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MASSLESS NEUTRINOS IN LEFT-RIGHT SYMMETRIC MODELS

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ABSTRACT

We present an alternative to the Yanagida-Gell-Mann-Ramond-Slansky method of obtaining small neutrino masses in spite of normal Dirac mass terms in left-right symmetric theories. In our scheme, the light neutrinos remain massless while there exist massive Dirac neutral leptons in each generation. The masses of these are determined by the scale of left-right symmetry breaking which must be large enough to prevent flavour-changing processes such as $\mu \rightarrow e\gamma$.

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1. - INTRODUCTION

In gauge models of the electroweak interactions such as $SO(10)$ ¹⁾ and $SU(2)_L \times SU(2)_R \times U(1)$ ²⁾, which contain right-handed partners for the usual left-handed neutrinos, it is expected that neutrinos acquire a Dirac mass of a magnitude similar to that of quarks and charged leptons. In $SO(10)$, for example, the neutrino Dirac mass matrix is necessarily related to the up quark mass matrix. In the simplest version of $SO(10)$ this relation is just an equality. In order to make such left-right symmetric models acceptable, it is necessary to explain the fact that neutrino masses are orders of magnitude smaller than those of the corresponding quarks and leptons.

Yanagida³⁾ and Gell-Mann, Ramond and Slansky⁴⁾, (YGRS), proposed a solution to this problem by giving the right-handed neutrino a Majorana mass M . The typical mass of the left-handed neutrino is then of order m_{quark}^2/M . Since the right-handed neutrino is an $SU(2)_L$ singlet, there is nothing to prevent a very large value for M . M is related to the breaking scale $m(W_R)$ of $SU(2)_R$ such that $M \lesssim m(W_R)$. In the standard breaking pattern of $SO(10)$ to $SU(5)$, the value of M is of the order of the grand unification mass scale $\sim 10^{15}$ GeV, so that even the heaviest of the left-handed neutrinos may end up extremely light ($\ll 1$ eV)⁴⁾. It was Witten's observation⁵⁾ that the YGRS scheme also works in the minimal $SO(10)$ model and that M arises only through higher order effects so that $M \sim \alpha^2 m(W_R)$. This yields considerably larger, but still acceptable, neutrino masses.

In contrast, if one wishes to assume much smaller values for $m(W_R)$, so as to produce interesting phenomenology⁶⁾, the resulting neutrino masses will exceed the cosmological bounds⁷⁾ for stable neutrinos^{*)}.

In this paper we present an alternative to the YGRS scheme. In our method the light neutrinos are massless and independent of the scale $m(W_R)$. It is thus possible to assume for $m(W_R)$ a scale much smaller than the grand unification mass.

*) It is possible that if the neutrinos are heavy enough they may be sufficiently unstable to satisfy cosmological and astrophysical limits. The most serious problem exists for ν_τ . An attempt to solve this in an $SU(2)_L \times SU(2)_R \times U(1)$ model has been given by Senjanovic and Masiero⁸⁾, but this solution involves some very artificial assumptions⁹⁾.

2. - A TOY MODEL

To exhibit the idea we consider a simple toy model without reference at first to any gauge group. There are three left-handed neutral fermions ν_L, n_L and s_L (with right-handed anti-particles ν_R^c, n_R^c, s_R^c) and two scalars ϕ and χ . The Yukawa coupling is taken to be

$$\mathcal{L}_Y = a \bar{\nu}_L n_R^c \phi + b \bar{s}_L n_R^c \chi + h.c. \quad (1)$$

The assumption of only two non-zero couplings among the five fields requires three global $U(1)$ symmetries. We are interested here in only one of them

$$\begin{aligned} \nu_L &\longrightarrow e^{i\alpha} \nu_L \\ s_L &\longrightarrow e^{i\alpha} s_L \\ n_L &\longrightarrow e^{-i\alpha} n_L \\ \phi, \chi &\longrightarrow \phi, \chi \end{aligned} \quad (2)$$

which remains unbroken once ϕ and χ develop non-zero vacuum expectation values (v.e.v.), $\langle\phi\rangle$ and $\langle\chi\rangle$. The other independent $U(1)s$ ^{*)} are broken by $\langle\phi\rangle$ and $\langle\chi\rangle$. If the breaking is not explicit in the scalar potential, there may be associated Goldstone bosons which could be made to decouple from the fermions by the Dine-Fischler-Srednicki trick¹⁰⁾, if so desired.

