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SCALING IN QUASI-ELASTIC ELECTRON SCATTERING ON NUCLEI

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ABSTRACT

The properties of quasi elastic electron scattering with respect to the scaling variable $y = \omega/q - q/2M$ are examined. It is shown that different sets of scattering experiments can indeed be related through the scaling variable.

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Recently, the possibility that in parallel with high energy physics, quasi-free electron scattering on nuclei may not be a separate function of the energy loss ω and of the momentum transfer q , but of the scaling variable $y = \omega/q - q/2M$ (M = nucleon mass) only, has been reconsidered¹⁾⁻³⁾.

These authors, using sum rules, relate scaling violation to a deviation of the mean value $\bar{y}$ from zero or alternatively in a displacement of the quasi-elastic peak from the free value $q^2/2M$. In this approach, a sum is performed over all final states and the total strength of the transition is expressed as the expectation value of a corresponding operator over the ground state.

In so doing one encounters the well-known problem that the apparent simplicity of the result has to be somewhat paid for by the accurate knowledge one has to have of the ground state wave function correlations.

The aim of this work is also to examine if and when scaling with respect to y sets in. We use, however, a description in which final states are explicitly calculated and a direct comparison with experiments of the differential cross-section is therefore possible. The basic ingredient of our approach is a picture which, although phenomenological, reproduces experimental cross-sections fairly well, of the ground and excited nuclear states with the Fermi gas model supplemented by the nucleon-nucleus optical potential.

The double differential electron-nucleus cross-section reads in the one-photon exchange approximation⁴⁾

$$\frac{1}{\sigma_M} \frac{d^2\sigma}{d\Omega' d\omega} = \left[\left(\frac{q^2 - \omega^2}{q^2} \right)^2 R_L(\omega, q) + \left(\frac{q^2 - \omega^2}{2q^2} + \frac{q^2}{2} \right) R_T(\omega, q) \right] \quad (1)$$

where σ_M stands for the Mott cross-section and

$$R_L(\omega, q) = \sum_n |\langle n | \sum_{j=1}^A \left[e_j - \frac{q^2}{8M^2} (2\mu_j - e_j) \right] e^{i\vec{q} \cdot \vec{r}_j} | 0 \rangle|^2 \delta(E_n - E_0 - \omega) \quad (2a)$$

$$R_T(\omega, q) = \sum_n |\langle n | \sum_{j=1}^A \frac{\mu_j}{2M} \vec{q} \times \vec{\sigma}_j e^{i\vec{q} \cdot \vec{r}_j} | 0 \rangle|^2 \delta(E_n - E_0 - \omega) \quad (2b)$$

where the nucleon form factors, irrelevant to the following discussion, have been omitted. In these expressions e_j is zero for neutrons and one for protons, μ_j is, respectively, -1.91 and 2.79 and the (small at $q \geq k_F$, $k_F \equiv$ Fermi momentum) convective part of the current has been neglected in R_T .

If the nucleus can be described by the Fermi gas model as an assembly of non-interacting nucleons (in particular without spin dependent forces), as well known, we can express⁵⁾ both R_L and R_T as a function of

$$f = \begin{cases} 2\omega M & \text{for } 0 \leq \omega \leq \omega_1 \\ k_F^2 - M^2 \left(\frac{\omega}{q} - \frac{q}{2M} \right)^2 & \omega_1 \leq \omega \leq \omega_2 \end{cases} \quad (3a)$$

$$(3b)$$

where

$$\omega_1 = \frac{q k_F}{M} - \frac{q^2}{2M}$$

$$\omega_2 = \frac{q k_F}{M} + \frac{q^2}{2M}$$

The previous expressions hold true only for $q < 2k_F$; for $q \geq 2k_F$ Eq. (3a) vanishes and $\omega_1 \rightarrow \omega_1$ in Eq. (3b).

It is therefore obvious that for large q 's, f and, in turn, qR_L and $1/q R_T$, are functions only of y and show therefore a scaling behaviour. In other words, they have the same value even for different ω 's and q 's, provided energy and momentum combine to yield the same y .

