1.5 followed by Section 1.6 which describes the Unitarity Triangle, a visual representation of the unitarity conditions of the quark mixing matrix and of $C\!P$-violation in the Standard Model. The methods for constraining the internal angles of the Unitarity Triangle, both indirectly by measuring the matrix elements and directly by measuring the angles, are presented in Section 1.7 along with the latest measurements. Finally, Section 1.8 gives the status of $C\!P$-violation measured in the charged B-meson decays that is the subject of this thesis.

1.1 Introduction

The Standard Model of particle physics unites three of the four fundamental forces of nature: the electromagnetic force, the weak force and the strong force, with gravity the notable exception. The Standard Model contains 12 fermions (spin 1/2), four vector
bosons (spin 1) and a scalar boson (spin 0). The scalar boson is the Higgs boson, the only undiscovered particle in the Standard Model; the production and study of the Higgs boson is one of the goals of the Large Hadron Collider (LHC). Three of the four vector bosons: the photon – $\gamma$, $W^\pm$ and $Z^0$, are the mediators of electroweak force (the combination of the electromagnetic and the weak forces) and the fourth is the gluon, $g$, which mediates the strong force. The 12 fermions are divided into six quarks and six leptons, each with its own flavour quantum number. The conversion of a particle to its anti-particle inverts some of its quantum numbers, including its flavour and charge. The possible interactions between the particles of the Standard Model are shown in Figure 1.1.

The quarks and leptons are divided into three generations each of which is made up of a doublet with masses increasing from generation I to III. This structure is shown in Table I.1 where the anti-fermions have not been shown but follow the same structure. In the lepton sector the doublets are made up of a charged lepton (electron – $e$, muon – $\mu$ and tau – $\tau$) and a neutral lepton (the neutrino). In the quark sector these doublets contain an up-type ($u$, $c$ and $t$) quark and a down-type ($d$, $s$ and $b$) quark with charges $+2/3\,e$ and $-1/3\,e$ (where $e$ is the electron charge) respectively. Quarks exists only as bound
### Table 1.1: The fermions of the Standard Model. Each fermion has an anti-particle counterpart. The masses are taken from reference [4].

<table>
<thead>
<tr>
<th>Generation</th>
<th>Flavour</th>
<th>Charge / e</th>
<th>Mass / MeV/c²</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>electron (e⁻)</td>
<td>-1</td>
<td>0.510989010 ± 0.000000013</td>
</tr>
<tr>
<td></td>
<td>electron neutrino (νₑ)</td>
<td>0</td>
<td>&lt; 0.000002</td>
</tr>
<tr>
<td>II</td>
<td>muon (μ⁻)</td>
<td>-1</td>
<td>105.658367 ± 0.000004</td>
</tr>
<tr>
<td></td>
<td>muon neutrino (ν₅)</td>
<td>0</td>
<td>&lt; 0.19</td>
</tr>
<tr>
<td>III</td>
<td>tau (τ⁻)</td>
<td>-1</td>
<td>1776.82 ± 0.16</td>
</tr>
<tr>
<td></td>
<td>tau neutrino (ν₅)</td>
<td>0</td>
<td>&lt; 18.2</td>
</tr>
</tbody>
</table>

(a) The quark content of the Standard Model.

(b) The lepton content of the Standard Model.
states known as hadrons; the hadrons are either a bound state of a quark/anti-quark pair, known as a meson, or of a bound state of three quarks (anti-quarks), known as a baryon.

The Standard Model is one of the most precise models in physics and has been tested to an exceptional level of accuracy. There are, however, some topics where the Standard Model cannot explain the visible universe; the observed asymmetry between the amount of matter and anti-matter in the Universe is underestimated by twelve orders of magnitude [5]. Cosmologists believe that there was an equal amount of matter and anti-matter at the beginning of the Universe; in 1967 Sakharov published the conditions that are required to generate the matter/anti-matter asymmetry observed in the Universe [6]. These are

- baryon number violation;
- \( \mathcal{C} \) and \( \mathcal{CP} \)-violation; and
- departure from thermal equilibrium.

\( \mathcal{C} \) and \( \mathcal{CP} \)-violation are the requirements that there is a difference between the properties of matter and anti-matter, and the amount of \( \mathcal{CP} \)-violation predicted by the Standard Model is insufficient to explain the observed matter/anti-matter asymmetry. Many models have been proposed that introduce increased amounts of \( \mathcal{CP} \)-violation through new physics, two examples are described in references [7,8]. Precision measurements of Standard Model observables provide valuable input to constrain, rule out or even confirm the predictions of these possible theories. In the following sections an explanation of \( \mathcal{CP} \)-violation and its place within the Standard Model is presented.

1.2 The \( \mathcal{C}, \mathcal{P} \) and \( \mathcal{T} \) Operators

Within the Standard Model everything is described theoretically by four-vectors e.g. \((ct, \vec{x})\) and \((E/c, \vec{p})\). The \( \mathcal{C}, \mathcal{P} \) and \( \mathcal{T} \) operators are defined by their effects on such four-vectors:

- **Parity (\( \mathcal{P} \))** corresponds to a flip of the particles spatial coordinates, \((t, \vec{x}) \rightarrow (t, -\vec{x})\);
- **Charge conjugation (\( \mathcal{C} \))** converts a particle into its anti-particle; and
- **Time reversal (\( \mathcal{T} \))** reverses the time coordinate of a particle, \((t, \vec{x}) \rightarrow (-t, \vec{x})\).
These are the three discrete symmetries present within the Standard Model and all are conserved within the strong and electromagnetic interactions. In 1956 it was suggested by Lee and Yang that there is no strong experimental evidence for parity conservation in the weak interaction and they proposed experiments to test for the non-conservation of parity in $\beta$ decays [9]. It was later shown experimentally by Wu et al. that parity is not conserved in the weak interaction through the $\beta$ decay of Cobalt-60 [10]. It was also found that $C$ is violated in the weak interaction by Garwin et al. [11]. In light of the non-conservation of parity Landau proposed that the combined operation of $CP$ is conserved in the weak interaction [12]. It was not until 1964 that Christenson et al. discovered $CP$-violation in the decay of neutral kaons [13]. Since then $CP$-violation has been extensively studied in kaon decays and, in 2001, $CP$-violation in the $B$-meson system was first measured by the B-factories [14][15].

1.3 Model Independent $CP$-violation

In this section the phenomenology of the mixing and decay of neutral mesons is presented. The ideas developed here are model independent and applicable to neutral meson systems such as $K^0$ (d$\bar{s}$), $D^0$ (c$\bar{u}$), $B^0_d$ (d$\bar{b}$) or $B^0_s$ (s$\bar{b}$), however the equations are all expressed in terms of the generic neutral $B$-meson, $B^0$, and its charge conjugate, $\bar{B}^0$, as it is $CP$-violation in the $B$-meson system that is the focus of this thesis.

1.3.1 Neutral $B$-meson Mixing

The two flavour eigenstates of the strong and electromagnetic interactions, $|B^0\rangle$ and $|\bar{B}^0\rangle$, have a common mass and opposite flavour. These two states are stable but may oscillate between one another, for example by the second order weak interaction mixing diagrams shown in Figure 1.2. The mass eigenstates, $|B^0_{L,H}\rangle$, are constructed from a superposition of the flavour eigenstates as

$$
|B^0_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle \\
|B^0_H\rangle = p|\bar{B}^0\rangle - q|B^0\rangle
$$

where $p$ and $q$ are complex constants with the normalisation condition $|p^2| + |q^2| = 1$. The sign of $q$ in the two equations is merely convention and the sign has been chosen to follow reference [16].
A beam of oscillating and decaying $B^0$ mesons is described in its rest frame by the two component wave function

$$\psi(t) = \psi_1(t) |B^0_H\rangle + \psi_2(t) |\overline{B}^0_L\rangle$$

(1.2)

where $t$ is the proper time. This evolves according to

$$i \frac{d}{dt} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

(1.3)

where the matrix $R$ may also be written as $R = M - \frac{i}{2} \Gamma$ [16]. The two eigenstates of $R$ in the $B^0$ meson system are defined as $\mu_H = m_H - \frac{i}{2} \Gamma_H$ and $\mu_L = m_L - \frac{i}{2} \Gamma_L$, with $\Delta m = m_H - m_L$ and $\Delta \Gamma = \Gamma_H - \Gamma_L$. The labels are arbitrary and in the $B^0$ meson system H and L standard for heavy and light respectively as the magnitude of the mass difference, $\Delta m$, is much larger than the difference in partial widths, $\Delta \Gamma$. The mass difference is positive by definition and it is the sign of $\Delta \Gamma$ that defines which of the two states has the shorter lifetime. In the kaon system, where $\Delta m$ is much smaller than $\Delta \Gamma$, the states are labelled L and S for long and short respectively, and $\Delta \Gamma$ is defined as positive. As $\Delta m$ is positive, $\Delta m = (5.292 \pm 0.009) / \text{ns}$, the long-lived state is heavier than the short-lived state [4].

The time evolution of the $|B^0_{L,H}\rangle$ states are defined by the two equations

$$|B^0_H(t)\rangle = e^{-i\mu_H t} |B^0_H\rangle = e^{-im_H t} e^{-\Gamma_H t/2} |B^0_H\rangle$$

$$|B^0_L(t)\rangle = e^{-i\mu_L t} |B^0_L\rangle = e^{-im_L t} e^{-\Gamma_L t/2} |B^0_L\rangle.$$ 

(1.4)

Figure 1.2: The two second order weak interaction $B^0 - \overline{B}^0$ mixing box diagrams.
By substituting in equation (1.1) the time evolution of the \(|B^0\rangle\) and \(|\bar{B}^0\rangle\) are found to be

\[
|B^0(t)\rangle = g_+(t) |B^0\rangle + \frac{q}{p} g_-(t) |\bar{B}^0\rangle
\]
\[
|\bar{B}^0(t)\rangle = \frac{p}{q} g_-(t) |B^0\rangle + g_+(t) |\bar{B}^0\rangle
\]

(1.5)

where

\[
g_\pm(t) = \frac{1}{2} \left[ e^{-i\mu_H t} \pm e^{-i\mu_L t} \right]
\]
\[
|g_\pm(t)|^2 = \frac{1}{4} \left[ e^{-\Gamma_H t} + e^{-\Gamma_L t} \pm 2e^{-\Gamma t} \cos(\Delta m t) \right]
\]

(1.6)

and $\Gamma = \frac{1}{2}(\Gamma_H + \Gamma_L)$. The third term in equation (1.6) defines the nature of the oscillations and the oscillation parameter, $x = \frac{\Delta m}{\Gamma}$, defines the magnitude of the oscillation frequency between the two states. In the $B^0_d$ system, $x_d = 0.774 \pm 0.008$, and in the $B^0_s$ system, $x_s = 26.2 \pm 0.5$ \cite{4}. The much larger value of $x$ in the $B^0_s$ system leads to a much more rapid oscillation between $B^0_s$ and $\bar{B}^0_s$. This is due to the larger mass difference, $\Delta m_s = (17.77 \pm 0.12) / \text{ps}$, than in the $B^0_d$ system, $\Delta m_d = (507 \pm 4) / \text{ns}$ \cite{4}.

### 1.3.2 Neutral B-meson Decay

When considering the decays of a $B^0$ or $\bar{B}^0$ into a final state $f$ there are two independent decay amplitudes

\[
A_f \equiv \langle f | T | B^0 \rangle
\]
\[
\bar{A}_f \equiv \langle f | T | \bar{B}^0 \rangle
\]

(1.7)

where $T$ is the transition matrix. The magnitudes of the two decay amplitudes, $|A_f|$ and $|\bar{A}_f|$, and the magnitude of the mixing parameters, $|p/q|$, are all observables along with the complex parameter, $\lambda_f$:

\[
\lambda_f \equiv \frac{q}{p} \frac{\bar{A}_f}{A_f}
\]

(1.8)

where $\bar{f}$ is the $\mathcal{CP}$-conjugate of $f$.

Using equation (1.6) it is possible to derive the probability per time interval at which a state that is initially a $B^0$ ($\bar{B}^0$) decays to the final state $f$ or $\bar{f}$ during the time interval
Charge Parity Violation in the B-meson System

These rates are given by

\[ \Gamma(B_0^0(t) \rightarrow f) = |A_f|^2 \left[ |g_+(t)|^2 + |g_-(t)|^2 + 2 \Re \left\{ \lambda_f g_+(t) g_-(t) \right\} \right] \] (1.9a)

\[ \Gamma(B_0^0(t) \rightarrow \bar{f}) = |\bar{A}_f|^2 \left[ |g_-(t)|^2 + |g_+(t)|^2 + 2 \Re \left\{ \bar{\lambda}_f g_+(t) g_-(t) \right\} \right] \] (1.9b)

\[ \Gamma(\bar{B}_0^0(t) \rightarrow f) = |\bar{A}_f|^2 \left[ |g_-(t)|^2 + |g_+(t)|^2 + 2 \Re \left\{ \bar{\lambda}_f g_+(t) g_-(t) \right\} \right] \] (1.9c)

\[ \Gamma(\bar{B}_0^0(t) \rightarrow \bar{f}) = |\bar{A}_f|^2 \left[ |g_+(t)|^2 + |\bar{\lambda}_f|^2 |g_-(t)|^2 + 2 \Re \left\{ \bar{\lambda}_f g_+(t) g_-(t) \right\} \right]. \] (1.9d)

With the formalisms of the mixing and decay of B-mesons defined above, the following section discusses the effects of CP-violation and how it manifests in these equations.

1.3.3 The Types of CP-violation

There are three types of CP-violation

- CP-violation in the decay amplitudes, known as direct CP-violation;
- CP-violation in the mixing, known as indirect CP-violation; and
- CP-violation in a phase mismatch between the mixing parameters and the decay amplitudes, known as interference CP-violation.

CP-violation in B-meson Decay

Direct CP-violation manifests as a difference in the decay rate of a particle to a final state \( (B^0 \rightarrow f) \) and the decay rate of the charge conjugate process \( (\bar{B}^0 \rightarrow \bar{f}) \). The decay amplitudes, defined in equation (1.7), contain contributions from both the weak interaction via quark mixing and the strong force via final state interactions. These amplitudes can be expressed as the sum of the different topological amplitudes decomposed into strong and weak phases, \( \delta \) and \( \phi \), respectively

\[ A = \sum_n A_n e^{i \phi_n} e^{i \delta_n} \quad \text{and} \quad \bar{A} = \sum_m \bar{A}_m e^{-i \phi_m} e^{i \bar{\delta}_m}. \] (1.10)

The weak phase is CP odd and changes sign between the transition amplitudes, and the strong phase is CP even and does not change. CP-violation arises when there is an interference between the weak and strong phases in two or more decay amplitudes.
resulting in the condition

\[ \frac{\bar{A}}{A} \neq 1. \]  
(1.11)

For charged B-mesons direct $CP$-violation is the only mechanism for $CP$-violation to arise. The direct $CP$-asymmetry, $A_{CP}^{dir}$, can be expressed via the difference in the rates between the $B^+$ and $B^-$ decays normalised to the total rate

\[ A_{CP}^{dir} = \frac{\Gamma(B^- \to f^-) - \Gamma(B^+ \to f^+)}{\Gamma(B^- \to f^-) + \Gamma(B^+ \to f^+)}. \]  
(1.12)

The existence of direct $CP$-violation in the kaon system was confirmed through the measurement of $\mathcal{R}_{K}\{\epsilon'/\epsilon\}$ \[17,18\] and has since been observed in the decay of B-mesons e.g. $A_{CP}^{dir}(B_d^0 \to K^+\pi^-)$ has been measured at LHCb as $-0.074 \pm 0.033 \pm 0.008$ \[19\] which is compatible with the current world average $-0.098 \pm 0.013$ \[4\].

$CP$-violation in B-meson Mixing

Indirect $CP$-violation occurs in neutral B-mesons and unlike direct $CP$-violation is independent of the final state. It arises when there is a difference in the mixing rates between $B^0 \to \bar{B}^0$ and $\bar{B}^0 \to B^0$. The condition for indirect $CP$-violation is

\[ \frac{q}{p} \neq 1 \]  
(1.13)

as seen in equation \[1.5\].

Indirect $CP$-violation in the B-meson system is small in the Standard Model due in part to the small value of the difference in decay rates of $B^0$ and $\bar{B}^0$. The $CP$-asymmetry in the mixing has been measured from semileptonic decays, $A_{sl}$, and has a current average of \[4\]

\[ A_{sl} = \frac{\Gamma(\bar{B}^0 \to \ell^+\nu_\ell X) - \Gamma(B^0 \to \ell^-\bar{\nu}_\ell X)}{\Gamma(\bar{B}^0 \to \ell^+\nu_\ell X) + \Gamma(B^0 \to \ell^-\bar{\nu}_\ell X)} = \begin{cases} -0.0005 \pm 0.0056 & B_d^0, \\ -0.0049 \pm 0.0038 & B_s^0. \end{cases} \]  
(1.14)
$\mathcal{CP}$-violation in the Interference Between B-meson Mixing and Decay

Generally $\mathcal{CP}$ is violated if $\lambda_f \neq \lambda_{\bar{f}}$ (see equation (1.8) for the definition of $\lambda_f$). So far two cases have been described, direct and indirect $\mathcal{CP}$-violation, which satisfy this condition. It is also possible for $\mathcal{CP}$-violation to occur when $|q/p| = 1$ and $|A_f/A_{\bar{f}}| = 1$ if

$$\Im \{\lambda_f\} \neq 0.$$  \hspace{1cm} (1.15)

This is known as interference $\mathcal{CP}$-violation.

The interference $\mathcal{CP}$-violation phase can be extracted by considering the time dependent rates for the transitions from a $B_0$ or $B_0^\ast$ to a final state $f$, obtained by writing equations (1.9a) and (1.9c) in the form

$$\Gamma(B_0(t) \to f) = \frac{|A_f|^2}{2} e^{-\Gamma t} [I_+ (t) + I_- (t)]$$  \hspace{1cm} (1.16)

$$\Gamma(\bar{B}_0(t) \to f) = \frac{|\bar{A}_f|^2}{2|\lambda_f|^2} e^{-\Gamma t} [I_+ (t) - I_- (t)]$$  \hspace{1cm} (1.17)

where the interference terms, $I_{\pm} (t)$, are

$$I_+ (t) = \left(1 + |\lambda_f|^2\right) \cosh \left(\frac{\Delta \Gamma t}{2}\right) - 2 \Re \{\lambda_f\} \sinh \left(\frac{\Delta \Gamma t}{2}\right)$$  \hspace{1cm} (1.18)

$$I_- (t) = \left(1 - |\lambda_f|^2\right) \cos (\Delta m t) + 2 \Im \{\lambda_f\} \sin (\Delta m t).$$  \hspace{1cm} (1.19)

The rates to the charge conjugate final states, $\bar{f}$, are obtained by substituting $\bar{\lambda}_f$ for $\lambda_f$ above. A time-dependent $\mathcal{CP}$-asymmetry, $A_{\mathcal{CP}}(t)$, can now be defined in terms of $\lambda_f$

$$A_{\mathcal{CP}}(t) = \frac{\Gamma(B_0(t) \to f) - \Gamma(\bar{B}_0(t) \to f)}{\Gamma(B_0(t) \to f) + \Gamma(\bar{B}_0(t) \to f)}$$

$$= \frac{(1 - |\lambda_f|^2) \cos (\Delta m t) + 2 \Im \{\lambda_f\} \sin (\Delta m t)}{(1 + |\lambda_f|^2) \cosh (\Delta \Gamma t/2) - 2 \Re \{\lambda_f\} \sinh (\Delta \Gamma t/2)}$$ \hspace{1cm} (1.20)

where the indirect $\mathcal{CP}$-violation is assumed to be small, i.e. $1 - |q/p| \sim 0$, which is true for the B-meson system. The equation is further simplified by assuming $\Delta \Gamma \sim 0$, which is true in the $B_0$ system. The coefficients of the cosine and sine terms, representing the direct $\mathcal{CP}$-violation and the interference $\mathcal{CP}$-violation, are labelled as $C$ and $S$ respectively.
and are given by
\[
C = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \quad \text{and} \quad S = \frac{2 \text{Im}\{\lambda_f\}}{1 + |\lambda_f|^2}.
\] (1.22)

A final simplification is obtained when the final state \( f \) is a \( \mathcal{CP} \) eigenstate. In this case \( |\lambda_f| = 1 \), \( C \) is zero and \( A_{\mathcal{CP}}(t) \) becomes
\[
A_{\mathcal{CP}}(t) = S \sin \Delta m t = \text{Im}\{\lambda_f\} \sin (\Delta m t).
\] (1.23)

In the Standard Model \( C \) is predicted to be zero for \( B^0_d \rightarrow J/\psi K^0_s \) and has been measured to be \( 0.018 \pm 0.021 \text{(stat.)} \pm 0.014 \text{(sys.)} \) by the Belle collaboration \[20\] and \( 0.024 \pm 0.020 \text{(stat.)} \pm 0.016 \text{(sys.)} \) by the BABAR collaboration \[21\], where the first errors are statistical (stat.) and the second systematic (sys.). In the same measurements the Belle and BABAR collaborations measured \( S = 0.642 \pm 0.031 \text{(stat.)} \pm 0.017 \text{(sys.)} \) \[20\] and \( S = 0.687 \pm 0.028 \text{(stat.)} \pm 0.012 \text{(sys.)} \) \[21\] respectively. This shows that interference \( \mathcal{CP} \)-violation is much larger than indirect \( \mathcal{CP} \)-violation in the \( B^0_d \) system.

### 1.4 \( \mathcal{CP} \)-violation in the Standard Model

The \( \mathcal{C} \) and \( \mathcal{P} \) symmetries are preserved naturally in many gauges theories, notably massless Quantum Chromodynamics (QCD) and Quantum Electrodynamics (QED), and even theories that are explicitly designed to violate parity are still \( \mathcal{CP} \) invariant. For example, the chiral gauge theory whose Lagrangian
\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}_L i\sigma D\psi_L
\] (1.24)
only allows left-handed Weyl fermions, \( \psi_L \), to interact with the gauge bosons, as the right-handed Weyl fermions are sterile, is \( \mathcal{CP} \) invariant. This implies that \( \mathcal{CP} \) is a natural symmetry of massless gauge theories and it is only possible to introduce \( \mathcal{CP} \)-violation into theories where mass has been introduced \[22\].

The Higgs mechanism of the Standard Model introduces a scalar field which couples to the fermion fields and gauge boson fields. The scalar (Higgs) sector generates a non-zero vacuum expectation value which spontaneously breaks the gauge invariance producing
the mass terms. The Lagrangian of the Standard Model is then written as

$$L = L_{\text{gauge}}(\psi_L, \psi_R, W, \phi) + L_{\text{Higgs}}(\phi) + L_{\text{Yukawa}}(\psi_L, \psi_R, \phi)$$

(1.25)

where the first term, $L_{\text{gauge}}$, is the kinetic term of the fields involved i.e. the left- and right-handed fermion fields, $\psi_L$ and $\psi_R$, the gauge bosons, $W$, and the scalar (Higgs) field, $\phi$, as well as their gauge interactions. The second term, $L_{\text{Higgs}}$, is the potential felt by the scalar fields, it is responsible for the non-zero value of the vacuum expectation value and gives rise to spontaneous symmetry breaking. The third, and final, term, $L_{\text{Yukawa}}$, describes the interaction between the fermionic and scalar fields, which after spontaneous symmetry breaking generates the fermion mass terms. In the Standard Model the gauge and Higgs terms are automatically $\mathcal{CP}$-invariant and it is therefore the Yukawa sector which must be the source of $\mathcal{CP}$-violation. The Yukawa term is

$$L_{\text{Yukawa}} = -\lambda^d_{ij} \bar{Q}^i_L \cdot \Phi d^j_R - (\lambda^d_{ij})^* \bar{d}^j_R \Phi^\dagger \cdot Q^i_L + \ldots$$

(1.26)

where the $i, j$ indices are generation labels, $Q^i_L$ is the SU(2)$_L$ quark doublet, $(u^i_L, d^i_L)$, $\Phi$ represents the Higgs doublet, $(\phi^+, \phi^0)$ and the $\ldots$ represents the up-type quarks. After a $\mathcal{CP}$ transformation the left (L) and right (R) indices, and the particle and anti-particle indices, are exchanged, resulting in

$$\mathcal{CP} : \bar{Q}^i_L \cdot \Phi d^j_R \rightarrow \bar{d}^j_R \Phi^\dagger \cdot Q^i_L.$$ 

(1.27)

If $\lambda$ are real then the Yukawa term is $\mathcal{CP}$-invariant. $\mathcal{CP}$-violation takes place in the scalar sector of the Standard Model and is introduced through complex Yukawa couplings.

The Yukawa couplings form a $3 \times 3$ mass matrix for the up and down type quarks, $u^{(\text{weak})}_i$ and $d^{(\text{weak})}_i$ respectively, which are eigenstates of the weak interaction. By diagonalising these matrices into the basis of the mass eigenstates rotation matrices, $U^{(u,d)}$, are defined

$$u^{(\text{weak})}_i = U^{(u)}_{ij} u^{(\text{mass})}_j \quad \text{and} \quad d^{(\text{weak})}_i = U^{(d)}_{ij} d^{(\text{mass})}_j.$$ 

(1.28)

Neutral weak interactions, $\bar{u}^{(\text{weak})}_i u^{(\text{weak})}_i \equiv \bar{u}^{(\text{mass})}_i u^{(\text{mass})}_i$, are unaffected as the transformation is unitary but charged weak interactions are

$$\bar{u}^{(\text{weak})}_i d^{(\text{weak})}_i \rightarrow \bar{u}^{(\text{mass})}_i \left( U^{(u)} \right)^\dagger U^{(d)} d^{(\text{mass})}_i.$$ 

(1.29)
The unitary matrix, $V_{\text{CKM}} \equiv (U^{(u)})^\dagger U^{(d)}$, provides the strength of the couplings between the up and down type quarks. This matrix is known as the Cabibbo-Kobayashi-Maskawa (CKM) matrix and it provides a complex phase which is the source of $CP$-violation in the Standard Model [23]. The $V_{\text{CKM}}$ matrix is summarised in the next section.

### 1.5 The CKM Matrix

The $V_{\text{CKM}}$ matrix, $V_{\text{CKM}}$, is formed from the $2 \times 2 CP$ invariant Cabibbo mixing matrix [24] plus a third generation to form the $3 \times 3 CP$-violating matrix [23]. The elements of the matrix, $V_{ij}$, represent the strength of the coupling between an up and a down type quark. The $\text{CKM}$ matrix is defined as

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

and the equivalent matrix for the anti-quark couplings is defined by the complex conjugate of the $\text{CKM}$ matrix elements, $V_{ij}^\ast$.

The $\text{CKM}$ matrix can be parametrised by three Euler angles of rotation and six complex phases, as it is a $3 \times 3$ unitary matrix. The quark fields of the Standard Model Lagrangian are invariant under rotations of the form $q \rightarrow q e^{i\phi}$ and five of the complex phases are “rotated away” by a redefinition the quark fields [22]. The remaining physical phase is the source of $CP$-violation in the Standard Model. The three Euler angles and the complex phase must be measured by experiment and are fundamental constants of nature. The Chau-Keung mixing angle representation [25] has become the convention for the expression of the $\text{CKM}$ matrix in terms of the three mixing angles, $\theta_{12}$, $\theta_{13}$ and
\[ \theta_{23}, \text{ and the complex phase, } \delta, \]

\[
\mathbf{V}_{\text{CKM}} = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & 0 & s_{13} e^{-i\delta} \\
0 & 1 & 0 \\
-s_{13} e^{-i\delta} & 0 & c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

(1.31a)

\[
= \begin{pmatrix}
c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\
-s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\
s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13}
\end{pmatrix}
\]

(1.31b)

where \( c_{ij} = \cos(\theta_{ij}) \) and \( s_{ij} = \sin(\theta_{ij}) \). The Chau-Keung formalisation shows the extension from two generations to three (the Cabibbo matrix is highlighted in equation (1.31a)).

A popular approximate parameterisation of the \( \text{CKM} \) matrix is the Wolfenstein parameterisation which is based on the strength of the couplings [26]. This allows the hierarchy of the quark strengths to be clearly expressed.

The Wolfenstein parameterisation defines four independent parameters

\[
\lambda \equiv s_{12} \equiv \sin(\theta_c), \quad A \equiv \frac{s_{23}}{s_{12}^2}, \quad \rho \equiv \frac{s_{13} \cos(\delta)}{s_{12} s_{23}} \quad \text{and} \quad \eta \equiv \frac{s_{13} \sin(\delta)}{s_{12} s_{23}}
\]

(1.32)

where \( \theta_c \) (\( \sin(\theta_c) = 0.2252 \pm 0.0009 \) [4]) is the Cabibbo angle [24]. The \( \text{CKM} \) matrix expressed in terms of \( \lambda \) is given by

\[
\mathbf{V}_{\text{CKM}}^{(3)} = \begin{pmatrix}
1 - \frac{1}{2} \lambda^2 & \lambda & A \lambda^3 (\rho - i\eta) \\
-\lambda & 1 - \frac{1}{2} \lambda^2 & A \lambda^2 \\
A \lambda^3 (1 - \rho - i\eta) & -A \lambda^2 & 1
\end{pmatrix} + \mathcal{O}(\lambda^4)
\]

(1.33)
to order $\lambda^3$ and

$$V^{(5)}_{\text{CKM}} = \begin{pmatrix}
1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda + \frac{1}{2}A^2\lambda^5(1 - 2(\rho + i\eta)) & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\
A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 + A\lambda^4\left(\frac{1}{2} - \rho - i\eta\right) & 1 - \frac{1}{2}A^2\lambda^4 \\
\end{pmatrix} + O(\lambda^6) \quad (1.34)$$

(1.36)

to order $\lambda^5$, where $\bar{\rho} = \rho(1 - \lambda^2/2)$ and $\bar{\eta} = \eta(1 - \lambda^2/2)$ [16].

### 1.6 The Unitarity Triangle

The unitarity of the [CKM] matrix can be expressed through six complex orthogonality conditions

\begin{align*}
\text{(db)} & \quad V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \\
& \quad O(\lambda^3) + O(\lambda^3) + O(\lambda^3) \\
\text{(sb)} & \quad V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0 \\
& \quad O(\lambda^4) + O(\lambda^2) + O(\lambda^2) \\
\text{(ds)} & \quad V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0 \\
& \quad O(\lambda) + O(\lambda) + O(\lambda^5) \\
\end{align*}

and

\begin{align*}
\text{(ut)} & \quad V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* = 0 \\
& \quad O(\lambda^3) + O(\lambda^3) + O(\lambda^3) \\
\text{(cb)} & \quad V_{cd}V_{td}^* + V_{cs}V_{ts}^* + V_{cb}V_{tb}^* = 0 \\
& \quad O(\lambda^4) + O(\lambda^2) + O(\lambda^2) \\
\text{(uc)} & \quad V_{ud}V_{cd}^* + V_{us}V_{cs}^* + V_{ub}V_{cb}^* = 0 \\
& \quad O(\lambda) + O(\lambda) + O(\lambda^5) \\
\end{align*}
where the quark pair in parentheses indicate which rows or columns are used in the calculation of the inner product and the $O(\lambda^n)$ indicated the magnitude of the term. These six conditions can each be represented as a triangle in the complex plane where each triangle has the same area, $J_{CP}/2$, due to the single complex parameter. The Jarlskog parameter $J_{CP} = 2.96^{+0.18}_{-0.17} \times 10^{-5}$, is a measure of the amount of $CP$-violation in the Standard Model and is required to be non-zero for $CP$-violation to be present.

The two triangles, (db) and (ut), have sides that are of comparable magnitude and are the subject of experimental investigation to confirm that the triangle is indeed a triangle. The other four triangles all have sides with unequal magnitudes making them difficult to measure experimentally. By choosing a phase convention such that $V_{cd}V_{cb}^*$ is real, normalising (db) and (ut) by $\left| V_{cd}V_{cb}^* \right| = \lambda^3$, the two triangles, (db) and (ut), are identical to $O(\lambda^3)$ but differ at $O(\lambda^5)$. These two triangles are presented in Figure 1.3.

The triangle in Figure 1.3(a) is known as the Unitarity Triangle and its angles are defined as

$$\alpha \equiv \phi_2 \equiv \arg \left( -\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right) = \arg \left( -\frac{1 - \rho - i\eta}{\rho + i\eta} \right)$$

$$\beta \equiv \phi_1 \equiv \arg \left( -\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right) = \arg \left( \frac{1}{1 - \rho - i\eta} \right)$$

$$\gamma \equiv \phi_3 \equiv \arg \left( -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right) = \arg (\rho + i\eta).$$

The inclusion of the $O(\lambda^5)$ terms introduces a complex phase in $V_{ts}$ such that the relationship between $\beta$ and $\gamma$ from Figure 1.3(a) and the equivalent angles, $\beta'$ and $\gamma'$, in Figure 1.3(b) is a difference of $\beta_s$

$$\beta' \equiv \arg \left( -\frac{V_{ts}V_{us}^*}{V_{td}V_{ud}^*} \right) = \beta + \beta_s$$

$$\gamma' \equiv \arg \left( -\frac{V_{tb}V_{ub}^*}{V_{ts}V_{us}^*} \right) = \gamma - \beta_s$$

$$\beta_s \equiv \arg \left( \frac{V_{ts}V_{tb}}{V_{cs}V_{cb}} \right).$$
Figure 1.3: The unitarity triangle representations of the conditions (ds) and (ut). The complex side lengths are expressed in terms of $V_{CKM}$ elements and $\lambda$. 
By taking the complex argument of the $V_{\text{CKM}}$ matrix and ignoring the small correction factor to $V_{cd}$ the following approximation is arrived at

$$\arg(V_{\text{CKM}}) = \begin{pmatrix} 0 & 0 & -\gamma \\ 0 & 0 & 0 \\ -\beta & \beta_s + \pi & 0 \end{pmatrix}.$$  \hspace{1cm} (1.39)

By comparison with third and fifth order Wolfenstein parameterisations, equations [1.33] and [1.34], of the CKM it is seen that

$$\beta = \tan^{-1}\left( \frac{\eta}{1 - \rho} \right)$$ \hspace{1cm} (1.40a)

$$\gamma = \tan^{-1}\left( \frac{\eta}{\rho} \right)$$ \hspace{1cm} (1.40b)

$$\beta_s = \tan^{-1}\left( \frac{\lambda^2 \eta}{1 + \lambda^2 (\rho - 1/2)} \right) \approx \lambda^2 \eta.$$ \hspace{1cm} (1.40c)

From these equations it is apparent that CP-violation enters $\beta$ and $\gamma$ at $O(\lambda^3)$, and at $O(\lambda^5)$ in $\beta_s$ for the $t \to s$ quark transition.

### 1.7 Constraining the CKM Parameters

The study of the quark flavours (often referred to as flavour physics) in the Standard Model is based upon the CKM matrix, however the CKM matrix angles cannot be predicted from theory and must be constrained experimentally. The angles can be measured indirectly via measurements of the matrix elements, $V_{ij}$, or they can be measured directly through CP-violating processes. The goal of flavour physics experiments is to overconstrain the unitarity triangle and to search for signs of new physics which could, for example, enter as new virtual particle contributions in processes involving quark loops, such as in the $B^0 - \bar{B}^0$ mixing diagrams in Figure 1.2.
1.7.1 Indirect Measurements

An overview of the current methods of measuring the elements of the \( \text{CKM} \) matrix, \( V_{ij} \), is given in the Particle Data Group (PDG) review of the \( \text{CKM} \) quark mixing matrix \cite{PDG}. In brief

- \( |V_{ud}| \) is measured using nuclear beta decay;
- \( |V_{us}| \) is measured using the semi-leptonic kaon decays;
- \( |V_{ub}| \) is measured using inclusive \( B \to X_u \ell \nu \) decays;
- \( |V_{cd}| \) is measured using semi-leptonic \( D \) decays or neutrino and anti-neutrino interactions;
- \( |V_{cs}| \) is measured using semi-leptonic \( D \) decays or leptonic \( D_s \) decays e.g. \( D \to K \ell \nu \) and \( D^+_s \to \mu^+ \nu_\mu \);
- \( |V_{cb}| \) is measured using exclusive and inclusive semi-leptonic \( B \) decays to charm;
- \( |V_{td}| \) and \( |V_{ts}| \) are measured from \( B^0 - \bar{B}^0 \) oscillation box diagrams, and rare \( K \) and \( B \) decays mediated by loop diagrams; and
- \( |V_{tb}| \) is measured using top quark decays.

Using these measurements the magnitude of the \( \text{CKM} \) matrix elements are \cite{PDG}

\[
|V_{\text{CKM}}| = \begin{pmatrix}
0.97425 \pm 0.00018 & 0.22543 \pm 0.00077 & 0.00354^{+0.00016}_{-0.00014} \\
0.22529 \pm 0.00077 & 0.97342^{+0.00021}_{-0.00019} & 0.04128^{+0.00058}_{-0.00029} \\
0.00858^{+0.00029}_{-0.00034} & 0.04054^{+0.00057}_{-0.000129} & 0.999141^{+0.000053}_{-0.000024}
\end{pmatrix}.
\]

These values are used to constrain the apex of the unitarity triangle, \((\bar{\rho}, \bar{\eta})\), and hence the internal angles of the unitarity triangle. The measurement of \( |V_{ub}| \) using semi-leptonic b-decays places a circular constraint centre at \((0, 0)\). A second circular band, this time centred around \((1, 0)\), comes from \( \Delta m_d \) and \( \Delta m_s \) measurements in \( B^0 - \bar{B}^0 \) oscillations. A final constraint comes from \( \epsilon_K = (2.01^{+0.59}_{-0.66}) \times 10^{-3} \) \cite{PDG}, a measure of the amount of \( C\bar{P} \)-violation in the kaon system, which is a hyperbola passing through the triangle apex. Figure \ref{fig:unitarity_triangle} presents the results of these constraints as 95\% confidence limit bands.
1.7.2 Direct Measurements

The internal angles of the unitarity triangle and the rotation angle, $\beta_s$, are also measured directly using $b$-decays. Figure 1.5 presents the current experimental constraints on the internal angles of the unitarity triangle using only the input from direct measurements. An overview of the current experimental constraints on the angles $\alpha$, $\beta$, $\gamma$ and $\beta_s$ follows.
Direct Measurement of $\beta$

A direct measurement of the angle $\beta$ can be extracted from the interference of the decays of $B^0$ and $B^0$ to a $CP$ eigenstate. In a decay where equation (1.23) holds i.e. $A_f = |\tilde{A}_f|$ and $C = 0$ leaving only the sine term, then using the approximation

$$\frac{q}{p} = \frac{V_{tb}^* V_{td}}{V_{tb}^* V_{td}} = e^{-i2\phi + O(\lambda^4)}$$

(1.41)

leads to the coefficient of the sine term being $S = -\eta_f \sin (2\phi)$. Here $\eta_f$ is the $CP$ eigenvalue of $f$ and $2\phi$ is the phase difference between the $B^0 \to f$ and $\bar{B}^0 \to f$ decay paths.

The “golden mode” for the measurement of the angle $\beta$ is the decay $B^0_d \to J/\psi K_S^0$ where the $b \to scc$ transitions are theoretically clean. These decays measure $S = \eta_f \sin (2\beta)$, with $\eta_f = -1$, and the latest measurements are from the Belle [20] and BABAR [21] collaborations. The current best fit value of $\sin (2\beta)$ is [28]

$$\sin (2\beta) = 0.689^{+0.023}_{-0.021}$$

$$\beta = (21.76^{+0.92}_{-0.82})^\circ$$

Direct Measurement of $\alpha$

The measurement of the angle $\alpha$ is performed using time dependent $CP$-asymmetries of $b \to u\bar{u}d$ dominated decay modes. A difficulty in using these modes arises as there is pollution from penguin contributions due to the $b \to d$ penguin amplitude having a different CKM phase to the $b \to u\bar{u}d$ tree level amplitudes and their magnitudes being of the same order, $\lambda$ [29]. Currently the angle $\alpha$ has been measured using $B \to \pi\pi$, $\rho\pi$ and $\rho\rho$ decays.

When using $B \to \pi\pi$ to extract the angle $\alpha$ large penguin contributions mean that $S$ from equation (1.23) is given by

$$S_{\pi^+\pi^-} = \sqrt{1 - C_{\pi^+\pi^-}^2} \sin (2\alpha + 2\Delta\alpha)$$

(1.42)

where $2\Delta\alpha$ is the phase difference between $e^{2i\gamma} \tilde{A}_{\pi^+\pi^-}$ and $A_{\pi^+\pi^-}$. The value of $\Delta\alpha$ can be extracted using an isospin analysis of $B^0 \to \pi^+\pi^-$, $B^0 \to \pi^0\pi^0$ and $B^+ \to \pi^+\pi^0$ [30].
There are 16 possible solutions for the angle $\alpha$ extracted using this method. The Belle collaboration has excluded the angle $\alpha$ in the range $11^\circ$ to $79^\circ$ at the 95% confidence level [31] and the BaBar collaboration has excluded the range $23^\circ$ and $67^\circ$ at the 90% confidence level [32].