The mass matrix may be written as

$$\begin{array}{c|ccc} & \nu_L & s_L & n_L \\ \hline \nu_R^c & 0 & 0 & a\langle\phi\rangle \\ s_R^c & 0 & 0 & b\langle\chi\rangle \\ n_R^c & a\langle\phi\rangle & b\langle\chi\rangle & 0 \end{array} \quad (3)$$

While this matrix is written in the Majorana form, one can interpret it by setting $n_R^c = \nu_R$ and recognizing ν_R as the right-handed partner of ν_L . $a\langle\phi\rangle$ is the standard Dirac mass term. The $U(1)$ symmetry (2) is then seen to be a lepton number symmetry with ν_L, s_L and n_R^c having lepton number 1. Diagonalizing the mass matrix we find a massless left-handed neutrino

^{*)} The other $U(1)s$ could be taken as
 $U(1)_2 : \nu_L \rightarrow e^{i\alpha} \nu_L, \phi \rightarrow e^{i\alpha} \phi$, others invariant
 $U(1)_3 : s_L \rightarrow e^{i\alpha} s_L, \chi \rightarrow e^{i\alpha} \chi$, others invariant.

$$\nu_1 = \cos \alpha |\nu_L\rangle - \sin \alpha |s_L\rangle \quad (4)$$

together with a Dirac neutrino, as expected from the conservation of lepton number. The Dirac neutrino is formed by n_R^c and

$$\nu_2 = \sin \alpha |\nu_L\rangle + \cos \alpha |s_L\rangle \quad (5)$$

where α in (4) and (5) is given by

$$\tan \alpha = a \langle \phi \rangle / b \langle \chi \rangle \quad (6)$$

and the Dirac mass is

$$(b^2 \langle \chi \rangle^2 + a^2 \langle \phi \rangle^2)^{1/2}.$$

In a left-right symmetric model for the weak interactions, we might identify ν_L with the electron neutrino, $\langle \phi \rangle$ with the breaking scale of $SU(2)_L \times U(1)$ and $\langle \chi \rangle$ with that of $SU(2)_R$. Typically $\langle \chi \rangle \gg \langle \phi \rangle$. $a \langle \phi \rangle$ will be of the order of the up quark mass m_u . If $a \approx b$, then $\tan \alpha \approx m_{WL}^c / m_{WR}$ and the Dirac mass is $\sim m_u m_{WR} / m_{WL}$. Thus, if m_{WL} / m_{WR} is not very small, then both ν_1 and ν_2 can be produced in, say, π decay. In order to exclude this unwanted ν_2 emission, b should be rather of the order of the gauge coupling constant. Then, ν_2 will be too heavy to be observed in the usual weak processes, and the only phenomenological consequence of this model is that the $e\nu$ coupling is reduced by a factor $\cos \alpha$ relative to the standard model. A quantitative analysis of this effect requires considering the case of several generations, to be discussed later. However, it is clear that as long as $\tan \alpha \lesssim 0.1$, the effect of the factor $\cos \alpha$ will be very hard to detect. Thus, if $a \langle \phi \rangle$ is of the order of the usual Dirac masses for heavy quarks (~ 10 GeV), $b \langle \chi \rangle$ need only be an order of magnitude larger; we have thus obtained a massless neutrino without introducing a superlarge mass scale.

3. - AN $SO(10)$ MODEL

To realize the above scheme within the framework of a gauge model, we use the group $SO(10)$. Our purpose is not to present a completely realistic model, but to illustrate how a mass matrix of the form of Eq. (3) might arise.