This also explains why this variable is expected to work better than Bjorken's $q^2/2M\omega$. Under these hypotheses, the differential cross-section can be written as

$$\begin{aligned} \frac{1}{\sigma_M} \frac{d^2\sigma}{d\Omega d\omega} &= \frac{3\pi}{k_F^3} \left[Z \left(\frac{q^2 - \omega^2}{q^2} \right)^2 \left(1 - (2\mu_p - 1) \frac{q^2}{8M^2} \right)^2 + \right. \\ &\quad \left. + \left(\frac{q^2 - \omega^2}{2q^2} + t_0^2 \frac{\sigma}{2} \right) (Z\mu_p^2 + N\mu_n^2) \frac{q^2}{2M^2} \right] \frac{M}{4\pi q} f(y) \end{aligned} \quad (4)$$

The physical meaning of scaling can be easily understood by remembering that the δ function of Eq. (2), implies in the non-relativistic limit $\omega = (\underline{k} + \underline{q})^2/2M - k^2/2M = q^2/2M + \underline{q} \cdot \underline{k}/M = q^2/2M + qk_{||}/M$, where $k_{||}$ stands for the nucleon momentum along the momentum transfer q . Therefore $y = \omega/q - q/2M \equiv k_{||}/M$ represents the longitudinal velocity. Hence scaling which is the consequence of the absence of interactions in the non-relativistic limit, can be attained at rather

low energy, i.e., before one expects the infinite momentum frame description (in which constituents are really free and the longitudinal momentum distribution is therefore the same for different kinematical conditions) to work. Of course, the above-mentioned simple Fermi gas picture is known to be inadequate. It has been shown⁶⁾ that a reasonably good, consistent picture of quasi-free electron scattering can be given both at low and high momentum transfer by substituting to the free particle energy E , $\tilde{E} = E + \text{Re } V_{\text{opt}}$ where $\text{Re } V_{\text{opt}}$ is the energy-dependent real part of the nucleon-nucleus optical potential.

Subsequently, the opening up of many-body channels, accounted for by the imaginary part of the nucleon-nucleus optical potential, has been considered^{7),8)}. Its inclusion amounts to modifying in addition the energy conserving δ function of the response function

$$\frac{1}{\pi} \text{Im} \frac{1}{\tilde{E}_n - \tilde{E}_0 - \omega - i\varepsilon} \quad \text{into} \quad \frac{1}{\pi} \text{Im} \frac{1}{\tilde{E}_n - \tilde{E}_0 - \omega + i \text{Im } V_{\text{opt}}(E_n - E_F)}$$

where E_F is the Fermi energy. The free Fermi gas model function $f(y)$ in Eq. (4) is therefore accordingly modified in both cases. As a result of our calculations we plot in Figs. 1 and 2 $f(y)/k_F^2$, respectively, with and without the imaginary part of the optical potential. The curves have been calculated at $k_F = 225 \text{ MeV/c}$, with the corresponding optical potential parametrization used in Ref. 8) for q values of 250, 400, 500 and 600 MeV/c.

The Coulomb energy which can be taken into account by properly redefining a shifted y variable, has been neglected. Similar conclusions can be drawn from the two sets of curves, although those with $\text{Im } V_{\text{opt}}$, upon which we focus, seem to be preferable from the comparison with experiments.

For $q = 250 \text{ MeV/c}$ a strong deviation from a universal $f(y)$ for negative y 's is apparent, as expected, since in that region (small energy loss) the Pauli principle is effective. As regards the other q values, all the curves are indeed peaked around $\bar{y} \approx 0.05$ in agreement with the sum rule calculations.

However, they tend towards a scaling behaviour at a remarkable level: as a matter of fact the curves for $q = 500$ and 600 MeV/c are practically indistinguishable and even for 400 MeV/c deviations are indeed rather small. We can therefore conclude that especially in the region (small ω 's or in turn negative y 's) where mesonic degrees of freedom, not included in this single particle Fermi gas picture, are of little importance, scaling is effective in quasi-elastic electron scattering. It would therefore be very interesting to

compare different sets of data in terms of the scaling variable y . In particular, the separate consideration of R_L and R_T might help in sorting out whether the spin dependence of nuclear forces, which does not enter out optical potential approach, is really an essential feature in exclusive quasi-elastic (e,e') . This will be considered in a forthcoming paper.

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FIGURE CAPTIONS

Fig. 1 : Scaling function $f(y)/k_F^2$, calculated in the Fermi gas model with the imaginary part of the nucleon optical potential, at constant momentum transfer $q = 250$ (crosses), 400 (dot-dashed), 500 (dashed) and 600 (continuous) MeV/c.

Fig. 2 : Same as in Fig. 1, without the imaginary optical potential.

Fig.1

Fig.2