The extraction of the angle $\alpha$ from $B \to \rho \rho$, from the Belle collaboration $\alpha = (91.7 \pm 14.9)^\circ$ [33] and the BaBar collaboration $\alpha = (92.4^{+6.0}_{-6.5})^\circ$ [34], uses a similar method to $B \to \pi \pi$ and benefits from a larger branching ratio and a larger penguin amplitude to tree amplitude ratio compared to this decay.

The final method discussed here uses a time dependent Dalitz analysis [35] of the $B^0 \to \pi^+\pi^-\pi^0$ which is dominated by the decay $B^0 \to \rho \pi$. Using this method a value for the angle $\alpha$ can be extracted which has only a single ambiguity, ($\alpha \to \alpha + \pi$), and analyses to measure the angle have been performed by the Belle collaboration, $68^\circ < \alpha < 95^\circ$ at the 68.3% confidence level [36], and the BaBar collaboration, $\alpha = (87^{+45}_{-13})^\circ$ at the 68.3% confidence level [37]. In both cases the mirror solution at $\alpha + 180^\circ$ have been ignored.

The current best fit value gives [28]

$$\alpha = (91.0 \pm 3.9)^\circ.$$  

Direct Measurement of $\gamma$

The measurement of the angle $\gamma$ differs from the other two as it can be measured using pure tree processes without any contributions from penguin diagrams. However, since the branching ratios of the decays involved are small, $\mathcal{O}(10^{-6})$, it is the least well constrained angle of the unitarity triangle.

The angle $\gamma$ is measured from the relative phases of the transitions $B \to D^0 K$ ($b \to c\pi s$) and $B \to \bar{D}^0 K$ ($b \to u\pi s$) when the $D^0$ and $\bar{D}^0$ decay to the same final state. From these decays the $B$ and $D$ amplitudes, the relative strong phases and the weak phase $\gamma$ can be extracted.

The Gronau, London and Wyler (GLW) method [38,39] considers D decays to $CP$ eigenstates but suffers from the ratio of the amplitudes

$$r_B = \frac{A(B^- \to \bar{D}^0 K^-)}{A(B^- \to D^0 K^-)} = \frac{A(B^+ \to \bar{D}^0 K^+)}{A(B^+ \to D^0 K^+)}$$ (1.43)
being very small. The Atwood, Dunietz and Soni (ADS) method \[40,41\] considers decays to non-$CP$ eigenstates and the amplitudes are of a similar magnitude. These two methods have been used to determine the values of the interference asymmetries and the latest measurements can be found in reference \[42\].

A method proposed by Giri, Grossman, Soffer and Zupan (GGSZ) \[43\] uses a Dalitz plot to extract $\gamma$ from multibody $D$ decays such as $D \rightarrow K_0^0\pi^+\pi^-$ and $D \rightarrow K_0^0K^+K^-$. The method considers both decays to $CP$ and flavour eigenstates, and analyses all the possible decay modes to extract the angle $\gamma$, the ratio of the interference amplitudes and the strong phase. The most precise determination of the angle $\gamma$ is found using this method, it has the advantage of much larger branching ratios (hence statistics) and only a two fold ambiguity ($\gamma \rightarrow \gamma + \pi$) over the GLW and ADS methods. Currently the Belle collaboration has extracted a value of $\gamma = (78.4^{+10.8}_{-11.6}\,(\text{stat.}) \pm 3.6\,(\text{syst.}) \pm 8.9\,(\text{model}))^\circ$ \[44\] and the BABAR collaboration a value of $\gamma = (68 \pm 14\,(\text{stat.}) \pm 4\,(\text{syst.}) \pm 3\,(\text{model}))^\circ$ \[45\]; the model error comes from the model of the different $D$ decays modes.

A combination of the results from these three methods gives a fitted value of \[28\]

$$\gamma = (67.2 \pm 3.9)^\circ.$$  

Another time dependent method to extract the angle $\gamma$ uses $B_0^s \rightarrow D_s^\mp K^\pm$ decays \[46\]. This channel is being studied by the LHCb collaboration and an initial measurement of the branching ratio has been made \[47\]

$$B(B_0^s \rightarrow D_s^\mp K^\pm) = \left(1.97 \pm 0.18\,(\text{stat.})^{+0.19}_{-0.20}\,(\text{syst.})^{+0.11}_{-0.10}(f_s/f_d)\right) \times 10^{-4}$$

where $f_s/f_d$ is the $B^0_s$ and $B_0^s$ production fraction ratio. A measurement of the angle $\gamma$ in this channel requires as an input a measurement of the $B^0_s$ mixing phase, $\beta_s$, which is described in the next section.

**Direct Measurement of $\beta_s$**

The decay $B^0_s \rightarrow J/\psi(\mu^+\mu^-)\phi(\rightarrow K^+K^-)$ is to a flavour non-specific final state that can be used to measure the $CP$ violating phase $\beta_s$. In the Standard Model the $CP$-violating phase in this decay is $\phi^{J/\psi\phi}_{B^0_s} \approx -2\beta_s$ where $\beta_s = \arg\left(V_{ts}V_{tb}^*/V_{cs}V_{cb}^*\right)$ \[51\]. This parameter is expected to be small in the Standard Model, $\beta_s = \eta \lambda^2 \approx O(0.03)$ \[51\] and hence a large $CP$-asymmetry in this decay channel is a sign of new physics \[52\].
Another possibility is to use the decay $B_s^0 \rightarrow J/\psi(\mu^+\mu^-)f_0(980)(\pi^+\pi^-)$ which proceeds via the same quark transition as $B_s^0 \rightarrow J/\psi\phi$ and it is a decay to a $CP$ eigenstate. This decay mode was first observed by the LHCb collaboration [53] and a first measurement of $\beta_s$ using this decay mode is given in reference [54]. This measurement is not yet included in the global average.

The value of $\beta_s$ has been measured most recently by the CDF, DO and LHCb collaborations [48–50]. These analyses measure several quantities including $\phi_s^{J/\psi\phi} = -2\beta_s$ and the difference between the partial width of the heavy and light $B_s^0$ meson states, $\Delta\Gamma_s$, which could also be affected by new physics contributions. A comparison between the latest results of the three experiments in the $\Delta\Gamma_s$$\phi_s^{J/\psi\phi}$ plane is shown in Figure 1.6, where LHCb measures

$$\phi_s^{J/\psi\phi} = (0.13 \pm 0.18(\text{stat.}) \pm 0.07(\text{sys.})) \text{ rad.}$$ (1.44)

These results are all consistent with the Standard Model and the current fitted result for $\beta_s$ is [28]

$$\beta_s = \left(0.01817^{+0.00087}_{-0.00083}\right) \text{ rad.}$$
1.7.3 Global Constraints from Direct and Indirect Measurements

A combined fit of the unitarity triangle has been performed, including both the direct and indirect measurements, and is presented in Figure 1.7. From this analysis the apex of the triangle has been fitted as

\[ \bar{\rho} = 0.144 \pm 0.025 \quad \text{and} \quad \bar{\eta} = 0.342^{+0.016}_{-0.015}. \]

The combination of the direct and indirect measurements is consistent with the Standard Model model, however they are not yet precise enough to rule out new physics scenarios entering via loop processes. The largest deviations between the Standard Model predicted values and the measured values occurs for the branching ratio of \( B^+ \to \tau^+\nu_\tau \).
and \( \sin(2\beta) \). \( \text{LHCb} \) will bring an increased precision to these measurements as its data sets increase in size and quickly surpass the B-factories and the Tevatron.

\section{1.8 Direct \( CP \)-violation in Charged B-meson Decays}

The decays of neutral B-mesons often have to disentangle \( CP \)-violation in the mixing and direct \( CP \)-violation. One simplification that has been discussed in Section 1.3.3 is to study B-decays to \( CP \) eigenstates as the direct \( CP \)-violating contribution is zero. An alternative is to use charged B-meson decays; this allows a clean measurement of direct \( CP \)-violation as \( B^+ \) and \( B^- \) cannot mix. As direct \( CP \)-violation requires an interference of the weak and strong phases of the decay amplitudes they would also seem ideal candidates to measure weak phases in the Standard Model. However, the decays of charged B-mesons tend to be dominated by the strong phase which is currently difficult to calculate and makes the separation of the two phases difficult.

The charged B-decays studied in this thesis are \( B^\pm \rightarrow J/\psi K^\pm \), \( B^\pm \rightarrow J/\psi \pi^\pm \), \( B^\pm \rightarrow \phi K^\pm \) and \( B^\pm \rightarrow \phi \pi^\pm \), where \( J/\psi \rightarrow \mu^+ \mu^- \) and \( \phi \rightarrow K^+ K^- \). These four decays proceed via four different quark transitions and probe different aspects of the Standard Model. The motivation for the study of each of the four decays, their Standard Model predictions and their sensitivity to physics beyond the Standard Model will now be discussed.

\subsection{1.8.1 Direct \( CP \)-violation in \( B^\pm \rightarrow J/\psi K^\pm \) Decays}

The direct \( CP \)-asymmetry in \( B^\pm \rightarrow J/\psi K^\pm \) decays arises through the interference of tree and penguin diagrams, such as those shown in Figure 1.8. The decay proceeds via the \( b \rightarrow s\bar{c}c \) quark transition which is traditionally associated with the extraction of the \( \text{CKM} \) angle \( \beta \) (see Section 1.7.2). The direct \( CP \)-asymmetry in this channel is measured via equation (1.12) and is a counting experiment, however the use of \( B^0 \) decays rather than \( B^- \) decays does not provide an (experimentally less complex) alternative to measure the \( \text{CKM} \) phase \( \beta \). The strong phase, arising from final state hadronic interactions, dominates the asymmetry and is not readily calculable.

One motivation for the study of the decay \( B^\pm \rightarrow J/\psi K^\pm \) is to provide an input to the "K–\( \pi \) puzzle". The "K–\( \pi \) puzzle" refers to the difference between the direct \( CP \)-asymmetry in the \( B^0 \rightarrow K^+ \pi^- \) and \( B^- \rightarrow K^- \pi^0 \) decay modes, which shows a significant deviation
from Standard Model predictions \cite{57}. Theoretical studies have been performed and these favour solutions where new physics enters via a contribution to the weak phase \cite{58,59}. A study by Hou, Nagashima and Soddu indicates that new physics introduced in this way would also have a significant contribution to the $CP$-asymmetry in the $b \to s\tau\bar{c}$ transition in $B^\pm \to J/\psi K^\pm$ decays \cite{56}. The Standard Model prediction for $A_{CP}^{\text{dir}}$ in $B^\pm \to J/\psi K^\pm$ is $\mathcal{O}(0.003)$ and values $\mathcal{O}(0.01)$ could be a sign of new physics \cite{56}.

The $B^\pm \to J/\psi K^\pm$ decay mode is well established and a summary of measurements that are currently used to determine the world average of the branching ratio, and the world average of the branching ratio, are given in Table \ref{table:1.2(a)}. The experimental status of the measurement of $A_{CP}^{\text{dir}}$ in the $B^\pm \to J/\psi K^\pm$ channel is presented in Table \ref{table:1.2(b)} and the current world average is compatible with the Standard Model prediction of no direct $CP$-violation.
Table 1.2: The current experimental status of $\mathcal{B}(B^\pm \to J/\psi K^\pm)$ and $A_{\text{CP}}^{\text{dir}}(B^\pm \to J/\psi K^\pm)$. Each of the listed measurements are used to calculate the world average.

(a) The current experimental status of $\mathcal{B}(B^\pm \to J/\psi K^\pm)$.

(b) The current experimental status of $A_{\text{CP}}^{\text{dir}}(B^\pm \to J/\psi K^\pm)$.

1.8.2 Direct $\mathcal{CP}$-violation in $B^\pm \to J/\psi \pi^\pm$ Decays

The tree and penguin diagrams for the $B^\pm \to J/\psi \pi^\pm$ decays are the same as those for $B^\pm \to J/\psi K^\pm$ where the outgoing s-type quark is substituted for a d-type quark; they are shown in Figure 1.8. The $B^\pm \to J/\psi \pi^\pm$ decays proceed via the $b \to d c\bar{c}$ quark transition and the branching ratio is suppressed by $\mathcal{O}(\lambda^2) \approx 5\%$ with respect to the $B^\pm \to J/\psi K^\pm$ mode, as seen by comparing the world average measurement of the branching ratios in Table 1.3(a) and Table 1.2(a).

The asymmetry is measured by counting the number of $B^- \to J/\psi \pi^-$ states and $B^+ \to J/\psi \pi^+$ and applying equation (1.12). The weak phase is provided by the CKM angle $\gamma$ although, like the $B^\pm \to J/\psi K^\pm$ decays, the strong phase is difficult to calculate and hence the channel does not readily lend itself to measurements of the angle $\gamma$ [70]. In the $B^\pm \to J/\psi \pi^\pm$ decays, unlike $B^\pm \to J/\psi K^\pm$, the weak and the strong phases are of the same order of magnitude.

The $B_d^0 \to D^+D^-$ decays have also been used to study the $b \to d\bar{c}c$ quark transitions and the initial measurements by the Belle and BABAR collaborations produced conflicting results. The Belle collaboration has measured a direct $\mathcal{CP}$-asymmetry of $-0.91 \pm 0.23 \pm 0.06$ [72] and the BABAR collaboration initially measured a value of $+0.11 \pm 0.35 \pm 0.06$ [73]. The Belle collaboration measurement shows a substantial $\mathcal{CP}$-asymmetry which is inconsistent with the Standard Model predictions [74] and also the BABAR...
collaboration’s measurement. The latest measurement from the BaBar collaboration (superseding the old value) is $-0.07 \pm 0.23 \pm 0.03$ 75 which is compatible with the Standard Model prediction of a very small amount of direct $C\!P$-violation. The input from the $B^\pm \rightarrow J/\psi \pi^\pm$ decays, which proceed via the same quark transitions, will provide additional constraints.

The experimental status of the measurement of the branching ratio and $A_{CP}^{\text{dir}}$ in the $B^\pm \rightarrow J/\psi \pi^\pm$ channel is presented in Table 1.3 The current world average of $A_{CP}^{\text{dir}}$ is compatible with the Standard Model prediction of direct $C\!P$-violation $\mathcal{O}(0.01)$ 70.

1.8.3 Direct $C\!P$-violation in $B^\pm \rightarrow \phi K^\pm$ Decays

The $B^\pm \rightarrow \phi K^\pm$ decay proceeds via a $b \rightarrow s\bar{s}s$ quark transition and cannot proceed via tree diagrams as the $b \rightarrow s$ transition cannot occur directly. Some diagrams for the
30 Charge Parity Violation in the B-meson System

<table>
<thead>
<tr>
<th>Channel</th>
<th>QCDF</th>
<th>pQCD</th>
<th>SCET1</th>
<th>SCET2</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^\pm \to \phi K^\pm$</td>
<td>$8.8^{+2.8+4.7}_{-2.7-3.6}$</td>
<td>$7.8^{+5.9}_{-1.8}$</td>
<td>$9.7^{+4.9+1.8}_{-3.9-1.5}$</td>
<td>$8.6^{+3.2+1.2}_{-2.7-1.0}$</td>
<td>$8.3 \pm 0.7$</td>
</tr>
<tr>
<td>$B^\pm \to \phi \pi^\pm$</td>
<td>$\approx 0.043$</td>
<td>$0.032^{+0.012}_{-0.004}$</td>
<td>$\approx 0.003$</td>
<td>$\approx 0.003$</td>
<td>$&lt; 0.24$</td>
</tr>
</tbody>
</table>

(a) The predicted \[78\] and experimental \[4\] branching ratios ($\times 10^{-6}$).

<table>
<thead>
<tr>
<th>Channel</th>
<th>QCDF</th>
<th>pQCD</th>
<th>SCET1</th>
<th>SCET2</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^\pm \to \phi K^\pm$</td>
<td>$0.6^{+0.1+0.1}_{-0.1-0.1}$</td>
<td>$1^{+0}_{-1}$</td>
<td>$0$</td>
<td>$0$</td>
<td>$-1 \pm 6$</td>
</tr>
<tr>
<td>$B^\pm \to \phi \pi^\pm$</td>
<td>$0$</td>
<td>$-8.0^{+0.9+1.5}_{-1.0-1.0}$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

(b) The predicted \[78\] and experimental \[4\] direct $CP$-asymmetries (%).

Table 1.4: A comparison of the predictions and measured values of the branching ratio and direct $CP$-asymmetries.

$B^\pm \to \phi K^\pm$ decays are given in Figure 1.9. As these decays cannot proceed via a direct transition they could be strongly influenced by generic new physics \[76\]. Fleischer and Mannel suggest three observables that could provide a “smoking-gun” signal for new physics in the $b \to sss$ quark transition using the isospin related decays $B^\pm \to \phi K^\pm$ and $B^0_\text{d} \to \phi K^0_\text{s} + c.c.$ (where “+ c.c.” indicates the charged conjugate decay) \[76\]. These variables compare the direct $CP$-asymmetries in the two processes and also the amplitudes. Unfortunately there are scenarios where the new physics effects may not be large enough to be distinguishable from the Standard Model effects. Gronau and Rosner have also shown that these decays can be part of an analysis of additional decays to probe the isospin structure of new physics \[77\].

The calculation of the branching ratio and direct $CP$-asymmetry in $B^\pm \to \phi K^\pm$ decays in the Standard Model is done using three different techniques: QCD factorisation (QCDF) \[78,79\], perturbative QCD (pQCD) \[80,81\] and soft collinear effective theory (SCET) \[82\]. A review of the predictions of these models is found in references \[78,83\] and a summary of the results is presented in Table 1.4. The predictions of $B(B^\pm \to \phi K^\pm)$ are in agreement with the experimental value. The precision of the measurement of $A^\text{dir}_{CP}(B^\pm \to \phi K^\pm)$ is not yet sufficient to test the theoretical predictions which due to the absence of amplitudes from tree diagrams are very small or zero.
1.8.4 Direct $C\mathcal{P}$-violation in $B^\pm \rightarrow \phi \pi^\pm$ Decays

The $B^\pm \rightarrow \phi \pi^\pm$ decay proceeds via a $b \rightarrow d\bar{s}s$ quark transition and the diagrams in Figure 1.9 show that the quark transition cannot occur directly, like $B^\pm \rightarrow \phi K^\pm$. Table 1.4 presents the theoretical calculations of $\mathcal{B}(B^\pm \rightarrow \phi \pi^\pm)$ and $A_{CP}^{\text{dir}}(B^\pm \rightarrow \phi \pi^\pm)$ using QCDF [78], pQCD [84] and SCET [82]. It is seen from Table 1.4(a) that $B^\pm \rightarrow \phi \pi^\pm$ has not yet been discovered and the limit on its branching ratio is an order of magnitude larger than the largest theoretical prediction. Mawlong, Mohanta and Giri predict a branching ratio of $4.45 \times 10^{-9}$ using QCDF [85] and this value agrees with the SCET calculations presented in Table 1.4(a). The values calculated using QCDF and pQCD presented in Table 1.4(a) are an order of magnitude larger than these and this is due to the inclusion of a contribution from $\omega - \phi$ mixing. If the $\phi$ contains a tiny $\omega$ component then the decay $B^\pm \rightarrow \omega \pi^\pm$, with a much larger branching ratio, could dominates the $B^\pm \rightarrow \phi \pi^\pm$ branching ratio. This enhances both the branching ratio and the $C\mathcal{P}$-asymmetry, which is predicted to be zero if this effect is not included [84,85]. This enhancement has the potential to mask any new physics contributions e.g. those presented in references [85,86]. Nonetheless, the discovery of this channel and a measurement of the direct $C\mathcal{P}$-asymmetry would provide valuable input to the theoretical calculations.

1.9 Summary

The Universe is predicted to have initially contained equal amounts of matter and anti-matter, and one of the three conditions required to produce the matter dominated Universe observed today is $C\mathcal{P}$-violation. There are three forms of $C\mathcal{P}$-violation: direct, indirect and interference between $C\mathcal{P}$-violation in the mixing and decay. $C\mathcal{P}$-violation in the Standard Model is introduced through a single complex phase which is present in the CKM matrix. The unitarity conditions of the CKM matrix are represented graphically by the Unitarity Triangle whose area is proportional to the amount of $C\mathcal{P}$-violation in the Standard Model. As the angles of the Unitarity Triangle are not predictable from theory they are being measured both directly and indirectly through the values of the element of the CKM matrix. A global fit of the direct and indirect measurements produces best fit values for the angles of the Unitarity Triangle which are in general in good agreement with the prediction of the Standard Model that the sum of the angles is $180^\circ$. There is some tension in the fitted value of the angle $\sin(2\beta)$ and its measured value. A measurement of the magnitude of the direct $C\mathcal{P}$-asymmetry in the $B^\pm \rightarrow J/\psi K^\pm$,
B^± \rightarrow J/\psi\pi^\pm, B^± \rightarrow \phi K^\pm and B^± \rightarrow \phi\pi^\pm decays provides an input into the “K-\pi puzzle”, the anomalous direct $\mathcal{CP}$-asymmetry measured by the Belle collaboration in $B^0_d \rightarrow D^+D^-$ decays and to provide an input into the calculation of the branching ratios and direct $\mathcal{CP}$-asymmetries in charged charmless $b$-decays respectively. The measurement of the branching ratio and $\mathcal{CP}$-asymmetry in these decay modes is the subject of this thesis.
Chapter 2

The LHC and the LHCb Experiment

The Large Hadron Collider beauty (LHCb) detector is a dedicated heavy flavour physics experiment studying the proton-proton collisions at the Large Hadron Collider (LHC). This chapter presents an overview of the LHC and its experiments in Sections 2.1 and 2.2 respectively. The LHC proton-proton collisions produce the full range of b-hadrons, as described in Section 2.3, and the LHCb detector is making precision measurements of these hadrons. The LHCb experiment is described in Section 2.4 which includes a summary of each sub-detector, the data acquisition and experiment control systems, and the trigger and stripping stages used to reduce the volume of data recorded by LHCb. The software used to reconstruct and analyse the data is presented in Section 2.5. Finally, the data taken during the 2010 run period is summarised in Section 2.6; it is this data that is analysed in this thesis.

2.1 The LHC

The LHC is a particle accelerator, 27 km in circumference built across the French-Swiss border, which collides two counter rotating proton beams. It is the flagship accelerator of the European Organization for Nuclear Research (CERN) laboratory and has a broad physics program. Part of this is a rich heavy flavour physics program and the LHCb detector is a specialised heavy flavour physics detector. The LHC is exploring TeV scale physics using two general purpose detectors, ATLAS and CMS. A secondary program at the LHC is the heavy ion mode which commences after the completion of the proton
running period and a specialised detector, \textbf{ALICE} is studying these collisions; the two
general purpose detectors are also taking data during these runs.

The protons at the \textbf{LHC} are extracted from a hydrogen bottle, the hydrogen from
this bottle is ionised, and then the electrons and protons are separated. The protons are
boosted in energy through a series of accelerators; a schematic of the accelerator chain is
shown in Figure 2.1. The first acceleration stage for the protons is a linear accelerator,
the LINAC2, which feeds the Proton Synchrotron (\textbf{PS}) booster. The \textbf{PS}
booster is a circular accelerator which injects the protons into the \textbf{PS}, CERN’s first synchrotron.
The \textbf{PS} has a maximum energy of 25 GeV and passes the protons into the penultimate
accelerator, the Super Proton Synchrotron (\textbf{SPS}). The \textbf{SPS} configures the proton beam
to have the proton bunch spacing required by the \textbf{LHC}. The \textbf{SPS} then fills the \textbf{LHC}
at a per beam energy of 450 GeV. The \textbf{LHC} boosts the protons up to a maximum energy
of 3.5 TeV per beam and collides them at four interaction points. During the 2010 data
taking period the beam parameters varied as the machine performance became better
understood. The majority of the data taken has a proton bunch spacing of 75 ns or
150 ns and a bunch collision rate of the order of 1 MHz. The design spacing and collision
rate are 25 ns and 30 MHz respectively, and the design energy per beam is 7 TeV.

The lead used in the heavy ion collisions follow a similar process with vapourised
lead fully ionised to provide the source ions. These are then accelerated by a linear
accelerator, the LINAC3, and are passed into the Low Energy Ion Ring (\textbf{LEIR}). From
here they follow the same path as the protons into the \textbf{PS} booster, the \textbf{PS}, the \textbf{SPS}
and finally the \textbf{LHC}. The \textbf{LHC} accelerates the ions to beam energies of 2.76 TeV per nucleon
and a total centre-of-mass energy of 1.15 PeV. The \textbf{LHC} has been described in full detail
in the \textbf{LHC} machine paper \cite{87}.

2.2 The LHC Experiments

The \textbf{LHC} has four main experiments to analyse the proton-proton or heavy-ion collisions
at four Interaction Points (\textbf{IPs}) situated in the straight sections of the ring. These four
experiments are described below.

\textbf{A Large Ion Collider Experiment (ALICE) \cite{88}} is situated at interaction point two,
\textbf{IP2}. It is designed to study the heavy ion runs, and the main aim of the experiment
is to make measurements of the expansion and cooling of the quark-gluon plasma produced in the heavy ion collisions.

A Toroidal Large ApparatuS (ATLAS) \[89\] is a general purpose detector situated at IP1. The ATLAS experiment is searching for new physics beyond the Standard Model at the TeV scale.

The Compact Muon Solenoid (CMS) \[90\] detector is installed at IP5 and is a general purpose detector. The physics programs of ATLAS and CMS are complementary, they utilise different technologies and independent analyses to help ensure the robustness of new discoveries.

The Large Hadron Collider beauty (LHCb) \[91\] detector is a single arm spectrometer located at IP8. The LHCb experiment is designed to make precision tests of the Standard Model primarily through the study of the decays of heavy flavour quarks. It is discussed in more detailed in Section 2.4.

In addition to these four main experiments, there are three other detectors sharing the interaction points of the primary detectors.
The LHC and the LHCb Experiment

Figure 2.2: Leading order Feynman diagrams for hard QCD scattering $b\bar{b}$ pair production.

(a) Gluon-gluon fusion.
(b) Quark-antiquark annihilation.

The Large Hadron Collider forward (LHCf) [92] detector is comprised of two detectors situated $\pm 140$ m from the collisions in the ATLAS cavern at IP1. The detectors measure the energy spectra of neutral particles produced at very forward angles. These measurements are used to calibrate models of hadron interactions in high energy, cosmic ray showers occurring in the upper atmosphere.

The Monopole and Exotics Detector At the LHC (MoEDAL) [93] is searching for magnetic monopoles and pseudo-stable massive highly ionising particles. MoEDAL is installed in the LHCb cavern and fixed to the walls and ceiling in the area surrounding the interaction point.

The TOTal cross-section, Elastic scattering and diffraction dissociation Measurement at the LHC (TOTEM) [94] consists of four detectors located inside the CMS detector volume and at $\pm 147$ m and $\pm 220$ m from the collision point at IP5. The detector has been designed to perform luminosity independent cross-section measurements and to study elastic and diffractive scattering.

2.3 B-production at the LHC

The proton-proton interactions at the LHC produce the full spectrum of $b$-hadrons, for example $B^+$ ($u\bar{b}$), $B^0$, $B^+_c$ ($c\bar{b}$), $\Lambda_b$ ($ub$) (and their charge conjugates), and are the most copious source of $b$-hadrons in the world. The main production mechanism for $b$-quarks is through hard QCD scattering, primarily through gluon-gluon fusion and
quark-antiquark annihilation; the leading order Feynman diagrams of which are presented in Figure 2.2 [95]. The momenta of the incoming partons are strongly asymmetric and are boosted with respect to the laboratory frame [96]. This results in a $b\bar{b}$ pair produced in a narrow cone around the beam axis, as illustrated in Figure 2.3, and a full $4\pi$ solid angle coverage is not required to study $B$-decays at the LHC.

The collisions which produce the $b\bar{b}$ quark pairs have a net charge and baryon number as a result of colliding two beams of charged matter. This leads to an excess of the number of produced particles compared to the number of produced anti-particles. For example, the colliding protons’ valence quarks are $u$ and $d$ type quarks which gives an excess of these types over their anti-partners. During the hadronisation phase a $\bar{b}$ quark is more likely to hadronise with an $u$ type quark into a $B^+$ than the $b$ with an $\bar{u}$ type quark to $B^-$ [97].

2.4 The LHCb Experiment

The LHCb detector is situated at IP8 and an aerial view of the surface building is shown in Figure 2.4. LHCb is designed to receive a luminosity of $2 \times 10^{32}$ cm$^{-2}$ s$^{-1}$, two orders
of magnitude lower than that at ATLAS and CMS. At this reduced luminosity LHCb receives only 0.74 proton-proton interactions per bunch crossing. The dependence of the number of proton-proton collisions on the luminosity is illustrated in Figure 2.5. During the 2010 data taking period LHCb operated with a larger number of proton-proton interactions per bunch crossing $\sim 2.5$. The LHC is running with an average number of proton-proton interactions per bunch crossing of 1.5 during the 2011 data taking period. This should allow LHCb to collect 1 fb$^{-1}$ of data by the end of 2011.

The estimated $b\bar{b}$ production cross-section at 14 TeV is 500 µb and, with a luminosity of $2 \times 10^{32}$ cm$^{-2}$ s$^{-1}$, $10^{12}$ $b\bar{b}$ pairs are expected in the nominal LHC year. The $b\bar{b}$ cross-section at this energy is extrapolated from measurements at the Tevatron \cite{98,99}. Initially these measurements were two orders of magnitudes larger than those predicted by theory, this difference has reduced with improved theoretical models. A first measurement of the $b\bar{b}$ production cross-section at a 7 TeV centre-of-mass energy has been made by the LHCb collaboration and found to be $(75.3 \pm 5.4 \pm 13.0)$ µb within the LHCb acceptance, which is in agreement with predictions of $(89.0^{+83\%}_{-44\%})$ µb and $(70.2^{+45\%}_{-38\%})$ µb from the MCFM and FONLL computer programs respectively \cite{100}. Using the Monte Carlo predictions from Pythia the cross-section was extrapolated to a $4\pi$ acceptance and found to be $(284 \pm 20 \pm 49)$ µb \cite{100}. It is an important cross check of these theoretical developments.
Figure 2.5: Poisson distributions for the probability of a given number of proton-proton interactions per beam crossing as a function of luminosity \[96\].
A schematic of the LHCb detector is shown in Figure 2.6. The interaction point is on the left and the $b\bar{b}$ pairs are produced in the positive $z$ direction, towards the right of the figure. LHCb utilises a right-handed coordinate system with positive $x$ going into the figure, referred to as the A-side, and the negative $x$ region is the C-side. An overview of the magnet, beam-pipe, sub-detectors, data acquisition and experiment control, and the trigger and stripping is given in this section, except the RICH sub-detector which is described in detail in Chapter 3.

The LHCb spectrometer covers a cone from $10-300$ mrad in the horizontal and $10-250$ mrad in the vertical, where $0$ mrad is along the beam axis. The lower limit of the acceptance is a restriction from the beam-pipe. A warm, dipole magnet bends charged particles in the horizontal, $x$, plane, which has an integrated field of $4$ Tm over $10$ m. The magnet is described in Section 2.4.2. LHCb has several tracking stations, described in Section 2.4.3 which perform precision measurements of the charged particles’ momenta using a variety of technologies. The first tracking sub-detector is the Vertex Locator (VELO); the particles created in the proton-proton interactions in each collision traverse the VELO, a silicon strip detector situated close to the interaction point, which provides precise reconstruction of the primary vertices and the secondary $b$- and $c$-hadron decay vertices. The Tracker Turicensis (TT) is situated immediately before the magnet and coupled with the Inner Tracker (IT), situated after the magnet, makes up the Silicon Tracker (ST). The Outer Tracker (OT) surrounds the IT and is built using straw tube technology. The decays of $b$-hadrons are frequently to hadronic final states that are topologically very similar and LHCb requires highly efficient separation of kaons and pions over a wide momentum range. There are two RICH detectors that provide this Particle IDentification (PID) in the momentum range of $2-100$ GeV/c, as well as proton separation at momenta less than $10$ GeV/c (vital for $b$-tagging). RICH-1 is situated between the VELO and IT sub-detectors, and RICH-2 between the IT/OT sub-detector and the first muon station. The RICH detectors are described in detail in Chapter 3. Further PID and energy measurements are provided by the calorimeters. LHCb has four sub-detectors that make up the calorimeter system, placed sequentially from the interaction point. These are the Scintillator Pad Detector (SPD), PreShower (PS), Electromagnetic Calorimeter (ECAL) and the Hadronic Calorimeter (HCAL), which provide identification of electrons, photons, hadrons, and their position and energy. The muon system is the final sub-detector and has alternating layers of sensors and iron filters. There is a single muon chamber between the RICH-2 detector and the calorimeters. A discussion of the calorimeter and muon systems is given in Section 2.4.4.
The data from each of the sub-detectors is collected, filtered and written to disk by the Data Acquisition (DAQ) system. This process is controlled and monitored by the Experiment Control System (ECS) and coordinated by the Timing and Fast Control (TFC) system. The DAQ, ECS and TFC systems are described in Section 2.4.5. The DAQ system supports a low latency, high bandwidth trigger system which is responsible for reducing the data rate from the 30 MHz collision frequency to 2 kHz, the rate at which the data is written to disk. The trigger consists of a hardware trigger, the Level-0 (L0), and a software trigger, the High Level Trigger (HLT) which is further divided into two stages HLT-1 and HLT-2; both the hardware and software trigger are described in Section 2.4.6. A final stage of data processing is the stripping, described in Section 2.4.7, which performs an offline reconstruction of the exclusive b- and c-decays used in physics analyses.

2.4.1 Beam-Pipe

The protons circulate in two beam-pipes, one for the clockwise beam and another for the anticlockwise beam, which maintain ultra-high vacuum conditions – a pressure of $10^{-10}$ mbar. Since LHCb reconstructs exclusive b- and c-hadron decays the beam-pipe passing through LHCb is made of low density materials, beryllium and aluminium,
and the bellows and flanges are kept out of LHCb’s acceptance. In the places where this has not been possible the cross-section of the bellows and flanges are kept to a minimum.

The LHCb beam-pipe starts with the interaction region where the beam-pipe is a thin aluminium foil that separates the VELO sensors from the ultra-high vacuum that the LHC requires for the circulating proton beams. As a result of this the VELO sensors operate in high vacuum conditions, $10^{-4}$ mbar [91]. The beam-pipe passing through the rest of the detector is made up of four sections, three beryllium sections and a stainless steel section. The first beryllium section, pictured in Figure 2.7(a), is welded to the aluminium exit window of the VELO and passes through RICH-1 and the TT to the magnet. It is a 1 mm thick cone with an angle of 25 mrad and contains a transition to the 10 mrad angle of the subsequent sections. The next two sections are also beryllium and vary in thickness from 1-2.4 mm, Figure 2.7(b) shows a photograph of section two which passes through the magnet. Section three passes through RICH-2, the tracking stations, M1 and part of the ECAL. The final section is 4 mm thick stainless steel which continues the 10 mrad cone. The support structures and connectors have all been designed with a minimum cross-section while retaining the necessary characteristics to either support the beam or maintain the ultra-high vacuum.

2.4.2 The LHCb Magnet

LHCb utilises a warm dipole magnet to produce a horizontal bending plane for charged particles. The magnet is in the form of two saddle-shaped coils inclined at a small angle
with respect to the $xz$ plane. This small angle means the magnet cross-section gets wider moving away from the interaction point in order to accommodate the detector acceptance. The magnet is pictured in Figure 2.8, where the two coils and their inclination are visible.

The magnet provides a field of 4 Tm over 10 m. Since the operation of both the [VELO] and the [RICH] sub-detectors require low fields, the fringe field of the magnet is kept to a minimum. The measured principal ($y$) component of the field strength along the beam axis, $z$, with $x = y = 0$, is shown in Figure 2.9 along with the modelled field strength. The nominal magnetic field direction is down (from positive $y$ to negative $y$) and this is the positive curve in Figure 2.9.

In order to account for any asymmetry in the detector, due to a variation in detection efficiency with spatial position, data is recorded with both the magnet polarity in the nominal down direction and in the up direction (from negative $y$ to positive $y$). A charged particle and its charge conjugate are deflected in opposite directions by the magnetic field. By reversing the magnet polarity the deflections are also reversed, allowing possible detector asymmetries to be accounted for in physics analyses.
Figure 2.9: The measured $y$-component of the field strength (circles) and the modelled field strength (solid line) for both magnet polarities. The positive curve is with the down polarity and the negative curve is with the up polarity.

2.4.3 The Tracking System

LHCb has four tracking sub-detectors: the Vertex Locator, the Tracker Turicensis\footnote{Formally the Trigger Tracker but it is no longer used in the L0 trigger and it was renamed keeping the same acronym. Turicensis is the Latin name of the Zurich institute that is heavily involved in the LT.}, the Inner Tracker and the Outer Tracker. The combination of an TT and OT detector plane is collectively called a T station. The hits from the sensitive elements of the detectors are combined to form tracks which, coupled with the magnetic field and material map, produce precision measurements of the momenta of charged particles.

The tracking strategy is divided into two parts, the track finding and track fitting. This strategy creates multiple track types depending on the detectors used in the reconstruction; the different track types are defined in Figure 2.10. The track finding starts by creating tracks in the VELO and the T stations. The main type of track used in physics analyses are the long tracks, which traverse all of the tracking stations. They are reconstructed in two ways: the first method extrapolates the VELO tracks in the positive $z$ direction, adding hits from the other tracking detectors, and the second method extrapolates the VELO and T station tracks towards each other. The final stage in the track finding adds
the \( T \) hits to these long tracks, reducing fakes and improving the momentum resolution. The downstream tracks are also used in many physics analyses where long-lived particles, such as secondary \( K^0_S \), decay outside of the \( \text{VELO} \). The track fitting is performed on all of the reconstructed tracks in both the upstream direction (toward the interaction point) and downstream direction (away from the interaction point). The final tracks have a momentum resolution of \( \Delta p/p = (0.35-0.55)\% \) depending on the track momentum, as measured with the data from the 2010 data taking period \([101]\).

The tracking is performing well and Figure 2.11(a) shows the reconstructed \( K^0_S \) decays using two oppositely charge pions from the long tracks. These are used to determine the tracking efficiency as a function of the transverse momentum. This agrees well with studies using the same algorithms on simulated events, as shown in Figure 2.11(b). A comprehensive overview of the tracking strategy is given in reference \([102]\) and the tracking procedure in reference \([103]\).

**Vertex Locator**

The Vertex Locator, the main tracking sub-detector of \( \text{LHCb} \), is a precision silicon-detector designed to reconstruct the primary vertices of the \( b \)- and \( c \)-hadrons produced in the proton-proton collisions. These travel on average 1 cm before decaying, resulting in the delicate \( \text{VELO} \) sensors being situated within a centimetre of the interaction point. This close proximity to the interaction point makes the \( \text{VELO} \) unique among the \( \text{LHC} \) experiments and presents many design challenges.
(a) The reconstructed $K^0_S$ mass using long tracks.

(b) The tracking efficiency using $K^0_S$ decays.

**Figure 2.11:** The tracking efficiency as a function of transverse momentum is calculated using the blue line in the $K^0_S$ mass plot as the numerator and black as the denominator.
The VELO is constructed from two overlapping halves, each consisting of 21 modules situated along the beam axis. Of these 21 modules, 6 are situated behind the nominal interaction point and 15 in front. The interaction point has a $z$-axis length of about 53 mm and this necessitates two upstream modules, i.e. at negative $z$ positions, used to veto pile-up events. The layout of the VELO modules is shown in the top half of Figure 2.12. An individual module consists of two silicon detectors mounted back-to-back, the readout electronics and the cooling elements. One of the silicon sensors on the module is called the $r$-sensor which consists of concentric semi-circles centred around the beam axis to measure the radial, $r$, position of the track; and the other is known as the $\phi$-sensor, with strips formed radially outwards from the beam and orthogonal to the $r$-sensor, to measure the angle, $\phi$, which is made with the $x$-axis in the $xy$ plane. In Figure 2.13(a) a view along the beam axis shows the central cut-out where the overlapping halve create the beam-pipe.

The VELO sensors are positioned nominally at 7 mm from the interaction point. This is so close that, during the ramping phase when the LHC beam energy is increased from 450 GeV to 3.5 TeV, the sensors would be within the LHC beam radius. Therefore,

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2Pile-up events in which multiple proton-proton collisions occur.
during injection, the VELO halves are retracted to 30 mm in the $x$-plane into the shadow of the beam-pipe \[104\].

The VELO modules are mounted within a vacuum vessel, 1.4 m long and 1.1 m in diameter, through which the beam passes. The LHC beam-pipe maintains a vacuum of $10^{-10}$ mbar and the VELO operates in a vacuum of $10^{-4}$ mbar. A custom designed corrugated aluminium foil box separates the beam and VELO vacua, and is known as the RF box. The corrugated design of the box allows for the overlap of the VELO modules when in the closed position. A schematic of the foil is shown in Figure 2.13(b). The foil box also serves to protect the VELO from radio frequency pickup and degradation from parasitic radio frequency coupling as the beam passes through the VELO.