The fields which enter the Yukawa interaction are the following:

ψ , the usual $\underline{16}$ fermion field
 s , a singlet fermion field
 ϕ , the usual $\underline{10}$ Higgs field
 χ , a $\underline{\overline{16}}$ Higgs field

The essential new feature of this set is the fermion singlet s . This choice is inspired by supersymmetry¹¹⁾, where singlets under the gauge group are needed in realistic models^{12)*}. We take the Yukawa coupling to be

$$\mathcal{L}_Y = a\psi\psi\phi + b\psi s\chi \quad (7)$$

$\mathcal{L}_Y$ has two global $U(1)$ s, characterized by the "charges" Q_A and Q_B which we choose to be

$$\begin{aligned} Q_A(\psi) = 1 \quad Q_A(\phi) = -2 \quad Q_A(s) = -1 \\ Q_B(s) = 1 \quad Q_B(\chi) = -1 \end{aligned} \quad (8)$$

and zero for the other fields. In contrast to the toy model, no unbroken $U(1)$ remains after ϕ and χ obtain vacuum expectation values. The reason for this difference to the toy model is that in $SO(10)$ ν_L and $n_L = \bar{\nu}_R$ are in the same irreducible representation.

We can, however, find a way of combining the global $U(1)$ with the gauged $U(1)$ s of $SO(10)$ to form a new conserved global $U(1)$, described by a charge K ^{**}). To do this we consider the $SU(2)_L \times SU(2)_R \times SU(4)$ decomposition of the representations we use. We have¹³⁾

$$\phi = (2, 2, 1) \oplus (1, 1, 6) \quad \psi = (2, 1, 4) \oplus (1, 2, \bar{4}) \quad \bar{\chi} = (2, 1, 4) \oplus (1, 2, \bar{4}) \quad (9)$$

The non-zero v.e.v.s of the model are $\langle\phi\rangle$, the usual Higgs from the $(2, 2, 1)$ piece of ϕ and $\langle\chi\rangle$, which comes from the $(1, 2, 4)$ piece of χ and breaks the left-right symmetry. We can choose K such that it is zero for the Higgs fields

*) Since singlets are neutral, the mechanism described here works only for neutral particles, like the Majorana mass method of YGRS.

**) This is similar to the method of finding the B-L conservation in the minimal $SU(5)$ model.

$$K = Q_B + 2(I_{3L} - I_{3R}). \quad (10)$$

The quantum number K plays the same role as the conserved $U(1)$ of the toy model since it has the values 1 for both s and ν_L which belongs to the $(2,1,4)$ of ψ and -1 for $n_L (= \bar{\nu}_R)$ which belongs to the $(1,2,4)$ of ψ .

But even if the global $U(1)$ s are broken by the Higgs Lagrangian (potential) such that K is not a good quantum number, the scheme works if we replace one of the global $U(1)$ s by a discrete subsymmetry. In particular, we replace $e^{i\alpha Q_B}$ by $e^{i\pi/2 Q_B}$ *). It then follows that we can replace $e^{i\alpha K}$ by

$$D_K = e^{i\pi K/2} = e^{i\pi(Q_B + 2(I_{3L} - I_{3R}))/2}. \quad (11)$$

Under this symmetry, the relevant fields of the theory transform as follows:

$$\begin{aligned} \phi(2,2,1)_o &\longrightarrow \phi(2,2,1)_o \\ \chi(1,2,4)_o &\longrightarrow \chi(1,2,4)_o \\ \chi(2,1,4)_o &\longrightarrow -\chi(2,1,4)_o \\ \nu_L &\longrightarrow i\nu_L \\ n_L(\bar{\nu}_R) &\longrightarrow -i n_L(\bar{\nu}_R) \\ S_L &\longrightarrow i S_L \end{aligned} \quad (12)$$

where the subscript o denotes the component developing a v.e.v. As long as $\langle \chi(2,1,4) \rangle$ is zero, the discrete operator D_K is a good symmetry and ensures, once again, a mass matrix of the form of Eq. (3). In view of the conserved symmetry D_K , $\langle \chi(2,1,3) \rangle = 0$ is a "natural" relation, e.g., stable under renormalization. Although the symmetry D_K plays the role of lepton number in forbidding Majorana masses for the neutrinos, there are allowed lepton-number violating processes with $|\Delta_L| = 4 \cdot n$, n being an integer, since D_K

