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SHORT DISTANCE STRUCTURE OF HADRONS IN SUPERSYMMETRIC QCD

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A B S T R A C T

Assuming supersymmetry to be a real symmetry in particle physics, we study the short distance internal structure of nucleons (structure functions at large Q^2). We determine all calculable quantities in the leading logarithm approximation and discuss the $N=1$ supersymmetry relations between them. Further relations are obtained by observing that the $N=1$ supersymmetric Lagrangian becomes $N=2$ supersymmetric if the quarks and squarks are (formally) put into the adjoint representation of $SU(3)$ colour.

From the calculated elements of the 4×4 matrix of the Q^2 evolution kernels and knowing the quark and gluon distributions at low Q^2 we are able to determine the quark, squark, gluon, and gluino distributions for Q^2 above the threshold for the production of supersymmetric particles. We find that in the asymptotic limit ($Q^2 \rightarrow \infty$) the quarks carry only $2/3$ of the nucleon's baryon number and the remaining $1/3$ is carried by the squarks. We also find that in this limit 0.32 of the longitudinal nucleon momentum is carried by gluons, 0.08 by gluinos, 0.36 by the quarks and 0.24 by the squarks.

1. INTRODUCTION

What do nucleons look like at short distances? This question can only be answered after the particle content of the theory of strong interactions and their short distance interactions are known. In QCD we know that the nucleon structure functions are described by quark and gluon distributions with well defined Q^2 evolutions. If one believes that supersymmetry [1] is a genuine symmetry, the particle contents of standard QCD will be enlarged by the supersymmetric partners of gluons (gluinos) and of quarks (squarks) [2]. The existence of such particles drastically changes the picture of the hadronic internal structure at short distances. In a realistic supersymmetric model the supersymmetry must be broken [3] in such a way that all successful phenomenological predictions of QCD and electroweak interactions are reproduced. Whatever the mechanism for the supersymmetry breaking (explicit [4], spontaneous [2,3] or dynamical [5]) the masses of the scalar quarks (squarks) must be larger than 17 GeV [6] and the mass of the fermionic gluons (gluinos) must be larger than 1-2 GeV [5,7-12]. Above these limits the hadronic structure will be significantly altered.

In this paper we give a quantitative description of the nucleon structure above the threshold for the production of squarks. We derive the quark and squark distributions using the known quark and gluon distributions at low Q^2 [13,14]. At high Q^2 these distributions may be very important in future experiments such as $p\bar{p} \rightarrow \mu^+\mu^-X$ for $Q^2 \sim (500)^2 \text{ GeV}^2$ or even for large p_T experiments. The quark, gluon, gluino and squark distributions at high Q^2 can be used to establish the existence of supersymmetric particles and hence the correctness of introducing supersymmetry into standard QCD or Grand Unified Theories.

In order to be able to make this kind of analysis we needed to calculate the 4×4 matrix for the Q^2 evolution kernel (the inverse Mellin transform of the 4×4 anomalous dimension matrix) in the leading logarithm approximation. These 16 quantities are related by $N = 1$ or $N = 2$ supersymmetry identities [1]. This is an example of the attractive supersymmetric relations between bosons and fermions.

In Section 2 we explain the formalism used in this paper and give the integro-differential equations for gluon, gluino, quark and squark distributions.

In Section 3 we give the analytic result of the 4×4 matrix of the Q^2 evolution kernel as well as the anomalous dimension matrix. We also derive the above-mentioned $N = 1$ and $N = 2$ supersymmetry relations. Quantities which are not gauge invariant violate the supersymmetry relations because of the introduction into the effective Lagrangian of a gauge fixing term which explicitly breaks supersymmetry. However, we show by explicit calculation that in the light cone axial gauge even gauge dependent quantities preserve the supersymmetry relations.

In Section 4 we study the quark and squark valence distributions, determining them as a function of Bjorken x and Q^2 (using the known valence quark distribution at low Q^2).

In Section 5 we study the gluon, gluino and "sea" quark and squark distributions. As in the case of the valence distributions we use a simple parameterization of the parton distributions at the squark threshold.

Finally, Section 6 is devoted to a general conclusion.