The main purpose of the VELO is to provide precision measurements of the Impact Parameter $\text{IP}$ of tracks emanating from the interaction point, and reconstruct primary and decay vertices. Studies with the data taken during 2010 compare the VELO performance with that expected from simulation \[104\]. The LHCb detector paper gives the parametrisation of the $\text{IP}$ resolution as $\sigma_{\text{IP}} = 14 + 35/p_T \mu m$ \[91\]. This can be compared to the resolutions extracted from the 2010 data: $\sigma_{\text{IP}} = 12.9 + 25.7/p_T \mu m$ and $\sigma_{\text{IP}} = 12.7 + 25.1/p_T \mu m$ in the $x$- and $y$-axes respectively as shown in Figures 2.14(a) and (b).
Silicon Trackers

The Tracker Turicensis and the Inner Tracker are silicon trackers utilising identical silicon detectors (albeit with different thicknesses) that share a common design of readout electronics, power distribution, detector control and monitoring. Both detectors are situated around the beam-pipe and the silicon provides high resolution, radiation tolerant sensors that are able to cope with high occupancies. Different constraints on the geometry have led to different detector layouts.

The TT is located between RICH-1 and the magnet, it has an active area of 8 m$^2$ and covers the full LHCb acceptance. The TT detects long-lived particles decaying outside of the VELO acceptance and low momentum particles that are swept out of the detector acceptance by the magnet. It is made up of four layers with each layer containing two half modules split such that one is above the beam-pipe and the other below. The silicon sensors within a half module have 512 strips and are grouped into sectors each of which has its own readout hybrid. The readout hybrids are used to readout the sectors, and also provide power and control signals. The sectors have either a 4-3 or 4-2-1 layout and are labelled K, L and M; the layout of a single TT layer is illustrated in Figure 2.15. The 4-2-1 half modules surround the beam-pipe where the occupancies are highest and the readout hybrids are placed at the edges of the detector layers minimising the amount of inactive material in the LHCb acceptance. Figure 2.16 is a photograph of the TT during installation and shows the exposed silicon sensors and their sector grouping on the left of the beam-pipe. In the upper left corner of the photograph the next layer is visible and...
Figure 2.15: Layout of the third TT layer. The image shows the \((x, u)\) orientation of this layer.

is rotated by 5° with respect to the front layer. The layers are arranged in two pairs, \((x, u)\) and \((v, x)\), where \(u\) and \(v\) indicate opposite rotations (of \(\pm 5°\) respectively) giving a stereo geometry to aid track reconstruction.

The TT is part of the three T stations: T1, T2 and T3, located between RICH-2 and the first muon chamber, M1. The TT forms the central part of the T stations in the region surrounding the beam-pipe. Although it covers only 1.5 % of the T station surface area, 20 % of the tracks pass through the detector, hence the requirement of a high-resolution radiation-hard detector. Each station contains four individual detector boxes surrounding the beam-pipe, which are staggered in \(z\) and overlapping in \(x\) to avoid acceptance gaps and aid alignment. The layout of the boxes is shown in Figure 2.17(a) where the overlaps are visible. A single sensor contains 384 strips and a group of sensors and their readout hybrids is known as a module. The boxes above and below the beam-pipe contain only one sensor in a module and the boxes either side of the beam-pipe contain two sensors per module, as shown in Figure 2.17(b). The readout hybrids in the modules are the same as used for the TT. Figure 2.18(a) shows a photograph of the TT modules being installed into a box and Figure 2.18(b) some modules.

Both the TT and the TT sub-detectors require signal-to-noise ratios above 10:1 in order to maintain high hit efficiencies \cite{91}. Performance studies with the 7 TeV data show
Figure 2.16: A photo of the TT during installation. Downstream is into the picture.
Figure 2.17: Layout of an inner tracker station.

Figure 2.18: Photographs of the IT box and sensors.
that the sub-detectors have signal-to-noise ratios of about 16:1. This high signal-to-noise ratio produces an excellent tracking efficiency, measured to be 99.3% for the TT and 99.8% for the IT using isolated, high momentum tracks [105].

**Outer Tracker**

The Outer Tracker [91] is the largest tracking sub-detector and covers almost all of the LHCb acceptance, extending out to 300 mrad (250 mrad) in the bending (non-bending) plane. The OT is composed of three stations, one surrounding each IT station. The stations, like the TT, are made up of four layers that have an x-u-v-x geometry. The u and v layers have a rotation of ±5° respectively, and the x layers have a 0° rotation, both with respect to the vertical. The OT surrounds the much smaller IT shown in Figure 2.19.
With a surface area of 29 m$^2$ the OT is much larger than the silicon detectors, resulting in a choice of straw drift-tubes as the sensor technology. A station layer is composed of two halves, split down the $y = 0$ line. Each half is composed primarily of F modules, which run the full length of the sub-detector, and a few shorter S modules, which surround the IT and are less than half the length of an F module; Figure 2.20 illustrates this layout. Both modules are made from two staggered rows, each row containing 64 drift tubes. In order to meet the requirement of 10\% occupancy, the F modules are split along the $x$ axis and contain 256 tubes. The splits are staggered between layers to minimise insensitive regions. The S modules are sufficiently short and do not require splitting; they contain 128 drift tubes.

Finally, a photograph of the T1 station is shown in Figure 2.21 looking downstream through the LHCB magnet (the full loop of which is visible in the foreground). Three of the IT boxes are visible in the centre and the OT layer surrounds it.

### 2.4.4 Particle Identification

Particle identification [106] is fundamental to LHCB as it is used to distinguish between the many hadronic and (semi-)-leptonic $b$- and $c$-hadron final states. The PID is provided by three sub-detectors: the RICH, the calorimeter system, and the muon system. The RICH detector is responsible for the identification of all charged hadrons. The calorimeters
identify electrons, photons and also hadrons, as well as being part of the hardware trigger system. The muon system is the final sub-detector and provides efficient muon hardware triggers and offline muon reconstruction.

The Calorimeter System

The calorimeter system \cite{107} is positioned after RICH2 and the first muon chamber, but before the main body of the muon system. The design of the calorimeter is driven by the requirement to provide a fast trigger for electrons, photons, neutral pions, hadrons and transverse energy measurements; and by the need to provide efficient \textit{PID} of neutral particles. The sub-detector is made up of four different components: the Scintillator Pad Detector, the PreShower, the Electromagnetic Calorimeter, and the Hadronic Calorimeter. Each of these detectors utilise the same principle of transmitting scintillator light using wavelength shifting fibres to photomultiplier tubes for readout; shown in Figure \ref{fig:CalorimeterDiagram}.

In order to maintain a uniform occupancy the calorimeter system is sub-divided into three regions of decreasing granularity moving out from the beam-pipe. The HCAL is the exception having only two regions. A photograph of the ECAL during installation, Figure \ref{fig:ECALInstallation}, illustrates this structure. The HCAL is shown in Figure \ref{fig:HCALInstallation}, although the two granularities are not visible.
(a) The ECAL and HCAL detector structure.

(b) A photograph of a scintillating tile being inserted into the HCAL. Taken by Maximilien Brice, 2005.

Figure 2.22: ECAL and HCAL layout.

(a) A photograph of the ECAL during installation.

(b) A photograph of the HCAL during installation.

Figure 2.23: Photographs of the ECAL and HCAL during installation.
The SPD and PS each consist of two single layers of scintillating tiles separated by a lead converter of 2.5 radiation lengths thick. Charge particles leave a signal in the SPD and all particles produce a shower in the lead converter which is measured by the PS. This provides additional information to distinguish between electrons, photons and hadrons in the L0 trigger decision.

The ECAL is a sampling calorimeter in the standard shashlik design. It uses interleaving layers of lead absorber plates and scintillator tiles that have wavelength shifting fibres running through them. The HCAL is also a sampling calorimeter that uses a steel absorber interleaved with scintillators and wavelength shifting fibres. The standard shashlik design of the ECAL has the lead absorber plates and scintillator tiles orientated in the $xy$ plane, perpendicular to the beam axis. The HCAL absorber and scintillators are orientated in the $yz$ plane such that they are parallel to the beam axis. The different layout of the two types of calorimeters is illustrated in Figure 2.22, which includes a photograph of the insertion of one of the scintillating tiles into the HCAL. The ECAL is 25 radiation lengths thick and provides complete containment of the photon showers. It has a design energy resolution of $\sigma_E/E = 10\%/\sqrt{E} \oplus 1\%$ (E in GeV). There is insufficient space in the cavern for the HCAL to be long enough to provide complete containment of the showers. The HCAL’s design energy resolution is $\sigma_E/E = (69 \pm 5)\%/\sqrt{E} \oplus (9 \pm 2)\%$ (E in GeV).

The calorimeters are calibrated and their performance measured with several different methods. Initially results from a test beam measured the energy resolution to be $\sigma_E/E = 8.5-9.5\%/\sqrt{E} \oplus 0.8\%$ (E in GeV) for the ECAL, where the exact value depends on the type of module. The HCAL’s energy resolution, $\sigma_E/E = (69 \pm 5)\%/\sqrt{E} \oplus (9 \pm 2)\%$ (E in GeV), is also taken from test beam data. Once installed in the LHCb cavern the calorimeters were initially calibrated and time aligned using cosmic rays. This initial calibration has since been superseded by the use of the data from the 2009 and 2010 data taking periods. The methods for calibrating the ECAL and HCAL are described in reference. The ECAL calibration method, requiring the most statistics (approximately 250 million events in the outer cells), reconstructs a photon pair around the $\pi^0$ mass in each ECAL cell and corrects the reconstructed peak to the $\pi^0$ mass. The primary HCAL calibration is from two $^{137}$Cs sources, using a system originally designed for the ATLAS TileCal. A summary of the calibration of the SPD and PS is found in reference. Calorimeter information has been used to reconstruct several resonances whose decay products rely on the calorimeter information, examples of these plots are shown in Figure 2.24 which reconstruct the $J/\psi(3097)$, $\eta(548)$ and $\omega(782)$. 
Figure 2.24: Invariant mass distributions reconstructed with calorimeter information.
The Muon System

The Muon System [91,96] provides a fast hardware trigger decision and muon identification. It is the furthest of LHCb's sub-detectors from the interaction point and is divided into 5 stations each covering 20-306 mrad (16-258 mrad) acceptance in the bending (non-bending) plane. The layout of the muon system is illustrated in Figure 2.25. Between the stations M1 and M2 are the calorimeters, and stations M2-M5 are separated by iron filters. These filters, including the calorimeters, remove all but the most penetrating muons. Each station is divided into four regions, labelled R1-4, which have a decreasing granularity moving out from the beam-pipe to keep the occupancy constant over the stations.

The first muon station, M1, is made from two detector technologies. In the inner most region, R1, radiation tolerant triple-Gas Electron Multipliers (triple-GEMs) are utilised [113] and the rest of R1, and the other muon stations, are made from Multi-Wire Proportion Chambers (MWPCs). The region around the beam-pipe, M1, which is in front of the calorimeters, receives the greatest flux of particles of all the muon stations' regions. The aging effects this flux would have on the MWPCs are difficult to suppress without a degradation in detector efficiency. The triple-GEMs are able to maintain their efficiency over the expected particle fluxes and lifetime of the sub-detector. In Figure 2.26 photographs of the muon chambers show the structure of M1, with the triple-GEMs around the beam-pipe in Figure 2.26(a) and a birds eye view of the gap between two stations, with the iron absorbers visible at the edge of the photograph, in Figure 2.26(b).

The operation of the muon stations in the hardware trigger requires a fast readout with high efficiency. The L0 trigger algorithms look for a coincidence in all five muon stations, requiring the efficiency in each station to be greater than 95%. Each station operates pairs of sensitive layers connected in an OR to improve efficiency, with the first station having only two layers to reduce the amount of material in front of the calorimeters, and the remaining stations have four layers OR’d into two pairs.

The muon identification algorithm in the hardware trigger, which is described in reference [114], requires a coincidence of hits in all five muon stations. The trigger algorithm uses each hit in M3 as a seed and attempts to fit a straight line through each muon station, the line is then extrapolated back to the interaction point, including a kick from the magnet, and the muon station hits are required to be in a field of interest around the extrapolated line. The muon identification algorithm that is run as part of a physics analysis is required to identify all muons over a wide momentum range. The algorithm
Figure 2.25: The muon stations and their iron/calorimeter filters.
(a) A photograph of the M1.

(b) A birds eye photograph of two adjacent muon stations with iron absorber visible at the sides.

Figure 2.26: Photographs of the muon stations.
providing muon identification in the early data is documented in reference [115] and its performance on the data taken during 2010 in reference [116]. The offline algorithm starts with long tracks and downstream tracks which are extrapolated from the final T station to each of the muon stations. A field of interest defined in the stations where the track is expected to pass, hits found in these regions are fitted as a muon track. The minimum momentum required of the tracks is 3 GeV, although tracks with a momentum less than 3.5 GeV and 4.5 GeV in M4 and M5 respectively are removed. This is because the probability of a muon reaching these stations with such a low momenta is negligible. This algorithm creates tracks with an ISMUON flag which is used as a basis to form more complex muon likelihoods and as a starting point for physics analyses [115].

The muon identification performance is quantified in data using \( \psi \rightarrow J/\psi \rightarrow \mu^+\mu^- \) to measure the muon efficiency and \( K_S^0 \rightarrow \pi^+\pi^- \) for the mis-ID rate [106]. The muon efficiencies from the offline analysis and after the L0 and HLT-1 trigger stages are presented in Figure 2.27. These results from the data taken during 2010 compare well with the most recent simulation and show the muon identification is performing well.

2.4.5 Data Acquisition and Experiment Control

The Online System [91, 96, 117] is the computing infrastructure at IP8 that is used to manage all of the detector control systems and support infrastructure; the readout and
The transportation of the data from the front-end electronics to permanent storage; and
the synchronisation of the data flow with respect to the hardware trigger and the LHC clock. It is typically grouped into three major systems: the Data Acquisition system, the Timing and Fast Control system, and the Experiment Control System. The control and monitoring of the support infrastructure is also part of the Online System. The architecture of the Online System is shown in Figure 2.28.

The Data Acquisition System

The DAQ system is responsible for transporting the event data from the front-end electronics through the infrastructure of the hardware and software triggers to the permanent storage in CERN’s central facilities. The signals from the sensor channels are read out and digitised by the front-end electronics, and then transmitted over optical links to the off-detector electronics – the Level-1 (L1) readout boards. The majority of sub-detectors use customised TELL1 boards for their off-detector readout electronics, the RICH uses UKL1 boards which are described in more detail in Chapter 3. Each

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3 Except the VELO which, due to space constraints and radiation levels, transmits analogue signals and the L1 boards perform the digitisation.
L1 board receives a subset of the sub-detector channels, processes the data from these
channels and writes it to the raw event banks. It is the raw event banks that the trigger
and offline reconstruction algorithms decode to access the sensor values. Each raw event
is written into a Multi-Event Package (MEP), a custom LHCb data format \[118\], and
each MEP is transmitted over Gigabit Ethernet (GBE) links to the event filter farm. The event filter farm consists of several thousand CPU cores running LHCb’s software
trigger, the HLT. If an event receives a positive decision from the HLT it is buffered by a
40 TB cache and from there sent to permanent storage. This buffer is sufficient to cache
the data in the cases where the peak event rate exceeds the transfer rate for a short
period of time. In addition to the event filter farm, the DAQ is responsible for two other
much smaller farms: the monitoring farm and the calibration farm \[119\]. The monitoring
farm uses triggered data and provides real time monitoring of the sub-detectors. The
 calibration farm receives events from dedicated calibration triggers. Each sub-detector
responds to these triggers in a different way, some not at all. The use of the calibration
triggers by the RICH sub-detector and the RICH monitoring algorithms are described in
Chapter 3.

The Timing and Fast Control System

The LHCb Timing and Fast Control system \[120,121\] is responsible for managing the
flow of data through the DAQ system. It distributes Timing, Trigger, and Control (TTC)
commands and manages the throttle signals from the DAQ systems. The throttle signals
are asserted when the data is not processed fast enough and a pause in triggered events
is required for the processing to clear some of the backlog. A simplified diagram of the
TFC system is shown in Figure 2.29.

The TFC system is controlled by the Readout Supervisor which is responsible for
managing the signals in the TFC network. LHCb has multiple Readout Supervisors
which allow for multiple, independent partitions to run in parallel. For example, the
RICH detector can take calibration data while the calorimeter and muon systems record
cosmic triggers for alignment. The TFC system has dedicated hardware to manage
the routing of the throttle and TTC signals between the Readout Supervisor and the
sub-detectors in an allocated partition, but there is only a single Readout Supervisor
connected to the L0 trigger decision, as shown in Figure 2.29. The other partitions
require trigger decisions generated by the Readout Supervisor. A block diagram with
two partitions running in parallel is illustrated in Figure 2.30.
Figure 2.29: An overview of the TTC system.
The operator interface to the LHCb detector is through the Experiment Control System [122]. It provides a single interface to all the Online Systems: the DAQ and TFC systems, the slow control systems such as the gas, cooling and power distribution systems, and the interface to the LHC machine. All of this is illustrated in Figure 2.31. Through a Graphical User Interface (GUI) complex configuration actions are performed using a minimal set of operator actions and warnings are presented when abnormal conditions arise. Behind the operator interface is a powerful Supervisory Control and Data Acquisition (SCADA) system maintaining the many detector subsystems. The ECS is presented in more detail in Chapter 3.

2.4.6 The Trigger

The purpose of the LHCb trigger is to provide an efficiently selected, enriched sample of b- and c-hadrons for physics analyses. The trigger reduces the 30 MHz collision frequency
to 2-3 kHz, which is the rate imposed by the maximum transfer rate to the offline storage and its capacity.

The trigger system consists of a single hardware trigger, the Level-0, and a two stage software trigger running in the event filter farm, the High Level Trigger. The trigger scheme used at the start of data taking is described in the LHCb detector paper [91]. The L0 operates several alleys which select the candidates using the full event information in the region of interest. The HLT-1 then confirms the L0 decision using more information and the HLT-2 performs inclusive and exclusive b- and c-hadron reconstruction with access to the full event. During the later stages of the 2010 data taking period the trigger strategy was revised in light of the higher number of interactions per bunch crossing. The L0 structure remains the same, however, the HLT-1 no longer confirms the L0 decision. The HLT-1 instead selects events with a single detached high transverse momentum track, typically hadron tracks, with further refinements for muon tracks. The HLT-2 performs inclusive reconstructions of the candidates passing the HLT-1. A presentation of the trigger and its performance during the first half of the 2010 run is presented in reference [123].
The LHC and the LHCb Experiment

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### Table 2.1: The main L0 alley thresholds for the most common trigger settings in the 2010 data. All the alleys have an SPD multiplicity cut of less than 900 applied.

<table>
<thead>
<tr>
<th>Alley</th>
<th>Threshold Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muon</td>
<td>( p_T &gt; 1,400 \text{ MeV/c} )</td>
</tr>
<tr>
<td>HighMuon</td>
<td>( p_T &gt; 2,400 \text{ MeV/c} )</td>
</tr>
<tr>
<td>DiMuon</td>
<td>( p_T &gt; 560 \text{ MeV/c} )</td>
</tr>
<tr>
<td></td>
<td>( p_T &gt; 480 \text{ MeV/c} )</td>
</tr>
<tr>
<td>Electron</td>
<td>( E_T &gt; 3,000 \text{ MeV} )</td>
</tr>
<tr>
<td>Hadron</td>
<td>( E_T &gt; 3,600 \text{ MeV} )</td>
</tr>
<tr>
<td>Photon</td>
<td>( E_T &gt; 3,200 \text{ MeV} )</td>
</tr>
</tbody>
</table>

---

The Level-0 Trigger

The L0 hardware trigger allows a total of 4 µs per trigger decision. The signal delays, the front-end electronics delays and the particle time of flight take 2 µs leaving 2 µs to make the trigger decision.

The L0 trigger operates several different alleys which are used to select candidates based on quantities that can be reconstructed by a single sub-detector. The VELO, the calorimeter system and the muon system are used to make the L0 decision. The VELO provides a pile-up veto; the calorimeters look for high transverse energy \( E_T \) hadronic and electromagnetic showers (with the ability to distinguish between electrons, photons and \( \pi^0 \)); and the muon system provides single- and di-muon triggers which look for high transverse momentum \( p_T \) tracks. A list of the main L0 trigger alleys and their thresholds are presented in Table 2.1. The alleys all have an SPD multiplicity cut applied in order to reject the events that contain a large number of candidates.

The efficiency of the L0 trigger has been measured with the data taken during 2010 using \( J/\psi \to \mu^+ \mu^- \) decays. The efficiency of a logical OR of all the L0 trigger lines is presented in Figure 2.32(a) and the efficiency of the logical OR of the muon lines in Figure 2.32(b). These measured efficiencies on the collision data compare well with the efficiencies from studies with simulated events.
The LHC and the LHCb Experiment

The High Level Trigger

The High Level Trigger has access to the full event information and could, in principle, perform a full offline analysis on each event. The input event rate to the HLT is 1 MHz and, since the event filter farm is not large enough to perform a detailed analysis of every event, the HLT algorithms typically sacrifice precision for speed. In order to further reduce the decision time for each stage only the minimum reconstruction is performed and care is taken to place the most efficient stages first in the trigger flow.

The division of the HLT into two components also speeds up the trigger, with HLT-1 and HLT-2 having different rate reduction strategies. The HLT-1 performs the reconstruction only in the region of interest and then performs cuts on simple signatures such as the transverse momentum or displacement of the track from the primary vertex. The performance of the logical OR of all the HLT-1 lines and the logical OR of all the muon HLT-1 lines are presented in Figures 2.33(a) and (b) respectively. Like the L0, the performance agrees with that expected from the simulated samples.

The HLT-1 reduces the event rate to approximately 40 kHz which is the input rate to the HLT-2. At this rate the HLT-2 can use the full event information to perform an inclusive reconstruction of the b- and c-hadrons, although the HLT-2 algorithms try to avoid accessing all the possible event information. Typically an algorithm will not access, and thus not calculate, any PID information utilising only track or vertex information to make the decision. Those algorithms wishing to utilise PID information
will typically require track reconstruction only in the final stages of the algorithm, thus on the minimum number of candidates. The HLT-2 reduces the output rate to 2 kHz.

### 2.4.7 The Stripping

The stripping performs the CPU intensive task of reconstructing the inclusive and exclusive b- and c-decays using the full reconstruction software \[124\]. The trigger system, despite its high rejection factor, still contains a large number of events with a low purity and the stripping increases the purity of specific channels. The physics analyses therefore do not need to access the triggered events directly and instead filter the stripping candidates significantly reducing the amount of CPU time required. The events rejected by the stripping, unlike the trigger, are not lost but the input to the stripping, the triggered data, is archived after approximately one year. The stripping is performed once or twice a year, allowing the stripping selections to evolve and the maximum amount of data to be available to the analyses.

Each of the LHCb physics analyses provides the stripping with a reconstruction line for an inclusive or exclusive decay and the stripping framework groups these exclusive decays into decay streams. For example, there is the DIMUON stream which contains the b-decays to muons, plus other muonic decay channels such as $J/\psi \rightarrow \mu^+\mu^-$. 

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**Figure 2.33:** LHCb HLT-1 trigger efficiency measured using $J/\psi \rightarrow \mu^+\mu^-$ decays selected from the data taken during 2010.
2.5 The LHCb Software

2.5.1 Gaudi

The LHCb software framework is implemented using the Gaudi package \cite{125}. Gaudi provides a flexible, stable framework that implements software solutions to many of the very different problems encountered in high energy physics. Two examples are the LHCb HLT which is optimised for CPU efficiency on a large server farm with low latency data access, compared to the physics analysis package which runs on a global batch system where different sites use a variety of technologies for their CPUs and data access.

2.5.2 Event Simulation

The generation and simulation of the signal and specific or general background samples is performed using the Gauss package \cite{126}. First Gauss runs the generation phase which runs Pythia \cite{127} and EvtGen \cite{128}. Pythia generates the underlying proton-proton collisions and performs the hadronisation of the generated particles, and EvtGen simulates the decay of the b-hadrons. Next Gauss simulates the interaction of the particles produced in the generation phase with the detector using Geant4 \cite{129}.

The digitisation stage is performed by Boole \cite{130}. Boole models the detector response to the hits, for example modelling the charge accumulated in the VELO sensors or the Cherenkov photons on the hybrid photodiodes (HPD). Furthermore, Boole simulates the response of the readout electronics and the L0 trigger, and, as a result, the output of the digitisation has the same format as the data recorded by LHCb. From this stage onwards common tools reconstruct and analyse the simulated events and data recorded by the LHCb experiment.

2.5.3 The Software Trigger

The software trigger, HLT, is run by a package called Moore \cite{131}. Moore runs on both the data recorded by LHCb or the simulated events after the Boole digitisation stage. Moore has two modes of running, rejection mode and pass through mode. When recording data Moore runs in rejection mode selecting only those events that pass the trigger lines adding information to the events as to what lines they passed. In pass
The LHC and the LHCb Experiment

through mode Moore does not reject events, it just adds the information to the events about the trigger decisions.

2.5.4 Reconstruction and Analysis

The data recorded by LHCb and the simulated events contain the same information and the Brunel package is used to reconstruct the events. Brunel performs track finding and fitting, creates clusters from the calorimeter information and runs the particle identification algorithms. This creates a **proto-particle** for each charged track or neutral cluster. Each proto-particle has associated with it a particle identification likelihood for each of the long-lived mass hypotheses: pion, kaon, muon and proton. Brunel also identifies the primary vertices in an event. The simulated events contain information about the simulated particles created in the generation phase, Brunel associates the proto-particles it has created with the simulated particle that generated the tracks and clusters in the detector simulation. This extra information allows analyses on simulated events to check the measured values against the value from the generated particle.

A physics analysis is performed by the DaVinci package. The DaVinci package is used to reconstruct particle decay chains e.g. $B^\pm \rightarrow J/\psi \rightarrow \mu^+\mu^-$ and to study their properties. An analysis takes the proto-particles in an event and creates a particle by assigning it one of the long-lived mass hypotheses. The properties of these particles can then be studied and they can be used to create other particles, whose properties are also available. By requiring particle properties to lie in certain ranges or have specific information associated with the particle, the particles can be filtered to select those most likely to match a decay chain.

2.5.5 Data Processing

The LHC produces an enormous amount of data and the World LHC Computing Grid (WLCG) is a data storage and analysis infrastructure, distributed across the globe, created to process the data produced by the four main LHC experiments. The resources are connected together via the Internet and managed by the Grid middleware. The Grid middleware is the software application which makes the global network of computers appear as a local batch system to the user. A user submits jobs, a description of the work to be performed and the resources required, to the Grid via the Grid middleware.
The LHCb computing model defines how LHCb uses the WLCG resources [135]. The Distributed Infrastructure with Remote Agent Control (DIRAC) and Gaudi/Athena and Grid Alliance (Ganga) applications are the interfaces for LHCb users to the WLCG resources. DIRAC interfaces to the WLCG middleware and is responsible for managing the resources allocated to LHCb by the WLCG, dealing with user authentication, and the input and output of user jobs. Ganga is the top level interface to DIRAC and manages user jobs, preparing them for submission to DIRAC and storing the output returned by DIRAC jobs. Through the Ganga interface the event simulation, reconstruction and analysis is controlled and recorded.

2.6 The 2010 Data Taking Period

The LHCb's first collisions were on the 23\textsuperscript{rd} November 2009, initially with an energy of 450 GeV per beam which increased to 1.18 TeV per beam. Collisions were recorded with these conditions (with a break over the Christmas period) until the 30\textsuperscript{th} March 2010 when the first proton-proton collisions at 3.5 TeV per beam were recorded. The 3.5 TeV collisions continued until the end of October and, after a short shutdown, the first heavy ion collisions commenced on the 4\textsuperscript{th} November [136]. A second period of 3.5 TeV proton-proton collisions commenced on 12\textsuperscript{th} April 2011 and heavy ion collisions are scheduled.
for the 14\textsuperscript{th} November. A further run is scheduled in 2012 before a long shut down to commission the LHC to its design energy of 7 TeV per beam \cite{137}. The 3.5 TeV per beam proton-proton collisions during 2010 were the LHC’s first high energy collisions and as the run progressed the amount of luminosity delivered increased exponentially. By the end of 2010 the LHC had delivered 42.15 pb\textsuperscript{−1} of integrated luminosity of which LHCb recorded 37.66 pb\textsuperscript{−1} (89 \%), shown in Figure 2.34.

The LHCb detector performed well during this first high energy run, operating at a higher number of collisions per bunch crossing, \( \mu \), than the designed 0.74. The peak \( \mu \) increased with increasing fill (time) and the majority of the luminosity was recorded with a peak \( \mu \) greater than 1.5, as seen by comparing Figures 2.34 and 2.35. Not all of the collisions that occur are recorded, the percentage of the collisions that occur that are recorded is known as the data taking efficiency. A breakdown of the data taking efficiency with the LHC fill number is given in Figure 2.36. As many of the sub-detectors require the proton-proton beams to be stable and colliding before they can turn on their sensors there is an inevitable period when data cannot be recorded. These detectors are those that require a high voltage to operate their sensors and can spontaneously turn off during a run. This causes small inefficiencies in the data taking which decreased as
experience was gained. The VELO can only move to its closed position once the LHC has started colliding protons as it monitors the collisions as it closes; during which time LHCb cannot record the data. The DAQ system causes data taking inefficiencies when the event rate becomes too high and the hardware triggers are suppressed. As the event filter farm has been deliberately increased in size as slowly as possible, to take advantage of the increase of computing power, there were larger inefficiencies at the start of the run. Again as the fill progressed the inefficiencies due to the DAQ system decreased. Due to the exponential increase of the luminosity the larger inefficiencies in the early fills caused a negligible data loss compared to the smaller inefficiencies in later fills.

2.7 Summary

The LHC is a proton-proton collider that has been running successfully for two years. The LHC has four main experiments situated around the collider and some smaller experiments situated in the caverns evacuated for the main experiments. One of the main detectors is LHCb which has been designed to study the b-hadrons, the full spectrum of which are produced by the proton-proton collisions at the LHC. The LHCb detector’s
performance matches the expectations from simulation and the DAQ has been successfully recording collisions with a greater occupancy than expected. The LHCb hardware and software trigger are run to reduce the number of events saved to disk by keeping only those that contain the signature of interesting events. The stripping is another stage designed to reduce the number of events available for the physics analyses; this performs the CPU intensive task of reconstructing the candidates of interest in the events and applying a further event selection. The WLCG is a computing infrastructure built to process the vast amounts of data that the LHC experiments record. The software that has been written to analyse and simulate the LHCb events is run on the WLCG and managed by the DIRAC and Ganga tools. Finally, the performance of LHCb during the 2010 data taking period was found to be excellent and the full 2010 data set is used for the physics analysis presented in this thesis.
Chapter 3

The LHCb RICH Detectors

This chapter describes the LHCb RICH sub-detector system which is used to identify charged particles. Section 3.1 describes the physical process that is used to identify the particles, the sub-detector itself, how these two are combined to identify the particles and the performance of the sub-detector. Section 3.2 describes the UKL1 board that is used to transfer the events from the RICH sub-detector to the event filter farm and the UKL1 control software that is implemented within the LHCb ECS. Finally, Section 3.3 describes the algorithms implemented to monitor the occupancies in the RICH sub-detector and to check for inactive regions.

3.1 Charged Particle Identification at LHCb

The identification of charged particles in LHCb is provided by two RICH detectors. Customised single photon detectors are used to detect Cherenkov light emitted as charged particles pass through different media. The different media allow the sub-detectors to cover a charged particle momentum range of 2 to 150 GeV/c.

3.1.1 The Cherenkov Effect

As a charged particle moves through a dielectric medium (at any speed) it polarises the region it passes through and, if the medium is an insulator, the medium restores itself to equilibrium through the emission of a photon. The photons emitted when the particle is moving faster than the speed of light in the medium interfere constructively.
and these photons continue to propagate. This is known as the Cherenkov effect and is illustrated geometrically in Figure 3.1.

The photons emitted have a constant angle with respect to the direction of particle motion forming a solid cone. The angle of emission with respect to the direction of motion is known as the Cherenkov angle, $\theta_C$, and is given by

$$\cos (\theta_C) = \frac{1}{n\beta}$$

(3.1)

where $n$ is the refractive index of the medium and $\beta = v_p/c$ with $v_p$ being the particle’s velocity and $c$ the speed of light in a vacuum.

### 3.1.2 The RICH System

The LHCb RICH detector is described in references 91, 96, 139. Its design is optimised for the detection of Cherenkov photons from charged hadrons (kaons, pions and protons) over a momentum range of 2 to 150 GeV/c. This covers the momentum range of the decay products of B-mesons and b-baryons, which peak around 50 GeV/c. The RICH-1 detector is positioned before the magnet and covers a momentum range of 2 to 60 GeV/c.
The technology and design principles are the same for both RICH-1 and RICH-2. Charged hadrons pass through a medium and radiate a cone of Cherenkov photons. These photons are focused by spherical mirrors and are then reflected out of the acceptance by flat mirrors to the photon detectors; the design of the optical system minimises the amount of material in the acceptance. The two main radiators are fluorocarbon gases at room temperature and pressure, used due to their low dispersion; RICH-1 uses $\text{C}_4\text{F}_{10}$ ($n=1.0014$) and RICH-2 uses $\text{CF}_4$ ($n=1.0005$) [140]. The gas radiators only provide a kaon-pion separation down to a momentum of 9.3 GeV/c and in RICH-1 a second radiator, a 5 cm wall of silica aerogel ($n=1.03$) [141], is situated at the start of the RICH-1 gas volume. It is the first radiator the particles traverse upon leaving the VELO. Schematic views and photographs of RICH-1 and RICH-2 are found in Figure 3.3.
Figure 3.3: Schematics and photographs of [RICH] 1 and [RICH] 2.
The LHCb RICH Detectors use a custom-built HPD [142], developed in collaboration with industry, to detect the Cherenkov photons focused by the optical system. The HPDs are vacuum devices with a photocathode layer deposited on the quartz entrance window of the vacuum tube. Photons incident on the window induce a photocurrent that is cross-focused and demagnified by up to a 20 kV accelerating potential onto a silicon pixel chip. A simple schematic of an HPD is shown in Figure 3.4(a) and a photograph in Figure 3.4(b). The silicon sensor and electronics are custom designed to meet the requirements of a 500 × 500 µm pixel size and a 25 ns readout rate [144]. The sensor and its electronics are mounted within the vacuum envelope of the HPD and are robust against the HPD manufacturing process. The silicon sensor was designed with the ALICE collaboration and it forms part of their tracking detector. The ALICE tracking detector requires a higher granularity (but lower readout rate) and a silicon sensor pixel is actually 62.5 × 500 μm. In LHCb the chip is operated in a mode where the high resolution dimension is reduced to the required 500 μm granularity by producing a single pixel from a group of eight pixels. This reduction in resolution, from 256 × 32 pixels to 32 × 32 pixels, is necessary to meet the 25 ns readout requirements of the RICH.

In the RICH detector there are 484 HPDs that are divided into four boxes. There are two boxes in RICH-1 and two in RICH-2. In RICH-1 each box contains 14 columns with 14 HPDs per column and the boxes are mounted above and below the gas volume. These are referred to as the top and bottom boxes respectively. A RICH-2 box has 18 columns with 16 HPDs per column and the boxes are located on either side of the gas volume, one on the A-side and the other on the C-side (left and right respectively when looking from the VELO to the muon sub-detector). The boxes are separated from the gas-radiator volumes by a quartz window and have their own nitrogen environment. The schematics presented in Figure 3.3 show the side view of RICH-1 and a birds eye view of RICH-2 so the HPD enclosures are always at the top and bottom of the image.

The data from the silicon sensors must be transferred at a high rate from the HPDs upon receipt of an L0 trigger. A custom board has been built to do this and it is known as the L0 board [145]. The L0 boards are the interface to the ECS for the HPDs, as well as providing biasing, monitoring and power-supply filtering. A pair of HPDs is connected to one L0 board and each L0 board has a pair of optical fibres to transmit the data, which it formats and error checks, to the next stage in the DAQ system, the UKL1 boards. A photograph of an L0 board is shown in Figure 3.5.

1 The development and manufacture of the HPDs was done in close collaboration with PHOTONIS [143]
Figure 3.4: The [RICH][HPD] detectors.

Figure 3.5: Photograph of an [L0] board.
3.1.3 Particle Identification

The Cherenkov angle is dependent on the velocity of the particle and the medium it is travelling through. The Cherenkov angle versus the momentum of the particle, for the different radiators types, forms bands for the different particle masses. The predicted distributions for the three RICH radiators, using simulated events, are shown in Figure 3.6. From this figure it is seen that the production of Cherenkov photons turns on at different momenta, depending on the particle mass, and that all tend to the same angle, saturate, when $\beta \sim 1$. From the figure it is also apparent that pions are identifiable down to 2 GeV/c using the gas radiators although little information is provided about the heavier kaons. The aerogel radiator is used in this low momentum region to separate the heavier species, such as the kaons and protons, although all aerogel tracks saturate at around 10 GeV/c. A plot of the reconstructed Cherenkov angle versus the momentum of the track have been produced for the two gas radiators in Figure 3.7 using the 2010 data [146]. The figures show a good agreement with the predicted distributions.

Due to the high event occupancy in LHCb a pattern recognition algorithm that considers all the tracks identified by the tracking system in a single global likelihood is used by the RICH system [147]. This provides an optimal solution as a large number of the Cherenkov rings overlap. The output of the PID algorithm is a likelihood that the track is an electron, pion, kaon, muon or proton. By taking the ratio of two of the track likelihoods for specific particle hypotheses, the likelihood that a track is one particle compared to another is found. The particle identification provides the logarithm of the ratio of a given particle hypothesis to the pion hypothesis, which can be expressed as the difference in two logarithms e.g.

$$\Delta LL_K - \pi = \log L_K / L_\pi = \log L_K - \log L_\pi.$$  

The difference with respect to any other particle hypothesis can be found by subtracting two $\Delta LL$s e.g.

$$\Delta LL_{K - \mu} = \Delta LL_K - \pi - \Delta LL_{\mu - \pi}.$$  

3.1.4 The RICH PID Performance

The RICH PID performance is monitored using a set of high statistics control channels that are isolated using only kinematic selections. These decays are $K^0_S \rightarrow \pi^+\pi^-$, $\Lambda \rightarrow p\pi^-$ + c.c., $\phi \rightarrow K^+K^-$ and $D^*(2010)^+ \rightarrow D^0 (\rightarrow K^-\pi^+)\pi^+ + c.c.$, where the c.c. implies the charge conjugate decay.$^2$

$^2$The $\phi \rightarrow K^+K^-$ is the exception as it cannot be isolated using only kinematic cuts and RICH information is used to select one of the kaons leaving the other unbiased for the performance monitoring.
Figure 3.6: Cherenkov angle versus particle momenta using simulated events.
Figure 3.7: Cherenkov angle versus particle momentum from the 2010 data [146].

Figure 3.8: The kaon efficiency versus momentum curves for two different $\Delta LL_{K-\pi}$ selections criteria. The full, red circles are more kaon-like tracks than the open red circles.
One of the criteria to monitor the RICH PID performance is to measure the number of the requested charged hadrons that remain after a selection based on the RICH $\Delta L_{K^{-}\pi}$ has been applied. The kaon selection efficiency versus kaon momentum is shown in Figure 3.8 using the $D^{*}(2010)^+ \rightarrow D^{0} (\rightarrow K^{-}\pi^{+}) \pi^{+} + c.c.$ decays. Both of the figures show the fraction of kaons selected versus hadron momentum and the fraction of pions that were misidentified as kaons for two different $\Delta L_{K^{-}\pi}$ selection criteria. In Figure 3.8(a) the curves are plotted using the 2010 data; in Figure 3.8(b) they are plotted using simulated events. A comparison between the two figures shows a good agreement between the data distributions and the simulated event distributions.

These decay channels are also used to calibrate the PID distributions for the hadrons in other decay channels. The analysis presented in this thesis uses the kaon and pion from the $D^{0}$ in the $D^{*}(2010)^+ \rightarrow D^{0} (\rightarrow K^{-}\pi^{+}) \pi^{+} + c.c.$ decays to provide calibrated kaon and pion $\Delta L_{K^{-}\pi}$ distributions.

3.2 The UKL1 Board

The UKL1 boards, designed and developed in Cambridge, are the RICH off-detector readout electronics. They are located in the LHCb experiment area approximately 100 m from the detector, behind the shielding wall, in a low radiation environment. The UKL1 technical manual describes in detail the UKL1 board components, operation and firmware [148].