*) This discrete (Z_4) symmetry forbids only terms such as $\phi\chi^2$ in the scalar potential, which can be shown to prevent any conserved quantity. Furthermore, this symmetry corresponds, in the $SU(2)_L \times SU(2)_R \times U(1)$ model, to the symmetry used by Senjanovic¹⁴⁾ to obtain the solution to the minimization of the scalar potential corresponding to our choice below. It is just the coupling $\phi\chi^2$ which is used by Witten⁵⁾ to generate the large Majorana mass for ν_R .

corresponds to the Z_4 group. For instance, the reaction $\nu_L + \nu_L \rightarrow \bar{\nu}_R + \bar{\nu}_R$ might occur. One could entertain the notion that other symmetries, such as B-L, are conserved only modulo a certain number.

The use of discrete symmetries to generate massless neutrinos has been discussed by many authors. In particular, $SU(2)_L \times SU(2)_R \times U(1)$ models, in which discrete symmetries are used to get a zero neutrino mass, have been presented by Branco and Senjanovic¹⁵⁾ and by Ramond and Reiss¹⁶⁾. Both these models involve an extension of the quark as well as of the lepton sector and neither is embodied in a grand unified theory like $SO(10)$. It is only in the case of such a theory that a normal Dirac mass term cannot be avoided such that the problem addressed by YGRS arises.

4. - DISCUSSION

The phenomenology of the model presented above clearly depends on the value of $\langle \chi \rangle$ which in turn is related to the pattern of symmetry breaking that reduces $SO(10)$ to $SU(3) \times SU(2) \times U(1)$. In the simplest pattern, $SO(10)$ is broken to $SU(5)$ by $\langle \chi \rangle$, the piece of the $\overline{16}$ which is a singlet under $SU(5)$. Clearly, in this case $\langle \chi \rangle$ is greater than or roughly equal to the grand unification mass GUM. In an alternative pattern, however, $SO(10)$ can be broken to $SU(2)_L \times SU(2)_R \times U(1)$ by a $\underline{54}$ or $\underline{210}$ of scalars^{*)} with $\langle \chi \rangle$ possibly much smaller than GUM, where $\langle \chi \rangle$ determines the mass scale $m(W_R)$ at which $SU(2)_R$ is broken¹⁷⁾. It is even possible to consider that $SU(2)_R$ is broken to $U(1)_R$ by a $(1,3,15)$ of $SU(2)_L \times SU(2)_R \times SU(4)$, and that $\langle \chi \rangle$ only breaks $U(1)_R$ ¹⁸⁾. Still another alternative is to have $SO(10)$ broken to $SU(5) \times U(1)$ by a $\underline{45}$ of scalars and subsequent breaking of $SU(5)$ to $SU(3) \times SU(2) \times U(1)$ with unbroken $U(1)_X$. χ decomposes under $SU(5) \times U(1)_X$ as follows:

$$\chi = (5, -3) \oplus (\overline{10}, 1) \oplus (1, 5)$$

The component which acquires a v.e.v. is the $(1,5)$, so that $\langle \chi \rangle$ is associated with the $U(1)_X$ breaking. This can occur at a very low mass scale.

So far, we have considered only one generation of neutrinos ν_L . The multi-generation version of (1) and (2) is

*) Additional Higgs scalars which couple only to the scalars with Yukawa couplings (and not to the fermions themselves) must obey appropriate transformation conditions under the symmetries used.

$$\mathcal{L}_Y = \psi A \psi \phi + s B \psi \chi \quad (13)$$

where A, B are coupling matrices in the space of the ψ_i and s_i ($i = \text{up, charm, top, ...}$). It is clear that we need as many s_i as ψ_i for otherwise we would get only one massless neutrino. The mass matrix of the neutrals given by (3) for one generation becomes a 9×9 (three generations) matrix

$$M = \begin{pmatrix} 0 & 0 & A\langle\phi\rangle \\ 0 & 0 & B\langle\chi\rangle \\ A\langle\phi\rangle & B\langle\chi\rangle & 0 \end{pmatrix} \quad (14)$$