2. (a) PARTICLE CONTENT OF SUPERSYMMETRIC QCD AND THEIR INTERACTIONS

The existence of a spin one half supersymmetry generator implies that every particle must have a "super" partner with spin shifted by a half unit [1,2]. Thus the gauge bosons (gluons) have Majorana fermions as partners. These are in the adjoint representation of $SU(3)$ and are called gluinos. Likewise left- and right-handed quarks have complex scalar partners which are called squarks (s-type and t-type associated with left- and right-handed quarks, respectively)

$$\text{Gluons } [G] \leftrightarrow \text{Gluons } [\lambda]$$

$$\text{Right-handed quarks } [q_R, \bar{q}_R] \leftrightarrow \text{squarks } [t, t^+]$$

$$\text{Left-handed quarks } [q_L, \bar{q}_L] \leftrightarrow \text{squarks } [s, s^+]$$

The various interactions between gluons, gluinos, quarks, and squarks are described by the following $N = 1$ supersymmetric, $SU(3)$ (colour) gauge invariant, Lagrangian

$$\begin{aligned}
 \mathcal{L} = & -\frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a - \frac{1}{2} \bar{\lambda}_a (i \not{D} \lambda)_a \\
 & - \bar{q}_i i \not{D} q_i - (D_\mu S_i)^\dagger D^\mu S_i - (D_\mu t_i)^\dagger D^\mu t_i \\
 & + ig\sqrt{2} \left\{ \bar{\lambda}_R^a S_i^\dagger T^a q_{Li} + \bar{\lambda}_L^a t_i^\dagger T^a q_{Ri} + h.c \right\} \\
 & - \frac{1}{2} g^2 (S_i^\dagger T^a S_i - t_i^\dagger T^a t_i)^2 + \text{mass terms}
 \end{aligned} \tag{2.1}$$

where "a" runs over the eight gluons and i runs over the number of flavours. A gauge fixing term plus a Faddeev-Popov ghost term must be added to the above Lagrangian. $G_{\mu\nu}^a$ is the gluon field strength

$$G_a^{\mu\nu} = \partial^\mu G^\nu - \partial^\nu G^\mu + g f^{abc} G_b^\mu G_c^\nu$$

and D_μ is the covariant derivative

$$D_{ij}^\mu = \partial_{ij}^\mu - ig G_\mu^a T_{ij}^a$$

where T^a are the generators of SU(3) in a given representation [for gluinos these are the structure constants of the group $(T^a)_{ij} = i f^{aij}$ and for quarks and squarks in the fundamental representation they are the Gell-Mann SU(3) matrices].

The presence of mass terms in the Lagrangian breaks supersymmetry. The mass spectrum [2-5,11] depends on the way the supersymmetry is broken. For exact supersymmetry the gluino is massless and the quarks have the same mass as their corresponding squarks. In practice the gluinos can have a Majorana mass [11] and the squarks have much larger masses than their corresponding quarks, because supersymmetry is broken. For sufficiently large momentum scales ($Q^2 \gg m_1^2$) the effects of these masses may be neglected and supersymmetry is effectively unbroken.

The Lagrangian density (2.1) contains neither auxiliary fields or unphysical degrees of freedom. This is because we have chosen the Wess-Zumino gauge [1] and integrated out all the auxiliary fields. But such a gauge choice still leaves the usual SU(3) gauge to be fixed. One way of doing this is to choose covariant gauges

$$\mathcal{L}_{G.F} = \frac{1}{2\alpha} (\partial_\mu G_a^\mu)^2 \tag{2.2}$$

with

$$\mathcal{L}_{\text{F.P.}} = \bar{n}_a (\partial^\mu D_\mu \eta)_a$$

However, this particular gauge explicitly breaks the supersymmetry since the supersymmetry partner of the longitudinal gluon $\partial^\mu G_\mu^a$ is not present in the gauge fixing. Such supersymmetry breaking does not affect gauge invariant quantities such as the anomalous dimensions of gauge invariant operators, but the naive supersymmetry relations (such as the equality of the wavefunction renormalization of members of the same supermultiplet) are in general violated in such a gauge.

In order to preserve the supersymmetry relations for all Green's functions (gauge invariant or otherwise) we must use the superfield formalism [1] or axial gauges ($n \cdot A = 0$, $n^2 \neq 0$) [15]. In this gauge all supersymmetry relations can be shown to be obeyed. This follows from the fact that the gluon propagator is a projector,

$$\eta^\mu \langle 0 | T (G_\mu^a(x) G_\nu^b(x)) | 0 \rangle = 0 \quad (2.3)$$

and so the unphysical fields $n^\mu G_\mu^a$ always decouple from Green's functions.

In Section 3 we will show by explicit calculation that these supersymmetry relations are also preserved in the light cone axial gauge ($n \cdot A = 0$, $n^2 = 0$). This gauge suffers from the problem of extra infrared divergences in some of the Green's functions, which have to be treated by a consistent regularization method.