The UKL1 boards are designed to perform data reduction and event formatting. A board is pictured in Figure 3.9(a) and the layout in Figure 3.9(b). The data is received on the left-hand side of both figures by three groups of 12 optical receivers, and then flows through to the right-hand side out to the readout network. Additional control signals are illustrated flowing in and out of the upper right corner of Figure 3.9(b) from the TFC network and the ECS. Each optical input corresponds to a single HPD, and, with 484 HPDs in RICH-1 and RICH-2, a minimum of 14 UKL1 boards are required. In practice, due to the high event occupancies and the asymmetry in the number of HPDs in RICH-1 and RICH-2, there are currently 11 UKL1 boards for RICH-1 and 13 for RICH-2. The high occupancy regions are shared between the UKL1 boards in order to efficiently use the available output bandwidth. The UKL1 boards maintain RICH-1 and RICH-2 as independent sub-detectors with no UKL1 board receiving HPD data from both.
Figure 3.9: The UKL board.
A UKL1 board has 36 input channels split over four Field Programmable Gate Arrays (FPGAs), known as ingress FPGAs, each of which processes nine channels. In the ingress FPGAs, the data from each HPD is checked for errors, optionally zero suppressed and formatted into the final HPD event bank. The ingress FPGAs are connected to a single FPGA known as the egress FPGA, which sequentially reads the multiplexed data from each ingress, and compiles and buffers the raw event bank. The egress FPGA is also the interface to the TFC via Timing Trigger and Control receiver (TTCrx) chip. Upon receipt of the appropriate command from the TFC, the egress FPGA builds the Multi-Event Packages (MEPs) by adding the number of requested events from the buffer into a single bank. The MEP is then transmitted via the Gigabit Ethernet (GBE) mezzanine using one of four bidirectional GBE ports to the event filter farm.

Due to the finite output bandwidth available to each UKL1 and the variable input event size, it is possible for the UKL1 boards to receive more data than they can transmit. The UKL1 boards contain several event buffers to help manage the data flow but in the event that the free space in these buffers drops below a configurable threshold, the UKL1 boards assert a throttle signal. This throttle signal is seen by the TFC network and L0 triggers are suppressed allowing the UKL1 buffers levels to reduce and the throttle signal to be deasserted.

Communication between the UKL1 boards and the ECS is over a local area network. Each UKL1 board has a Credit Card PC (CCPC), a credit-card sized embedded PC which provides the interface to the components on the UKL1 boards, e.g. the FPGAs or GBE card. The CCPC server monitors the components, publishes the values of their registers and the ECS then subscribes to these values. The ECS sends updated configuration settings to the CCPC server which then writes the new values to the requested component.

3.2.1 The Experiment Control System Architecture

The LHCb ECS is a Supervisory Control and Data Acquisition system. A SCADA system provides a supervisory layer allowing access to hardware through their controllers, monitoring through trending and alarms, archiving, access control and a means to automate the system. A SCADA system is also ideally highly scalable, cross-platform,

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3 The zero suppression algorithm is designed to reduce event sizes by encoding the position of the pixels containing a signal in the event bank. This algorithm is optional and is automatically disabled above a certain occupancy when it no longer reduces the event size.
The LHCb RICH Detectors

flexible, extensible and has built in redundancy. After the evaluation of many commercial
SCADA systems [154] the “Prozeßvisualisierungs- und Steuerungs-System” (PVSS) is
the commercial SCADA system in which the majority of the controls for the LHC and
its main experiments are implemented [155]. Due to its success in this area it is used by
many of CERN’s other facilities and experiments [156].

The Joint Controls Project (JCOP) group is a collaboration between CERN and
the LHC experiments, and are responsible for the common components of the ECS
systems [157]. The JCOP group performed the initial evaluation of the commercial
SCADA systems and chose PVSS. It is a JCOP maintained PVSS framework that
provides the core features required by all the experiments [158] and in addition to the
framework a series of guidelines are provided for those implementing detector subsystem
controls to ensure a coherent system [159,160]. The framework also ensures that
independent ECS systems can communicate, such as in the case of the LHC machine
and the individual experiments. In addition to the JCOP framework, PVSS provides
a powerful integrated development environment which provides a drag and drop GUI
building, tools to write small scripts for GUI interactivity and to write larger libraries,
graphical access to internal data structures, and a powerful set of libraries and drivers
for performing common computing and communication tasks.

The LHCb ECS is organised into a hierarchical tree structure. At the top of the
hierarchy there is a single node, the parent, which is connected to multiple nodes, the
children, each of which may be connected to further nodes. There are two types of node
in the tree: the Device Units (DU), which represent hardware or software objects, and
the Control Units (CU), which are logical groupings of CUs or DUs. In LHCb there
are DUs for each L0 and L1 board, all the voltage channels, the instances of the HLT
processes in the event filter farm and many others. The tree structure allows these to be
collated into logical groupings and control by CU.

The ECS hierarchy is modelled as a Finite State Machine (FSM); each node has a
predefined set of states and each state has its own set of actions. The interaction with
the hierarchy is through a GUI, two windows of which are illustrated in Figure 3.10.
A minimalist view showing only the nodes, each of which is expandable to show their
children, is shown in Figure 3.10(a). Right clicking on any node opens its interface,
allowing direct interactions with the node, and Figure 3.10(b) shows the top-level LHCb
CU interface. Control of the hierarchy’s FSM is done primarily through the System’s
State widget in Figure 3.10(b). Clicking the state (in this case RUNNING) brings up a drop
down list of actions, the selected action is then propagated to each of the Sub-Systems.
which change state once the action has been completed. By changing the options in this window different predefined run modes are selected. The sub-systems are grouped according to purpose e.g. the [DAQ](#) domain contains all the [L0](#) and [L1](#) DUs. There are also top-level domains for each of [LHCb](#)’s sub-detectors, listed at the bottom of Figure 3.10(b). In order to run [LHCb](#), all the resources associated with a sub-detector are taken by the [LHCb](#) domain. It is possible to release an individual sub-detector (or any [CU](#) or [DU](#)) and this allows the individual sub-detectors to operate independently of the [LHCb](#) domain and each other.
The UKL1 control system is built atop the LHCb PVSS framework which adds LHCb specific libraries to the JCOP framework [161]. The project provides the structures and code to interface with the FSM and a GUI to install, monitor and configure the project manually. The typical interaction with the project is in automated responses to FSM actions propagating down the tree. The GUI is used by experts primarily for modifying the default behaviour and debugging errors.

The FSM framework component [162] is the PVSS interface to the State Management Interface (SMI++) framework [163] and it is used by all experiments that have adopted the JCOP framework. SMI++ is a C++ implementation of the State Manager language used by some of pre-LHC era experiments [164]. Within the framework the states and actions for an FSM type are defined and instances of this type are then instantiated as nodes in the FSM tree. The UKL1 boards are part of the LHCb DAQ domain and a UKL1 CU node is created for the RICH-1 and RICH-2 detectors from the DAQ domain type, defined in reference [165]. The CUs pass the actions they receive onto their children and report their state, which is formed by a combination of the state of their children. Part of the RICH-2 FSM structure, along with the UKL1 CU and some DUs are shown in Figure 3.11(a). The UKL1 framework component creates the UKL1 FSM tree from a RICH specific CU, all the UKL1 boards it discovers on the network, the FSMConfDB object and the Hugin from the TFC system. Once created the interface to the UKL1 CU is provided by the panel in Figure 3.11(b).

The Hugin board is used by the TFC system to perform the logical OR of the L1 throttle signals and propagate them to the Readout Supervisor. The Hugin is represented in the FSM by the yyhugin nodes, where yy is a sub-detector identifier e.g. r2 for RICH-2. The Hugin node allows an operator to see the state of the Hugin and each connected UKL1 throttle signal. The RICH2_L1_ConfDB node is an instance of the DAQ_Domain_v1 type defined in the FSMConfDB JCOP framework component, the component that manages the cache of configuration settings [166]. A group of configuration settings are known as a recipe and the recipes are stored in the Configuration DataBase (ConfDB), a database external to PVSS which has its own JCOP component [167]. When the UKL1 CU receives an action it first issues it to the FSMConfDB node. The FSMConfDB node loads, if necessary, the recipe from the ConfDB into the PVSS recipe cache. Once done it transitions to a ready state which prompts the UKL1 CU to send the action to the
The UKL1 DU. The UKL1 DU upon receipt of an action retrieves the specified recipe from the cache, applies it to the UKL1 board and then transitions to the appropriate state.

The UKL1 DUs are a custom type whose states and actions are defined by the UKL1 CU. A graphical interface to the UKL1 board’s configuration registers and their corresponding setting in the ConfDB is provided. The main UKL1 DU window allows access to all of the UKL1 registers, however due to their large number they are logically grouped into windows containing only a subset of the registers. The main window uses three drop down boxes to allow access to the registers depending on their type: status, configuration or recipe. The window displaying the status registers for the ingress FPGA are shown in Figure 3.12(a) and are displayed by selecting the appropriate option from the Hardware Status drop down list. Several of the registers are highlighted in green indicating they are in a good state but they would change colour in the event of the register going into an error state, with the colours defined by the JCOP framework. The Hardware Configuration windows, an example of which is pictured in Figure 3.12(b), are read-only windows that show the current values of the configuration registers for the FPGA and GBE mezzanine. The status and configuration windows monitor the registers and update their values when they change; this is controlled through the Start monitoring, Stop...
(a) The UKL1DU ingress status panel.

Figure 3.12: The UKL1DU interface.
The LHCb RICH Detectors

Figure 3.12: The UKL1DU interface.

(b) The UKL1DU FPGA configuration view.

(c) The UKL1DU FPGA configuration editor.
monitoring and Monitoring config buttons. The Hardware Configuration window contains an additional button Hardware config. Clicking this button opens the window in Figure 3.12(c) which allows write access to each of the registers. The action buttons, e.g. Apply, are labelled according the JCOP guidelines so they are familiar to those experienced with the ECS but not necessarily the UKL1 project. The Hardware config and Apply buttons are only enabled if the board is part of the user’s tree and if the UKL1 is not in the RUNNING state. It is possible to force these controls to enable but this requires further actions.

The Recipe configuration drop down box opens the recipe editor interface, shown in Figure 3.13(a), which allows the UKL1 specific recipes to be edited. The editor windows are the same as for the hardware configuration and are accessed through the Edit recipe drop down list. The fwUkl1 library implements a fwUkl1_load and a fwUkl1_save function for getting and setting the recipe settings, and a fwUkl1_read and a fwUkl1_write function for getting and setting the hardware registers. The configuration interface chooses the appropriate function sets depending on the open mode. The project also provides an interface for managing the recipes at a CU level, the window for this is shown in Figure 3.13(b). As most of the recipe settings are the same for all UKL1s the mass recipe creator creates a recipe for a single UKL1 board and then copies to all the other boards (or a selected subset).

The UKL1 project takes the user settings and translates these into the format that is recognised by UKL1 register(s) or unpacks the register(s) read from the UKL1 board for display. The fwHw LHCb framework component is responsible for creating and managing the PVSS representation of UKL1 board’s registers [168]. The UKL1 project writes and reads from the UKL1 board’s registers using fwHw provided functions. The fwHw library uses the fwCCPC library to transfer the data from PVSS to the UKL1 board [169] by communicating with the CCPC server using the Distributed Information Management (DIM) application [170]. DIM, like SMI++, has been inherited from the pre-LHC era experiments and it provides a distributed infrastructure for systems to communicate. Applications publish to the DIM namespace and others subscribe to them, with DIM providing error recover and redundancy. The PVSS DIM libraries, fwDIM, are used to find and connect to the CCPC server of each UKL1 board [171]. Using the definition of the UKL1 board from the fwHw type the fwCCPC component configures the CCPC server using the fwDIM libraries. A separate DIM server is provided for RICH-1 and RICH-2. A graphical overview of the UKL1 project, illustrating all the links described, is found in Figure 3.14.
The LHCb RICH Detectors

(a) The recipe configuration panel for a UKL board.

(b) The mass recipe creator panel.

Figure 3.13: The per UKL and per CU recipe interface.
3.3 The RICH Monitoring

The [RICH] monitoring software is a collection of algorithms and tools running in two dedicated CPU farms: the monitoring farm and the calibration farm \[172\]. The monitoring farms receive events that have been sent to the event filter farm and have passed both the hardware and software triggers. The calibration farm receives events that have been triggered using dedicated calibration triggers which do not pass through the event filter farm. The package is responsible for monitoring the data quality from the [RICH] detectors and the [RICH] monitoring algorithms report the output of their analysis as either histograms or counters which are displayed using the [LHCb] Histogram Presenter. The histogram presenter is a tool developed by [LHCb] to display the histograms and counters from the monitoring algorithms and typically these overlay reference histograms showing the expected distributions. The histograms are also capable of generating alerts if they indicate exceptional conditions. An example of a [RICH] monitoring Presenter page displaying a histogram that has generated an alert is shown in Figure 3.15. The author was responsible for the [RICH] monitoring pages during the [RICH] commissioning and very early start up phase of the [LHC].
**Figure 3.15:** The Histogram Presenter. In this screenshot an alert has been raised and is visible in the bottom left.
3.3.1 Panoptes

The software package that contains the RICH monitoring algorithms, tools and their configuration scripts is called Panoptes and the algorithms are used to monitor the events from the HPDs. The events are processed on a “best effort” basis with as many events sent to the algorithms as the available resources and algorithm processing time allows. The algorithms build up a histogram of a time slice and, in order that the data accumulated over a long data taking period does not wash out anomalous conditions, the analysis buffers are reset periodically and only set time ranges are analysed. Most of the detector safety is done by PVSS projects monitoring voltages, temperatures, currents, etc. with automatic hardware actions as a last resort. However, Panoptes is able to provide additional information about the HPD data itself and, for example, during the ramping of the HPD high voltage it monitors the event occupancy for signs of HPD faults. In these cases an operator must pay close attention to monitoring output as Panoptes, except in a few specific cases, has no automatic influence on the ECS.

The Commissioning And Monitoring Error Reporting Application (CAMERA) [173], developed in Cambridge, is an instant messaging application used for reporting errors from Panoptes. In the LHCb online environment two CAMERA servers run, one for each RICH sub-detector. The Panoptes monitoring algorithms submit messages to a CAMERA server that consist of any number of histograms or blocks of text plus a brief summary. The CAMERA GUI client connects to the selected CAMERA server and the server pushes out the summary messages to the client. The client user then selects a message summary and the extra detail is retrieved and displayed. The CAMERA client’s main window is shown in Figure 3.16(a) and the algorithms reporting to CAMERA are listed on the left with the colour representing the message severity. Selecting an algorithm filters the summary messages on the right and clicking one of the summaries opens the full message (if there is one) in a separate window. An example is shown in Figure 3.16(b) where two histograms display the HPD’s test pattern and some additional summary text provides a description of the error.

CAMERA differs from the standard histogram reporting tools as it is not continuously producing histograms for monitoring, instead it only alerts a user when something notable happens. This made the tool very useful during commissioning where knowledge of the HPD occupancies must be known on short time intervals to look for spikes. It allowed Panoptes to become more effective in performing detector safety monitoring. During normal physics data taking the algorithms typically receive kHz event rates and the
Figure 3.16: The CAMERA GUI.
The LHCb RICH Detectors

algorithms are coded with care not to overload the servers and operators with messages every time a tiny anomaly occurs.

During the RICH commissioning CAMERA operated using a collection of bespoke scripts and hardcoded configuration settings in the Panoptes package. The work to integrate these into the online system was part of the author’s work on the RICH monitoring. This early version of the CAMERA setup also led to the empirical determination of the maximum number of files that a single directory can contain.

The Hit Map Monitor Algorithm

The HitMapsMonitor algorithm runs in the monitoring farm and was important during the commissioning phase of the RICH sub-detectors as it allowed experts to see an image for each of the HPDs to monitor their occupancy. The author rewrote the algorithm to provide automated monitoring of the HPD occupancies via the HPDCountTool. The HPDCountTool monitors each of the seen HPDs calculating the average occupancy over the last \( n \) events and if the current occupancy differs from the moving average by more than \( x \) sigma an error is flagged. Both \( n \) and \( x \) are configurable. The HitMapsMonitor keeps track of the number of errors seen and reports them to CAMERA when they exceed a threshold. The HotPixelTool is used by the HitMapsMonitor to look for pixels that fire in more than \( y \% \) of events, where \( y \) is configurable. This tool is not part of the author’s work. The HitMapsMonitor histogram output was updated to produce, in addition to the full resolution two-dimensional hit maps for the individual HPDs and each RICH box, a low resolution RICH box hit map and a one bin hit map for each HPD. Each of these histograms are enabled individually, with most disabled by default, and in Figure 3.17 the HitMapsMonitor Presenter page is shown. The histograms provide additional checks for error patterns that are not known to the monitoring algorithms.

The choice of a moving average with a configurable number of events is due to the wide range of expected occupancy for each HPD. With no high voltage applied to the HPD the sensors have a large occupancy. As the voltage is ramped (over the course of tens of minutes) the occupancy decreases to almost zero and, finally, the occupancy during data taking depends on the HPD’s position within the sub-detector. The ramping and dark count occupancies are dependent on the particular HPD. While the expected occupancy varies dramatically, the rate of change of occupancy should be small and similar for each HPD. The moving average analysis effectively limits the rate at which the occupancy can climb, where too steep a gradient could cause damage to the HPD. A
Figure 3.17: The Presenter page for the hit map monitor. In addition to the automated monitoring problems such as the “barcode” pattern are clearly visible.
hard (configurable) upper limit on the \text{HPD} occupancy is also enforced by the tool. Too high an occupancy could damage the sensor and cause the \text{HPD} to draw an abnormally large current.

**The Test Pattern Monitor Algorithm**

The \text{TFC} system is capable of sending specific calibration triggers which the \text{L0} boards respond to in different ways. One of these triggers causes the \text{L0} to inject charge into
Figure 3.19: The \texttt{CAMERA} message window for the \texttt{TestPatternMonitor} algorithm.

the pixel chips generating a test pattern in the \texttt{HPD}. The test pattern used is selected via the \texttt{L0} configuration recipe. The three tests patterns, shown in Figure 3.18, are

\textbf{Corner pixels} a symmetric pattern with each of the four corner pixels turned on;

\textbf{Rows} a symmetric pattern about the column axis but asymmetric about the row axis where the pixel rows 0, 3, 7, 15, 19, 23, 26 and 31 are turned on; and

\textbf{Half diagonal} an asymmetric pattern about the row and column axis where the pixels (0,0), (1,1), (2,2), \ldots (16,16) are turned on creating a diagonal line from the corner to the centre of the pixel chip.

The \texttt{TestPatternMonitor} algorithm, running in the calibration farm, implements an analysis of the test patterns to detect errors and produces histograms for visual monitoring. The algorithm monitors the efficiency of the test pattern, the test pattern occupancy divided by the expected test pattern occupancy; and the number of dead \texttt{HPD}s, the \texttt{HPD}s where the test pattern occupancy drops below a configurable value. The test pattern efficiency is also tracked for stability by monitoring the moving average; large deviations from the moving average are considered to be an error. The \texttt{CAMERA} tool is used to report these numbers at set intervals of events and a full \texttt{CAMERA} message is shown in Figure 3.19. The \texttt{TestPatternMonitor} algorithm also produces profile histograms projecting the two-dimensional test pattern efficiency and the dead \texttt{HPD} map onto the \texttt{RICH} box rows and columns. The Presenter page for \texttt{RICH-2} is shown in Figure 3.20.

\section*{3.4 Summary}

The \texttt{RICH} sub-detector uses the Cherenkov effect to produce photons in a cone with a radius dependent on the particle velocity. These photons are focused into a ring and detected by the \texttt{HPD}s. The \texttt{RICH} reconstruction algorithms produce a likelihood that
Figure 3.20: The TestPatternMonitor algorithm’s Presenter page.
an identified particle in the detector matches one of the charged particle hypotheses: 
electron, kaon, muon, pion or proton. The particle hypothesis is typically expressed as 
the difference of the logarithm of the chosen particle likelihood with respect to the pion 
likelihood, the most common particle in an event, e.g. $\Delta L_{K-\pi}$. The RICH sub-detectors 
performed well during the 2010 data taking period and the measured performance agrees 
well with the expectations from simulated events. The author wrote and maintained the 
UKL1 control software, which forms part of the LHCb ECS. The RICH sub-detectors 
are monitored by the Panoptes software package and the author wrote the algorithms 
to provide an automated monitoring of the HPD occupancy and the efficiency of HPD 
calibration patterns. These algorithms continue to run as part of the standard Panoptes 
monitoring tools.
Chapter 4

Event Reconstruction and Selection

The trigger and stripping, described in Chapter 2, are used to select events that are most likely to contain a B-candidate for analysis, significantly reducing the number of background events. Despite this the output data sets have too low a purity for a full analysis and a further event selection is required to produce the data sets that have a high enough purity. This chapter describes the optimisation of an event selection, using simulated events, for the signal decay modes $B^\pm \rightarrow J/\psi K^\pm$, $B^\pm \rightarrow J/\psi \pi^\pm$, $B^\pm \rightarrow \phi K^\pm$ and $B^\pm \rightarrow \phi \pi^\pm$, where the $J/\psi$ and $\phi$ are reconstructed as $J/\psi \rightarrow \mu^+\mu^-$ and $\phi \rightarrow K^+K^-$ respectively. The simulated events produced to perform the selection optimisation are described in Section 4.1 and the reconstruction of the B-meson candidates from these events is described in Section 4.2. The optimisation procedure and the results are described in Section 4.3. The performance of the optimised event selection is evaluated in Section 4.4 and then it is used to predicted the yields expected from each of the four channels at different LHC centre-of-mass collision energies and LHCb data set sizes in Section 4.5.

4.1 The Simulated Events

The simulated events used for the optimisation of the event selection were generated using a group of settings referred to as MC09. These settings match the planned 2009 LHC start up conditions with a centre-of-mass energy of 10 TeV.\footnote{This run did not happen due an accident with the LHC magnets which resulted in a massive helium leak causing a delay of approximately one year in the start up.}
Table 4.1: Summary of the generated simulated samples.

<table>
<thead>
<tr>
<th>Decay</th>
<th>Generated Events</th>
<th>Generator Efficiency / %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^\pm \to J/\psi (\to \mu^+\mu^-) K^\pm$</td>
<td>1,107,512</td>
<td>16.30 ± 0.07</td>
</tr>
<tr>
<td>$B^\pm \to J/\psi (\to \mu^+\mu^-) \pi^\pm$</td>
<td>1,097,026</td>
<td>15.74 ± 0.13</td>
</tr>
<tr>
<td>$B^\pm \to \phi (\to K^+K^-) K^\pm$</td>
<td>1,044,740</td>
<td>18.91 ± 0.15</td>
</tr>
<tr>
<td>$B^\pm \to \phi (\to K^+K^-) \pi^\pm$</td>
<td>1,114,398</td>
<td>18.58 ± 0.15</td>
</tr>
<tr>
<td>$B^\pm \to J/\psi (\to \mu^+\mu^-) X$</td>
<td>1,110,523</td>
<td>18.49 ± 0.11</td>
</tr>
<tr>
<td>$B^\pm \to \phi (\to K^+K^-) X$</td>
<td>1,009,871</td>
<td>33.13 ± 0.24</td>
</tr>
<tr>
<td>Inclusive b$b$</td>
<td>51,730,913</td>
<td>43.21 ± 0.29</td>
</tr>
<tr>
<td>Minimum bias</td>
<td>10,581,931</td>
<td>–</td>
</tr>
</tbody>
</table>

A total of eight samples were generated and simulated with the MC09 settings. The four signal decays: $B^\pm \to J/\psi K^\pm$ and $B^\pm \to J/\psi \pi^\pm$, where the $J/\psi$ decays as $J/\psi \to \mu^+\mu^-$, and $B^\pm \to \phi K^\pm$ and $B^\pm \to \phi \pi^\pm$, where the $\phi$ decays as $\phi \to K^+K^-$, were generated by forcing one of the $b\bar{b}$ pair to decay to one of these signal decays. The $B^\pm \to J/\psi X$ and $B^\pm \to \phi X$ samples, where $X$ is anything (including multiple particles), provide events for some of the more common backgrounds. Two large generic background samples were also produced. The inclusive b$b$ sample was generated by requiring that the event contains a $b\bar{b}$ pair and the minimum bias sample required only a proton-proton collision to occur. For all samples, except the minimum bias sample, the requirement that the $b\bar{b}$ pair decayed within the LHCb acceptance was made. This was done at the generator level and is known as a generator cut. Table 4.1 lists the number of events generated for each sample along with the efficiency of the generator cut. The inclusive b$b$ sample was stripped, due to the large number of events generated, keeping only those events where positive $L_0$ and HLT-1 decisions were found. There are 13,154,821 stripped b$b$ events available, reducing the number of events by approximately a quarter.

### 4.2 Event Reconstruction

In order to build the $B$-candidates from either the data or simulated events, two long-lived charged particles are selected from all the particles in the event and are assigned the muon mass hypothesis, if reconstructing a $J/\psi$, or the kaon mass hypothesis, if reconstructing
Event Reconstruction and Selection

a $\phi$. This forms a new particle, which is combined with another long-lived charged particle from the event to make the B-candidate. The charged particle is called the bachelor hadron, $h$, and can be assigned either a kaon or pion mass hypothesis. For both $B^\pm \rightarrow J/\psi h^\pm$ and $B^\pm \rightarrow \phi h^\pm$ the dominant decay is to a kaon and it is therefore reconstructed with the kaon mass hypothesis. Assigning the kaon mass hypothesis to a reconstructed track that is in reality a lighter pion causes the reconstructed $B^+$ to have a larger invariant mass.

As each event contains of $O(200)$ reconstructed particles the reconstruction of all the possible three particle combinations leads to multiple B-candidates per event. The combination of uncorrelated particles produces a combinatorial background, the reduction of which is the main goal of the event selection as it would otherwise swamp the signal candidates. As a first step the number of B-candidates are reduced with an initial selection, the preselection which significantly speeds up the optimisation of the event selection. The preselection is summarised in Table 4.2.

In both the $B^\pm \rightarrow J/\psi K^\pm$ and $B^\pm \rightarrow \phi K^\pm$ reconstruction the first stage is to select the intermediate resonance. The mass of the track pair are required to be within three sigma of the nominal mass for that resonance $[4]$, where sigma is the width of the Gaussian resonance peak and is given by the resolution of the LHCb detector (which is larger than the intrinsic width of the $J/\psi$ or $\phi$). This removes a significant number of background candidates. For both resonance candidates a loose requirement on the decay vertex error, the $\chi^2$ per number of Degrees of Freedom (nDoF), from the vertex fit is applied. This removes the candidates that do not appear to originate from the same point. The tracks used to build the $J/\psi$, assigned the muon mass hypothesis, are required to have hits in the muon chambers. This is the minimum requirement for a track to be considered a muon in LHCb and the tracks are flagged as ISMUON (Section 2.4.4). The tracks used to build the $\phi$ are assigned the kaon mass hypothesis and a minimum requirement on the likelihood of the track being a kaon over pion, $\Delta LL_{K-\pi}$, is applied (Section 3.1.3). To first order all tracks in an event are pions, hence the requirement of kaon-like tracks with respect to a pion is made. As there are a large number of tracks that fulfill this criteria a final loose requirement is made on the $\chi^2$ per nDoF from the track fit (Section 2.4.3). As the optimised selection must select both kaon and pions for the bachelor hadron, cuts which bias the selected particles to one mass hypothesis are avoided. The subsequent analysis of the selected candidates uses the PID information, primarily from the RICH detectors as described in Section 3.1.3 and hence the momentum requirement on the bachelor...
### Table 4.2: Event Preselection

<table>
<thead>
<tr>
<th>Particle</th>
<th>Cut Description</th>
<th>Cut Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^\pm$</td>
<td>mass window</td>
<td>$\pm 500 \text{ MeV}/c^2$</td>
</tr>
<tr>
<td>$J/\psi$</td>
<td>mass window</td>
<td>$3\sigma/\pm 33 \text{ MeV}/c^2$</td>
</tr>
<tr>
<td></td>
<td>vertex $\chi^2/\text{nDoF}$</td>
<td>$&lt; 25$</td>
</tr>
<tr>
<td>$\mu$ pair</td>
<td>identified as muon</td>
<td>ISMUON</td>
</tr>
<tr>
<td>$h^\pm$</td>
<td>momentum in range</td>
<td>(2 to 100) GeV/c</td>
</tr>
</tbody>
</table>

(a) The $B^\pm \rightarrow J/\psi h^\pm$ preselection.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Cut Description</th>
<th>Cut Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^\pm$</td>
<td>mass window</td>
<td>$\pm 500 \text{ MeV}/c^2$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>mass window</td>
<td>$3\sigma/\pm 13 \text{ MeV}/c^2$</td>
</tr>
<tr>
<td></td>
<td>vertex $\chi^2/\text{nDoF}$</td>
<td>$&lt; 25$</td>
</tr>
<tr>
<td>K pair</td>
<td>track $\chi^2/\text{nDoF}$</td>
<td>$&lt; 10$</td>
</tr>
<tr>
<td></td>
<td>momentum in range</td>
<td>(2 to 100) GeV/c</td>
</tr>
<tr>
<td></td>
<td>minimum $\Delta L_{K-\pi}$</td>
<td>$&gt; -5$</td>
</tr>
<tr>
<td>$h^\pm$</td>
<td>momentum in range</td>
<td>(2 to 100) GeV/c</td>
</tr>
</tbody>
</table>

(b) The $B^\pm \rightarrow \phi h^\pm$ preselection.
hadron is that it is within the momentum range over which the RICH operates. Finally a wide mass window is applied on the B-candidate around the nominal $B^\pm$ invariant mass which provides enough combinatorial background statistics for the optimisation of the event selection. The topology of the reconstructed b-decays is shown in Figure 4.1.

### 4.3 Event Selection Optimisation

#### 4.3.1 Optimisation Method

The selection optimised is a cut-based event selection. A list of variables which allow signal and background to be distinguished are chosen and a candidate is classified as signal if it lies within the specified ranges of all the variables. In order to test the quality of the selection, the number of signal and background candidates passing the event selection are determined and the metric, $S/\sqrt{S+B}$, is calculated. The selection with the largest value of the metric is the optimum.

---

2This is visualised with two variables by plotting one variable on the $x$-axis and the other on the $y$-axis. Drawing lines at $x=$lower limit, $x=$upper limit and similarly for $y$ will create a rectangle within which the candidates are classified as signal. This visualisation can trivially be expanded to three-dimensions and less trivially visualised in n-dimensions.
A trivial method to optimise a cut-based event selection would be to quantise each of the variables and then test every combination of the ranges. This is computationally too expensive, for example with ten variables each with fifteen points where the ranges can be set requires $10^{15}$ computations of the metric. The optimisation of the event selection is therefore performed using two programs developed to reduce the search time by reducing the number of times the metric is calculated. The two programs have been used in other analyses in LHCb; the use of two independent programs is done as a cross-check of the minimum found by the other. The first program, an Automated Method for Choosing Cuts (AMCC) [174], was developed for the study of the forward-backward asymmetry in $B^0_d \rightarrow K^0\mu^+\mu^-$ decays [175] and also in the measurement of the CKM angle $\gamma$ using $B^\pm \rightarrow D (\rightarrow K^0_S\pi^+\pi^-) K^\pm$ decays [176]. The second program, a Cut Recursive OPtimiser (CROP) [177], was originally used to produce an optimised event selection for the RICH calibration decay $D_s \rightarrow \phi\pi^+$ and has been used by the LHCb $\beta_s$ analysis working group. Details about the implementation of the two methods are found in references [175] and [177] for AMCC and CROP respectively.

The two programs must be given a selection of variables and possible selection values to produce the optimised event selection. The starting point for the choice of the selection of variables is the topology of the $b$-decays, see Figure 4.1. The $B$-candidate and the resonances are short-lived particles; due to their large momenta and the closeness of the VELO to the primary vertex their decay vertices are within the VELO sub-detector volume. This allows precision measurements of the particle decay vertex to be made.
Figures 4.2 and 4.3 show a schematic of the decay of the B-candidate and the decay of the resonance; the vertex separation is labelled. The first variable chosen to be part of the optimisation variables is the Primary-Vertex Decay-Vertex (PVDV) separation $\chi^2$, whose lower limit is allowed to vary. The PVDV separation $\chi^2$ is formed by adding all the tracks from the decay vertex to the primary vertex and recalculating the vertex $\chi^2$. A large vertex $\chi^2$ indicates that the combination of tracks form a poor vertex and hence the tracks from the $B^\pm$ or the resonance are not likely to originate from the primary vertex. Another variable used is the IP $\chi^2$ with respect to the primary vertex; the IP is the distance of closest approach between the primary vertex and the momentum vector of the particle, as illustrated in Figure 4.3. The IP $\chi^2$ is calculated by projecting the decay vertex to the primary vertex and reevaluating the decay vertex. As the B-candidate is produced at the primary vertex its IP $\chi^2$ is expected to be small; the upper limit of the B-candidate IP $\chi^2$ is provided to the optimisers. The resonance originates from the B-candidate and its IP $\chi^2$ should be large; a lower limit on the resonance IP $\chi^2$ is provided to the optimisers. For the resonances this rejects the prompt component, those $J/\psi$ or $\phi$ that are produced in the primary vertex. The upper limit on the vertex $\chi^2$ per nDoF of the B-candidate and the resonance, both already used in the preselection presented in Section 4.2, forms another selection variable. Finally, the B-candidate is expected to have a high transverse momentum (momentum component perpendicular to the LHCb $z$-axis) compared to the background and the minimum value of the B-candidate and its daughters’ transverse momentum all form selection variables.

The variables considered for the charged particles are divided into two categories: how likely the fitted track is to be a track and how likely the assigned particle hypothesis
Table 4.3: Hadronisation fractions of a $b\bar{b}$ pair to $B$-mesons and $b$-baryons [4].

<table>
<thead>
<tr>
<th>$b$-hadron</th>
<th>Production Fraction / %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^+$</td>
<td>$40.0 \pm 1.2$</td>
</tr>
<tr>
<td>$B^0_d$</td>
<td>$40.0 \pm 1.2$</td>
</tr>
<tr>
<td>$B^0_s$</td>
<td>$11.4 \pm 1.2$</td>
</tr>
<tr>
<td>$\Lambda_b$</td>
<td>$8.6 \pm 2.1$</td>
</tr>
</tbody>
</table>

is to be correct. In order to test the quality of the track fit three variables are considered: the track $\chi^2$ per nDoF of the fit, which comes from the track fitting algorithm; the track probability $\chi^2$, which is the p-value of the track $\chi^2$ distribution; and the ghost track probability, which are tracks made up of pseudo-random combinations of tracking detector sensor hits [178]. For the muon or kaon candidates, from the $J/\psi$ or $\phi$ respectively, the log-likelihood difference to the pion mass hypothesis, $\Delta LL_{\mu-\pi}$ and $\Delta LL_{K-\pi}$, respectively are used to distinguish them from the predominately pion candidates in the event. No particle identification requirement is made on the bachelor hadron since both kaons and pions are required.

Optimised selections using both AMCC and CROP are produced with the simulated $B^\pm \to J/\psi K^\pm$ and $B^\pm \to J/\psi \pi^\pm$ samples as the signal, and the simulated minimum bias samples as the background. Another selection is produced with the same signal samples and the simulated inclusive $b\bar{b}$ sample as the background. The signal samples are filtered so they contain only reconstructed decay chains where each particle is associated to the correct simulated particle and the simulated particles form the correct signal chain; this is known as truth-matching. The background samples are filtered to remove the reconstructed decay chains that are identified as signal using truth-matching. This is repeated using the simulated $B^\pm \to \phi K^\pm$ and $B^\pm \to \phi \pi^\pm$ as signal samples.

Each of the input samples is weighted to ensure the correct normalisation. The weight for a signal sample is given by

$$w_{signal} = \frac{2 \times \sigma_{b\bar{b}} \times h(b \to B^+) \times \mathcal{B}(B^+ \to f) \times \epsilon_{gen}}{N_{gen}}$$

where $\sigma_{b\bar{b}}$ is the $b\bar{b}$ cross-section, evaluated as $(336 \pm 6) \mu$b using the Pythia event generator; $h (b \to B^+)$ is the fraction of $b$-quarks that hadronise to $B^+$, taken from
### Table 4.4: The branching ratios of the signal decays, including the decays of the resonances to their specific final states [4]. As only an upper limit has been set on the $B^\pm \rightarrow \phi\pi^\pm$ branching ratio the Standard Model prediction taken from reference [85] is used.

<table>
<thead>
<tr>
<th>Decay</th>
<th>Branching Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J/\psi \rightarrow \mu^+\mu^-$</td>
<td>$(5.93 \pm 0.06) \times 10^{-2}$</td>
</tr>
<tr>
<td>$\phi \rightarrow K^+K^-$</td>
<td>$(4.93 \pm 0.06) \times 10^{-1}$</td>
</tr>
<tr>
<td>$B^\pm \rightarrow J/\psi K^\pm$</td>
<td>$(1.014 \pm 0.034) \times 10^{-3}$</td>
</tr>
<tr>
<td>$B^\pm \rightarrow J/\psi\pi^\pm$</td>
<td>$(4.9 \pm 0.4) \times 10^{-5}$</td>
</tr>
<tr>
<td>$B^\pm \rightarrow \phi K^\pm$</td>
<td>$(8.3 \pm 0.7) \times 10^{-6}$</td>
</tr>
<tr>
<td>$B^\pm \rightarrow \phi\pi^\pm$</td>
<td>$4.45 \times 10^{-9}$</td>
</tr>
</tbody>
</table>

Table 4.3: $B(B^+ \rightarrow f)$ is the branching fraction of $B^+$ to the signal decay, $f$, and is taken from Table 4.4. $\epsilon_{gen}$ is the generator efficiency, from Table 4.1 and $N_{gen}$ is the number of generated events, also from Table 4.1. The factor of two is due to both $B^+$ and $B^-$ being able to decay to a signal candidate. The weight of the simulated inclusive $b\bar{b}$ sample is given by

$$w_{b\bar{b}} = \frac{\sigma_{b\bar{b}}}{N_{gen}}$$

and the weight of the simulated minimum bias sample is $w_{\text{minbias}} = 1/N_{gen}$. The samples are divided into training and validation samples, the training sample is used to optimise the event selection and the validation sample used as an independent sample to verify the metric.

For the initial optimisation run all of the selection variables, each with a large range and low resolution, are given to both AMCC and CROP. From this initial run the variables which reject no background candidates are removed and those selection variables which are kept have their resolution increased around the optimum value. AMCC and CROP are then rerun. This process is repeated gradually until the optimum metric is found. For this method of selecting the variables and their range a comparison between the two methods is advantageous. By using two independent methods to optimise the selections and explore the phase space, the cuts rejected and the ranges to explore are less biased by the optimisation method.
4.3.2 Optimisation Results

The optimised event selections produced by AMCC and CROP are given in Tables 4.5 and 4.6. Four event selections were optimised using the simulated $B^\pm \rightarrow J/\psi K^\pm$ and $B^\pm \rightarrow J/\psi \pi^\pm$, or $B^\pm \rightarrow \phi K^\pm$ and $B^\pm \rightarrow \phi \pi^\pm$, as the signal samples and the simulated minimum bias or inclusive $b\bar{b}$ samples as the background. Since the event selection optimisation using the minimum bias samples chose cuts that are a subset of the inclusive $b\bar{b}$ events selection it is the inclusive $b\bar{b}$ selection that is presented in the tables. If a value from the event selection optimised on the minimum bias sample is tighter than the inclusive $b\bar{b}$ event selection then it is given in brackets.

Table 4.5 shows that for the $B^\pm \rightarrow J/\psi h^\pm$ event selection the two optimisation programs produce (almost) identical event selections for the three charged particles and that the event selections for the $B^\pm$ and $J/\psi$ candidates are very similar. AMCC produces an event select that cuts harder on the $B^\pm$ PVDV separation $\chi^2$ than that produced by CROP which instead cuts harder on the $J/\psi$ vertex separation $\chi^2$. These small differences provide confidence that the two optimisation programs produced a good selection and that they are close to the global maximum. Table 4.6 shows that for the $B^\pm \rightarrow \phi h^\pm$ event selection the two selections differ significantly, the event selection produced by AMCC is tighter than the one produced by CROP. The requirement to select three hadronic tracks makes the separation of signal and background much harder. A stronger reliance of the transverse momentum of both the $B^\pm$ and its daughter tracks, when compared the $B^\pm \rightarrow J/\psi h^\pm$ selection, is apparent. Through the investigation of the optimised selection metrics and the observable distributions after the event selections has been applied a decision is made on which metric to use.