It follows from the general discussion in the Appendix that this yields three massless neutrinos ν_1, ν_2, ν_3 and three heavy Dirac neutrinos N_1, N_2, N_3 . The masses of the heavy neutrinos are given approximately by the eigenvalues of $B\langle\chi\rangle$ and thus scale with $m(W_R)$. The low-energy phenomenology depends on $A\langle\phi\rangle/B\langle\chi\rangle$. If A and B are similar in magnitude (that is their elements) the low-energy effects depend on the (small) parameter

$$\varepsilon = \frac{\langle\phi\rangle}{\langle\chi\rangle} \approx \frac{m(W_L)}{m(W_R)} \quad (15)$$

For the two generation case, which illustrates the phenomenology, we write explicitly

$$\begin{aligned} \nu_{eL} &= \cos\beta \nu_1 + \sin\beta (\cos\gamma N_1 + \sin\gamma N_2) \\ \nu_{\mu L} &= \cos\delta (\sin\alpha \nu_1 + \cos\alpha \nu_2) \\ &\quad + \sin\delta (\sin\gamma' N_1 - \cos\gamma' N_2) \end{aligned} \quad (16)$$

where $\nu_{eL}, \nu_{\mu L}$ are the weak, and ν_1, ν_2, N_1, N_2 are the mass eigenstates with $m(\nu_1) = m(\nu_2) = 0$. ν_1 and ν_2 may thus be chosen arbitrarily; the absence of ν_2 in ν_{eL} defines our choice.

From diagonalizing M we know that β, δ are of order ϵ , but γ, γ' depend on the relationship between A and B . By rotating the s_i and n_i , which are insensitive to the $SU(2)$ interactions, we can bring B into diagonal form. We can then distinguish two cases:

a) A can be diagonalized by just rotating the v_L . (This holds if $[AA^\dagger, BB^\dagger] = 0$.) In this case $\gamma = \gamma'$. If furthermore A is already diagonal, then $\gamma = \gamma' = 0$ and no generational mixing occurs.

b) A cannot be diagonalized by rotating only the v_L . Then $\gamma \neq \gamma'$.

From the orthogonality of v_{eL} and $v_{\mu L}$ we have

$$\sin \alpha = \tan \delta \tan \beta \sin (\gamma - \gamma')$$

$\alpha = 0$ for case a) above, $\alpha \neq 0$ for the general case b). A non-zero value of $\sin \alpha$ corresponds to the phenomenon that the zero mass neutrinos produced in beta decay are not orthogonal to those produced in muon decay. This phenomenon is well known in the neutrino mixing formalism. Thus a beam of v_μ from π decay would have a probability of behaving like v_{eL} given by

$$|\langle \tilde{v}_{eL} | \tilde{v}_{\mu L} \rangle|^2 = \cos^2 \beta \cos^2 \delta \sin^2 \alpha = \sin^2 \beta \sin^2 \delta \sin^2 (\gamma - \gamma')$$

where $\tilde{v}_{iL}$ is the zero mass piece of v_{iL} and we assume the N_i to be too heavy to be produced. This could be observed in neutrino oscillation experiments, although it is not a "true" oscillation since there is no space-time dependence. Oscillation experiments¹⁹⁾ give a weak limit of 10^{-2} on the above parameter, e.g., $\beta^2 \delta^2 < 10^{-2}$.

There are also modifications to the usual universality conditions. Comparing muon and beta decay, $\cos \theta_c$ is reduced from its usual value by the factor $\cos \delta$. The ratio $\Gamma(\pi \rightarrow e\nu)/\Gamma(\pi \rightarrow \mu\nu)$ is reduced from its usual value by $\cos^2 \beta / \cos^2 \delta$.