(b) PARAMETERIZATION OF THE STRUCTURE FUNCTIONS

In the leading logarithm approximation the factorization theorem and the renormalization group equation can be combined into the following equations for the Q^2 evolution of structure functions [14,16]

$$\begin{aligned} F_2^s(x, Q^2) = & x e_{q_i^2} \left[q_i(x, Q^2) + \bar{q}_i(x, Q^2) + S_i(x, Q^2) + \bar{S}_i(x, Q^2) \right. \\ & \left. + t_i(x, Q^2) + \bar{t}_i(x, Q^2) \right] \end{aligned} \quad (2.4)$$

where i runs over all flavours whose mass threshold is below Q^2 and where e_{qi} are the electric charges of flavour i quarks (or antiquarks) and squarks. In the leading logarithm approximation gluons and gluinos do not couple to the electromagnetic or weak currents.

The Q^2 dependence of these parton densities (quarks, squarks, gluons and gluinos) is controlled by the following set of integrodifferential equations

$$\begin{aligned}
 Q^2 \frac{d}{dQ^2} G(x, Q^2) &= \frac{\alpha(Q^2)}{8\pi} \left[P_{GG} \otimes G + P_{G\lambda} \otimes \lambda + P_{Gq} \otimes \sum_i (q_i + \bar{q}_i) + P_{Gs} \otimes \sum_i (s_i + \bar{s}_i + t_i + \bar{t}_i) \right] \\
 Q^2 \frac{d}{dQ^2} \lambda(x, Q^2) &= \frac{\alpha(Q^2)}{8\pi} \left[P_{\lambda G} \otimes G + P_{\lambda\lambda} \otimes \lambda + P_{\lambda q} \otimes \sum_i (q_i + \bar{q}_i) + P_{\lambda s} \otimes \sum_i (s_i + \bar{s}_i + t_i + \bar{t}_i) \right] \\
 Q^2 \frac{d}{dQ^2} q_i(x, Q^2) &= \frac{\alpha(Q^2)}{8\pi} \left[\frac{1}{2n_f} P_{qG} \otimes G + \frac{1}{2n_f} P_{q\lambda} \otimes \lambda + P_{qq} \otimes q_i + P_{qs} \otimes (s_i + t_i) \right] \quad (2.5) \\
 Q^2 \frac{d}{dQ^2} s_i(x, Q^2) &= \frac{\alpha(Q^2)}{8\pi} \left[\frac{1}{4n_f} P_{sG} \otimes G + \frac{1}{4n_f} P_{s\lambda} \otimes \lambda + \frac{1}{2} P_{sq} \otimes q_i + P_{ss} \otimes s_i \right] \\
 Q^2 \frac{d}{dQ^2} t_i(x, Q^2) &= \frac{\alpha(Q^2)}{8\pi} \left[\frac{1}{4n_f} P_{tG} \otimes G + \frac{1}{4n_f} P_{t\lambda} \otimes \lambda + \frac{1}{2} P_{tq} \otimes q_i + P_{ts} \otimes t_i \right]
 \end{aligned}$$

with similar equations to the last two for antiquarks and antisquarks. The kernels P_{ij} are functions of x and the parton densities for gluons (G), gluinos (λ), quarks (q_i) or squarks (s_i, t_i) are functions of x and Q^2 . The convolution " $\otimes$ " is defined by

$$A \otimes B = \int_x^1 \frac{dy}{y} A(y) B\left(\frac{x}{y}\right) \quad (2.6)$$

In the absence of squarks one can see that the quantities $q_i - q_j$ or $q_i - \bar{q}_i$ have Q^2 evolutions which only depend on themselves. This is because these quantities cannot mix with gluons or gluinos since $q_i - q_j$ is a flavour non-singlet and $q_i - \bar{q}_i$ is odd under charge conjugation. This is not the case in the presence of squarks since the squarks have the same quantum numbers as the quarks (except for spin). In this case the quantities $q_i + s_i - (q_j + s_j)$ or $q_i + s_i - (\bar{q}_i + \bar{s}_i)$ have Q^2 evolutions which depend on themselves

$$Q^2 \frac{d}{dQ^2} (q_i + s_i - (q_j + s_j)) = \frac{\alpha(Q^2)}{8\pi} [(P_{qq} + P_{sq}) \otimes (q_i + s_i - (q_j + s_j))] \quad (2.7a)$$

and

$$Q^2 \frac{d}{dQ^2} (q_i + s_i - (\bar{q}_i + \bar{s}_i)) = \frac{\alpha(Q^2)}{8\pi} [(P_{qq} + P_{sq}) \otimes (q_i + s_i - (\bar{q}_i + \bar{s}_i))] \quad (2.7b)$$

[here and henceforth we write $s(x, Q^2)$ to be the identical distribution of s-type or t-type squarks]. These equations are derived using the relation (see Section 3)

$$P_{qq} + P_{qs} = P_{sq} + P_{ss} \quad (2.8)$$

which follows from supersymmetry.