Figures 4.4 and 4.5 show the distributions of the $B^\pm$ invariant mass and bachelor hadron $\Delta L_{K-\pi}$ variables for the AMCC event selection, CROP event selection and the truth-matched $B^\pm \rightarrow J/\psi h^\pm$ signal decays passing the preselection. The $B^\pm \rightarrow J/\psi h^\pm$ histograms (Figure 4.4) show that the two distributions are very similar and a loss in signal efficiency is apparent when compared to the true matched signal distributions. Figure 4.5 compares the AMCC and CROP event selections with the truth-matched $B^\pm \rightarrow \phi K^\pm$ signal decays. These two event selections clearly have different signal efficiencies and the background is higher in the sidebands than in Figure 4.4. CROP has a higher signal efficiency but has a lower purity.
<table>
<thead>
<tr>
<th>Particle</th>
<th>Cut Description</th>
<th>AMCC Cut Value</th>
<th>CROP Cut Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^\pm$</td>
<td>mass window</td>
<td>$\pm 500 \text{ MeV/c}^2$</td>
<td>$\pm 500 \text{ MeV/c}^2$</td>
</tr>
<tr>
<td></td>
<td>vertex $\chi^2/ \text{nDoF}$</td>
<td>&lt; 9</td>
<td>&lt; 10</td>
</tr>
<tr>
<td></td>
<td>$\text{PVDV} \chi^2$</td>
<td>&gt; 300</td>
<td>&gt; 200</td>
</tr>
<tr>
<td>$J/\psi$</td>
<td>mass window</td>
<td>$\pm 3\sigma/33 \text{ MeV/c}^2$</td>
<td>$\pm 3\sigma/33 \text{ MeV/c}^2$</td>
</tr>
<tr>
<td></td>
<td>vertex $\chi^2/ \text{nDoF}$</td>
<td>&lt; 10</td>
<td>&lt; 10</td>
</tr>
<tr>
<td></td>
<td>$\text{PVDV} \chi^2$</td>
<td>&gt; 200</td>
<td>&gt; 275</td>
</tr>
<tr>
<td></td>
<td>$\text{IP} \chi^2$</td>
<td>&gt; 19</td>
<td>&gt; 19</td>
</tr>
<tr>
<td>$\mu$ pair</td>
<td>identified as muon</td>
<td>ISMUON</td>
<td>ISMUON</td>
</tr>
<tr>
<td></td>
<td>$\mu^\pm$ track $\chi^2/ \text{nDoF}$</td>
<td>&lt; 4.8</td>
<td>&lt; 4.8</td>
</tr>
<tr>
<td></td>
<td>$\mu^\pm$ $</td>
<td>\Delta p_T</td>
<td>$</td>
</tr>
<tr>
<td></td>
<td>$\mu^\pm$ $\Delta LL_{\mu-\pi}$</td>
<td>&gt; −1.5</td>
<td>&gt; −1.5</td>
</tr>
<tr>
<td>$h^\pm$</td>
<td>momentum in range</td>
<td>(2 to 100) GeV/c</td>
<td>(2 to 100) GeV/c</td>
</tr>
<tr>
<td></td>
<td>$p_T$</td>
<td>&gt; 1.6 GeV/c</td>
<td>&gt; 1.6 GeV/c</td>
</tr>
<tr>
<td></td>
<td>track ghost probability</td>
<td>&lt; 0.9</td>
<td>&lt; 0.8</td>
</tr>
</tbody>
</table>

Table 4.5: A comparison between the optimised event selection from AMCC and CROP for the $B^\pm \to J/\psi h^\pm$ reconstruction.
<table>
<thead>
<tr>
<th>Particle</th>
<th>Cut Description</th>
<th>AMCC Cut Value</th>
<th>CROP Cut Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>B±</td>
<td>mass window</td>
<td>±500 MeV/c²</td>
<td>±500 MeV/c²</td>
</tr>
<tr>
<td></td>
<td>PVDV $\chi^2$</td>
<td>&gt; 350</td>
<td>&gt; 270</td>
</tr>
<tr>
<td></td>
<td>$p_T$</td>
<td>&gt; 3 GeV/c</td>
<td>&gt; 2.6 (3) GeV/c</td>
</tr>
<tr>
<td>$\phi$</td>
<td>mass window</td>
<td>±3$\sigma$/13 MeV/c²</td>
<td>±3$\sigma$/13 MeV/c²</td>
</tr>
<tr>
<td></td>
<td>vertex $\chi^2$/ nDoF</td>
<td>&lt; 13</td>
<td>&lt; 11.5</td>
</tr>
<tr>
<td></td>
<td>PVDV $\chi^2$</td>
<td>&gt; 80</td>
<td>&gt; 50</td>
</tr>
<tr>
<td></td>
<td>IP $\chi^2$</td>
<td>&gt; 4.5</td>
<td>&gt; 7</td>
</tr>
<tr>
<td>K pair</td>
<td>K± momentum in range</td>
<td>(2 to 100) GeV/c</td>
<td>(2 to 100) GeV/c</td>
</tr>
<tr>
<td></td>
<td>K± $p_T$</td>
<td>&gt; 1.4 GeV/c</td>
<td>&gt; 0.6 (0.9) GeV/c</td>
</tr>
<tr>
<td></td>
<td>K± $\Delta L_{K-\pi}$</td>
<td>&lt; 1.8(1.5)</td>
<td>&lt; 1.5</td>
</tr>
<tr>
<td>h±</td>
<td>sum K± track ghost probability</td>
<td>&lt; 1.8(1.5)</td>
<td>&lt; 1.5</td>
</tr>
<tr>
<td></td>
<td>track probability $\chi^2$</td>
<td>&gt; 0(0.05)</td>
<td>&gt; 0</td>
</tr>
<tr>
<td></td>
<td>momentum in range</td>
<td>(2 to 100) GeV/c</td>
<td>(2 to 100) GeV/c</td>
</tr>
<tr>
<td></td>
<td>$p_T$</td>
<td>&gt; 1.9 GeV/c</td>
<td>&gt; 1.5 (1.8) GeV/c</td>
</tr>
<tr>
<td></td>
<td>track ghost probability</td>
<td>&lt; 0.8</td>
<td>&lt; 0.7</td>
</tr>
</tbody>
</table>

Table 4.6: A comparison between the optimised event selections from AMCC and CROP for the $B^\pm \rightarrow \phi h^\pm$ reconstruction.
Figure 4.4: The $B^\pm$ invariant mass and bachelor hadron distributions showing a comparison between the AMCC (red circles), CROP (blue squares) and truth-matched signal (black triangles) event selections applied to the $B^\pm \to J/\psi K^\pm/\pi^\pm$ simulated samples.
Figure 4.5: The $B^\pm$ invariant mass and bachelor hadron distributions showing a comparison between the AMCC (red circles), CROP (blue squares) and truth-matched signal (black triangles) event selections applied to the $B^\pm \rightarrow \phi K^\pm / \pi^\pm$ simulated samples.
The four metrics: signal efficiency, background rejection, $S/\sqrt{S+B}$ and $B/S$, are tabulated in Table 4.7. The signal efficiency is calculated using the number of truth-matched signal candidates passing the optimised event selection divided by the number passing the preselection. The background rejection is one minus the background selection efficiency, which is defined as the number of truth-matched background candidates passing the optimised event selection divided by the number passing the preselection. Table 4.7(a) shows the metrics using the AMCC and CROP optimised $B^\pm \to J/\psi h^\pm$ event selections. The AMCC optimised event selection has a higher optimisation metric and higher signal efficiency so this selection is chosen as the event selection for $B^\pm \to J/\psi h^\pm$. The cut values from the AMCC optimised event selection are taken primarily from the inclusive $b\bar{b}$ values, except where the minimum bias optimised event selection chooses a tighter value then that is used. Table 4.7(b) gives the $B^\pm \to \phi h^\pm$ optimised event selections from AMCC and CROP. The CROP optimisation produces an event selection with lower $S/\sqrt{S+B}$ but better signal efficiency. In this case the background rejection is sufficient for the CROP event selection and the higher signal efficiency it produces is more desirable than the purer AMCC event selection. For these reasons the CROP optimisation is used for the $B^\pm \to \phi h^\pm$ event selection. As with the $B^\pm \to J/\psi h^\pm$ event selection, the inclusive $b\bar{b}$ cut values are used unless the minimum bias cut values are tighter, in which case they are used.

<table>
<thead>
<tr>
<th>Optimiser</th>
<th>Signal Efficiency / %</th>
<th>Background Rejection / %</th>
<th>$B/S$</th>
<th>$S/\sqrt{(S+B)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMCC</td>
<td>18.07 ± 0.04</td>
<td>&gt; 99.99998</td>
<td>0.0957 ± 0.0008</td>
<td>389.9 ± 0.4</td>
</tr>
<tr>
<td>CROP</td>
<td>17.53 ± 0.04</td>
<td>&gt; 99.99998</td>
<td>0.1095 ± 0.0009</td>
<td>381.7 ± 0.4</td>
</tr>
</tbody>
</table>

(a) $B^\pm \to J/\psi h^\pm$ performance metrics.

<table>
<thead>
<tr>
<th>Optimiser</th>
<th>Signal Efficiency / %</th>
<th>Background Rejection / %</th>
<th>$B/S$</th>
<th>$S/\sqrt{(S+B)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMCC</td>
<td>14.5 ± 0.2</td>
<td>&gt; 99.999</td>
<td>14,300 ± 200</td>
<td>0.698 ± 0.008</td>
</tr>
<tr>
<td>CROP</td>
<td>25.0 ± 0.2</td>
<td>&gt; 99.991</td>
<td>73,800 ± 700</td>
<td>0.403 ± 0.004</td>
</tr>
</tbody>
</table>

(b) $B^\pm \to \phi h^\pm$ performance metrics.

Table 4.7: The performance metrics for AMCC and CROP optimised event selections.
In the $B^\pm \rightarrow J/\psi h^\pm$ case the use of two optimisation programs, which produced consistent outputs, shows that the chosen selection is near the global maximum. The muons are relatively easy to identify due to their weak interaction with matter and placing a mass window around the narrow $J/\psi$ resonance is a powerful rejection criteria. This initial, highly efficient, signal peak with a large number of rejected candidates makes it easier for the selection cuts to separate the signal candidates, which is done primarily through vertex separation cuts, from the background candidates. In the $B^\pm \rightarrow \phi h^\pm$ reconstruction the use of two optimisation programs reveals the difficulty in selecting three hadron tracks from predominantly hadron tracks. The programs performed well but they could not converge to a similar answer, which is believed to be due to a large number of local minima. Here having two programs allowed this difficult phase space to be explored independently and by comparing their output at different stages they were able to direct each other to an optimum selection.

### 4.4 Selection Performance

The AMCC optimised event selection and the CROP optimised event selection are chosen for the $B^\pm \rightarrow J/\psi h^\pm$ candidate selection and the $B^\pm \rightarrow \phi h^\pm$ candidate selection respectively. In both cases two additional selection criteria are included. The first removes bachelor hadron candidates that cannot be separated using the RICH particle identification; this requires the HASRICH flag to be true. The second is the track $\chi^2$ per $n\text{DoF}$; the LHCb tracking group provide an efficiency for a track $\chi^2$ per $n\text{DoF}$ of less than five and this track quality cut is applied to all tracks, replacing the track quality cuts (track $\chi^2$ per $n\text{DoF}$, track probability $\chi^2$ and the track ghost probability) in the optimised event selection.

The candidates passing the $B^\pm \rightarrow J/\psi h^\pm$ event selection are shown in Figure 4.6 and show a significant increase in purity compared to the preselection. The distributions from the $B^\pm \rightarrow J/\psi h^\pm$ event selection compare well to the truth-matched signal distributions and a comparison of the two shows that the event selection has passed a high fraction of the total signal candidates. The $B^\pm \rightarrow \phi h^\pm$ event selection is presented in Figure 4.7 and the distributions contain more background candidates than the $B^\pm \rightarrow J/\psi h^\pm$ event selection although the background rejection is still high.
Figure 4.6: The preselection (red circles), event selection (blue squares) and truth-matched signal selection (black triangles) B± invariant mass and bachelor hadron distributions from the B± → J/ψK±/π± simulated samples.
Figure 4.7: The preselection (red circles), event selection (blue squares) and truth-matched signal selection (black triangles) $B^\pm$ invariant mass and bachelor hadron distributions from the $B^\pm \rightarrow \phi K^\pm/\pi^\pm$ simulated samples.
Table 4.8: The final event selections. The preselection cuts are not shown as they are by definition 100%.

### (a) The $B^\pm \to J/\psi K^\pm$ event selection.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Cut Description</th>
<th>Cut Description</th>
<th>$B^\pm \to J/\psi K^\pm$</th>
<th>$B^\pm \to J/\psi\pi^\pm$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^\pm$</td>
<td>vertex $\chi^2/\text{nDoF}$</td>
<td>$&lt; 9$</td>
<td>98.53 ± 0.03</td>
<td>98.24 ± 0.03</td>
</tr>
<tr>
<td></td>
<td>PVDV $\chi^2$</td>
<td>$&gt; 300$</td>
<td>75.4 ± 0.1</td>
<td>76.0 ± 0.1</td>
</tr>
<tr>
<td>$J/\psi$</td>
<td>vertex $\chi^2/\text{nDoF}$</td>
<td>$&lt; 10$</td>
<td>98.93 ± 0.03</td>
<td>98.90 ± 0.03</td>
</tr>
<tr>
<td></td>
<td>PVDV $\chi^2$</td>
<td>$&gt; 200$</td>
<td>72.6 ± 0.1</td>
<td>72.7 ± 0.1</td>
</tr>
<tr>
<td></td>
<td>IP $\chi^2$</td>
<td>$&gt; 19$</td>
<td>70.7 ± 0.1</td>
<td>72.0 ± 0.1</td>
</tr>
<tr>
<td>$\mu$ pair</td>
<td>$\mu^\pm p_T$</td>
<td>$&gt; 1.4$ GeV/c</td>
<td>47.4 ± 0.1</td>
<td>47.5 ± 0.1</td>
</tr>
<tr>
<td></td>
<td>$\mu^\pm \Delta L_{\mu-\pi}$</td>
<td>$&gt; -1.5$</td>
<td>99.07 ± 0.02</td>
<td>99.03 ± 0.03</td>
</tr>
<tr>
<td>$h^\pm$</td>
<td>$p_T$</td>
<td>$&gt; 1.6$ GeV/c</td>
<td>62.2 ± 0.1</td>
<td>64.0 ± 0.1</td>
</tr>
<tr>
<td></td>
<td>has RICH</td>
<td>HASRICH</td>
<td>98.17 ± 0.03</td>
<td>98.23 ± 0.03</td>
</tr>
</tbody>
</table>

### (b) The $B^\pm \to \phi h^\pm$ event selection.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Cut Description</th>
<th>Cut Description</th>
<th>$B^\pm \to \phi K^\pm$</th>
<th>$B^\pm \to \phi\pi^\pm$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^\pm$</td>
<td>PVDV $\chi^2$</td>
<td>$&gt; 270$</td>
<td>73.4 ± 0.1</td>
<td>73.9 ± 0.1</td>
</tr>
<tr>
<td></td>
<td>$p_T$</td>
<td>$&gt; 3$ GeV/c</td>
<td>67.1 ± 0.2</td>
<td>66.9 ± 0.2</td>
</tr>
<tr>
<td>$\phi$</td>
<td>vertex $\chi^2/\text{nDoF}$</td>
<td>$&lt; 11.5$</td>
<td>97.74 ± 0.05</td>
<td>97.74 ± 0.05</td>
</tr>
<tr>
<td></td>
<td>PVDV $\chi^2$</td>
<td>$&gt; 50$</td>
<td>72.8 ± 0.1</td>
<td>73.3 ± 0.1</td>
</tr>
<tr>
<td></td>
<td>IP $\chi^2$</td>
<td>$&gt; 7$</td>
<td>88.3 ± 0.1</td>
<td>88.7 ± 0.1</td>
</tr>
<tr>
<td>$K$ pair</td>
<td>$K^\pm p_T$</td>
<td>$&gt; 0.9$ GeV/c</td>
<td>75.2 ± 0.1</td>
<td>75.5 ± 0.1</td>
</tr>
<tr>
<td></td>
<td>$K^\pm \Delta L_{K-\pi}$</td>
<td>$&gt; 0$</td>
<td>97.59 ± 0.05</td>
<td>97.70 ± 0.05</td>
</tr>
<tr>
<td>$h^\pm$</td>
<td>$p_T$</td>
<td>$&gt; 1.8$ GeV/c</td>
<td>77.3 ± 0.1</td>
<td>77.9 ± 0.1</td>
</tr>
<tr>
<td></td>
<td>has RICH</td>
<td>HASRICH</td>
<td>98.74 ± 0.04</td>
<td>98.79 ± 0.04</td>
</tr>
</tbody>
</table>

For more information, see event reconstruction and selection.
Table 4.9: The event selection performance metrics.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Signal</th>
<th>Background</th>
<th>B/S</th>
<th>S/√S + B</th>
</tr>
</thead>
<tbody>
<tr>
<td>B± → J/ψh±</td>
<td>17.87 ± 0.06</td>
<td>&gt; 99.9998</td>
<td>0.928 ± 0.005</td>
<td>188.3 ± 0.4</td>
</tr>
<tr>
<td>B± → φh±</td>
<td>26.3 ± 0.3</td>
<td>&gt; 99.99</td>
<td>138,000 ± 2,000</td>
<td>0.191 ± 0.003</td>
</tr>
</tbody>
</table>

The final B± → J/ψK± and B± → J/ψπ± event selections are listed in Tables 4.8(a) and 4(b) respectively. These two tables list the efficiency of each of the event selection cuts, where the efficiency of an individual cut is calculated by applying only that cut to the events that pass the preselection. The efficiency of each cut for the simulated B± → J/ψK± and B± → J/ψπ± samples are almost the same and this is also true for the simulated B± → φK± and B± → φπ± samples. Four of the cut variables are plotted, comparing the simulated B± → J/ψK±, B± → J/ψπ± and inclusive b̅b distributions, in Figure 4.8. Similar plots, comparing the simulated B± → φK±, B± → φπ± and inclusive b̅b samples, are found in Figure 4.9. All show the differences between the signal and background distributions for the truth-matched candidates only.

The performance metrics for the final event selections are given in Table 4.9. These are slightly different to the equivalent metrics in Table 4.7 due to the necessary addition of the HASRICH and track \( \chi^2 \) per nDoF cuts which reduces the sample slightly with no significant gain in background rejection. There is a large difference between the B/S and S/√S + B metrics for the B± → J/ψh± and B± → φh± event selections. This shows that the B± → φh± has a lower background rejection than the B± → J/ψh± which, due to the much larger number of background candidates, more than compensates for the increased signal efficiency of the B± → φh± event selection.

The final metric is the total efficiency of the event selection. The total efficiency takes into account all the possible losses of signal candidates and is used to calculate the expected signal yields. The total efficiency of the optimised event selection is given by

\[
\epsilon_{\text{total}} = \epsilon_{\text{gen}} \times \epsilon_{\text{presel/gen}} \times \epsilon_{\text{sel/presel}}
\]

where \( \epsilon_{\text{gen}} \) is the generator efficiency from Table 4.1, \( \epsilon_{\text{presel/gen}} \) is the number of truth-matched signal candidates remaining after the preselection divided by the total number of generated candidates (Table 4.1); and \( \epsilon_{\text{sel/presel}} \) is the number of truth-matched signal
Figure 4.8: A subset of the $B^{\pm} \to J/\psi h^{\pm}$ event selection variables. The distributions are for truth-matched signal and background candidates. The simulated $B^{\pm} \to J/\psi K^{\pm}$ samples are the red circles, the simulated $B^{\pm} \to J/\psi \pi^{\pm}$ samples are the blue squares and the simulated inclusive $b\bar{b}$ samples the black triangles. The lines indicate the limit of the selection criteria for this variable and the arrows the portion of the distribution kept by the selection criteria.
Figure 4.9: A subset of the $B^\pm \rightarrow \phi h^\pm$ event selection variables. The distributions are for truth-matched signal and background candidates. The simulated $B^\pm \rightarrow \phi K^\pm$ samples are the red circles, the simulated $B^\pm \rightarrow \phi \pi^\pm$ samples are the blue squares and the simulated inclusive $b\bar{b}$ samples the black triangles. The lines indicate the limit of the selection criteria for this variable and the arrows the portion of the distribution kept by the selection criteria.
Table 4.10: Total event selection efficiency.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Generator Efficiency, $\epsilon_{\text{gen}} / %$</th>
<th>Preselection Efficiency, $\epsilon_{\text{presel/gen}} / %$</th>
<th>Selection Efficiency, $\epsilon_{\text{sel/presel}} / %$</th>
<th>Total Efficiency, $\epsilon_{\text{total}} / %$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^\pm \to J/\psi K^\pm$</td>
<td>16.30 ± 0.07</td>
<td>14.04 ± 0.03</td>
<td>17.85 ± 0.10</td>
<td>0.408 ± 0.003</td>
</tr>
<tr>
<td>$B^\pm \to J/\psi \pi^\pm$</td>
<td>15.74 ± 0.13</td>
<td>13.13 ± 0.03</td>
<td>18.3 ± 0.1</td>
<td>0.378 ± 0.004</td>
</tr>
<tr>
<td>$B^\pm \to \phi K^\pm$</td>
<td>18.91 ± 0.15</td>
<td>9.19 ± 0.03</td>
<td>26.3 ± 0.1</td>
<td>0.457 ± 0.005</td>
</tr>
<tr>
<td>$B^\pm \to \phi \pi^\pm$</td>
<td>18.58 ± 0.15</td>
<td>8.64 ± 0.03</td>
<td>26.4 ± 0.1</td>
<td>0.424 ± 0.004</td>
</tr>
</tbody>
</table>

4.5 Expected Yields

The expected yields are calculated from the simulated samples, described in Section 4.1 using

$$N = 2 \times \int L dt \times \sigma_{b \bar{b}} \times h(b \to B^+) \times B(B^+ \to f) \times \epsilon_{\text{tot}}$$  \hspace{1cm} (4.1)$$

where $\int L dt$ is the integrated luminosity; $\sigma_{b \bar{b}}$ is the $b \bar{b}$ cross-section; $h(b \to B^+)$ is the fraction of $b$-quarks that hadronise to $B^+$, taken from Table 4.3; $B(B^+ \to f)$ is the branching fraction of $B^+$ to the signal decay, $f$, and is calculated from Table 4.4; $\epsilon_{\text{tot}}$ is the total efficiency, from Table 4.10. The factor of two is introduced as both the $b$- or $\bar{b}$-quark can hadronise and decay to a signal candidates.

The cross-section is dependent on the centre-of-mass energy and the simulated samples were generated using a centre-of-mass energy of 10 TeV. The cross-sections have been determined using the Pythia event generator at centre-of-mass energies of 7 TeV, 10 TeV and 14 TeV to be $(0.457 \pm 0.048)$ mb, $(0.699 \pm 0.010)$ mb and $(1.04 \pm 0.01)$ mb respectively. The cross-section at a centre-of-mass energy of 7 TeV has been measured by LHCb as $(0.284 \pm 0.020 \pm 0.049)$ mb \footnote{LHCb measurement}. In order to calculate the yields presented in Table 4.11 the cross-section at a given centre-of-mass energy is scaled by the ratio of the
measured 7 TeV cross-section and the 7 TeV cross-section determined using Pythia. The expected yields for these three cross-sections are given in Table 4.11 for the integrated luminosities of 37 pb$^{-1}$, the amount of data collected during the 2010 data taking period; 1 fb$^{-1}$, the amount of data expected to be taken by the end of the 2011 data taking period; 2 fb$^{-1}$, the amount of data collected in a nominal LHC year (Section 2.4); and 10 fb$^{-1}$, the amount of data to be taken before an upgrade of the experiment.

With the yields given in Table 4.11 it is expected that the 2010 data set will provide abundant $B^\pm \to J/\psi K^\pm$ events, and an observation of the $B^\pm \to J/\psi \pi^\pm$ and $B^\pm \to \phi K^\pm$ decays. The data set collected by the end of 2011 should contain sufficient statistics to measure the $CP$-asymmetries in all decays except the $B^\pm \to \phi \pi^\pm$ which will require all the LHCb statistics to make a measurement.

Table 4.11: The expected yields for the signal channels, errors are statistical.

<table>
<thead>
<tr>
<th>Channel</th>
<th>37 pb$^{-1}$</th>
<th>1 fb$^{-1}$</th>
<th>2 fb$^{-1}$</th>
<th>10 fb$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{b\bar{b}} - 7$ TeV</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B^\pm \to J/\psi K^\pm$</td>
<td>$2,040 \pm 20$</td>
<td>$55,300 \pm 400$</td>
<td>$110,500 \pm 800$</td>
<td>$(553 \pm 4) \times 10^3$</td>
</tr>
<tr>
<td>$B^\pm \to J/\psi \pi^\pm$</td>
<td>$92 \pm 1$</td>
<td>$2,490 \pm 30$</td>
<td>$4,990 \pm 50$</td>
<td>$24,900 \pm 300$</td>
</tr>
<tr>
<td>$B^\pm \to \phi K^\pm$</td>
<td>$157 \pm 2$</td>
<td>$4,240 \pm 40$</td>
<td>$8,480 \pm 90$</td>
<td>$42,400 \pm 400$</td>
</tr>
<tr>
<td>$B^\pm \to \phi \pi^\pm$</td>
<td>$0.16 \pm 0.03$</td>
<td>$4.3 \pm 0.1$</td>
<td>$8.5 \pm 0.2$</td>
<td>$42.7 \pm 0.6$</td>
</tr>
<tr>
<td>$\sigma_{b\bar{b}} - 10$ TeV</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B^\pm \to J/\psi K^\pm$</td>
<td>$3,130 \pm 20$</td>
<td>$84,500 \pm 600$</td>
<td>$(169 \pm 1) \times 10^3$</td>
<td>$(845 \pm 6) \times 10^3$</td>
</tr>
<tr>
<td>$B^\pm \to J/\psi \pi^\pm$</td>
<td>$141 \pm 2$</td>
<td>$3,810 \pm 40$</td>
<td>$7,630 \pm 80$</td>
<td>$38,100 \pm 400$</td>
</tr>
<tr>
<td>$B^\pm \to \phi K^\pm$</td>
<td>$240 \pm 3$</td>
<td>$6,490 \pm 70$</td>
<td>$13,000 \pm 100$</td>
<td>$64,900 \pm 700$</td>
</tr>
<tr>
<td>$B^\pm \to \phi \pi^\pm$</td>
<td>$0.24 \pm 0.03$</td>
<td>$6.5 \pm 0.2$</td>
<td>$13.1 \pm 0.3$</td>
<td>$65.4 \pm 0.8$</td>
</tr>
<tr>
<td>$\sigma_{b\bar{b}} - 14$ TeV</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B^\pm \to J/\psi K^\pm$</td>
<td>$4,650 \pm 30$</td>
<td>$125,800 \pm 900$</td>
<td>$(252 \pm 2) \times 10^3$</td>
<td>$(1,258 \pm 9) \times 10^3$</td>
</tr>
<tr>
<td>$B^\pm \to J/\psi \pi^\pm$</td>
<td>$210 \pm 2$</td>
<td>$5,670 \pm 60$</td>
<td>$11,300 \pm 100$</td>
<td>$56,700 \pm 600$</td>
</tr>
<tr>
<td>$B^\pm \to \phi K^\pm$</td>
<td>$357 \pm 4$</td>
<td>$9,650 \pm 100$</td>
<td>$19,300 \pm 200$</td>
<td>$(96.5 \pm 1.0) \times 10^3$</td>
</tr>
<tr>
<td>$B^\pm \to \phi \pi^\pm$</td>
<td>$0.36 \pm 0.04$</td>
<td>$9.7 \pm 0.2$</td>
<td>$19.5 \pm 0.3$</td>
<td>$97 \pm 1$</td>
</tr>
</tbody>
</table>
4.6 Summary

An event selection has been optimised for the $B^\pm \to J/\psi h^\pm$ decays and $B^\pm \to \phi h^\pm$ decays using signal and background events simulated with the LHCb software. The optimisation was done using two independent optimisation programs, AMCC and CROP, both of which have been used on other analyses within the LHCb experiment. The AMCC optimised event selection was chosen for the $B^\pm \to J/\psi h^\pm$ event selection and the CROP optimised event selection was chosen for the $B^\pm \to \phi h^\pm$ event selection after a study of the performance of the different event selections. The expected yields of the signal decays $B^\pm \to J/\psi K^\pm$, $B^\pm \to J/\psi \pi^\pm$, $B^\pm \to \phi K^\pm$ and $B^\pm \to \phi \pi^\pm$ were then evaluated at different cross-sections and integrated luminosities. An initial observation of the $B^\pm \to J/\psi \pi^\pm$ and $B^\pm \to \phi K^\pm$ decays is expected from 2010 data taking period along with abundant $B^\pm \to J/\psi K^\pm$ candidates. By the end of the 2011 data taking period there should be sufficient statistics to measure the $CP$-asymmetries in all decays, except $B^\pm \to \phi \pi^\pm$ which will require all the LHCb statistics to make a measurement. The discovery of this channel is possible with smaller data sets.
Chapter 5

Observed Yields in the 2010 Data

This chapter presents the application of the $B^\pm \to J/\psi h^\pm$ and $B^\pm \to \phi h^\pm$ event selections to the events collected during the 2010 data taking period at a centre-of-mass energy of 7 TeV. First the 2010 data set is summarised in Section 5.1 and a set of $B^\pm \to J/\psi K^\pm$ and $B^\pm \to J/\psi \pi^\pm$ simulated events produced with newer Monte Carlo settings are summarised in Section 5.2. The $B^\pm \to J/\psi h^\pm$ and $B^\pm \to \phi h^\pm$ event selections are reviewed in Section 5.3 in light of the additional selection criteria from the stripping lines. Section 5.3 also uses the updated $B^\pm \to J/\psi K^\pm$ and $B^\pm \to J/\psi \pi^\pm$ simulated events to evaluate the performance of the $B^\pm \to J/\psi h^\pm$ event selection and evaluate the expected $B^\pm \to J/\psi K^\pm$ and $B^\pm \to J/\psi \pi^\pm$ yields. Finally, in Section 5.4 an extended maximum likelihood fit is used to determine the number of $B^\pm \to J/\psi K^\pm$, $B^\pm \to J/\psi \pi^\pm$, $B^\pm \to \phi K^\pm$ and $B^\pm \to \phi \pi^\pm$ candidates in the 2010 data set.

5.1 The Data Sets

The 2010 data corresponds to a total integrated luminosity of $\sim 37$ pb$^{-1}$ and is divided into two, approximately equal sized, samples with opposite magnet polarities. The data has been triggered, reconstructed and stripped (as described in Section 2.5) and the $B^\pm \to J/\psi h^\pm$ and $B^\pm \to \phi h^\pm$ candidates are required to pass specific trigger and stripping lines. The $B^\pm \to J/\psi h^\pm$ candidates are required to pass the LOMuon, Hlt1TrackMuon and Hlt2DiMuonUnbiasedJPs1 lines which had the same settings for all the candidates passing the event selection. The $B^\pm \to \phi h^\pm$ candidates are required to pass the L0Hadron, Hlt1TrackAllL0 and Hlt2TopoOSTF2Body lines. The threshold values of the cuts varied
as the data taking progressed and the small changes are considered a source of systematic uncertainty in the $B^\pm \to \phi h^\pm$ sample.

The data was reconstructed and stripped using a consistent set of software versions and calibration settings referred to as Reco08-Stripping12b. The candidates are selected using DaVinci v28r2p2 with the conditions database tag head-20101112 and the detector description database tag head-20101026. The output of the stripping is to several streams containing similar stripping lines and this analysis uses the DIMUON stream’s $Bu2JpsiKNoPIDDetached$ line and the BHADRON stream’s $B2twobody$ line for the $B^\pm \to J/\psi h^\pm$ and $B^\pm \to \phi h^\pm$ samples respectively. The integrated luminosity recorded for each stream is shown in Table 5.1. The $Bu2JpsiKNoPIDDetached$ line contains candidates reconstructed as $B^\pm \to J/\psi K^\pm$ and these candidates are filtered, applying the $B^\pm \to J/\psi h^\pm$ event selection. The $B2twobody$ line contains $B^\pm$ candidates that have been reconstructed as $B^\pm \to \phi \pi^\pm$ i.e. with the pion mass hypothesis for the bachelor hadron track. The $B^\pm \to \phi K^\pm$ candidates are reconstructed from the BHADRON stream by selecting only the events where a $B^\pm \to \phi \pi^\pm$ candidate is saved by the $B2twobody$ line and then reconstructing the $B^\pm \to \phi K^\pm$ decay candidates from this event. These candidates are filtered using the $B^\pm \to \phi h^\pm$ event selection.

### Table 5.1: The integrated luminosity recorded for each stream by polarity.

<table>
<thead>
<tr>
<th>Stream</th>
<th>Magnet Down / pb$^{-1}$</th>
<th>Magnet Up / pb$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIMUON</td>
<td>18.51 ± 1.85</td>
<td>17.93 ± 1.79</td>
</tr>
<tr>
<td>BHADRON</td>
<td>18.50 ± 1.85</td>
<td>17.99 ± 1.80</td>
</tr>
</tbody>
</table>

5.2 The Simulated Events

For the 2010 data taking period new simulated samples, referred to as MC10, have been generated with settings consistent with the 2010 data taking period. These settings were finalised before the start of the run and hence are not tuned to the recorded data. The samples have been flagged with the trigger settings which correspond to approximately half of the 2010 data set and the Reco08-Stripping12b stripping decisions. The samples used in this chapter are presented in Table 5.2 where it is seen that samples for the
Observed Yields in the 2010 Data

<table>
<thead>
<tr>
<th>Decay</th>
<th>Polarity</th>
<th>Generated Events</th>
<th>Generator Efficiency / %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^\pm \rightarrow J/\psi (\rightarrow \mu^+\mu^-) K^\pm$</td>
<td>Down</td>
<td>1,023,988</td>
<td>15.57 ± 0.14</td>
</tr>
<tr>
<td>Up</td>
<td>1,005,586</td>
<td>15.29 ± 0.14</td>
<td></td>
</tr>
<tr>
<td>$B^\pm \rightarrow J/\psi (\rightarrow \mu^+\mu^-) \pi^\pm$</td>
<td>Down</td>
<td>1,317,990</td>
<td>15.06 ± 0.08</td>
</tr>
<tr>
<td>Up</td>
<td>1,315,086</td>
<td>14.94 ± 0.08</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2: A summary of the generated MC10 samples.

<table>
<thead>
<tr>
<th>Trigger Decision</th>
<th>Magnet Down / %</th>
<th>Magnet Up / %</th>
</tr>
</thead>
<tbody>
<tr>
<td>L0MuonDecision_TOS</td>
<td></td>
<td>L0Global_TIS</td>
</tr>
<tr>
<td>Hlt1TrackMuonDecision_TOS</td>
<td></td>
<td>L0Global_TIS</td>
</tr>
<tr>
<td>Hlt2DiMuonUnbiasedJPsiDecision_TOS</td>
<td></td>
<td>L0Global_TIS</td>
</tr>
</tbody>
</table>

(a) MC10 magnet up and down $B^\pm \rightarrow J/\psi K^\pm$ samples.

<table>
<thead>
<tr>
<th>Trigger Decision</th>
<th>Magnet Down / %</th>
<th>Magnet Up / %</th>
</tr>
</thead>
<tbody>
<tr>
<td>L0MuonDecision_TOS</td>
<td></td>
<td>L0Global_TIS</td>
</tr>
<tr>
<td>Hlt1TrackMuonDecision_TOS</td>
<td></td>
<td>L0Global_TIS</td>
</tr>
<tr>
<td>Hlt2DiMuonUnbiasedJPsiDecision_TOS</td>
<td></td>
<td>L0Global_TIS</td>
</tr>
</tbody>
</table>

(b) MC10 magnet up and down $B^\pm \rightarrow J/\psi \pi^\pm$ samples.

Table 5.3: The trigger decision efficiencies on the selected candidates.

$B^\pm \rightarrow J/\psi K^\pm$ and $B^\pm \rightarrow J/\psi \pi^\pm$ channels have been generated. The simulated samples are processed with DaVinci v28r2p2, the conditions database tag sim-20101210-vc-md100 (sim-20101210-vc-mu100) for the magnet down (up) sample and the detector description database tag head-20101206 for both magnet polarities.

5.3 The Event Selection and Expected Yields

The first stage of the event selection is to select the events passing the required trigger and stripping lines, presented in Section 5.1. In addition to the Triggered On Signal (TOS)
events, those events triggered by the required lines, an event is selected if it passes the trigger stage through a line independent of the require line. These are Triggered Independent of Signal (TIS) events and are unbiased by any selection criteria. The efficiency of each trigger stage has been evaluated on simulated \( B^\pm \to J/\psi K^\pm \) and \( B^\pm \to J/\psi \pi^\pm \) MC10 samples; the efficiencies are presented in Tables 5.3(a) and (b). It is seen that the efficiencies for the two signal samples and each magnet polarity are very similar.

The event selection presented in Table 4.8(a) is applied to MC10 simulated \( B^\pm \to J/\psi K^\pm \) and \( B^\pm \to J/\psi \pi^\pm \) events and the efficiency of the stripping selection on these event is presented in Table 5.4. The efficiency of a specific cut in the stripping selection is determined from the number truth-matched signal candidates passing a stripping selection cut divided by the number of truth-matched signal candidates passing the \( B^\pm \to J/\psi h^\pm \) event selection. The stripping selection has a reduced mass window compared to the \( B^\pm \to J/\psi h^\pm \) event selection, reducing the number of background candidates to a minimum. The \( B^\pm \) candidate is reconstructed by setting the mass of the \( J/\psi \) candidate to its nominal value [4], which improves the resolution of the \( B^\pm \) candidate, further reducing the effect of the tightened mass window. A lifetime cut on the \( B^\pm \) is present in the stripping selection but not the event selection. As this is highly correlated with the Primary-Vertex Decay-Vertex separation \( \chi^2 \) in the event selection the reduction
### Table 5.4:

The $B_{s}^{\pm}K^{0}_{s}\pi^{0}$ efficiencies on the $B_{s}^{\pm} \rightarrow J/\psi K^{\pm}$ selected candidates. An efficiency of 100 % indicates that the stripping cut is less than or equal to the selection cut and is by definition 100 %.

**Observed Yields in the 2010 Data**

<table>
<thead>
<tr>
<th>Particle</th>
<th>Cut Description</th>
<th>Value</th>
<th>Magnet Down / %</th>
<th>Magnet Up / %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{s}^{\pm}$</td>
<td>mass $&gt; 5, 100 \text{ MeV}/c^2$</td>
<td>99.93 ± 0.01</td>
<td>99.91 ± 0.01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>mass $&lt; 5, 550 \text{ MeV}/c^2$</td>
<td>99.998 ± 0.002</td>
<td>99.990 ± 0.005</td>
<td></td>
</tr>
<tr>
<td></td>
<td>proper lifetime $&gt; 0.15 \text{ ps}$</td>
<td>99.75 ± 0.02</td>
<td>99.80 ± 0.02</td>
<td></td>
</tr>
<tr>
<td></td>
<td>vertex $\chi^2/\text{nDoF}$</td>
<td>&lt; 10</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>$J/\psi$</td>
<td>mass window $\pm 80 \text{ MeV}/c^2$</td>
<td>100</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td></td>
<td>vertex $\chi^2/\text{nDoF}$</td>
<td>&lt; 16</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>$\mu$ pair</td>
<td>minimum $\Delta LL_{\mu-\pi}$</td>
<td>&gt; 0</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>max. track $\chi^2/\text{nDoF}$</td>
<td>&lt; 5</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>both identified as muon ISMUON</td>
<td></td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>$h^{\pm}$</td>
<td>transverse momentum $&gt; 500 \text{ MeV}/c$</td>
<td>100</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td></td>
<td>track $\chi^2/\text{nDoF}$</td>
<td>&lt; 5</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

(a) MC10 magnet up and down $B_{s}^{\pm} \rightarrow J/\psi K^{\pm}$ samples.

(b) MC10 magnet up and down $B_{s}^{\pm} \rightarrow J/\psi \pi^{\pm}$ samples.
in signal event selection efficiency is small. The remaining cuts in the stripping selection are either looser than or equal to the event selection cuts and are by definition 100%. A comparison between the stripping selection and the event selection applied to the events passing the stripping selection is presented in Figures 5.1(a) and (b) for the magnet down and magnet up samples respectively. It can be seen that the addition of the event selection significantly reduces the number of background events relative to the number of signal events.
Figure 5.2: A comparison of the $B^\pm$ invariant mass distribution from the 2010 data set with the B2twobody stripping selection applied (blue squares), and the B2twobody stripping and $B^\pm \to \phi h^\pm$ event selection applied (red circles).

A comparison of the B2twobody stripping selection and the $B^\pm \to \phi h^\pm$ event selection is presented in Table 5.5. The MC10 signal samples for the $B^\pm \to \phi K^\pm$ and $B^\pm \to \phi \pi^\pm$ channels have not yet been produced so an analysis of the selections using truth-matching is not possible. The $B^\pm$ invariant mass distributions for the events passing the stripping selection, and the events passing the stripping and event selection are shown in Figures 5.2(a) and (b) for the magnet down and up data respectively. Applying the B2twobody selection and the $B^\pm \to \phi h^\pm$ event selection does not appear to have improved the number of background candidates relative to the B2twobody selection only. As the statistics are low in this channel little gain is seen by applying the event selection and all of the events passing the stripping selection are selected for further analysis.