The most severe limits on ϵ come from flavour-violating processes like $\mu \rightarrow e\gamma$. Considering only the left-handed gauge couplings, one gets

$$B = \frac{\Gamma(\mu \rightarrow e\gamma)}{\Gamma(\mu \rightarrow e\nu\nu)} = \frac{3\alpha}{32\pi} \sin^2 \beta \sin^2 \delta \times \quad (17)$$

$$\times \left\{ \left[F\left(\frac{m_l^2}{m_W^2}\right) - F(0) \right] \cos \gamma \sin \gamma' - \left[F\left(\frac{m_2^2}{m_W^2}\right) - F(0) \right] \sin \gamma \cos \gamma' \right\}^2$$

where m_1, m_2 are the masses of N_1, N_2 and $m_W = m(W_L)$. The function $F(x)$ is given in many papers²⁰⁾. The GIM cancellation is illustrated by the factors $[F(x) - F(0)] \approx x$ for small x , but for large values of the m_i there is no GIM, since $F(\infty) - F(0) = 2$. To satisfy the present experimental limit of $B < 2 \cdot 10^{-10}$ ²¹⁾, we need

$$\beta^2 \delta^2 \lesssim 10^{-8}$$

$$\varepsilon \lesssim 10^{-2}$$

if γ and γ' are unconstrained. If $\gamma = \gamma'$, Eq. (17) becomes

$$\left\{ \frac{1}{2} \sin 2\gamma \left[F\left(\frac{m_1^2}{m_W^2}\right) - F\left(\frac{m_2^2}{m_W^2}\right) \right] \right\}^2$$

$$\approx 9 \sin^2 2\gamma \left(\frac{m_W^2}{m_1^2 m_2^2} \right)^2 \left(m_2^2 \ln \left(\frac{m_1^2}{m_W^2} \right) - m_1^2 \ln \left(\frac{m_2^2}{m_W^2} \right) \right)^2 \quad (18)$$

where the last equality assumes $m_1, m_2 \gg m_W$. Thus, there is an extra suppression factor of $(m_W^2/m_N^2)^2 \approx \varepsilon^4$, even if m_1 and m_2 are not degenerate. Then, the limit on ε might be raised to 10^{-1} , corresponding to a right-handed mass scale of 1 TeV. When this suppression of the left-handed couplings is effective, the right-handed couplings are comparable. These give

$$B = \frac{3\alpha}{32\pi} \left(\frac{m(W_L)}{m(W_R)} \right)^4 (U_{e1} U_{\mu 1})^2 \left[F\left(\frac{m_1^2}{m_{WR}^2}\right) - F\left(\frac{m_2^2}{m_{WR}^2}\right) \right]^2 \quad (19)$$

where $U_{e1}, U_{\mu 1}$ are the right-handed couplings of N_1 to e and μ , and we have neglected corrections of relative order δ or β . This is of the same order as Eq. (17), if $\gamma = \gamma'$ and $\beta^2 \delta^2$ (and $1/2 \sin 2\gamma$) are replaced by $(m(W_L)/m(W_R))^4$ (and $U_{e1} \cdot U_{\mu 1}$).

To summarize, we have given in this paper an alternative to the YGRS scheme of overcoming the normal Dirac mass terms for neutrinos which occur in grand unified models with left-right symmetry. It requires extension of the fermion sector by one neutral lepton for each of the generations. The resulting mass spectrum contains three massless left-handed neutrinos and three massive

Dirac neutrinos. The mass scale $m(W_R)$ of left-right symmetry breaking determines the mass of the Dirac particles. The most severe limit on it comes from $\mu \rightarrow e\gamma$, and under reasonable assumptions the minimal value of $m(W_R)$ is 10 to 100 times $m(W_L)$. Finally, we would like to point out that our construction can be directly implemented in a supersymmetric version of $SO(10)$ ²²⁾. In fact, in such models our scheme is a natural option, since any realistic supersymmetric model contains singlets, and because radiatively produced masses (needed in Witten's construction) are small²³⁾.

