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DISTORTION EFFECTS IN QUARK-ELASTIC ELECTRON SCATTERING ON NUCLEI

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ABSTRACT

The Fermi gas model treatment of quasi-elastic electron scattering is reconsidered. The absorptive many body effects in the outgoing channel incorporated in the imaginary nucleon nucleus optical potential are explicitly introduced in the response function. The cross-section is slightly distorted towards higher energies in better agreement with experiment. The standard treatment, however, holds essentially true.

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Quasi-elastic electron scattering has been the object of considerable interest both from the experimental and from the theoretical point of view.

Recently, it has been shown¹⁾ that a unified picture of this quasi-free inclusive process can be given within the simple framework of the Fermi gas model both at low $q \approx k_F$ and at high $q \approx 2k_F$ ($k_F \equiv$ Fermi momentum) momentum transfer q .

The basic input of that analysis lies in the prescription of the nucleon single particle energy spectrum. The nucleon is assumed to move in an average potential, represented by the real part of the nucleon-nucleus optical potential, which adds to the free particle kinetic energy. Even in such a treatment the process proceeds through the energy-conserving creation of a particle-hole pair. The same thing holds true in microscopic attempts to calculate^{2),3)} the effective interaction by summing up ring diagrams. In these descriptions one therefore disregards the energy uncertainty due to the finite lifetime of the excited state.

It is the purpose of this paper to improve upon the previous Fermi gas model treatment of quasi-elastic electron scattering by proper consideration of distortion effects incorporated in the imaginary part of the optical potential.

To motivate its introduction, let us recall that in the process the virtual photon interacts with a nucleon inside the Fermi sea ($k < k_F$) and ejects it, because of the Pauli principle, above it ($k > k_F$). If one wants to introduce some average nuclear interaction U in this description, one is forced to admit that if $k < k_F$, U is the average nuclear potential, whereas if $k > k_F$ then U represents the interaction of an additional nucleon in nuclear matter, i.e., the optical potential seen by the added particle. This potential also has an imaginary part, which accounts for the coupling of the nucleon motion to the internal degrees of freedom, leading with increasing energy to complicated many-nucleon configurations. This causes a reduction of the probability of the single-particle states.

It has to be stressed, however, that the loss of flux from the one-nucleon channel is compensated for in the inclusive cross-section by the contribution of these many-particle channels so that the net effect will just amount to a distortion.

We will therefore describe the process of quasi-elastic electron scattering by using a Fermi gas model supplemented by the optical potential for the nucleon, both below the Fermi sea (only real part) and above (real and imaginary parts).

A similar programme has also been carried out in Ref. 4). That calculation is, on the one hand, more refined since it treats finite nuclei starting from the DWKA approximation and, on the other hand, in what concerns the energy domain and the exclusive quasi-elastic cross-section we are interested in, somewhat redundant. We prefer to keep to the Fermi gas model which has the non-trivial advantage of a fair over-all agreement with experiment, within a reasonable calculational simplicity. To exemplify, we consider Coulomb scattering in which only the longitudinal response function enters. The differential cross-section then reads²⁾

$$\frac{1}{\sigma_M} \frac{d^2\sigma}{d\Omega' d\omega} = \frac{3\pi Z}{k_F^3} \operatorname{Im} \Pi(\omega, q) =$$

$$= \frac{1}{\pi} \operatorname{Im} \left\{ \sum_n \frac{\langle 0 | \rho^\dagger(-q) | n \rangle \langle n | \rho(-q) | 0 \rangle}{E_n - E_0 - \omega - i\varepsilon} + \frac{\langle 0 | \rho(q) | n \rangle \langle n | \rho(q) | 0 \rangle}{E_n - E_0 + \omega - i\varepsilon} \right\} \quad (1)$$

where σ_M stands for the Mott cross-section

$$\rho(q) \approx \sum_{i=1}^A \frac{1+\tau_i^3}{2} e^{-i q \cdot r_i}$$

$|0\rangle$ and $|n\rangle$ are the ground and excited nuclear states, respectively, and ω and q are the energy and momentum transferred to the nucleus in the one-photon exchange approximation. In the Fermi gas model

$$\operatorname{Im} \Pi(\omega, q) = \operatorname{Im} 2 \int \frac{d^3k}{(2\pi)^3} \Theta(|q+k| - k_F) \Theta(k_F - k) \cdot$$

$$\left(\frac{1}{E_{q+k} - E_k - \omega - i\varepsilon} + \frac{1}{E_{q+k} - E_k + \omega - i\varepsilon} \right) \quad (2)$$