The total efficiency, $\epsilon_{\text{total}}$, for the $B^\pm \to J/\psi K^\pm/\pi^\pm$ magnet up and down samples is given by the product of the generator efficiency, $\epsilon_{\text{gen}}$, the event selection efficiency on the generated events, $\epsilon_{\text{sel/gen}}$, the stripping selection efficiency on the events passing the event selection, $\epsilon_{\text{strip/sel}}$, and the trigger selection efficiency on the events passing the stripping selection, $\epsilon_{\text{trig/strip}}$,

$$\epsilon_{\text{total}} = \epsilon_{\text{gen}} \times \epsilon_{\text{sel/gen}} \times \epsilon_{\text{strip/sel}} \times \epsilon_{\text{trig/strip}}.$$  

The efficiencies for the $B^\pm \to J/\psi K^\pm/\pi^\pm$ magnet up and down samples are presented in Table 5.6. By comparing Tables 5.6(a) and (b) it is apparent that the ratio of the
efficiencies of the $B^\pm \rightarrow J/\psi K^\pm$ and $B^\pm \rightarrow J/\psi \pi^\pm$ channels is not unity. The selection efficiencies are systematically higher for the $B^\pm \rightarrow J/\psi K^\pm$ events compared to the $B^\pm \rightarrow J/\psi \pi^\pm$ events, for both the magnet up and down samples. The difference in the kaon and pion mass leads to a difference in the kinematics of the two decays. These differences are small but are statistically significant and must be taken into account in the subsequent analysis, either directly or as a systematic error. The selection efficiencies of the magnet down samples are systematically higher than for the magnet up samples. For the $B^\pm \rightarrow J/\psi K^\pm$ magnet up and down samples $(0.490 \pm 0.005) - (0.478 \pm 0.005) = (0.012 \pm 0.007)$ which is compatible with zero to two standard deviations. Only the difference in the generator efficiencies is not compatible with zero to one standard deviation. For the $B^\pm \rightarrow J/\psi \pi^\pm$ magnet up and down samples $(0.454 \pm 0.003) - (0.382 \pm 0.003) = (0.072 \pm 0.004)$ which shows a significant deviation from zero; this is due to the different in the selection efficiency on the generated events. This is unexpected as, unlike in the LHC collisions, a reversal of the magnet polarity in the simulation should be the only change and the behaviour of the particles should be “mirrored”. This is not expected to cause any significant change in the particle kinematics nor acceptance, which should affect the $B^\pm \rightarrow J/\psi K^\pm$ sample in a similar way.

The total efficiencies in Table 5.6 are combined with the integrated luminosity, the cross-section, the hadronisation fraction and the nominal branching fractions using equation (4.1) to calculate the expected yields. The cross-section at the current running conditions has been measured by LHCb and extrapolated using Pythia to a $4\pi$ rad acceptance [100]. It is this cross-section, $(284 \pm 20 \pm 49) \mu b$, that is used to calculate the yields, the luminosity used is 18.2 pb$^{-1}$, the average luminosity recorded per magnet polarity, and the branching ratios are all taken from Table 4.4. The expected yields are presented in Table 5.6 which are systematically higher than those calculated in Section 4.5. This is due to a higher selection efficiency from those cuts that form the preselection.

5.4 The Observed Yields

An estimate of the observed yields in the 2010 data set of the $B^\pm \rightarrow J/\psi K^\pm$, $B^\pm \rightarrow J/\psi \pi^\pm$, $B^\pm \rightarrow \phi K^\pm$ and $B^\pm \rightarrow \phi \pi^\pm$ decays is made by applying an additional $\Delta LL_{K^-\pi}$ selection criteria to the selected events and performing an extended maximum likelihood fit.
### Table 5.6: The $B^{\pm} \to J/\psi h^\pm$ event selection efficiency evaluated on the MC10 events and the expected yields in the magnet up and down data sets.

<table>
<thead>
<tr>
<th>Efficiency</th>
<th>/ %</th>
<th>Magnet Down</th>
<th>Magnet Up</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generator</td>
<td>$\epsilon_{\text{gen}}$</td>
<td>15.57 ± 0.14</td>
<td>15.29 ± 0.14</td>
</tr>
<tr>
<td>Selection/Generator</td>
<td>$\epsilon_{\text{sel/gen}}$</td>
<td>3.98 ± 0.02</td>
<td>3.96 ± 0.02</td>
</tr>
<tr>
<td>Stripping/Selection</td>
<td>$\epsilon_{\text{strip/sel}}$</td>
<td>99.68 ± 0.03</td>
<td>99.72 ± 0.03</td>
</tr>
<tr>
<td>Trigger/Stripping</td>
<td>$\epsilon_{\text{trig/strip}}$</td>
<td>79.4 ± 0.2</td>
<td>79.1 ± 0.2</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>$\epsilon_{\text{total}}$</td>
<td>0.490 ± 0.005</td>
<td>0.478 ± 0.005</td>
</tr>
<tr>
<td><strong>Yield</strong></td>
<td>18.2 pb$^{-1}$</td>
<td>1,210 ± 10</td>
<td>1,180 ± 10</td>
</tr>
</tbody>
</table>

(a) The simulated $B^\pm \to J/\psi K^\pm$ events.

<table>
<thead>
<tr>
<th>Efficiency</th>
<th>/ %</th>
<th>Magnet Down</th>
<th>Magnet Up</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generator</td>
<td>$\epsilon_{\text{gen}}$</td>
<td>15.06 ± 0.08</td>
<td>14.94 ± 0.08</td>
</tr>
<tr>
<td>Selection/Generator</td>
<td>$\epsilon_{\text{sel/gen}}$</td>
<td>3.85 ± 0.02</td>
<td>3.26 ± 0.02</td>
</tr>
<tr>
<td>Stripping/Selection</td>
<td>$\epsilon_{\text{strip/sel}}$</td>
<td>99.16 ± 0.04</td>
<td>99.24 ± 0.04</td>
</tr>
<tr>
<td>Trigger/Stripping</td>
<td>$\epsilon_{\text{trig/strip}}$</td>
<td>79.1 ± 0.2</td>
<td>79.0 ± 0.2</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>$\epsilon_{\text{total}}$</td>
<td>0.454 ± 0.003</td>
<td>0.382 ± 0.003</td>
</tr>
<tr>
<td><strong>Yield</strong></td>
<td>18.2 pb$^{-1}$</td>
<td>54.4 ± 0.6</td>
<td>45.9 ± 0.5</td>
</tr>
</tbody>
</table>

(b) The simulated $B^\pm \to J/\psi \pi^\pm$ events.
The $B^\pm \to J/\psi h^\pm$ and $B^\pm \to \phi h^\pm$ selected data sets are both divided into two independent data sets by applying an additional selection criteria to the bachelor hadron. A cut of $\Delta LL_{K^{-}\pi} > 5$ is used to select candidates where the bachelor hadron is kaon-like and a cut of $\Delta LL_{K^{-}\pi} < -5$ to select pion-like bachelor hadrons. These two data sets contain either the $B^\pm \to J/\psi K^\pm (B^\pm \to \phi K^\pm)$ signal candidates or the $B^\pm \to J/\psi \pi^\pm (B^\pm \to \phi \pi^\pm)$ signal candidates. Using the MC10 simulated events the signal efficiency of these two cuts is $(90.4 \pm 0.1)\%$ and $(83.2 \pm 0.2)\%$ for the kaon and pion selections respectively. Both have a less than 2% contamination by the other hadron.

The signal candidate yield is extracted using an extended maximum likelihood fit to the $B^\pm$ invariant mass observable. The maximum likelihood method is a powerful method for determining an unknown set of parameters, $\vec{p}$, in an unbinned data set recording a set of observables, $\vec{x}$. If the distribution of the $i^{th}$ event is described by the normalisable function, $F$, then the likelihood, $L$, is defined as

$$L(\vec{p}) = \prod_{i=0}^{n} F(x_i; \vec{p}).$$

In the likelihood framework the normalisable function, $F(\vec{x}; \vec{p})$, represents a Probability Distribution Function (PDF) and the likelihood is the product of the probability of each event. By varying the parameters, $\vec{p}$, to maximise the per event probability, the optimum value of each parameter is determined.

The total PDF, $F$, may be constructed from the sum of multiple PDFs, where each represents a contributing process. In this case the product of each PDF and the fraction that function contributes is summed to make the total likelihood e.g. $F = f_G \cdot G + (1-f_G) \cdot H$. The fraction parameter, $f$, is part of the fit parameter set, $\vec{p}$, and the sum of the fractions is constrained to one. In an extended maximum likelihood fit the constraint on the total number of events is removed and an additional parameter added, representing the production rate. The likelihood function is rewritten as the product of a Poisson distribution to observe the total number of events and the original likelihood. The number of events determined for each of the functions by the two methods is identical, but the extended maximum likelihood fit is not constrained by the sample size and returns larger, uncorrelated errors for the event yields.

When using the extended maximum likelihood method as described above the likelihood function is maximised and the unknown parameters fitted. The canonical method
to fit the unknowns is to minimise the negative log-likelihood function

\[ l = -\log (L) = - \sum_{i=0}^{n} \log (F(x_i; \vec{p})) \]

and in this analysis a generic minimisation program, MINUIT \[179\], is used to return the fitted parameters and their errors. The MINUIT is package is integrated into the ROOT framework \[180\] and a dedicated package, RooFit, provides a toolkit for modelling the expected distribution of events in a physics analysis \[181\].

The extended maximum likelihood function that is used to extract the signal yield is

\[ F(m; N_{\text{sig}}, N_{\text{bkg}}, \mu, \sigma, p1) = N_{\text{sig}} \cdot F_{\text{sig}}(m; \mu, \sigma) + N_{\text{bkg}} \cdot F_{\text{bkg}}(m; p1) \]

where \( m \) is the \( B^\pm \) invariant mass, \( N_{\text{sig}} \) and \( N_{\text{bkg}} \) are the number of signal and background candidates, and \( F_{\text{sig}} \) and \( F_{\text{bkg}} \) are normalisable functions representing the signal and background candidates with free parameters \( \mu \) and \( \sigma \), and \( p1 \) respectively.

The signal function, \( F_{\text{sig}} \), is modelled by a Gaussian function defined as

\[ \text{Gaussian}(m; \mu, \sigma) = \exp \left( -\frac{1}{2} \left( \frac{m - \mu}{\sigma} \right)^2 \right) \]

where \( \mu \) is the mean of the Gaussian function and \( \sigma \) is its width. The background function, \( F_{\text{bkg}} \), is modelled by a first order Chebychev polynomial \[182\]. This is a straight line with a gradient \( p1 \) and is defined as

\[ \text{Chebychev}(m; p1) = p1 \cdot \left( -1 + 2 \cdot \frac{m - m_{\text{min}}}{m_{\text{max}} - m_{\text{min}}} \right) + 1 \]

where \( m_{\text{min}} \) and \( m_{\text{max}} \) are the minimum and maximum values, respectively, of the \( B^\pm \) invariant mass in the data set.

The number of \( B^\pm \rightarrow J/\psi K^\pm \) and \( B^\pm \rightarrow J/\psi \pi^\pm \) candidates extracted using the extended maximum likelihood fit are \( 3,370^{+60}_{-59} \) and \( 180^{+28}_{-27} \) respectively. Both of these values are larger than those predicted in Table 5.6. The distributions of the variables in the simulated samples have not been tuned to the distributions measured in the data and the differences between these values are not unexpected. The extended maximum likelihood fits are shown in Figures 5.3(a) and (b) where the total function and its two components are plotted along with the data sets. The fit to the data with the \( \Delta LL_{K-\pi} < -5 \) selection applied shows a reasonable match but the low statistics limit the precision of
Figure 5.3: The extended maximum likelihood fits to determine the signal yields in the 2010 data sets with a $\Delta L_{L_{K\pi}}$ selection criteria applied to the bachelor hadron. The data sets contain both the magnet up and down events. The solid lines are the total function (blue) and its components (Gaussian in light blue and Chebychev in grey), and the points are the data sets.
the fit. The nominal ratio of the branching fraction, $\mathcal{B}(B^\pm \to J/\psi \pi^\pm)/\mathcal{B}(B^\pm \to J/\psi K^\pm)$, is approximately 5% [1] which is in agreement with the ratio of the yields extracted.

The number of $B^\pm \to \phi K^\pm$ and $B^\pm \to \phi \pi^\pm$ candidates extracted using the extended maximum likelihood fit are $121^{+14}_{-13}$ and $0 \pm 1$. Figure 5.3(c) shows that the Gaussian function provides a good representation of the $B^\pm \to \phi K^\pm$ signal candidates. There are no $B^\pm \to \phi \pi^\pm$ candidates, as expected, and the straight line provides a good representation of the background candidates as seen in Figure 5.3(d).

### 5.5 Summary

LHCb recorded a total of 37 pb$^{-1}$ of data in 2010, of which approximately half was magnet up and the other half magnet down. The $B^\pm \to J/\psi h^\pm$ and $B^\pm \to \phi h^\pm$ event selections presented in Chapter 4 were applied to the data. Due to the low statistics of the $B^\pm \to \phi h^\pm$ data sets and the high purity of the $B^\pm$ twobody stripping selection the stripping selection was used as the $B^\pm \to \phi h^\pm$ event selection. Using the latest simulated events the $B^\pm \to J/\psi h^\pm$ event selection performance was re-evaluated and the expected yields for the two different magnet polarities were calculated. Finally, an extended maximum likelihood fit was used to extract the four signal yields. These were not consistent with expected yields from the simulated events, however as the simulation has not been tuned to the data this is not unexpected. The ratio of the $B^\pm \to J/\psi \pi^\pm$ and $B^\pm \to J/\psi K^\pm$ yields is consistent with the nominal ratio of their branching fractions, and the $B^\pm \to \phi \pi^\pm$ candidates are consistent with zero. This shows the extended likelihood fits performed as expected.
Chapter 6

Method to Extract the \( \mathcal{CP} \)-asymmetry in Charged B-meson Decays

In this chapter a method is presented to extract the ratio of yields of the signal channels, \( \frac{N(B^+ \to J/\psi \pi^\pm)}{N(B^+ \to J/\psi K^\pm)} \) and \( \frac{N(B^\pm \to \phi \pi^\pm)}{N(B^\pm \to \phi K^\pm)} \), and the \( \mathcal{CP} \)-asymmetry present in the \( B^\pm \to \phi K^\pm \) and \( B^\pm \to \phi \pi^\pm \) decay modes using the \( B^\pm \to J/\psi K^\pm \) and \( B^\pm \to J/\psi \pi^\pm \) decay modes, respectively, as control channels. Section 6.1 presents the method to extract the \( \mathcal{CP} \)-asymmetry from the \( B^\pm \to \phi K^\pm \) and \( B^\pm \to \phi \pi^\pm \) decays modes and the expected sensitivity of these measurements with different integrated luminosities is presented in Section 6.2. An extended maximum likelihood fit is presented in Section 6.3 which can be used to extract the ratio of yields and the measured asymmetries from the \( B^\pm \to J/\psi h^\pm \) and \( B^\pm \to \phi h^\pm \) samples. Using the MC09 simulated samples the extended maximum likelihood fit is validated in Section 6.4.

6.1 Extraction of the \( \mathcal{CP} \)-asymmetry

The condition required for direct \( \mathcal{CP} \)-violation, \( A^\text{dir}_{\mathcal{CP}} \), to be present in charged B-meson decays is the observation of a difference in the rate of \( B^+ \) and \( B^- \) decaying to the final states \( f^+ \) and \( f^- \) respectively (see equation (1.12)). The measured asymmetry, \( A_{\text{meas}} \), is given by

\[
A_{\text{meas}} = \frac{N(B^- \to f^-) - N(B^+ \to f^+)}{N(B^- \to f^-) + N(B^+ \to f^+)} \quad (6.1)
\]
where its uncertainty, $\sigma_{A_{\text{meas}}}$, is given by the total number of $B^\pm \to f^\pm$ decays, $N_{\text{tot}}$,

$$\sigma^2_{A_{\text{meas}}} = 1/N_{\text{tot}} \quad (6.2)$$

where $N_{\text{tot}} = N(B^- \to f^-) + N(B^+ \to f^+)$. In order to relate the measured asymmetry, $A_{\text{meas}}$, to the $CP$-asymmetry, $A_{\text{CP}}$ ($\equiv A_{\text{dir}}^{CP}$), it is necessary to introduce a parameter, $r$, representing the polluting asymmetries, $A_r$, that contribute to the measured asymmetry

$$r = \frac{N_b \cdot f(b \to B^-) \cdot \epsilon_{h^-}}{N_{\bar{b}} \cdot f(\bar{b} \to B^+) \cdot \epsilon_{h^+}} \quad (6.3a)$$

$$A_r = \frac{(1 - r)}{(1 + r)} \quad (6.3b)$$

where $N_{b/\bar{b}}$ is the number of $b$ or $\bar{b}$ quarks produced; $f(b/\bar{b} \to B^-/B^+)$ is the hadronisation fraction; and $\epsilon_{h^-}$ is the detection efficiency of the negative kaon or pion, and $\epsilon_{h^+}$ of the positive kaon or pion. The polluting asymmetries have contributions from

- the production asymmetry which is the difference in the number $b$ and $\bar{b}$ quarks that are able to hadronise to $B^+$ and $B^-$ respectively; and
- the detection efficiency which is caused by differences in the detection efficiency of the positive and negative hadrons.

The $CP$-asymmetry is then defined as

$$A_{\text{CP}} = A_r + A_{\text{meas}} \quad (6.4)$$

There are seven different asymmetries present in the four signal channels $B^\pm \to J/\psi K^\pm$, $B^\pm \to J/\psi\pi^\pm$, $B^\pm \to \phi K^\pm$ and $B^\pm \to \phi\pi^\pm$. Each of the channels has its own $CP$-asymmetry; there are two detector asymmetries, one for the kaons and one for the pions, which are common to a pair of channels; and there is a $B^\pm$ production asymmetry common to all. The control channels $B^\pm \to J/\psi K^\pm$ and $B^\pm \to J/\psi\pi^\pm$ are used to extract the production and detector asymmetries using

$$A_r^K = A_{\text{CP}}(B^\pm \to J/\psi K^\pm) - A_{\text{meas}}(B^\pm \to J/\psi K^\pm) \quad (6.5a)$$

$$A_r^\pi = A_{\text{CP}}(B^\pm \to J/\psi\pi^\pm) - A_{\text{meas}}(B^\pm \to J/\psi\pi^\pm) \quad (6.5b)$$
where the world averages of $A_{\text{CP}}(B^\pm \to J/\psi K^\pm)$ and $A_{\text{CP}}(B^\pm \to J/\psi \pi^\pm)$ are used as an input \cite{4}. The $\mathcal{CP}$-asymmetries in $B^\pm \to \phi K^\pm$ and $B^\pm \to \phi \pi^\pm$ are then extracted using the measured values of $A^K_r$ and $A^\pi_r$ from $B^\pm \to J/\psi K^\pm$ and $B^\pm \to J/\psi \pi^\pm$ respectively.

### 6.2 Sensitivity

The expected sensitivity of the $\mathcal{CP}$-asymmetry measurement in the $B^\pm \to \phi K^\pm$ and $B^\pm \to \phi \pi^\pm$ decay modes is determined using the calculated yields for the signal decay modes presented in Section 4.5. The calculation uses values of $37 \text{ pb}^{-1}$, $1 \text{ fb}^{-1}$, $2 \text{ fb}^{-1}$ and $10 \text{ fb}^{-1}$ for the integrated luminosity and a value of $7 \text{ TeV}$ for the centre-of-mass energy. This corresponds to the collision data recorded in the 2010 data taking period and the collision data expected from the current 2011 data taking period.

The error on the $\mathcal{CP}$-asymmetry, determined from equation (6.4), is

$$
\sigma^2_{A_{\text{CP}}} = \sigma^2_{A_{\text{meas}}} + \sigma^2_{A_{\text{r}}} \quad (6.6)
$$

where $\sigma$ is used to represent the error on the subscripted quantity. In order to determine the error in the $\mathcal{CP}$-asymmetry for $B^\pm \to \phi K^\pm/\pi^\pm$ the error in the polluting asymmetry measured using the $B^\pm \to J/\psi K^\pm/\pi^\pm$ control channels must therefore be known and is given by

$$
\sigma^2_{A_{\text{r}}} = \sigma^2_{A_{\text{meas}}} + \sigma^2_{A_{\text{CP}}} \quad (6.7)
$$

where the world average value for $\sigma_{A_{\text{CP}}}$ is used \cite{4}. In both equations (6.6) and (6.7) $\sigma^2_{A_{\text{meas}}}$ is given by equation (6.2) and is dependent only upon the total yield of that channel.

The statistical errors calculated for the polluting asymmetries using the yields presented in Table 4.11 and at a centre-of-mass energy of $7 \text{ TeV}$ are shown in Table 6.1. With $37 \text{ pb}^{-1}$ of data, corresponding to the data taken during 2010, the measurement of the polluting asymmetry is statistically limited by the measured asymmetry in $B^\pm \to J/\psi K^\pm$ and $B^\pm \to J/\psi \pi^\pm$. With $1 \text{ fb}^{-1}$ of collision data sufficient statistics are collected such that the measurement of $\sigma_{A_{\text{r}}}$ is statistically limited by the current world average of the measured value of the $\mathcal{CP}$-asymmetry in $B^\pm \to J/\psi K^\pm$ and $B^\pm \to J/\psi \pi^\pm$. 
Method to Extract the $\mathcal{CP}$-asymmetry in Charged B-meson Decays

Table 6.1: The expected statistical sensitivity of the polluting asymmetry. The measured yields in Section 5.4 are used to calculate the measured asymmetry sensitivity for $37 \text{ pb}^{-1}$ and the expected yields from Section 4.5 are used to calculate the measured asymmetry sensitivity for $1 \text{ fb}^{-1}$, $2 \text{ fb}^{-1}$ and $10 \text{ fb}^{-1}$. The world averages are used for the $\mathcal{CP}$-asymmetry.

<table>
<thead>
<tr>
<th>$\mathcal{L}dt$</th>
<th>Channel</th>
<th>$\sigma_{\mathcal{CP}}$</th>
<th>$\sigma_{\text{meas}}$</th>
<th>$\sigma_{A_r}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$37 \text{ pb}^{-1}$</td>
<td>$B^\pm \to J/\psi K^\pm$</td>
<td>0.007</td>
<td>–</td>
<td>0.017</td>
</tr>
<tr>
<td></td>
<td>$B^\pm \to J/\psi \pi^\pm$</td>
<td>0.07</td>
<td>–</td>
<td>0.075</td>
</tr>
<tr>
<td>$1 \text{ fb}^{-1}$</td>
<td>$B^\pm \to J/\psi K^\pm$</td>
<td>0.007</td>
<td>0.0043</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>$B^\pm \to J/\psi \pi^\pm$</td>
<td>0.07</td>
<td>0.020</td>
<td>–</td>
</tr>
<tr>
<td>$2 \text{ fb}^{-1}$</td>
<td>$B^\pm \to J/\psi K^\pm$</td>
<td>0.007</td>
<td>0.0030</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>$B^\pm \to J/\psi \pi^\pm$</td>
<td>0.07</td>
<td>0.014</td>
<td>–</td>
</tr>
<tr>
<td>$10 \text{ fb}^{-1}$</td>
<td>$B^\pm \to J/\psi K^\pm$</td>
<td>0.007</td>
<td>0.0013</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>$B^\pm \to J/\psi \pi^\pm$</td>
<td>0.07</td>
<td>0.0063</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 6.2: The expected statistical sensitivity of the $\mathcal{CP}$-asymmetry. The polluting asymmetries sensitivities are taken from Table 6.1. The measured yields in Section 5.4 are used to calculate the measured asymmetry sensitivity for $37 \text{ pb}^{-1}$ and the expected yields from Section 4.5 are used to calculate the measured asymmetry sensitivity for $1 \text{ fb}^{-1}$, $2 \text{ fb}^{-1}$ and $10 \text{ fb}^{-1}$. The world averages for the $\mathcal{CP}$-asymmetries are shown for comparison.

<table>
<thead>
<tr>
<th>$\mathcal{L}dt$</th>
<th>Channel</th>
<th>$\sigma_{A_r}$</th>
<th>$\sigma_{\text{meas}}$</th>
<th>$\sigma_{\mathcal{CP}}$</th>
<th>$\sigma_{A_r}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$37 \text{ pb}^{-1}$</td>
<td>$B^\pm \to \phi K^\pm$</td>
<td>0.019</td>
<td>–</td>
<td>0.091</td>
<td><strong>0.093</strong></td>
</tr>
<tr>
<td></td>
<td>$B^\pm \to \phi \pi^\pm$</td>
<td>0.10</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$1 \text{ fb}^{-1}$</td>
<td>$B^\pm \to \phi K^\pm$</td>
<td>0.0082</td>
<td>0.015</td>
<td>–</td>
<td><strong>0.017</strong></td>
</tr>
<tr>
<td></td>
<td>$B^\pm \to \phi \pi^\pm$</td>
<td>0.073</td>
<td>0.48</td>
<td>–</td>
<td>0.49</td>
</tr>
<tr>
<td>$2 \text{ fb}^{-1}$</td>
<td>$B^\pm \to \phi K^\pm$</td>
<td>0.0076</td>
<td>0.011</td>
<td>–</td>
<td><strong>0.013</strong></td>
</tr>
<tr>
<td></td>
<td>$B^\pm \to \phi \pi^\pm$</td>
<td>0.071</td>
<td>0.34</td>
<td>–</td>
<td>0.35</td>
</tr>
<tr>
<td>$10 \text{ fb}^{-1}$</td>
<td>$B^\pm \to \phi K^\pm$</td>
<td>0.0071</td>
<td>0.0049</td>
<td>–</td>
<td><strong>0.0086</strong></td>
</tr>
<tr>
<td></td>
<td>$B^\pm \to \phi \pi^\pm$</td>
<td>0.070</td>
<td>0.15</td>
<td>–</td>
<td>0.17</td>
</tr>
</tbody>
</table>

The world averages are used for the $\mathcal{CP}$-asymmetry 

[1]
The calculation of the $CP$-asymmetry error in the $B^\pm \to \phi K^\pm/\pi^\pm$ channel at different integrated luminosities is presented, along with the current world averages of the $CP$-asymmetry, in Table 6.2. It is seen that with the 2010 data set, $37 \text{ pb}^{-1}$, the $CP$-asymmetry is dominated by the statistical error in the measured $B^\pm \to \phi K^\pm$ asymmetry and is above the current world average. With the expected 2011 data set, $1 \text{ fb}^{-1}$, LHCb can make a world’s best measurement in this channel; the dominant error is still the measured asymmetry in the $B^\pm \to \phi K^\pm$ channel. For the $B^\pm \to \phi \pi^\pm$ channel, the yields expected with $1 \text{ fb}^{-1}$ and $2 \text{ fb}^{-1}$ of data (Table 4.11) are insufficient to measure the $CP$-asymmetry, this will require the full $10 \text{ fb}^{-1}$ LHCb data set.

### 6.3 Likelihood Fit Procedure

This section describes an extended likelihood fit that is used to extract the ratio of yields, $N(B^\pm \to J/\psi\pi^\pm)/N(B^\pm \to J/\psi K^\pm)$ and $N(B^\pm \to \phi\pi^\pm)/N(B^\pm \to \phi K^\pm)$, and the measured asymmetries in the signal channels: $B^\pm \to J/\psi K^\pm$, $B^\pm \to J/\psi\pi^\pm$, $B^\pm \to \phi K^\pm$ and $B^\pm \to \phi\pi^\pm$, using the $B^\pm \to J/\psi h^\pm$ and $B^\pm \to \phi h^\pm$ selected events. The asymmetry and yields of the backgrounds in these samples are also extracted.

#### 6.3.1 Likelihood function Definition

The likelihood function used to extract the ratio of yields and asymmetry is defined as

$$F(m, \Delta LL_{K-\pi}, \alpha; r_{\text{obs}}, N_{\text{sig}}^K, N_{\text{comb}}^K, N_{\text{comb}}^\pi, N_{\text{mt}}^K, A_{\text{sig}}^K, A_{\text{sig}}^\pi, A_{\text{comb}}^K, A_{\text{comb}}^\pi, \vec{p}) =$$

$$N_{\text{sig}} \cdot \left[ \frac{1}{1 + r_{\text{obs}}} \cdot \frac{1 - q A_{\text{sig}}^K}{2} \cdot F_{\text{sig}}^K (m, \Delta LL_{K-\pi}, \alpha; \vec{p}_{\text{sig}}^K) + \frac{r_{\text{obs}}}{1 + r_{\text{obs}}} \cdot \frac{1 - q A_{\text{sig}}^\pi}{2} \cdot F_{\text{sig}}^\pi (m, \Delta LL_{K-\pi}, \alpha; \vec{p}_{\text{sig}}^\pi) \right]$$

$$+ N_{\text{comb}}^K \cdot \frac{(1 - q A_{\text{comb}}^K)}{2} \cdot F_{\text{comb}}^K (m, \Delta LL_{K-\pi}, \alpha; \vec{p}_{\text{comb}}^K)$$

$$+ N_{\text{comb}}^\pi \cdot \frac{(1 - q A_{\text{comb}}^\pi)}{2} \cdot F_{\text{comb}}^\pi (m, \Delta LL_{K-\pi}, \alpha; \vec{p}_{\text{comb}}^\pi)$$

$$+ N_{\text{mt}}^K \cdot \frac{(1 - q A_{\text{mt}}^K)}{2} \cdot F_{\text{mt}}^K (m, \Delta LL_{K-\pi}, \alpha; \vec{p}_{\text{mt}}^K)$$

(6.8)
where

$F$ is the total fit function;

$F_X^Y$ are the fit functions, the subscript $(X)$ indicates the shape it represents (sig, comb, mt) and the superscript $(Y)$ whether it is the kaon or pion $\Delta L L_{K-\pi}$ distribution that is used to form the fit;

$m$ is the $B^\pm$ invariant mass observable;

$\Delta L L_{K-\pi}$ is the bachelor hadron particle identification observable;

$\alpha$ is a kinematic observable defined as the energy of the intermediate resonance over the magnitude of the momentum of the bachelor hadron, e.g. $E(J/\psi)/|\vec{p}(K)|$;

$q$ is the charge of the $B$-candidate i.e. $B^+ = +1$ and $B^- = -1$;

$\vec{p}$ are the floating parameters in the fit function, $F$;

$\vec{p}_X^Y$ are the floating parameters in the fit functions, $F_X^Y$;

$r_{\text{obs}}$ is the observed ratio of the yield of the signal bachelor pion candidates over the yield of the signal bachelor kaon candidates;

$N_X^Y$ is the yield extracted for shape identified by the subscript and superscript labels;

and

$A_X^Y$ is the measured asymmetry, $A_{\text{meas}}$, for the shape represented by the subscript and the bachelor hadron $\Delta L L_{K-\pi}$ distribution represented by the superscript.

Each fit function, $F_X^Y$, is a three-dimensional conditional function which is the same for the $B^+$ and $B^-$ candidates. It is generically expressed as

$$F \left( m, \Delta L L_{K-\pi}, \alpha; \vec{p} \right) = G(m; \vec{p}(\alpha)) \cdot H \left( \Delta L L_{K-\pi} \right) \cdot I(\alpha)$$

and consists of the three components. $G(m; \vec{p}(\alpha))$ is the likelihood function that describes the $B^\pm$ invariant mass distribution and is used to separate the signal decays from the combinatorial background; it does not have a strong separation power between the kaon candidates and the pion candidates. $H \left( \Delta L L_{K-\pi} \right)$ is the likelihood function that describes the $\Delta L L_{K-\pi}$ distribution and separates the kaon sample from the pion sample; it has less power to separate the signal candidates from the background candidates. Finally, the $B^\pm \rightarrow J/\psi \pi^\pm$ ($B^\pm \rightarrow \phi \pi^\pm$) invariant mass distribution is a function of the $\alpha$ observable;
$I(\alpha)$ is included to provide the total likelihood function with information about the $\alpha$ observable \cite{183}. This three-dimensional likelihood function allows the three components to be separated and the physics parameters to be determined. The three likelihood functions $G$, $H$ and $I$, are now described.

**The $B^\pm$ Invariant Mass Function: $G(m, \vec{p}|\alpha)$**

The three contributions to the $B^\pm$ invariant mass distribution are the $B^\pm \rightarrow J/\psi K^\pm$ or $B^\pm \rightarrow \phi K^\pm$ signal channel, the $B^\pm \rightarrow J/\psi \pi^\pm$ or $B^\pm \rightarrow \phi \pi^\pm$ signal channel, and the combinatorial background.

The signal decays, $B^\pm \rightarrow J/\psi K^\pm$ and $B^\pm \rightarrow \phi K^\pm$, are reconstructed with the correct mass hypothesis for the bachelor hadron, which as a resonance decay takes the form of a Gaussian function. This Gaussian function has a mean corresponding to the $B^\pm$ invariant mass, $\mu$, and a width, $\sigma$, dependent on the LHCb detector resolution. The bachelor kaon looses energy through the emission of photons which are not reconstructed thereby creating a tail on the low-mass side of the Gaussian function’s mean. The tail begins $a \cdot \sigma$ from the Gaussian mean, where $a$ is a number which determines where the tail starts. The shape of the tail is controlled by a parameter $n$. This function is known as a Crystal Ball \cite{184}. The decays also exhibit a small exponential tail on the high-mass side of the Gaussian mean which is also described by a Crystal Ball function but with a negative $a$. This double Crystal Ball (DCB) function is defined as

$$
DCB \left( m; \mu, \sigma, a_{low}, n_{low}, a_{high}, n_{high} \right) =
\begin{cases}
\left( \frac{n_{low}}{n_{low} \sigma} \right)^{n_{low}} \cdot \exp \left( -\frac{1}{2} a_{low}^2 \right) \\
\exp \left( -\frac{1}{2} \left( \frac{m-\mu}{\sigma} \right)^2 \right) - a_{low} \cdot \exp \left( -\frac{1}{2} a_{low}^2 \right) \\
\left( \frac{n_{high}}{n_{high} \sigma} \right)^{n_{high}} \cdot \exp \left( -\frac{1}{2} a_{high}^2 \right) \\
\exp \left( -\frac{1}{2} \left( \frac{m-\mu}{\sigma} \right)^2 \right) - a_{high} \cdot \exp \left( -\frac{1}{2} a_{high}^2 \right)
\end{cases}
\right.
$$

where low is used to indicate the Crystal Ball parameters that control the low-mass tail and high the Crystal Ball parameters that control the high-mass tail. The central region is a Gaussian function as defined in Section 5.4.

The signal decays $B^\pm \rightarrow J/\psi \pi^\pm$ and $B^\pm \rightarrow \phi \pi^\pm$ are described by Crystal Ball functions for which the low-mass tail is generated by the emission of photons from the bachelor pion, as for the $B^\pm \rightarrow J/\psi K^\pm$ and $B^\pm \rightarrow \phi K^\pm$ decays. The assignment of the kaon mass
to the pion during reconstruction creates a high-mass tail and shifts the reconstructed \( B^\pm \) invariant mass to a higher value. The high-mass tail and shift are determined by the kinematics of the decay and the shifted mean of the Crystal Ball, \( \mu_{\text{shifted}} \), is given by

\[
\mu_{\text{shifted}} = \sqrt{\mu^2 + (1 + \alpha) \cdot (M^2_K - M^2_\pi)}
\]

(6.11)

where \( \alpha \) is the ratio of the energy of the resonance divided by the magnitude of the hadron momentum, e.g. \( E_{J/\psi} / |\vec{p}_h| \); \( M_K \) and \( M_\pi \) are the kaon and pion masses respectively; and the \( \mu \) is fixed to be the same as in the kaon double Crystal Ball function. The Crystal Ball (CB) function is then

\[
\text{CB}(m; \mu_{\text{shifted}}, \sigma, a, n|\alpha) = \begin{cases} 
\frac{(n/|a|-|a| - \frac{m-\mu_{\text{shifted}}}{\sigma})^{n}}{(n/|a|-|a| - \frac{m-\mu_{\text{shifted}}}{\sigma})^{n}} & m-\mu_{\text{shifted}} < -|a| \\
\exp \left(-\frac{1}{2} \left(\frac{m-\mu_{\text{shifted}}}{\sigma}\right)^2\right) & m-\mu_{\text{shifted}} \geq -|a| 
\end{cases}
\]

(6.12)

The combinatorial background is modelled by a first order Chebychev polynomial which is defined in Section 5.4.

The \( \Delta LL_{K-\pi} \) Function: \( H(\Delta LL_{K-\pi}) \)

The \( \Delta LL_{K-\pi} \) is used primarily to separate \( B^\pm \rightarrow J/\psi K^\pm \) from \( B^\pm \rightarrow J/\psi \pi^\pm \) or \( B^\pm \rightarrow \phi K^\pm \) from \( B^\pm \rightarrow \phi \pi^\pm \) for which the \( B^\pm \) invariant mass distribution provides very little separation. To do this the distributions of the bachelor hadron’s \( \Delta LL_{K-\pi} \) must be known for the signal channels. In the simulated signal samples this is done by selecting the truth-matched signal candidates and the remaining candidates are the background distribution. In a sample without information from the generated event, or where a simulated \( \Delta LL_{K-\pi} \) signal distribution cannot be used, calibrated kaon and pion \( \Delta LL_{K-\pi} \) distributions are reweighted to the phase space of the signal samples.

The \( \Delta LL_{K-\pi} \) distributions for true kaons and true pions are determined from the kaon and pion of the \( D^0 \) decay in a \( D^*(2010)^\pm \rightarrow D^0 (\rightarrow K^\mp \pi^\pm) \pi^\pm \) calibration sample. These calibration distributions can be transformed to match the phase-space of another decay channel by reweighting the calibration kaon or pion candidates. The method uses the set of variables that determine the \( \Delta LL_{K-\pi} \) distribution to bin the calibration and
signal data sets; the weight for the \(i^{th}\) bin is given by

\[
W_i = \frac{N_i^{\text{sig}}}{N_i^{\text{cal}}} \times \frac{N_{\text{cal}}^{\text{total}}}{N_{\text{sig}}^{\text{total}}} \quad (6.13)
\]

where \(N_i^{\text{sig/cal}}\) is the number of signal or calibration candidates in the \(i^{th}\) bin and \(N_{\text{total}}^{\text{sig/cal}}\) is the total number of signal or calibration candidates. The weight is assigned to each of the calibration kaon (pion) candidates depending on the bin they are located within. The signal \(\Delta L L_{K^{-}\pi}\) distribution is determined by summing over the calibration candidates applying the candidate’s weight, such that the value of the signal \(\Delta L L_{K^{-}\pi}\) for the \(j^{th}\) candidate in the \(i^{th}\) bin is

\[
H_{ij}^{\text{sig}}(\Delta L L_{K^{-}\pi}) = W_i H_{ij}^{\text{cal}}(\Delta L L_{K^{-}\pi}). \quad (6.14)
\]

The \(\Delta L L_{K^{-}\pi}\) distribution for the signal candidates is then the weighted calibration distribution. Ideally the binning is chosen such that the distribution of the signal \(\Delta L L_{K^{-}\pi}\) and calibration \(\Delta L L_{K^{-}\pi}\) in a given bin match.

In this chapter the simulated signal samples, Table 4.1, and a simulated calibration sample are used to validate the reweighting procedure. The simulated signal samples have had the \(B^{\pm} \rightarrow J/\psi h^{\pm}\) or the \(B^{\pm} \rightarrow \phi h^{\pm}\) event selection applied and use truth-matching to remove the remaining background candidates. A simulated \(D^*(2010)^{\pm} \rightarrow D^{0} (\rightarrow K^{\mp}\pi^{\pm}) \pi^{\pm}\) calibration sample consisting of 10,518,931 events has a kinematic selection applied and then uses truth-matching to remove the remaining background events.

The kinematic variables used to reweight the calibration samples are the momentum, transverse momentum and pseudo-rapidity (where the pseudo-rapidity is defined as \(\eta = -\log (\tan^{-1}(p_T/\vec{p}/2)))\). The number of bins and range of each of the variables used is presented in Table 6.3. A comparison between the kaon and pion weighting variables from the simulated samples are shown in Figures 6.1 and 6.2. The \(\Delta L L_{K^{-}\pi}\) distribution for the combinatorial background is taken as the \(\Delta L L_{K^{-}\pi}\) distribution of the background candidates removed from the signal samples by truth-matching.

A distribution was created using momentum (\(\vec{p}\)), transverse momentum (\(p_T\)) and pseudo-rapidity (\(\eta\))

\[
f(\vec{p}, p_T, \eta) = \vec{p} + p_T + \eta \quad (6.15)
\]
Method to Extract the $C\P$-asymmetry in Charged B-meson Decays

Figure 6.1: A comparison of the truth-matched simulated $B^\pm \to J/\psi K^\pm$ (red circles) and $B^\pm \to J/\psi \pi^\pm$ (blue squares) signal sample’s weighting variables.
Method to Extract the $CP$-asymmetry in Charged B-meson Decays

(a) Momentum distributions.

(b) Ratio of the $B^\pm \rightarrow \phi K^\pm$ to $B^\pm \rightarrow \phi \pi^\pm$ momentum distribution.

(c) Transverse momentum distributions.

(d) Ratio of the $B^\pm \rightarrow \phi K^\pm$ to $B^\pm \rightarrow \phi \pi^\pm$ transverse momentum distribution.

(e) Pseudo-rapidity distributions.

(f) Ratio of the $B^\pm \rightarrow \phi K^\pm$ to $B^\pm \rightarrow \phi \pi^\pm$ pseudo-rapidity distribution.