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APPENDIX

In this Appendix we prove the following theorem: if in a model with n (left-handed) neutrinos, m are of type A and m' are of type B ($m + m' = n$, $m \geq m'$ for definiteness) and the number $n(A) - n(\bar{A}) - [n(B) - n(\bar{B})]$ ($\bar{A}$, $\bar{B}$ are the antiparticles) is conserved, then:

- a) $m - m'$ neutrinos are massless
- b) there are m' Dirac neutrinos

Proof: The neutrino mass matrix for n neutrinos written in Majorana form is an $n \times n$ matrix linking the n left-handed neutrinos ν_{Li} to the corresponding n right-handed antineutrinos ν_{Rj}^c . Under the assumption of the theorem, the mass matrix has the form

$$\begin{pmatrix} O & R^T \\ R & O \end{pmatrix}$$

where R is an $m' \times m$ matrix. R contains m column vectors of length m' ; so only m' are independent, and by an orthogonal transformation among the columns $m' - m$ vectors can be transformed into null vectors. The resulting matrix has the form

$$\begin{pmatrix} O & O & O \\ O & O & S^T \\ O & S & O \end{pmatrix}$$

where S is an $m' \times m'$ matrix. Thus we see that the first $m' - m$ states are massless. The remaining $2m' \times 2m'$ matrix has the standard Dirac form. An orthogonal transformation of S yields

$$\begin{pmatrix} O & O & O \\ O & \bigcirc & m_1 \dots m_{m'} \\ O & m_1 \dots m_{m'} & \bigcirc \end{pmatrix}$$

where $m_1, \dots, m_{m'}$ are the masses of the Dirac neutrinos.

In a previous discussion of Majorana mass matrices²⁴⁾ it is stated that the existence of m' Dirac neutrinos corresponds to m' conservation laws. Our result indicates that one conservation law provides a sufficient condition for the existence of m' Dirac neutrinos. It is possible, a posteriori, to find m'

"conservation laws" by defining a different lepton number for each of the Dirac particles. These, however, may be called pseudo-conservation laws in the sense that weak radiative corrections might upset them. But as long as the original single conservation law is respected by the weak interactions, the theorem holds even with radiative corrections included. They simply change R and correspondingly the composition of the zero mass states and the different Dirac states.