We will assume, in terms of $U = V_{\text{opt}}(E) = V + iW$

$$E_k = \frac{k^2}{2M} + V_k, \quad E_{q+k} = \frac{(q+k)^2}{2M} + V_{q+k} + iW_{q+k}$$

where the parametrization, at the Fermi momentum $k_F = 225 \text{ keV/c}$ we are using, is roughly

$$V = V_0 + V_1 E = -40.2 + 0.32 E \quad E < 40 \text{ Mev}$$

$$= -\frac{1960}{30+E} \quad E \geq 40 \text{ Mev}$$

and

$$W = - \sqrt{\frac{(q+k)^2}{2M} - \frac{k_F^2}{2M}} \quad (3)$$

These forms are to be taken as phenomenological. They are numerically in essential agreement with nuclear matter calculations⁵⁾ and with scattering experiments analyses. The introduction of W in E_{q+k} only, corresponds to the previous statement that only above the Fermi sea can we have absorption away from the one-nucleon channel. If we let $W \rightarrow 0$ we immediately obtain

$$I_{\text{Im}} \Pi(\omega, q) = \int \frac{d^3k}{(2\pi)^2} \Theta(|q+k| - k_F) \Theta(k_F - k) \cdot \delta\left(\frac{(q+k)^2}{2M} + V_{q+k} - \frac{k^2}{2M} - V_k - \omega\right) \quad (4)$$

As mentioned above, from this expression one easily gets the two limiting cases. For small q 's we have

$$\delta\left(\frac{(q+k)^2}{2M} (1+V_1) - \frac{k^2}{2M} (1+V_1) - \omega\right)$$

i.e., the effective mass ($M^* = M/(1+V_1)$) approximation. For high q 's, where $V \approx 0$,

$$\delta\left(\frac{(q+k)^2}{2M} - \frac{k^2}{2M} - V_0 - \frac{k^2}{2M} V_1 - \omega\right)$$

i.e., a separation energy essentially given by the ground state expectation value of $V_0 + k^2/2M V_1$. The effect of the imaginary part of the optical potential on the Fermi gas model response can be seen in Figs. 1 and 2 where we plot $(4\pi q/Mk_F^2) \text{Im}\Pi(\omega, q)$ as a function of the energy loss ω for the fixed values of the momentum transfer $q = 250$ and 500 MeV/c. The continuous line corresponds to Eqs (2) and (3) (imaginary part) whereas the dashed line corresponds to Eq. (4) (no imaginary part). The differences between the two curves are apparent. In essential agreement with Ref. 4), the effect of the imaginary potential is to make the curve more asymmetric, pushing some of the strength towards higher energies, due to multinucleon removal. This seems to tend towards a better agreement with experiments, both as regards the shape around the peak and the filling in of the dip before the Δ resonance⁶⁾. The effects are, however, at most at a 10% level, essentially confirming the energy-conserving Fermi gas description based on the modified single-particle spectrum.

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REFERENCES

- 1) F.A. Brieva and A. Dellafiore, Nucl. Phys. A292 (1977) 445.
- 2) A.L. Fetter and J.D. Walecka, Quantum Theory of Many-Particle Systems, McGraw-Hill, New York (1971).
- 3) W.M. Alberico, M. Ericson and A. Molinari, Nucl. Phys. A379 (1982) 429.
- 4) Y. Horikawa, F. Lenz and N.C. Mukhopadhyay, Phys. Rev. C22 (1980) 1680.
- 5) F.A. Brieva and J.R. Rook, Nucl. Phys. A291 (1977) 299 and references therein.
- 6) T.W. Donnelly, J.W. van Orden, T. de Forest Jr. and W.C. Hermans, Phys. Lett. 76B (1978) 393.

FIGURE CAPTIONS

Figure 1 : Plot of the quantity $(4\pi q/Mk_F)\text{Im}\Pi(\omega, q)$ versus the energy loss ω , calculated in the Fermi gas model with (continuous line) and without (dashed line) the imaginary part of the nucleon optical potential, at constant momentum transfer $q = 250 \text{ MeV/c}$.

Figure 2 : Same as in Fig. 1, at $q = 500 \text{ MeV/c}$.

$q=250\text{MeV}/c$

- Figure 1 -

$q=500\text{MeV}/c$

- Figure 2 -