Figure 6.2: A comparison of the truth-matched simulated $B^\pm \rightarrow \phi K^\pm$ (red circles) and $B^\pm \rightarrow \phi \pi^\pm$ (blue squares) signal sample’s weighting variables.
Method to Extract the $\mathcal{CP}$-asymmetry in Charged B-meson Decays

<table>
<thead>
<tr>
<th>Variable</th>
<th>Channel</th>
<th>Number bins</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta LL_{K^{-}\pi}$</td>
<td>$B^\pm \to J/\psi K^\pm$</td>
<td>83</td>
<td>-60</td>
<td>99</td>
</tr>
<tr>
<td></td>
<td>$B^\pm \to J/\psi \pi^\pm$</td>
<td>47</td>
<td>-99</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>$B^\pm \to \phi K^\pm$</td>
<td>83</td>
<td>-35</td>
<td>95</td>
</tr>
<tr>
<td></td>
<td>$B^\pm \to \phi \pi^\pm$</td>
<td>135</td>
<td>-98</td>
<td>21</td>
</tr>
<tr>
<td>Momentum</td>
<td>$B^\pm \to J/\psi K^\pm$</td>
<td>35</td>
<td>6.3 GeV</td>
<td>98.1 GeV</td>
</tr>
<tr>
<td></td>
<td>$B^\pm \to J/\psi \pi^\pm$</td>
<td>36</td>
<td>3.8 GeV</td>
<td>98.1 GeV</td>
</tr>
<tr>
<td></td>
<td>$B^\pm \to \phi K^\pm$</td>
<td>35</td>
<td>6.3 GeV</td>
<td>98.1 GeV</td>
</tr>
<tr>
<td></td>
<td>$B^\pm \to \phi \pi^\pm$</td>
<td>36</td>
<td>3.9 GeV</td>
<td>98.1 GeV</td>
</tr>
<tr>
<td>Transverse momentum</td>
<td>$B^\pm \to J/\psi K^\pm$</td>
<td>19</td>
<td>1.0 GeV</td>
<td>7.4 GeV</td>
</tr>
<tr>
<td></td>
<td>$B^\pm \to J/\psi \pi^\pm$</td>
<td>32</td>
<td>1.0 GeV</td>
<td>7.1 GeV</td>
</tr>
<tr>
<td></td>
<td>$B^\pm \to \phi K^\pm$</td>
<td>19</td>
<td>1.0 GeV</td>
<td>7.4 GeV</td>
</tr>
<tr>
<td></td>
<td>$B^\pm \to \phi \pi^\pm$</td>
<td>32</td>
<td>1.0 GeV</td>
<td>7.1 GeV</td>
</tr>
<tr>
<td>Pseudo-rapidity</td>
<td>$B^\pm \to J/\psi K^\pm$</td>
<td>55</td>
<td>2.0</td>
<td>4.9</td>
</tr>
<tr>
<td></td>
<td>$B^\pm \to J/\psi \pi^\pm$</td>
<td>52</td>
<td>1.8</td>
<td>4.8</td>
</tr>
<tr>
<td></td>
<td>$B^\pm \to \phi K^\pm$</td>
<td>55</td>
<td>2.0</td>
<td>4.9</td>
</tr>
<tr>
<td></td>
<td>$B^\pm \to \phi \pi^\pm$</td>
<td>52</td>
<td>1.8</td>
<td>4.8</td>
</tr>
</tbody>
</table>

Table 6.3: The reweighting variables used for the simulated sample reweighting.
(a) The $\Delta L_{K-\pi}$ distribution for the weighted calibration sample (red circles), the signal sample (blue squares) and the calibration sample (black triangles).

(b) The residuals of the calibration and weighted calibration $\Delta L_{K-\pi}$ distributions.

(c) The residuals of the signal and weighted calibration $\Delta L_{K-\pi}$ distributions.

Figure 6.3: The results of the weighting of the $f(p, p_T, \eta)$ distribution using the simulated $B^\pm \rightarrow J/\psi K^\pm$ sample.
Method to Extract the $CP$-asymmetry in Charged B-meson Decays

where the input momentum, transverse momentum and pseudo-rapidity distributions were taken from the simulated $B^\pm \rightarrow J/\psi K^\pm$ sample for the signal and the simulated $D^*(2010)^\pm \rightarrow D^0 (\rightarrow K^{+}\pi^{\pm})\pi^{\pm}$ sample for the calibration. The signal and calibration samples were then binned in $\vec{p}$, $p_T$ and $\eta$ and the candidate weights calculated for each bin. The calibration $f(\vec{p}, p_T, \eta)$ distribution is weighted to produce the signal $f(\vec{p}, p_T, \eta)$ distribution and the results are shown in Figure 6.3. A comparison between the known signal distribution and the weighted calibration distribution is shown in Figure 6.3(a), which also shows the source calibration distribution. The residuals of the calibration to weighted and the signal to weighted distributions are shown in Figures 6.3(b) and (c) respectively. The spread in the distribution of the residuals from the known signal distribution and the reweighted signal distribution shown in Figure 6.3(c) indicates that the two distributions match and the reweighting is successful.

The weighting procedure is run with each of the simulated signal samples, which have been selected as described above. Applying the reweighting procedure to these samples does not produce the expected $\Delta LL_{K^{+}\pi^{-}}$ signal distributions; an additional selection criteria, which removes all the bachelor hadron candidates that passed through the RICH aerogel radiator, has been applied to both the signal and calibration samples. The cut removes both signal and background candidates, reducing the selected data set by approximately 80%, and is therefore not included in the final event selection. The results of the weighting procedure are shown in Figures 6.4 to 6.7 for each of the simulated signal decay modes. The results are similar across all the samples and the figures all show a convincing match between the weighted $\Delta LL_{K^{+}\pi^{-}}$ distribution and the truth-matched signal $\Delta LL_{K^{+}\pi^{-}}$ distribution. The residual plots all support the conclusion of weighted distribution matching the signal distribution.

The weighting method was first proposed for use in LHCb with a $B^\pm \rightarrow h^+h^-$ analysis using $D^*(2010)^\pm \rightarrow D^0 (\rightarrow K^{+}\pi^{\pm})\pi^{\pm}$ decays as the calibration sample [186]. It was later adopted, and further developed, for use in the measurement of the CKM angle $\gamma$ using $B^\pm \rightarrow DK^{\pm}$ decays [187]. The weighting procedure described here is inspired by the original three-dimensional weighting using the ROOT framework first implemented in reference [186] and developed in reference [187]. The version presented here uses the RooFit libraries to implement an n-dimensional weighting procedure and is the authors own work. The reweighting procedure developed in reference [187] is also based on the RooFit libraries. A comparison between the two showed an excellent agreement in all four signal channels.
Method to Extract the $C\!P$-asymmetry in Charged B-meson Decays

(a) The $\Delta L_{K-\pi}$ distribution for the weighted calibration sample (red circles), the signal sample (blue squares) and the calibration sample (black triangles).

(b) The residuals of the calibration and weighted calibration $\Delta L_{K-\pi}$ distributions. The data points that are precisely zero indicate no entries in this bin.

(c) The residuals of the signal and weighted calibration $\Delta L_{K-\pi}$ distributions. The data points that are precisely zero indicate no entries in this bin.

Figure 6.4: The results of the weighting of the $B^\pm \rightarrow J/\psi K^\pm$ MC09 signal sample.
Method to Extract the $C\!P$-asymmetry in Charged B-meson Decays

Figure 6.5: The results of the weighting of the $B^{\mp} \to J/\psi\pi^{\mp}$ MC09 signal sample.
Method to Extract the $C\not\!P$-asymmetry in Charged B-meson Decays

(a) The $\Delta L L_{K-\pi}$ distribution for the weighted calibration sample (red circles), the signal sample (blue squares) and the calibration sample (black triangles).

(b) The residuals of the calibration and weighted calibration $\Delta L L_{K-\pi}$ distributions. The data points that are precisely zero indicate no entries in this bin.

(c) The residuals of the signal and weighted calibration $\Delta L L_{K-\pi}$ distributions. The data points that are precisely zero indicate no entries in this bin.

Figure 6.6: The results of the weighting of the $B^\pm \rightarrow \phi K^\pm$ MC09 signal sample.
164 Method to Extract the $C\!P$-asymmetry in Charged B-meson Decays

(a) The $\Delta LL_{K-\pi}$ distribution for the weighted calibration sample (red circles), the signal sample (blue squares) and the calibration sample (black triangles).

(b) The residuals of the calibration and weighted calibration $\Delta LL_{K-\pi}$ distributions. The data points that are precisely zero indicate no entries in this bin.

(c) The residuals of the signal and weighted $\Delta LL_{K-\pi}$ calibration distributions. The data points that are precisely zero indicate no entries in this bin.

Figure 6.7: The results of the weighting of the $B^\pm \rightarrow \phi\pi^\pm$ MC09 signal sample.
The $\Delta LL_{K-\pi}$ likelihood functions used in the overall likelihood, equation (6.8), are formed by a non-parametric estimation of the reweighted $\Delta LL_{K-\pi}$ distributions. This is done using an adaptive kernel density estimation \[ H(\Delta LL_{K-\pi}) = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{h_i} K\left(\frac{x-x_i}{h_i}\right) \] (6.16)

where $x_i$ is the $i^{th}$ entry of $n$ in the data set, $h_i$ is the bandwidth for the $i^{th}$ entry and $K$ is the kernel density estimator. The kernel density estimator used is a Gaussian function, whose integral is one and width is $h_i$, and a Gaussian function is created for each of the $n$ entries in the data set. The width of the Gaussian function is specified for each entry and is adapted to the local density of the entries. If there is a high density of entries then the width of the Gaussian is small; a low density of entries and the Gaussian width is large to smooth out any statistical fluctuations. The $\Delta LL_{K-\pi}$ function is produced by performing a sum over all the Gaussian functions forming a function for the $\Delta LL_{K-\pi}$ distribution which is independent of the choice of bandwidth or analogously with a histogram independent of the choice of the bin width.

The Kinematic Function: $I(\alpha)$

The shift in the $B^\pm$ invariant mass distribution when a $B^\pm \rightarrow J/\psi \pi^\pm$ decay is reconstructed with the kaon mass hypothesis for the bachelor pion is calculated using the approximation given in equation (6.11). The purely kinematic variable $\alpha$ is defined for the $B^\pm \rightarrow J/\psi h^\pm$ decays as $\alpha = E_{J/\psi}/|\vec{p}_h|$ where $E_{J/\psi}$ is the $J/\psi$ energy and $\vec{p}_h$ is the hadron momentum. Similarly for the $B^\pm \rightarrow \phi h^\pm$ decays the $\alpha$ observable is defined as $\alpha = E_{\phi}/|\vec{p}_h|$.

The distributions for the $\alpha$ observable in each signal channel is determined by using truth-matching to select the true signal candidates from the $B^\pm \rightarrow J/\psi K^\pm$, $B^\pm \rightarrow J/\psi \pi^\pm$, $B^\pm \rightarrow \phi K^\pm$ and $B^\pm \rightarrow \phi \pi^\pm$ simulated samples. An adaptive kernel density estimation, described in Section 6.3.1, is then used to produce $I(\alpha)$ from the $\alpha$ distributions in these truth-matched samples.

6.3.2 The Likelihood Fit

The overall fit is validated using the truth-matched MC09 simulated signal samples. The invariant mass, $\Delta LL_{K-\pi}$ and $\alpha$ distributions are fitted simultaneously; the resulting projections and histograms of the data sets are shown in Figures 6.8 to 6.11.
Method to Extract the $\mathcal{CP}$-asymmetry in Charged B-meson Decays

(a) The projected likelihood function and the $B^\pm$ invariant mass observable.

(b) The residuals of the projected likelihood function to the $B^\pm$ invariant mass observable.

(c) The projected likelihood function and the $\Delta L_{K^-\pi}$ observable.

(d) The residuals of the projected likelihood function to the $\Delta L_{K^-\pi}$ observable.

(e) The projected likelihood function and the $\alpha$ observable.

(f) The residuals of the projected likelihood function to the $\alpha$ observable.

Figure 6.8: A fit of the full likelihood function (blue line) to the truth-matched simulated $B^\pm \rightarrow J/\psi K^\pm$ sample (black points).
Method to Extract the $\mathcal{CP}$-asymmetry in Charged B-meson Decays

(a) The projected likelihood function and the $B^\pm$ invariant mass observable.

(b) The residuals of the projected likelihood function to the $B^\pm$ invariant mass observable.

(c) The projected likelihood function and the $\Delta L_{\bar{K}^0\pi}$ observable.

(d) The residuals of the projected likelihood function to the $\Delta L_{\bar{K}^0\pi}$ observable.

(e) The projected likelihood function and the $\alpha$ observable.

(f) The residuals of the projected likelihood function to the $\alpha$ observable.

Figure 6.9: A fit of the full likelihood function (blue line) to the truth-matched simulated $B^\pm \to \phi K^\pm$ sample (black points).
**Table 6.4:** The fitted values of the parameters for the different signal samples.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>$(5,277.0 \pm 0.2) \text{ MeV}/c^2$</td>
<td>$\mu$</td>
<td>$(5,278.0 \pm 0.3) \text{ MeV}/c^2$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$(15.7 \pm 0.2) \text{ MeV}/c^2$</td>
<td>$\sigma$</td>
<td>$(15.4 \pm 0.2) \text{ MeV}/c^2$</td>
</tr>
<tr>
<td>$a_{\text{low}}$</td>
<td>$2.61 \pm 0.10$</td>
<td>$a_{\text{low}}$</td>
<td>$2.39^{+0.10}_{-0.09}$</td>
</tr>
<tr>
<td>$n_{\text{low}}$</td>
<td>$1.0 \pm 0.1$</td>
<td>$n_{\text{low}}$</td>
<td>$1.1 \pm 0.1$</td>
</tr>
<tr>
<td>$a_{\text{high}}$</td>
<td>$2.7 \pm 0.2$</td>
<td>$a_{\text{high}}$</td>
<td>$2.2 \pm 0.1$</td>
</tr>
<tr>
<td>$n_{\text{high}}$</td>
<td>$1.8^{+0.5}_{-0.4}$</td>
<td>$n_{\text{high}}$</td>
<td>$2.8^{+0.5}_{-0.4}$</td>
</tr>
</tbody>
</table>
Method to Extract the $\mathcal{CP}$-asymmetry in Charged B-meson Decays

(a) The projected likelihood function and the $B^\pm$ invariant mass observable.

(b) The residuals of the projected likelihood function to the $B^\pm$ invariant mass observable.

(c) The projected likelihood function and the $\Delta L_{K^-\pi}$ observable.

(d) The residuals of the projected likelihood function to the $\Delta L_{K^-\pi}$ observable.

(e) The projected likelihood function and the $\alpha$ observable.

(f) The residuals of the projected likelihood function to the $\alpha$ observable.

Figure 6.10: A fit of the full likelihood function (blue line) to the truth-matched simulated $B^\pm \rightarrow J/\psi\pi^\pm$ sample (black points).
Method to Extract the $C\bar{P}$-asymmetry in Charged B-meson Decays

(a) The projected likelihood function and the $B^{\pm}$ invariant mass observable.

(b) The residuals of the projected likelihood function to the $B^{\pm}$ invariant mass observable.

(c) The projected likelihood function and the $\Delta LL_{K^-\pi}$ observable.

(d) The residuals of the projected likelihood function to the $\Delta LL_{K^-\pi}$ observable.

(e) The projected likelihood function and the $\alpha$ observable.

(f) The residuals of the projected likelihood function to the $\alpha$ observable.

Figure 6.11: A fit of the full likelihood function (blue line) to the truth-matched simulated $B^{\pm} \rightarrow \phi \pi^{\pm}$ sample (black points).
the other fitted parameters are fixed to the values from the first stage and the measured asymmetries are then extracted using the likelihood fit.

6.4.1 The Ratio of Yields

To extract the ratio of yields from the simulated $B^\pm \rightarrow J/\psi h^\pm$ and $B^\pm \rightarrow \phi h^\pm$ samples a simplified version of the likelihood function, defined in equation (6.8), is used. The likelihood function is simplified so as not to depend on the charge, such that the asymmetry parameters are not considered. The simulated $B^\pm \rightarrow J/\psi h^\pm$ and $B^\pm \rightarrow \phi h^\pm$ samples are created by merging the $B^\pm \rightarrow J/\psi K^\pm$ and $B^\pm \rightarrow J/\psi \pi^\pm$ or the $B^\pm \rightarrow \phi K^\pm$ and $B^\pm \rightarrow \phi \pi^\pm$ MC09 samples. The merged data sets are weighted according to the ratio of the branching fractions presented in Table 4.4. In the two fits the tails of the double Crystal Ball function and the Crystal Ball function are fixed to the values given in Table 6.4 and, in the $B^\pm \rightarrow \phi h^\pm$ fit, the parameter $\sigma$ of the $B^\pm \rightarrow \phi \pi^\pm$ Crystal Ball is fixed to the value given in Table 6.4(d).

The fit results presented in Figure 6.12 for the $B^\pm \rightarrow J/\psi h^\pm$ sample, show that the projected $B^\pm$ invariant mass, $\Delta LL_{K-\pi}$ and $\alpha$ likelihood functions agree well with the truth-matched signal candidates from the simulated events. The residuals for the $B^\pm$ invariant mass, $\Delta LL_{K-\pi}$ and $\alpha$ observable likelihood function projections show a good fit between the histograms and the projections. The values of the fitted parameters are given in Table 6.5(a). The ratio of the two weighted simulated samples, 0.044, and the value extracted from the fit, 0.043 ± 0.002, are in very good agreement. The total input signal and background weighted yields are 14,972.9 and 431.5 respectively; again the signal and background yields extracted from the fit, 14,964 ± 120 and 460 ± 25 respectively, are compatible with the input values.

The fit result presented in Figure 6.13 for the $B^\pm \rightarrow \phi h^\pm$ sample, show that the projected $B^\pm$ invariant mass, $\Delta LL_{K-\pi}$ and $\alpha$ likelihood functions agree well with the truth-matched signal candidates from the simulated events. In this case, due to the small ratio of expected yields, this only indicates that the $B^\pm \rightarrow \phi K^\pm$ likelihood function fits well. The values of the fitted parameters are presented in Table 6.5(a). The ratio of the two signal channels is 0.001 compared to the extracted ratio, $(3.3 \pm 0.9) \times 10^{-6}$. The total signal yield, 903$^{+31}_{-30}$, agrees well with input value, 904.1, and the background yield, 258 ± 17, is also in good agreement with the input value, 256.2. The theoretical branching ratio used in the construction of the weighted data set, $4.45 \times 10^{-9}$, is the lowest prediction.
Method to Extract the $C\P$-asymmetry in Charged B-meson Decays

(a) The projected likelihood function and the $B^\pm$ invariant mass observable.

(b) The residuals of the projected likelihood function to the $B^\pm$ invariant mass observable.

(c) The projected likelihood function and the $\Delta L_{K^\mp\pi}$ observable.

(d) The residuals of the projected likelihood function to the $\Delta L_{K^\mp\pi}$ observable.

(e) The projected likelihood function and the $\alpha$ observable.

(f) The residuals of the projected likelihood function to the $\alpha$ observable.

**Figure 6.12:** A fit to extract the ratio of yields (blue line). In Figures (a), (c) and (e) the component likelihood projections, the $B^\pm \rightarrow J/\psi K^\pm$ histograms (blue squares) and the $B^\pm \rightarrow J/\psi\pi^\pm$ histograms (red circles) are shown.
Figure 6.13: A fit to extract the ratio of yields (blue line). In Figures (a), (c) and (e) the component likelihood projections, the $B^\pm \to \phi K^\pm$ histograms (blue squares) and the $B^\pm \to \phi \pi^\pm$ histograms (red circles) are shown.
found in the literature [85]. The current experimental limit on the branching ratio is $< 2.4 \times 10^{-7}$ at the 90% confidence level [189] which would give an $r_{\text{obs}}$ value of $< 0.1$. Using the $B^{\pm} \to J/\psi h^{\pm}$ sample it has therefore been demonstrated that the likelihood function can successfully extract a ratio of $O(10^{-2})$ which would allow an improvement of an order of magnitude on the current experimental limit. A value of the ratio of yields of $O(10^{-2})$ for $N(B^{\pm} \to \phi \pi^{\pm})/N(B^{\pm} \to \phi K^{\pm})$ would also lead to a discovery of $B^{\pm} \to \phi \pi^{\pm}$ if the new Standard Model predictions of the branching ratio [78,82,84] are correct or new physics models provide an enhancement of the $B^{\pm} \to \phi \pi^{\pm}$ branching ratio [85].

### 6.4.2 The Measured Asymmetries

The measured asymmetries in the simulated $B^{\pm} \to J/\psi h^{\pm}$ samples are extracted using the likelihood function defined in equation (6.8). The weighted simulated samples used are the same as for the $B^{\pm} \to J/\psi h^{\pm}$ ratio of yields fit. The projections of the fitted likelihood function are presented in Figure 6.14 and the extracted parameters in Table 6.6(a). The results are consistent with the input asymmetries of $-0.009, 0.003$ and $0.066$ for the kaon, pion and background components respectively. The relatively large error on $A_{\text{meas}}^{\pi}$ indicates the difficulty in fitting the relatively small contribution of the $B^{\pm} \to J/\psi \pi^{\pm}$ candidates to the sample.

The measured asymmetries for the simulated $B^{\pm} \to \phi h^{\pm}$ data set are extracted using the likelihood function defined in equation (6.8) but to the unweighted combination
Method to Extract the CP-asymmetry in Charged B-meson Decays

(a) The projected likelihood function and the $B^\pm$ invariant mass observable.

(b) The residuals of the projected likelihood function to the $B^\pm$ invariant mass observable.

(c) The projected likelihood function and the $\Delta L_{K-\pi}$ observable.

(d) The residuals of the projected likelihood function to the $\Delta L_{K-\pi}$ observable.

(e) The projected likelihood function and the $\alpha$ observable.

(f) The residuals of the projected likelihood function to the $\alpha$ observable.

**Figure 6.14:** A fit to extract the measured asymmetry (blue line). In Figures (a), (c) and (e) the component likelihood projections, the $B^\pm \to J/\psi K^\pm$ histograms (blue squares) and the $B^\pm \to J/\psi \pi^\pm$ histograms (red circles) are shown.
176 Method to Extract the $\mathcal{CP}$-asymmetry in Charged $B$-meson Decays

(a) The projected likelihood function and the $B^\pm$ invariant mass observable.

(b) The residuals of the projected likelihood function to the $B^\pm$ invariant mass observable.

(c) The projected likelihood function and the $\Delta LL_{K\pi}$ observable.

(d) The residuals of the projected likelihood function to the $\Delta LL_{K\pi}$ observable.

(e) The projected likelihood function and the $\alpha$ observable.

(f) The residuals of the projected likelihood function to the $\alpha$ observable.

Figure 6.15: A fit to extract the measured asymmetry (blue line). In Figures (a), (c) and (e) the component likelihood projections, the $B^\pm \to \phi K^\pm$ histograms (blue squares) and the $B^\pm \to \phi \pi^\pm$ histograms (red circles) are shown.
Table 6.6: The fitted values of the parameters on the combined MC09 samples.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A^K_{\text{meas}}$</td>
<td>$-0.009 \pm 0.008$</td>
<td>$A^K_{\text{meas}}$</td>
<td>$0.0007 \pm 0.02$</td>
</tr>
<tr>
<td>$A^\pi_{\text{meas}}$</td>
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<td>$A^\pi_{\text{meas}}$</td>
<td>$-0.003 \pm 0.02$</td>
</tr>
<tr>
<td>$A^{\text{bkg}}_{\text{meas}}$</td>
<td>$0.11 \pm 0.06$</td>
<td>$A^{\text{bkg}}_{\text{meas}}$</td>
<td>$-0.01 \pm 0.02$</td>
</tr>
</tbody>
</table>

(a) The extracted parameters for the $B^\pm \to J/\psi X$ sample.

(b) The extracted parameters for the $B^\pm \to \phi X$ sample.

of the $B^\pm \to \phi K^\pm$ and $B^\pm \to \phi \pi^\pm$ MC09 signal samples. The unweighted combined simulated samples are used to increase the ratio of the $B^\pm \to \phi K^\pm$ and $B^\pm \to \phi \pi^\pm$ samples in order to demonstrate the extraction of the measured asymmetry from the $B^\pm \to \phi \pi^\pm$ samples. The signal likelihoods have their double Crystal Ball parameters and Crystal Ball parameters fixed to those in Table 6.4. The other signal and background parameters, except the ratio of yields, are fixed to those in Table 6.5. The ratio of yields is extracted using a likelihood fit to the unweighted simulated $B^\pm \to \phi h^\pm$ sample (with the double Crystal Ball parameters and Crystal Ball parameters fixed). The extracted ratio, $1.06 \pm 0.02$, is consistent with the input value, 1.03. The projections of the fitted likelihood function are presented in Figure 6.15 and the extracted parameters in Table 6.6(b). The results are consistent with the input asymmetries of 0.001, $-0.006$, and $-0.006$ for the kaon, pion and background components respectively.

### 6.5 Summary

A method to extract the $C\!P$-asymmetries present in the $B^\pm \to \phi K^\pm$ and $B^\pm \to \phi \pi^\pm$ decay modes using the $B^\pm \to J/\psi K^\pm$, $B^\pm \to J/\psi \pi^\pm$ decay modes, respectively, as control channels has been presented. The method uses an extended maximum likelihood fit to extract the ratio of yields, $N(B^\pm \to \phi K^\pm)/N(B^\pm \to J/\psi K^\pm)$ and $N(B^\pm \to \phi \pi^\pm)/N(B^\pm \to \phi K^\pm)$, and the measured asymmetries in the four signal channels. The measured asymmetry and the nominal $C\!P$-asymmetry in the control channels are used to determine the polluting asymmetry. This polluting asymmetry is used, along with the measured asymmetry extracted using the likelihood fit, to calculate the $C\!P$-asymmetry in the $B^\pm \to \phi K^\pm$ and $B^\pm \to \phi \pi^\pm$ decay modes. Using the 37 pb$^{-1}$ data set taken during 2010 a first
measurement of the $\mathcal{CP}$-asymmetry in the $B^\pm \to \phi K^\pm$ decay mode is possible. With the 1 fb$^{-1}$ data set expected from the 2011 data taking period a world’s best measurement of the $\mathcal{CP}$-asymmetry in the $B^\pm \to \phi K^\pm$ channel could be made. With the full LHCb data set of 10 fb$^{-1}$ the first measurement of the $\mathcal{CP}$-asymmetry in the $B^\pm \to \phi \pi^\pm$ decay mode can be made. The likelihood function has been demonstrated to successfully extract the $\mathcal{CP}$-asymmetries and ratio of yields from a simulated $B^\pm \to J/\psi h^\pm$ sample but suffered from the limited statistics of the $B^\pm \to \phi \pi^\pm$ decay mode in the simulated $B^\pm \to \phi h^\pm$ sample. The likelihood fit has been demonstrated to extract a ratio of yields of $\mathcal{O}(10^{-2})$ which more recent theoretical calculations of the $B^\pm \to \phi \pi^\pm$ branching ratio predict $^{78,82,84}$. 
Chapter 7

Measurement of the Ratio of Branching Fractions Using 2010 Data

This chapter extracts the ratio of branching fractions, \( \mathcal{B}(B^± → J/ψπ^±)/\mathcal{B}(B^± → J/ψK^±) \) and \( \mathcal{B}(B^± → φπ^±)/\mathcal{B}(B^± → φK^±) \), from the 2010 data using the extended maximum likelihood fit developed in Chapter 6. The extended maximum likelihood fit is described in Section 7.1 which contains details of the method used to constrain some of the parameters of the invariant mass function, and the methods used to determine the \( \Delta LL_{K−π} \) and kinematic functions. The extended likelihood fit is used to measure the ratio of yields, \( N(B^± → J/ψπ^±)/N(B^± → J/ψK^±) \) and \( N(B^± → φπ^±)/N(B^± → φK^±) \), and the results of the likelihood fit applied to the 2010 data are presented in Section 7.2. Once the ratio of yields are measured they are corrected for and the efficiencies are determined from the simulated samples. Section 7.3 details the method to calculate the efficiency correction factor and presents its value. By measuring the ratio of yields of two topologically similar channels many of the systematic uncertainties cancel in the ratio; those that do not are evaluated in Section 7.4. The final results are presented in Section 7.5.

7.1 The Likelihood Fit to the 2010 Data

The extended maximum likelihood fit, which is defined in equation (6.8) but simplified so as not to depend on the charge, is used to measure the ratio of yields, \( N(B^± → J/ψπ^±)/N(B^± → J/ψK^±) \) and \( N(B^± → φπ^±)/N(B^± → φK^±) \). The combinato-
The background is separated into two components, the first with a kaon $\Delta L L_{K-\pi}$ function and the second with a pion $\Delta L L_{K-\pi}$ function, and a new background, the missing track background, is also included with a kaon $\Delta L L_{K-\pi}$ function. The function to be minimised is

$$F(m, \Delta L L_{K-\pi}; r_{\text{obs}}, N_{\text{sig}}, N^K_{\text{comb}}, N^K_{\text{mt}}, \vec{p}) =$$

$$N_{\text{sig}} \cdot \left[ \frac{1}{1 + r_{\text{obs}}} \cdot F^K_{\text{sig}}(m, \Delta L L_{K-\pi}; \alpha; \vec{p}_{\text{sig}}) + \frac{r_{\text{obs}}}{1 + r_{\text{obs}}} \cdot F^K_{\text{π}}(m, \Delta L L_{K-\pi}; \alpha; \vec{p}_{\text{π}}) \right]$$

$$+ N^K_{\text{comb}} \cdot F^K_{\text{comb}}(m, \Delta L L_{K-\pi}; \alpha; \vec{p}_{\text{comb}}) + N^K_{\text{mt}} \cdot F^K_{\text{mt}}(m, \Delta L L_{K-\pi}; \alpha; \vec{p}_{\text{mt}})$$

(7.1)

where

- $F$ is the total fit function;
- $F^X_Y$ are the fit functions, the subscript (X) indicates the shape it represents (sig, comb or mt) and the superscript (Y) whether it is a kaon or pion $\Delta L L_{K-\pi}$ that is used to form the fit;
- $m$ is the $B^{\pm}$ invariant mass observable;
- $\Delta L L_{K-\pi}$ is the bachelor hadron kaon particle identification observable;
- $\alpha$ is the kinematic observable defined as the energy of the resonance over the magnitude of the momentum of the bachelor hadron, e.g. $E(J/\psi)/|\vec{p}(K)|$;
- $\vec{p}$ are the floating parameters in the fit function, $F$;
- $\vec{p}^X_Y$ are the floating parameters in the fit functions, $F^X_Y$;
- $r_{\text{obs}}$ is the observed ratio of the yield of the signal pion candidates over the yield of the signal kaon candidates; and
- $N^X_Y$ is the yield extracted for the shape identified by the subscript and superscript labels.

Each fit function, $F^X_Y$, is a three-dimensional unnormalised probability distribution is generically expressed as

$$F(\vec{p}) = G(m; \vec{p}; \alpha) \cdot H(\Delta L L_{K-\pi}) \cdot I(\alpha)$$

(7.2)
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>((5,280.00 \pm 0.03)) MeV/c^2</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>((8.10 \pm 0.03)) MeV/c^2</td>
</tr>
<tr>
<td>( a_{\text{low}} )</td>
<td>(2.10 \pm 0.03)</td>
</tr>
<tr>
<td>( n_{\text{low}} )</td>
<td>(1.85 \pm 0.06)</td>
</tr>
<tr>
<td>( a_{\text{high}} )</td>
<td>(2.02 \pm 0.03)</td>
</tr>
<tr>
<td>( n_{\text{high}} )</td>
<td>(3.0 \pm 0.1)</td>
</tr>
</tbody>
</table>

(a) The \( B^\pm \rightarrow J/\psi K^\pm \) results.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>((5,280.00 \pm 0.04)) MeV/c^2</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>((10.41 \pm 0.03)) MeV/c^2</td>
</tr>
<tr>
<td>( a )</td>
<td>(2.38 \pm 0.02)</td>
</tr>
<tr>
<td>( n )</td>
<td>(1.04 \pm 0.03)</td>
</tr>
</tbody>
</table>

(b) The \( B^\pm \rightarrow J/\psi \pi^\pm \) results.

Table 7.1: The fitted values of the parameters for the \( B^\pm \rightarrow J/\psi K^\pm \) and \( B^\pm \rightarrow J/\psi \pi^\pm \)
thruth-matched signal samples.

which is the same as described in Chapter 6. Each of the functions \( G, H \) and \( I \), and the shapes they represent are now described.

7.1.1 The Mass Function: \( G(m; \vec{p}|\alpha) \)

The signal mass functions are a double Crystal Ball function and a single Crystal Ball function with a mean dependent on the \( \alpha \) observable, as described in Section 6.3.1. The tails of these Crystal Ball functions are fixed using the values obtained from the simulated samples. The exponential tails on the double and single Crystal Ball functions for the \( B^\pm \rightarrow J/\psi K^\pm \) and \( B^\pm \rightarrow J/\psi \pi^\pm \) samples, respectively, are fixed to the values extracted from a maximum likelihood fit to the truth-matched \( B^\pm \rightarrow J/\psi K^\pm \) and \( B^\pm \rightarrow J/\psi \pi^\pm \) MC10 samples. The projections of the fits in the three observables are presented in Figures 7.1 and 7.2 where it is seen that the fits agree well with the simulated samples. The values of the parameters that are fixed are given in Table 7.1. The \( B^\pm \rightarrow \phi K^\pm \) and \( B^\pm \rightarrow \phi \pi^\pm \) parameters are fixed from the fits to the MC09 truth-matched samples given in Tables 6.4(b) and (d).

The combinatorial background, \( \text{comb} \), is described by a first order Chebychev function as defined in Section 6.3.1. There are two combinatorial background functions, one for the kaon candidates and one for the pion candidates. These two functions have independent parameters as there is not expected to be any correlation between the two samples.
Figure 7.1: A fit of the full likelihood function (blue line) to the truth-matched MC10 $B^\pm \rightarrow J/\psi K^\pm$ sample (black points).
(a) The projected likelihood function and the $B^\pm$ invariant mass observable.

(b) The residuals of the projected likelihood function to the $B^\pm$ invariant mass observable.

(c) The projected likelihood function and the $\Delta L_{K-\pi}$ observable.

(d) The residuals of the projected likelihood function to the $\Delta L_{K-\pi}$ observable.

(e) The projected likelihood function and the $\alpha$ observable.

(f) The residuals of the projected likelihood function to the $\alpha$ observable.

Figure 7.2: A fit of the full likelihood function (blue line) to the truth-matched MC10 $B^\pm \rightarrow J/\psi \pi^\pm$ sample (black points).
The missing track background, $mt$, is from $B^\pm \rightarrow J/\psi X$ decays, where the $X$ represents anything including multiple particles. The majority of these decays are expected to be $B^\pm \rightarrow J/\psi K^*(892)(\rightarrow K\pi)$ where either the kaon or pion track is not reconstructed. If the kaon is not reconstructed then the loss of mass is sufficient for the majority of the events to fall below the lower mass limit and their contribution to the data set is negligible. The missing track background therefore consists of candidates where the kaon is reconstructed and the pion is lost; this is modelled by an Argus function 

$$G^{mt}(m;m_0,c,p) = m\left(1 - (m/m_0)^2\right)^p \exp\left(c\left(1 - (m/m_0)^2\right)\right).$$

The Argus function is an exponential, controlled by a slope parameter $c$, that curves down to a cut-off mass, $m_0$. A parameter $p$ is used to control the power of the cut-off slope. The cut-off mass is linked to the amount of mass lost in the missing track, given approximately by the $B^\pm$ invariant mass minus the missing track’s mass, and provides an upper limit on the distribution.

### 7.1.2 The $\Delta LL_{K-\pi}$ Function: $H(\Delta LL_{K-\pi})$

The weighting procedure, detailed in Section 6.3.1, is applied to the 2010 data set to produce the $\Delta LL_{K-\pi}$ distributions for use in the extended maximum likelihood fit. The stripping provides a CALIBRATION stripping stream and from this stream the $D^*(2010)^\pm \rightarrow D^0 (\rightarrow K^{\mp}\pi^{\pm}) \pi^\pm$ decays are selected using only kinematic cuts. The $D^0$ kaon (pion) daughter is used to provide the reference kaon (pion) candidates to use for the reweighting procedure. The kaon data set contains a total of 761,334 candidates and the pion data set a total of 762,200 candidates. The calibration samples are binned using the bachelor hadron track kinematic variables: momentum, transverse momentum and pseudo-rapidity, and the number of tracks in the event. The number of bins for each variable is

$$(\vec{p}, p_T, \eta, \text{number tracks}) = (18 \times 1 \times 4 \times 3).$$

Due to the low statistics in the signal samples, the number of bins used is much lower than that in Chapter 6.

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1 This lead to the $bkg$ terms being renamed to $comb$ in equation (7.1) compared to equation (6.8) for clarity.
Measurement of the Ratio of Branching Fractions Using 2010 Data

Figure 7.3: The weighted calibration distribution for the 2010 data set (red circles). The original calibration distribution (black triangles) and the signal distributions (blue squares) are also shown.

(a) The kaon calibration sample and the $B^{\pm} \rightarrow J/\psi h^{\pm} \Delta L_{K_{S} \pi}^{-}$ distribution.

(b) The pion calibration sample and the $B^{\pm} \rightarrow J/\psi h^{\pm} \Delta L_{K_{S} \pi}^{-}$ distribution.

(c) The kaon calibration sample and the $B^{\pm} \rightarrow \phi h^{\pm} \Delta L_{K_{S} \pi}^{-}$ distribution.

(d) The pion calibration sample and the $B^{\pm} \rightarrow \phi h^{\pm} \Delta L_{K_{S} \pi}^{-}$ distribution.
The results of the weighting procedure are shown in Figure 7.3; each plot shows a comparison of the calibration, the signal and the weighted calibration $\Delta LL_{K-\pi}$ distributions for the $B^{\pm} \rightarrow J/\psi h^{\pm}$ and $B^{\pm} \rightarrow \phi h^{\pm}$ data sets. In all the figures the weighted distributions represents the $\Delta LL_{K-\pi}$ distribution for the specific hadron species and the fraction of this distribution in the signal distribution is determined by the likelihood fit. An exact match between the weighted and signal distributions is therefore not expected due to the presence of both hadron species in the signal distribution.

The weighted kaon [PID] distribution for the $B^{\pm} \rightarrow J/\psi h^{\pm}$ sample is shown in Figure 7.3(a). The calibration procedure has the correct effect; it removes the pion contribution and alters the shape of the peak in the region 0 to 30. The weighted pion distribution in Figure 7.3(b) correctly removes the kaon contribution and, in the region 0 to 20, the distribution is strongly peaked, like the signal sample in the same region. It is difficult to draw any strong conclusions from Figures 7.3(c) and (d) due to the low statistics in the $B^{\pm} \rightarrow \phi h^{\pm}$ data set but it appears to have a similar effect as in the $B^{\pm} \rightarrow J/\psi h^{\pm}$ samples.

Since the weighting procedure appears to suffer from similar effects encountered in Section 6.3.1, the procedure is repeated removing all the tracks that passed through the RICH aerogel radiator, as in Section 6.3.1 and the results are shown in Figure 7.4. A much better agreement between the shapes of the weighted distributions and the signal distributions is seen. For example, the tail of the pion distribution in Figure 7.4(b) now matches the tail of the signal distribution. The removal of the tracks with RICH aerogel radiator information represents a significant loss of statistics and the number of candidates in the $B^{\pm} \rightarrow \phi h^{\pm}$ data set becomes insufficient to make a measurement. For this reason the $\Delta LL_{K-\pi}$ likelihood functions are created from the data sets, using the non-parametric method described in Section 6.3.1 that includes those candidates that passed through the RICH aerogel radiator. This introduces a systematic error from the shape of the weighted $\Delta LL_{K-\pi}$ distributions into the fit.

### 7.1.3 The Kinematic Function: $I(\alpha)$

The functional form of the $\alpha$ observable is determined using a non-parametric method (described in Section 6.3.1) to extract the shape of the distribution from the truth-matched signal and background MC09 $B^{\pm} \rightarrow J/\psi K^{\pm}$, $B^{\pm} \rightarrow J/\psi \pi^{\pm}$, $B^{\pm} \rightarrow \phi K^{\pm}$ and $B^{\pm} \rightarrow \phi \pi^{\pm}$ samples. The use of the simulated samples to extract the $\alpha$ distributions is an approximation as they do not exactly match the distributions from the data. A
Figure 7.4: The weighted calibration distribution with the no RICH aerogel tracks selection criteria applied to the 2010 data set (red circles). The original calibration distribution (black triangles) and the signal distributions (blue squares) with the no RICH aerogel selection criteria applied are also shown.
7.2 The Ratio of Yields

The fit is performed on the $B^\pm \to J/\psi h^\pm$ data, combining the magnet up and magnet down data sets, to give a total integrated luminosity of $(36.44 \pm 3.64) \text{ pb}^{-1}$. The fit
Figure 7.6: A fit of equation (7.1) (blue line) to the $B^\pm \rightarrow J/\psi h^\pm$ data (black points). The component likelihood projections are also shown.
Parameter | Value
---|---
μ | (5, 279.0 ± 0.2) MeV/c²
σ^K | (10.7 ± 0.2) MeV/c²
σ^π | (10.3^{+1.0}_{-0.9}) MeV/c²
p1^K | 0.2 ± 0.2
p1^π | −0.13 ± 0.06
m₀ | 5, 202 ± 26
| −51 ± 120
p | 4 ± 3
N^K_{comb} | 386 ± 44
N^π_{comb} | 932 ± 40
N^K_{mt} | 260^{+27}_{−26}
N^s_{ig} | 4, 492 ± 71
r^{J/ψ}_{obs} | **0.047 ± 0.005**

Table 7.2: The fitted values of the parameters to the \( B^± \to J/ψ h^± \) data.

extracts the ratio of the yields, \( r^{J/ψ}_{obs} \equiv N(B^± \to J/ψ π^±)/N(B^± \to J/ψ K^±) \), and the values of the fitted parameters are presented in Table 7.2. The extracted value of \( r^{J/ψ}_{obs} \) is

\[
r^{J/ψ}_{obs} = \frac{N(B^± \to J/ψ π^±)}{N(B^± \to J/ψ K^±)} = 0.047 ± 0.005
\]

where the error is statistical only. The projections of the fitted functions in each observable are presented in Figure 7.6; all show a good agreement between the fitted function and the data.