REFERENCES

- 1) H. Georgi, in *Particles and Fields*, AIP Press, New York (1975);
H. Fritzsch and P. Minkowski, *Ann. Phys.* 93 (1975) 193;
H. Georgi and D.V. Nanopoulos, *Nucl. Phys.* B155 (1976) 52.
- 2) R.N. Mohapatra and J.C. Pati, *Phys. Rev. D* 11 (1975) 566, 2558;
R.N. Mohapatra, in *"New Frontiers in High Energy Physics"*, eds. A. Perlmutter
and L. Scott, Plenum Press (1978) p. 337.
- 3) T. Yanagida, in *Proc. of the Workshop on Unified Theory and Baryon Number in
the Universe*, eds. O. Sawada and A. Sugamoto, KEK (1979).
- 4) M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, eds. P. van Nieuwenhuizen
and D. Freedman, North Holland (1979) p. 317.
- 5) E. Witten, *Phys. Lett.* 91B (1980) 81.
- 6) R.N. Mohapatra and G. Senjanovic, *Phys. Rev. D* 23 (1981) 165;
T.G. Rizzo and G. Senjanovic, *Phys. Rev. D* 24 (1981) 704.
- 7) S.S. Gerstein and Ya. B. Zeldovich, *Zh. Eksp. Teor. Fiz. Pisma Red.* 4
(1966) 174;
R. Cowsick and J. McClelland, *Phys. Rev. Lett.* 29 (1972) 669.
- 8) M. Roncadelli and G. Senjanovic, *Phys. Lett.* 107B (1981) 59.
- 9) P.B. Pal, to be published.
- 10) M. Dine, W. Fischler and M. Srednicki, *Phys. Lett.* 104B (1981) 199.
- 11) P. Fayet and S. Ferrara, *Phys. Rep.* C32 (1977) 249;
J. Wess and J. Bagger, *Princeton Lectures* (1981);
E. Witten, *Erice Lectures* (1981).
- 12) E. Witten, *Nucl. Phys.* B185 (1981) 513; *Phys. Lett.* 105B (1981) 267;
M. Dine, W. Fischler and M. Srednicki, *Nucl. Phys.* B184 (1981) 575;
M. Dine and M. Srednicki, *Nucl. Phys.* B202 (1982) 238;
S. Dimopoulos, S. Raby and F. Wilczek, *Phys. Rev. D* 24 (1981) 1681;
D. Dimopoulos and S. Raby, *Nucl. Phys.* B192 (1981) 353;
S. Dimopoulos and H. Georgi, *Nucl. Phys.* B193 (1981) 150;
N. Sakai, *Zeit. f. Physik C* 11 (1982) 153;
H.P. Nilles and S. Raby, *Nucl. Phys.* B198 (1982) 102;
C. Nappi and B. Ovrut, *Phys. Lett.* 110B (1982) 215;
M. Dine and W. Fischler, *Phys. Lett.* 110B (1982) 227;
C. Nappi and B. Ovrut, *Phys. Lett.* 113B (1982) 175;
L. Alvarez-Gaumé, M. Claudson and M. Wise, Harvard preprint HUTP-81/A063;
L.E. Ibáñez and G.G. Ross, *Phys. Lett.* 105B (1981) 439; *Phys. Lett.* 110B
(1982) 215;
M.B. Einhorn and D. Jones, *Nucl. Phys.* B196 (1981) 475;
R. Barbieri, S. Ferrara and D.V. Nanopoulos, *Zeit. f. Physik C* 13 (1982)
276 and CERN preprint TH-3305 (1982);
J. Ellis, L. Ibáñez and G.G. Ross, *Phys. Lett.* 113B (1982) 283;
S. Dimopoulos and S. Raby, Los Alamos preprint 82 (1982);
S. Polchinsky and L. Susskind, Stanford University preprint (1982);
C.S. Aulakh and R.N. Mohapatra, CCNY preprint HEP-82-4 (1982).

- 13) See, for instance:
R. Slansky, Phys. Rep. 79C (1981) 1.
- 14) G. Senjanovic, Nucl. Phys. B153 (1979) 334.
- 15) G.C. Branco and G. Senjanovic, Phys. Rev. D18 (1978) 1621.
- 16) P. Ramond and D.B. Reiss, Phys. Lett. 80B (1978) 87.
- 17) For possible breaking patterns of $SO(10)$, see, for example:
M. Yasué, Phys. rev. D24 (1981) 1005;
R.N. Mohapatra and B. Sakita, Phys. Rev. D21 (1980) 1062;
H. Georgi and D.V. Nanopoulos, Nucl. Phys. B159 (1979) 16;
Q. Shafi, M. Sondermann and Ch. Wetterich, Phys. Lett. 92B (1980) 304.
- 18) E.W. Robinett and J.L. Rosner, Phys. Rev. D25 (1982) 3036.
- 19) N.J. Baker et al., Neutrino '81, Hawaii.
- 20) W.J. Marciano and A.I. Sanda, Phys. Lett. 67B (1977) 303;
S.T. Petrov, Yad. Fiz. 25 (1977) 641 [Sov. J. Nucl. Phys. 25 (1977) 340];
R.E. Shrock, Phys. Rev. D16 (1977) 1444;
T.P. Cheng and L.F. Li, Phys. Rev. Lett. 45 (1980) 1908;
E. Ma and A. Pramudita, Phys. Rev. D24 (1981) 1410.
- 21) J.D. Bowman et al., Phys. Rev. Lett. 42 (1979) 556.
- 22) K. Tamvakis and D. Wyler, unpublished.
- 23) L. Ibáñez, CERN preprint TH-3352 (1982);
E. Jurczek, Diplomarbeit ETH-Zürich (1982).
- 24) L. Wolfenstein, Nucl. Phys. B186 (1981) 147.