The fit is performed on the \( B^± \to φ h^± \) data, combining the magnet up and magnet down data sets, to give a total integrated luminosity of \((36.44 ± 3.64) \text{ pb}^{-1}\). The fit extracts the ratio of the yields, \( r^{φ}_{obs} \equiv N(B^± \to φ π^±)/N(B^± \to φ K^±) \), and the values of the fitted parameters are presented in Table 7.3. The projections of the fitted functions in each observable are presented in Figure 7.7; all show a good agreement between the fitted function and the data. In the fit the width of the \( B^± \to φ π^± \) Crystal Ball is taken to be the width of the \( B^± \to φ K^± \) double Crystal Ball multiplied by the ratio of their widths from Tables 6.4(b) and [d]. The \( B^± \to φ π^± \) Crystal Ball width is not allowed to float in the fit as the low number of expected candidates results in instabilities in the fit.
The projected likelihood function and the $B^{\pm}$ invariant mass observable.

The residuals of the projected likelihood function to the $B^{\pm}$ invariant mass observable.

The projected likelihood function and the $\Delta LL_{K^{\pm} - \pi}$ observable.

The residuals of the projected likelihood function to the $\Delta LL_{K^{\pm} - \pi}$ observable.

The projected likelihood function and the $\alpha$ observable.

The residuals of the projected likelihood function to the $\alpha$ observable.

Figure 7.7: A fit of equation (7.1) (blue line) to the $B^{\pm} \to \phi h^{\pm}$ data (black points). The component likelihood projections are also shown.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>$(5, 281 \pm 2) \text{ MeV/c}^2$</td>
</tr>
<tr>
<td>$\sigma^K$</td>
<td>$(21^{+3}_{-2}) \text{ MeV/c}^2$</td>
</tr>
<tr>
<td>$p1^K$</td>
<td>$-0.7 \pm 0.4$</td>
</tr>
<tr>
<td>$p1^\pi$</td>
<td>$0 \pm 0.08$</td>
</tr>
<tr>
<td>$N^K_{\text{comb}}$</td>
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<td>$N^\pi_{\text{comb}}$</td>
<td>$49^{+11}_{-10}$</td>
</tr>
<tr>
<td>$N_{\text{sig}}$</td>
<td>$146^{+15}_{-14}$</td>
</tr>
<tr>
<td>$r_{\phi}^{\text{obs}}$</td>
<td>$0.01 \pm 0.04$</td>
</tr>
</tbody>
</table>

Table 7.3: The fitted values of the parameters to the $B^\pm \to \phi h^\pm$ data.

Since the ratio of yields, $r_{\phi}^{\text{obs}}$, is expected to be zero in this data set a frequentist upper confidence interval is set at the $90 \%$ confidence level using a likelihood ratio method [4]. The ratio of yields is found to be

\[ r_{\phi}^{\text{obs}} = \frac{N(B^\pm \to \phi \pi^\pm)}{N(B^\pm \to \phi K^\pm)} < 0.116 \quad \text{at the 90 \% confidence level.} \]

### 7.3 The Efficiency Correction

The ratio of the measured yields, $r_{\phi}^{J/\psi \phi}$ ($r_{\phi}^{\text{obs}}$), extracted from the data is the number of $B^\pm \to J/\psi \pi^\pm$ ($B^\pm \to \phi \pi^\pm$) candidates in the data set divided by the number of $B^\pm \to J/\psi K^\pm$ ($B^\pm \to \phi K^\pm$) candidates in the data set. The ratio of branching fractions, $r_{B}^{J/\psi \phi} = B(B^\pm \to J/\psi \pi^\pm)/B(B^\pm \to J/\psi K^\pm)$ ($r_{B}^{\phi} = B(B^\pm \to \phi \pi^\pm)/B(B^\pm \to \phi K^\pm)$), is extracted from the number of candidates observed divided by the acceptance (generator) efficiency ($\epsilon_{\text{gen}}$), the reconstruction efficiency ($\epsilon_{\text{rec}}$), the trigger efficiency ($\epsilon_{\text{trig}}$), the stripping efficiency ($\epsilon_{\text{strip}}$) and the event selection efficiency ($\epsilon_{\text{evt}}$) by

\[ r_{B} = r_{\phi}^{\text{obs}} \times \frac{\epsilon_{\text{gen}}^K \cdot \epsilon_{\text{rec}}^K \cdot \epsilon_{\text{trig}}^K \cdot \epsilon_{\text{strip}}^K \cdot \epsilon_{\text{evt}}^K}{\epsilon_{\text{gen}}^\pi \cdot \epsilon_{\text{rec}}^\pi \cdot \epsilon_{\text{trig}}^\pi \cdot \epsilon_{\text{strip}}^\pi \cdot \epsilon_{\text{evt}}^\pi} \quad (7.3) \]

where a superscript K or $\pi$ is used to indicate which sample the efficiency is calculated for.
The reconstruction, trigger, stripping and event selection efficiencies are calculated using the simulated events where the terms $\epsilon_{\text{rec}}$, $\epsilon_{\text{trig}}$, $\epsilon_{\text{strip}}$ and $\epsilon_{\text{evt}}$ reduce to a single selection efficiency, $\epsilon_{\text{sel}}$. Equation (7.3) becomes

$$r_{B} = r_{\text{obs}} \times \frac{\epsilon_{\text{gen}} \cdot \epsilon_{\text{K}}^{K}}{\epsilon_{\text{gen}} \cdot \epsilon_{\text{K}}^{\pi}}.$$  

(7.4)

The efficiencies for the $B^{\pm} \rightarrow J/\psi K^{\pm}$ candidates are calculated using the magnet up and magnet down simulated samples independently and then combined using a weighted average with the sample integrated luminosity, $\int L \, dt$, as the weight. The weighted efficiency, $\epsilon$, is therefore given by

$$\epsilon = \frac{\epsilon_{\text{up}} \cdot \int L \, dt_{\text{up}} + \epsilon_{\text{down}} \cdot \int L \, dt_{\text{down}}}{\int L \, dt_{\text{up}} + \int L \, dt_{\text{down}}},$$

where $\epsilon_{\text{up/down}}$ are the efficiencies calculated on the magnet up or down samples. The weighted generator efficiencies for the $B^{\pm} \rightarrow J/\psi K^{\pm}$ and $B^{\pm} \rightarrow J/\psi \pi^{\pm}$ simulated events are given in Table 7.4(a) and the weighted selection efficiencies in Table 7.4(b).

The simulated samples from which the efficiency corrections are calculated for the $B^{\pm} \rightarrow \phi K^{\pm}$ and $B^{\pm} \rightarrow \phi \pi^{\pm}$ are only simulated for the magnet down polarity. The generator efficiency, $\epsilon_{\text{gen}}$, and selection efficiency, $\epsilon_{\text{sel}}$, are taken from Tables 4.1 and 4.10.
respectively. These efficiencies are used directly in equation (7.4) to calculate the correction ratio.

### 7.4 Systematic Uncertainties

By measuring the ratio of branching fractions many of the systematic effects cancel due to the similarities in the topologies of the decays. However, there are systematic uncertainties in the measurements that need to be accounted. They fall into two categories: the finite statistics of the simulated samples used to calculate the correction factor and the parametrisation used in the likelihood fit. The systematic uncertainty from the finite size of a simulated sample is taken from the statistical error in the efficiency used to correct the ratio of yields. The systematic uncertainties from the fit parametrisation are calculated as follows:

- the combinatorial background Chebychev function is substituted for an exponential function removing the assumption that the background is linear over the narrow mass window;
- the signal double Crystal Ball function is replaced with a Gaussian function to remove the dependence on the value of the tail parameters fixed from the simulated samples;
- the likelihood fit is performed without the missing track function;
- a two-dimensional likelihood fit, dependent only on the invariant mass and the \( \Delta LL_{K-\pi} \), with the signal Crystal Ball replaced with a double Crystal Ball. The double Crystal Ball function has a floating mean whose high-mass tails are fixed to values extracted from the simulated events and the low-mass tails are fixed to the same values as for the single Crystal Ball; and
- apply the no RICH aerogel track selection criteria to the data sets and perform the fit with the weighted calibration distribution with this selection criteria applied.

The two dominant sources of error are from the use of the simulated shapes for the \( \alpha \) distributions and the limitations of the \( \Delta LL_{K-\pi} \) weighting.

The simulation systematic uncertainties arise from the statistical uncertainty in the generator ratio and the selection ratio. The systematic uncertainties in the likelihood fit parametrisation are calculated by performing the likelihood fit with one of the
specified modifications and then subtracting the extract value of the ratio of yields, \( r_{\text{sys}} \), from the original ratio of yields, \( r_{\text{obs}} \). The difference is divided by the original ratio of yields to calculate the fractional uncertainty, \((r_{\text{sys}} - r_{\text{obs}})/r_{\text{obs}}\). The individual systematic uncertainties are then combined in quadrature to calculate the total fractional systematic uncertainty on the measurement. When calculating each of the systematic uncertainties for the ratio of branching fractions, \( \mathcal{B}(B^{\pm} \to J/\psi \pi^{\pm})/\mathcal{B}(B^{\pm} \to J/\psi K^{\pm}) \), the values of the parameters of the Argus function are fixed to those values presented in Table 7.2 since the likelihood fit becomes unstable with some of the alternate parameterisations. The individual fractional systematic uncertainties, the total fractional systematic uncertainty and the total systematic uncertainty for the \( B^{\pm} \to J/\psi h^{\pm} \) analysis are presented in Table 7.5. The confidence limit calculated for the ratio of branching fractions \( \mathcal{B}(B^{\pm} \to \phi \pi^{\pm})/\mathcal{B}(B^{\pm} \to \phi K^{\pm}) \) is based on statistics only.

<table>
<thead>
<tr>
<th>Source of uncertainty</th>
<th>Fractional uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kaon signal as Gaussian</td>
<td>-0.07</td>
</tr>
<tr>
<td>Combinatorial background as exponential</td>
<td>-0.006</td>
</tr>
<tr>
<td>No missing track component</td>
<td>0.01</td>
</tr>
<tr>
<td>No RICH aerogel tracks</td>
<td>0.4</td>
</tr>
<tr>
<td>No alpha observable and pion signal as double Crystal Ball</td>
<td>0.4</td>
</tr>
<tr>
<td>Ratio of the generator efficiencies</td>
<td>0.008</td>
</tr>
<tr>
<td>Ratio of the selection efficiencies</td>
<td>0.01</td>
</tr>
<tr>
<td>Fractional uncertainty</td>
<td>0.6</td>
</tr>
<tr>
<td><strong>Absolute systematic uncertainty</strong></td>
<td><strong>0.03</strong></td>
</tr>
</tbody>
</table>

*Table 7.5:* The systematic uncertainties for \( r_{B}^{J/\psi} \).
7.5 Results

The ratio of branching fractions, \( \mathcal{B}(B^\pm \to J/\psi\pi^\pm)/\mathcal{B}(B^\pm \to J/\psi K^\pm) \), is calculated using equation (7.4) and the values from Tables 7.2 and 7.4:

\[
\frac{r_{\text{obs}}}{r_{\text{gen}}} = \frac{0.047 \pm 0.005(\text{stat.}) \pm 0.03(\text{syst.})}{1.050 \pm 0.008} = 1.04 \pm 0.01
\]

where the first uncertainty is statistical (stat.) and the second systematic (syst.) in \( r_{\text{obs}} \), and the uncertainty in \( r_{\text{gen and}} r_{\text{sel}} \) is statistical only. The ratio of branching fractions, \( r_{B^\pm} \), is found to be

\[
r_{B^\pm} = \frac{\mathcal{B}(B^\pm \to J/\psi\pi^\pm)}{\mathcal{B}(B^\pm \to J/\psi K^\pm)} = 0.051 \pm 0.005(\text{stat.}) \pm 0.03(\text{syst.}).
\]

The limit on the ratio of branching fractions, \( \mathcal{B}(B^\pm \to \phi\pi^\pm)/\mathcal{B}(B^\pm \to \phi K^\pm) \), is calculated using equation (7.4), and the values from Tables 4.1, 4.10 and 7.3:

\[
r_{\text{obs}} < 0.116 \text{ at the } 90 \% \text{ confidence level}
\]

\[
\epsilon_{\phi}^{\text{gen}} = 18.19 \pm 0.15
\]

\[
\epsilon_{\phi}^{\text{sel}} = 0.457 \pm 0.005
\]

\[
\epsilon_{\pi}^{\text{gen}} = 18.58 \pm 0.15
\]

\[
\epsilon_{\pi}^{\text{sel}} = 0.424 \pm 0.004
\]

where the uncertainties are statistical only. The ratio of branching fractions, \( r_{B^\pm} \), is found to be

\[
r_{B^\pm} = \frac{\mathcal{B}(B^\pm \to \phi\pi^\pm)}{\mathcal{B}(B^\pm \to \phi K^\pm)} < 0.13 \text{ at the } 90 \% \text{ confidence level.}
\]

7.6 Summary

The ratio of the branching fractions, \( \mathcal{B}(B^\pm \to J/\psi\pi^\pm)/\mathcal{B}(B^\pm \to J/\psi K^\pm) \), has been measured from the analysis of a \((36.44 \pm 3.64) \text{ pb}^{-1} \) data set collected by LHCb during 2010 at a centre-of-mass energy of \( \sqrt{s} = 7 \text{ TeV} \). The total number of observed candidates is
202 ± 22 \( B^\pm \to J/\psi \pi^\pm \) and 4,290 ± 71 \( B^\pm \to J/\psi K^\pm \). The extracted ratio of branching fractions, \( r_{J/\psi}^{obs} \), is

\[
r_{J/\psi}^{obs} = \frac{B(B^\pm \to J/\psi \pi^\pm)}{B(B^\pm \to J/\psi K^\pm)} = 0.051 \pm 0.005 \text{(stat.)} \pm 0.03 \text{(syst.)}.
\]

This ratio has been measured by other experiments \cite{62,63,191}, most recently by the CDF collaboration \cite{185}. In addition the LHCb collaboration has also published a first measurement of the ratio using a one-dimensional extended maximum likelihood fit to the \( B^\pm \) invariant mass distribution. The weighted \( \Delta LL_{K^-\pi} \) distributions were used to divide the data sets into a kaon and pion samples from which the \( B^\pm \to J/\psi \pi^\pm \) and \( B^\pm \to J/\psi K^\pm \) yields were extracted. The result is \( r_{J/\psi}^{obs} = 0.0394 \pm 0.0039 \text{(stat.)} \pm 0.0017 \text{(syst.)} \) measured from 175 ± 17 \( B^\pm \to J/\psi \pi^\pm \) and 5,026 ± 73 \( B^\pm \to J/\psi K^\pm \) candidates \cite{192}. This result and the result presented in this thesis are within 2\( \sigma \) of each other. Both results are compatible with the current world average, \( 0.049 \pm 0.004 \) \cite{4}.

An upper limit on the ratio of branching fractions, \( \mathcal{B}(B^\pm \to \phi \pi^\pm)/\mathcal{B}(B^\pm \to \phi K^\pm) \), has been set from the analysis of a \((36.44 \pm 3.64) \text{ pb}^{-1}\) data set collected by LHCb using 146\( ^{+15}_{-14} \) signal candidates as

\[
r_{\phi}^{obs} = \frac{\mathcal{B}(B^\pm \to \phi \pi^\pm)}{\mathcal{B}(B^\pm \to \phi K^\pm)} < 0.13 \text{ at the 90 \% confidence level.}
\]

The current limit on the branching ratio of \( B^\pm \to \phi \pi^\pm \) has been set by BABAR to be \(< 2.4 \times 10^{-7} \) at the 90 \% confidence level \cite{189}, where the upper limit is from a Bayesian credible interval \cite{4}. Using the world average for the branching ratio of \( B^\pm \to \phi K^\pm \), \((8.3 \pm 0.7) \times 10^{-6} \), an upper limit on \( r_{\phi}^{obs} \) is \(< 0.0029 \) at the 90 \% confidence level \cite{4}.

With the 1 \( \text{fb}^{-1} \) data set, expected from the 2011 data taking period, and improvements in the simulation and \( \Delta LL_{K^-\pi} \) reweighting procedure it is anticipated that LHCb could produce a competitive measurement of \( \mathcal{B}(B^\pm \to \phi \pi^\pm)/\mathcal{B}(B^\pm \to \phi K^\pm) \).
Chapter 8

Conclusions

This thesis presented the work performed while I was a member of the LHCb collaboration and is comprised of two major components. The first is the software that was written to control and monitor the off-detector readout electronics for the RICH sub-detector, and the algorithms that form part of the Panoptes monitoring package. The second is the analysis of four charged B-meson decays using the 37 pb⁻¹ data set and simulated events.

The LHCb experiment is comprised of millions of elements that are configured and monitored using an experiment control system implemented in an industrial SCADA system, PVSS. One of those elements is the UKL1 board which is the off-detector readout board for the RICH sub-detector. The UKL1 boards are responsible for collating the events from each HPD, performing event formatting, zero suppression, error detection and transmitting the formatted events to the event filter farm. The software presented in this thesis was written to provide the ability to configure and monitor either a single or a group of UKL1 boards. The UKL1 software utilised many tools that allowed the project to communicate with the rest of the experiment control system and the UKL1 hardware. The software was installed in time for the commissioning of the RICH sub-detector and has been in use ever since.

In addition to the monitoring of the detector, and its readout and support infrastructure, through the experiment control system the performance of the detector is also monitored through the real-time analysis of a subset of the events saved to disk. The RICH sub-detector uses the Panoptes software package to monitor the sub-detector physics performance and the state of the HPDs. Two monitoring algorithms were presented, one which monitors the number of hits per event for each HPD to detect sudden changes in occupancy, and a second which is used as part of the calibration to compare
the recorded hit pattern for each HPD to the requested preset pattern. Both of these algorithms were utilised during the sub-detector commissioning and are still in use as part of the standard monitoring configuration.

The measurement of the $CP$-asymmetries in $B^\pm \to J/\psi K^\pm$, $B^\pm \to J/\psi \pi^\pm$, $B^\pm \to \phi K^\pm$ and $B^\pm \to \phi \pi^\pm$ decays (where $J/\psi \to \mu^+\mu^-$ and $\phi \to K^+K^-$) provides a test of the Standard Model predictions in these channels. The first stage of the analysis was to develop an event selection for the $B^\pm \to J/\psi h^\pm$ and $B^\pm \to \phi h^\pm$ decays, where the $h$ could be either a kaon or pion. The event selection was developed using two optimisation programs with simulated signal and background events as an input. The optimum event selection was chosen based on the values of $S/\sqrt{S+B}$ (where $S$ is the number of signal candidates and $B$ the number of background candidates) and the efficiency of selecting signal candidates. Using the chosen event selection the expected yields were calculated for each signal decay using selection efficiencies determined using the simulated signal events. The two event selections were then applied to the $37 \text{ pb}^{-1}$ data sets collected by the LHCb experiment during the 2010 data taking period at a centre-of-mass energy of 7 TeV. Using the RICH particle identification as a selection criteria the two data sets were divided into kaon and pion samples, and an estimate of the yields of each of the signal channels in the data was made. This was found to be larger than that predicted using the simulated events. As the simulation has not been tuned to the recorded data and differences in the distributions between the data and simulated events make an accurate comparison difficult.

Using the simulated events a method to extract the ratio of the yields, $r_{J/\psi}^B$ and $r_{\phi}^B$, and the measured asymmetry between the $B^+$ and $B^-$ signal decay modes and was developed. This method used a three-dimensional extended likelihood fit to extract the ratio of yields and the measured asymmetries, where the three dimensions are the $B^\pm$ invariant mass, the bachelor hadron $\Delta L_{K-\pi}$ and a kinematic variable $\alpha$. The bachelor hadron $\Delta L_{K-\pi}$ distribution utilises the powerful kaon/pion separation provided by the RICH sub-detector and a reweighting method to determine the kaon and pion $\Delta L_{K-\pi}$ distribution for the $B^\pm \to J/\psi h^\pm$ ($B^\pm \to \phi h^\pm$) events was developed and validated. The reweighting was not able to correctly reconstruct the $\Delta L_{K-\pi}$ distribution when testing on simulated events if all the bachelor hadron candidates were considered. By selecting only those bachelor hadron candidates that had no information provided by the RICH aerogel radiator, the reweighting was shown to work correctly. The method was then shown to correctly extract the ratio of yields, $N(B^\pm \to J/\psi \pi^\pm)/N(B^\pm \to J/\psi K^\pm)$ and $N(B^\pm \to \phi \pi^\pm)/N(B^\pm \to \phi K^\pm)$, and the measured charge asymmetry in each component.
of the extended maximum likelihood fit. In order to determine the CP-asymmetry in the signal channel the production and detector asymmetries must be determined. The presented method used $B^\pm \to J/\psi K^\pm$ and $B^\pm \to J/\psi \pi^\pm$ as control channels to extract the CP-asymmetry from the $B^\pm \to \phi K^\pm$ and $B^\pm \to \phi \pi^\pm$ channels respectively. The expected yields were used to show that with 1 fb$^{-1}$ of data\cite{LHCb} can match the statistical precision of the current world average of the $B^\pm \to \phi K^\pm$ measurement, however it requires the 10 fb$^{-1}$ data set to measure the $B^\pm \to \phi \pi^\pm$ CP-asymmetry, assuming the Standard Model branching ratio predicted in reference \cite{85}.

Finally, from the 37 pb$^{-1}$ data set a measurement of the ratio of branching fractions $\mathcal{B}(B^\pm \to J/\psi \pi^\pm)/\mathcal{B}(B^\pm \to J/\psi K^\pm)$ was made and a 90% confidence limit on the ratio of branching $\mathcal{B}(B^\pm \to \phi \pi^\pm)/\mathcal{B}(B^\pm \to \phi K^\pm)$ was set. This was done using the three-dimensional extended maximum likelihood fit where the $\Delta LL_{K\pi}$ kaon and pion distributions were calibrated using data only. The extended maximum likelihood fits returned the ratio of the signal yields; this was corrected using the ratio of the kaon and pion selection efficiencies determined from the simulated events. The measured ratio of branching fractions $\mathcal{B}(B^\pm \to J/\psi \pi^\pm)/\mathcal{B}(B^\pm \to J/\psi K^\pm)$ was determined to be

$$\frac{\mathcal{B}(B^\pm \to J/\psi \pi^\pm)}{\mathcal{B}(B^\pm \to J/\psi K^\pm)} = 0.051 \pm 0.005\text{(stat.)} \pm 0.03\text{(syst.)}$$

which is consistent the \cite{LHCb} measurement, $0.0394 \pm 0.0039\text{(stat.)} \pm 0.0017\text{(syst.)}$ \cite{192}, to within 2$\sigma$ and within 1$\sigma$ of the world average, $0.049 \pm 0.004$ (where the error is statistical only) \cite{4}. The limit on the ratio of branching fractions, $\mathcal{B}(B^\pm \to \phi \pi^\pm)/\mathcal{B}(B^\pm \to \phi K^\pm)$, was found to be

$$\frac{\mathcal{B}(B^\pm \to \phi \pi^\pm)}{\mathcal{B}(B^\pm \to \phi K^\pm)} < 0.13 \text{ at the 90\% confidence level.}$$

The current world average is $< 0.0029$ at the 90\% confidence level \cite{4} which is significantly lower than the value presented here.
Colophon

This thesis was made in βTεΧ 2\textregistered using the \texttt{hepthesis} class [193].
Acronyms

**ADS**  Atwood, Dunietz and Soni
**ALICE**  A Large Ion Collider Experiment
**AMCC**  Automated Method for Choosing Cuts
**ATLAS**  A Toroidal Large ApparatuS
**CAMERA**  Commissioning And Monitoring Error Reporting Application
**CCPC**  Credit Card PC
**CERN**  European Organization for Nuclear Research
**CKM**  Cabibbo-Kobayashi-Maskawa
**CMS**  Compact Muon Solenoid
**ConfDB**  Configuration DataBase
**CROP**  Cut Recursive OPtimiser
**CU**  Control Unit
**DAQ**  Data Acquisition
**DIM**  Distributed Information Management
**DIRAC**  Distributed Infrastructure with Remote Agent Control
**DU**  Device Unit
**ECAL**  Electromagnetic Calorimeter
**ECS**  Experiment Control System
**E_{T}**  transverse energy
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<th>Acronym</th>
<th>Description</th>
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<td>pseudo-rapidity</td>
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<tr>
<td>FPGA</td>
<td>Field Programmable Gate Array</td>
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<tr>
<td>FSM</td>
<td>Finite State Machine</td>
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<td>Ganga</td>
<td>Gaudi/Athena and Grid Alliance</td>
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<tr>
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<td>Graphical User Interface</td>
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<td>OT</td>
<td>Outer Tracker</td>
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<td>Definition</td>
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<tr>
<td>QED</td>
<td>Quantum Electrodynamics</td>
</tr>
<tr>
<td>RICH</td>
<td>Ring- Imaging Cherenkov</td>
</tr>
<tr>
<td>SCADA</td>
<td>Supervisory Control and Data Acquisition</td>
</tr>
<tr>
<td>SCET</td>
<td>soft collinear effective theory</td>
</tr>
<tr>
<td>SMI++</td>
<td>State Management Interface</td>
</tr>
<tr>
<td>SPD</td>
<td>Scintillator Pad Detector</td>
</tr>
<tr>
<td>SPS</td>
<td>Super Proton Synchrotron</td>
</tr>
<tr>
<td>ST</td>
<td>Silicon Tracker</td>
</tr>
<tr>
<td>TFC</td>
<td>Timing and Fast Control</td>
</tr>
<tr>
<td>TIS</td>
<td>Triggered Independent of Signal</td>
</tr>
<tr>
<td>TOS</td>
<td>Triggered On Signal</td>
</tr>
<tr>
<td>TOTEM</td>
<td>TOTal cross-section, Elastic scattering and diffraction dissociation Measurement at the LHC</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>triple-GEM</td>
<td>triple-Gas Electron Multiplier</td>
</tr>
<tr>
<td>TT</td>
<td>Tracker Turicensis</td>
</tr>
<tr>
<td>TTC</td>
<td>Timing, Trigger, and Control</td>
</tr>
<tr>
<td>TTCrx</td>
<td>Timing Trigger and Control receiver</td>
</tr>
<tr>
<td>VELO</td>
<td>Vertex Locator</td>
</tr>
<tr>
<td>WLCG</td>
<td>World LHC Computing Grid</td>
</tr>
</tbody>
</table>
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List of Figures


1.2 $\bar{B}^0 - B^0$ mixing box diagrams. ......................................................... 6

1.4 The unitarity triangle fit using indirect measurements only. ......................... 20

1.5 The unitarity triangle fit using direct measurements only. .......................... 20

1.6 Comparison of the $B^0_s$ mixing angle results. ........................................ 24

1.7 The unitarity triangle fit using both direct and indirect measurements. .......... 25

1.8 Standard Model diagrams for $B^\pm \to J/\psi K^\pm/\pi^\pm$. ............................. 27

1.9 Standard Model diagrams for $B^\pm \to \phi K^\pm/\pi^\pm$. ............................. 29

2.1 Schematic of the CERN accelerator complex. ............................................. 35

2.2 Leading order Feynman diagrams for hard QCD scattering $b\bar{b}$ pair production. ................................................................. 36

2.3 Predicted $b\bar{b}$ production polar angle at the LHC simulated by the Pythia event generator. ................................................................. 37

2.4 Aerial view of IP8. Image courtesy of Margot Frenot, 2000. ......................... 38

2.5 Number of proton-proton interactions vs. luminosity. ............................... 39

2.6 Layout of the LHCb detector. ................................................................. 41

2.8 LHCb magnet. ................................................................. 43

2.9 LHCb magnetic field variation in $z$, both polarities. .................................. 44
2.10 LHCb track types .............................................. 45
2.11 Tracking efficiency as a function of transverse momentum .................. 46
2.12 VELO module layout ............................................ 47
2.13 The VELO Photo taken by Maximilien Brice, 2006 .............................. 48
2.14 Impact parameter resolution Vs inverse transverse momentum ............... 49
2.15 Layout of the third [TT] layer .................................. 50
2.16 Photo of the TT during installation ................................... 51
2.17 Layout of the [TT] .................................................. 52
2.18 Photographs of the [TT] ........................................... 52
2.19 Schematic of the tracking stations ..................................... 53
2.20 Schematic of an [OT] layer .......................................... 54
2.21 Photograph of T1 station ........................................... 55
2.22 ECAL and HCAL layout ........................................... 56
2.23 Photographs of the [ECAL] and [HCAL] during installation. Taken by Maximilien Brice, 2004 .................................................. 56
2.24 Invariant mass distributions reconstructed with calorimeter information .... 58
2.25 The muon stations and their iron/calorimeter filters ............................. 60
2.26 Photographs of the muon stations ..................................... 61
2.27 Muon performance with early data ...................................... 62
2.28 Architecture of the readout network ................................... 63
2.29 An overview of the [TFC] system .................................... 65
2.30 Multiple partitions in the [TFC] system .................................. 66
2.31 Scope of the [ECS] system ........................................... 67
2.32 L0 trigger efficiency measured on the data taken during 2010 ................. 69
2.33 L0xHLT-1 trigger efficiency measured on the data taken during 2010 .......... 70
LIST OF FIGURES 225

2.34 The integrated luminosity delivered by the LHC and collected by LHCb during the 2010 run. ................................. 73

2.35 The peak $\mu$ per fill during the 2010 data taking period. ......... 74

2.36 The breakdown of the efficiency per LHC fill. ....................... 75


3.2 A plot of polar angle vs momentum for the RICH detectors. ........ 79

3.3 Schematics and photographs of RICH-1 and RICH-2 .................. 80

3.4 The RICH HPD detectors. ................................................ 82

3.5 Photograph of an L0 board. .............................................. 82

3.6 Cherenkov angle versus particle momenta using simulated events. .... 84

3.7 Cherenkov angle versus particle momentum from the 2010 data. ....... 85

3.8 The kaon efficiency versus momentum curves for two different selections criteria. ........................................ 85

3.9 The UKL1 board. ......................................................... 87

3.10 The top-level LHCb node PVSS panels. .............................. 90

3.11 The UKL1 FSM tree and CU panel. .................................... 92

3.12 The UKL1 DU interface. ................................................. 93

3.12 The UKL1 DU interface. ................................................. 94

3.13 The per UKL1 and per CU recipe interface. ......................... 96

3.14 Illustration of the UKL1 project. ........................................ 97

3.15 The Histogram Presenter. ................................................. 98

3.16 The CAMERA GUI .......................................................... 100

3.17 The Presenter page for the hit map monitor. ......................... 102

3.18 The TestPatternMonitor hit maps. ...................................... 103
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.19</td>
<td>The <strong>CAMERA</strong> message window for the <strong>TestPatternMonitor</strong> algorithm.</td>
</tr>
<tr>
<td>3.20</td>
<td>The <strong>TestPatternMonitor</strong> algorithm’s Presenter page.</td>
</tr>
<tr>
<td>4.1</td>
<td>Topology of a signal b-decay.</td>
</tr>
<tr>
<td>4.2</td>
<td>Schematic of the B⁺ variables provided to the optimisers.</td>
</tr>
<tr>
<td>4.3</td>
<td>Schematic of the J/ψ variables provided to the optimisers.</td>
</tr>
<tr>
<td>4.4</td>
<td>A comparison of the <strong>AMCC</strong> and <strong>CROP</strong> event selections on the B⁺ → J/ψK⁺/π⁺ simulated samples.</td>
</tr>
<tr>
<td>4.5</td>
<td>A comparison of the <strong>AMCC</strong> and <strong>CROP</strong> event selections on the B⁺ → φK⁺/π⁺ simulated samples.</td>
</tr>
<tr>
<td>4.6</td>
<td>The B⁺ invariant mass and bachelor hadron ΔLL_K₋π distribution after the B⁺ → J/ψh⁺ event selection.</td>
</tr>
<tr>
<td>4.7</td>
<td>The B⁺ invariant mass and bachelor hadron ΔLL_K₋π distribution after the B⁺ → φh⁺ event selection.</td>
</tr>
<tr>
<td>4.8</td>
<td>A subset of the B⁺ → J/ψh⁺ event selection variables.</td>
</tr>
<tr>
<td>4.9</td>
<td>A subset of the B⁺ → φh⁺ event selection variables.</td>
</tr>
<tr>
<td>5.1</td>
<td>The B⁺ → J/ψh⁺ invariant mass distributions from the 2010 data set.</td>
</tr>
<tr>
<td>5.2</td>
<td>The B⁺ → φh⁺ invariant mass distributions from the 2010 data set.</td>
</tr>
<tr>
<td>5.3</td>
<td>Extended maximum likelihood fits to determine the signal yields from the 2010 data set.</td>
</tr>
<tr>
<td>6.1</td>
<td>A comparison of the B⁺ → J/ψK⁺ and B⁺ → J/ψπ⁺ weighting variables.</td>
</tr>
<tr>
<td>6.2</td>
<td>A comparison of the B⁺ → φK⁺ and B⁺ → φπ⁺ weighting variables.</td>
</tr>
<tr>
<td>6.3</td>
<td>The results of the weighting of the f(p_T,η) distribution using the simulated B⁺ → J/ψK⁺ sample.</td>
</tr>
<tr>
<td>6.4</td>
<td>The results of the weighting of the B⁺ → J/ψK⁺ MC09 signal sample.</td>
</tr>
<tr>
<td>6.5</td>
<td>The results of the weighting of the B⁺ → J/ψπ⁺ MC09 signal sample.</td>
</tr>
</tbody>
</table>
6.6  The results of the weighting of the $B^\pm \to \phi K^\pm$ MC09 signal sample. . . . 163
6.7  The results of the weighting of the $B^\pm \to \phi \pi^\pm$ MC09 signal sample. . . . 164
6.8  A fit of the full likelihood function to the truth-matched $B^\pm \to J/\psi K^\pm$ MC09 signal sample. .................................................. 166
6.9  A fit of the full likelihood function to the truth-matched $B^\pm \to \phi K^\pm$ MC09 signal sample. .................................................. 167
6.10 A fit of the full likelihood function to the truth-matched $B^\pm \to J/\psi \pi^\pm$ MC09 signal sample. .................................................. 169
6.11 A fit of the full likelihood function to the truth-matched $B^\pm \to \phi \pi^\pm$ MC09 signal sample. .................................................. 170
6.12 A fit to extract the ratio of yields from the combined $B^\pm \to J/\psi K^\pm$ and $B^\pm \to J/\psi \pi^\pm$ MC09 signal samples. .................................................. 172
6.13 A fit to extract the ratio of yields from the combined $B^\pm \to \phi K^\pm$ and $B^\pm \to \phi \pi^\pm$ MC09 signal samples. .................................................. 173
6.14 A fit to extract the $B^\pm \to J/\psi K^\pm$ and $B^\pm \to J/\psi \pi^\pm$ measured asymmetry from the MC09 signal samples. .................................................. 175
6.15 A fit to extract the $B^\pm \to \phi K^\pm$ and $B^\pm \to \phi \pi^\pm$ measured asymmetry from the MC09 signal samples. .................................................. 176
7.1  A fit of the full likelihood function to the truth-matched $B^\pm \to J/\psi K^\pm$ MC10 signal sample. .................................................. 182
7.2  A fit of the full likelihood function to the truth-matched $B^\pm \to J/\psi \pi^\pm$ MC10 signal sample. .................................................. 183
7.3  The weighted calibration distributions for the 2010 data set. .................. 185
7.4  The weighted calibration distributions for the 2010 data set with no RICH aerogel tracks. .................................................. 187
7.5  A comparison of the magnet up collision data and simulated samples for the $\alpha$ observable. .................................................. 188
7.6  A fit of equation (7.1) to the $B^\pm \to J/\psi h^\pm$ data. .................. 189
7.7 A fit of equation (7.1) to the $B^\pm \rightarrow \phi h^\pm$ data. . . . . . . . . . . . . . . . . 191
List of Tables

1.1 The fermions of the Standard Model. ........................................ 3
1.2 Current experimental status of $B(B^+ \to J/\psi K^\pm)$ and $A_{\text{CP}}^{\text{dir}} (B^\pm \to J/\psi K^\pm)$. 28
1.3 Current experimental status of $B(B^\pm \to J/\psi \pi^\pm)$ and $A_{\text{CP}}^{\text{dir}} (B^\pm \to J/\psi \pi^\pm)$. 29
1.4 A comparison of the predictions and measured values of the branching ratio and direct $\mathcal{CP}$-asymmetries. ........................................ 30
2.1 The main L0 alley thresholds for the most common trigger settings in the 2010 data. ..................................................... 68
4.1 Generated simulated samples. .................................................. 108
4.2 The $B^\pm \to J/\psi h^\pm$ and $B^\pm \to \phi h^\pm$ preselection. .................... 110
4.3 $b$-hadron hadronisation fractions. ........................................ 114
4.4 The branching ratios of signal channels from $B^+$ decays. ............. 115
4.5 AMCC and CROP optimised event selections for $B^\pm \to J/\psi h^\pm$. ........ 117
4.6 AMCC and CROP optimised event selections for $B^\pm \to \phi h^\pm$. ........... 118
4.7 The AMCC and CROP optimised event selection metrics. .................. 121
4.8 The final event selections. ................................................... 125
4.9 The event selection performance metrics. .................................... 126
4.10 Total event selection efficiency. ............................................ 129
4.11 The expected yields for the signal channels. ................................ 130
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1</td>
<td>The integrated luminosity recorded for each stream by polarity.</td>
<td>134</td>
</tr>
<tr>
<td>5.2</td>
<td>A summary of the generated MC10 samples.</td>
<td>135</td>
</tr>
<tr>
<td>5.3</td>
<td>The trigger decision efficiencies on the selected candidates.</td>
<td>135</td>
</tr>
<tr>
<td>5.4</td>
<td>The (B_{u2JpsiKNoPIDDetached} ) efficiencies on the (B^\pm \rightarrow J/\psi h^\pm ) selected candidates.</td>
<td>137</td>
</tr>
<tr>
<td>5.5</td>
<td>A comparison between the (B_{2twobody} ) stripping selection and the (B^\pm \rightarrow \phi h^\pm ) event selection.</td>
<td>138</td>
</tr>
<tr>
<td>5.6</td>
<td>Performance of the (B^\pm \rightarrow J/\psi h^\pm ) event selection evaluated on the MC10 samples.</td>
<td>141</td>
</tr>
<tr>
<td>6.1</td>
<td>The expected statistical sensitivity of the polluting asymmetry.</td>
<td>150</td>
</tr>
<tr>
<td>6.2</td>
<td>The expected statistical sensitivity of the (CP)-asymmetry.</td>
<td>150</td>
</tr>
<tr>
<td>6.3</td>
<td>The reweighting variables used for the simulated sample reweighting.</td>
<td>158</td>
</tr>
<tr>
<td>6.4</td>
<td>The fitted values of the parameters for the different signal samples.</td>
<td>168</td>
</tr>
<tr>
<td>6.5</td>
<td>The fitted values of the parameters on the combined MC09 samples.</td>
<td>174</td>
</tr>
<tr>
<td>6.6</td>
<td>The fitted values of the parameters on the combined MC09 samples.</td>
<td>177</td>
</tr>
<tr>
<td>7.1</td>
<td>The fitted values of the parameters for the (B^\pm \rightarrow J/\psi K^\pm ) and (B^\pm \rightarrow J/\psi \pi^\pm ) truth-matched signal samples.</td>
<td>181</td>
</tr>
<tr>
<td>7.2</td>
<td>The fitted values of the parameters to the (B^\pm \rightarrow J/\psi h^\pm ) data.</td>
<td>190</td>
</tr>
<tr>
<td>7.3</td>
<td>The fitted values of the parameters to the (B^\pm \rightarrow \phi h^\pm ) data.</td>
<td>192</td>
</tr>
<tr>
<td>7.4</td>
<td>Weighted averages for the (B(B^\pm \rightarrow J/\psi \pi^\pm)/B(B^\pm \rightarrow J/\psi K^\pm) ) calculation.</td>
<td>193</td>
</tr>
<tr>
<td>7.5</td>
<td>The systematic uncertainties for (r_{B^\pm}^{J/\psi} ).</td>
<td>195</td>
</tr>
</tbody>
</table>