We close this section by defining some useful combinations of quark and squark distributions

$$(i) \text{ Valence quarks } q_v(x, Q^2) = \sum_{i=1}^{n_f} (q_i(x, Q^2) - \bar{q}_i(x, Q^2)) \quad (2.9a)$$

$$(ii) \text{ Valence squarks } S_v(x, Q^2) = \sum_{i=1}^{n_f} (s_i(x, Q^2) - \bar{s}_i(x, Q^2)) \quad (2.9b)$$

$$(iii) \text{ Total quark and antiquark distribution } q_t(x, Q^2) = \sum_{i=1}^{n_f} (q_i(x, Q^2) + \bar{q}_i(x, Q^2)) \quad (2.9c)$$

$$(iv) \text{ Total squark and antisquark distribution } S_t(x, Q^2) = \sum_{i=1}^{n_f} (s_i(x, Q^2) + \bar{s}_i(x, Q^2)) \quad (2.9d)$$

The Q^2 evolution of q_v , s_v , q_t , s_t , G and λ are given by the equations

$$Q^2 \frac{d}{dQ^2} q_v = \frac{\alpha(Q^2)}{8\pi} (P_{qq} \otimes q_v + P_{qs} \otimes S_v) \quad (2.10a)$$

$$Q^2 \frac{d}{dQ^2} S_v = \frac{\alpha(Q^2)}{8\pi} (P_{sq} \otimes q_v + P_{ss} \otimes S_v) \quad (2.10b)$$

The quantities q_v and s_v do not mix with gluons and gluinos. The other quantities mix with each other and their Q^2 evolution is described by the 4×4 matrix equation

$$Q^2 \frac{d}{dQ^2} \begin{pmatrix} G \\ \lambda \\ q_t \\ S_t \end{pmatrix} = \frac{\alpha(Q^2)}{8\pi} \begin{pmatrix} P_{GG} & P_{G\lambda} & P_{Gq} & P_{Gs} \\ P_{\lambda G} & P_{\lambda\lambda} & P_{\lambda q} & P_{\lambda s} \\ P_{qG} & P_{q\lambda} & P_{qq} & P_{qs} \\ P_{sG} & P_{s\lambda} & P_{sq} & P_{ss} \end{pmatrix} \otimes \begin{pmatrix} G \\ \lambda \\ q_t \\ S_t \end{pmatrix} \quad (2.11)$$

In the next section we give analytic expressions for the 16 elements of the kernel matrix and discuss supersymmetry relations between them.

3. (a) ANALYTIC EXPRESSIONS FOR $P_{ij}(x)$ AND SUPERSYMMETRIC RELATIONS

Our results for $P_{ij}(x)$ are shown in Table 1. They are obtained from the inverse Mellin transforms of the anomalous dimensions of twist 2 operators in the Wilson operator product expansion on the light cone. P_{ij} are gauge invariant quantities, so that there are some supersymmetry relations between different kernels. Since the Lagrangian is $N = 1$ supersymmetric (i.e. gluons and gluinos are members of the same vector supermultiplet and quarks and squarks are members of the same matter supermultiplet) the quantities

$$V(x, Q^2) = G(x, Q^2) + \lambda(x, Q^2) \quad (3.1a)$$

and

$$\Phi(x, Q^2) = q_t(x, Q^2) + S_t(x, Q^2) \quad (3.1b)$$

are invariant under supersymmetry transformations and this leads to the Q^2 evolution equations

$$Q^2 \frac{d}{dQ^2} V = \frac{\alpha(Q^2)}{8\pi} [P_{VV} \otimes V + P_{V\Phi} \otimes \Phi] \quad (3.2a)$$

$$Q^2 \frac{d}{dQ^2} \Phi = \frac{\alpha(Q^2)}{8\pi} [P_{\Phi V} \otimes V + P_{\Phi\Phi} \otimes \Phi] \quad (3.2b)$$

where P_{VV} , $P_{V\Phi}$, $P_{\Phi V}$, $P_{\Phi\Phi}$ are the inverse Mellin transforms of elements of the anomalous dimension matrix of the two supersymmetric invariant operators O_N^V and O_N^Φ , where

$$O_N^Y = \frac{i^N}{N!} \left[G_a^{\mu_1 \mu_2} D^{\mu_2} \dots D^{\mu_{N-1}} G_{\mu_N}^a + \bar{\lambda}_a \gamma^{\mu_1} D^{\mu_2} \dots D^{\mu_N} \lambda^a \right] \quad (3.3a)$$

$$O_N^\Phi = \frac{i^N}{N!} \left[\bar{q}_i \gamma^{\mu_1} D^{\mu_2} \dots D^{\mu_N} q_i + S_i^\dagger D^{\mu_1} \dots D^{\mu_N} S_i + t_i^\dagger D^{\mu_1} \dots D^{\mu_N} t_i \right] \quad (3.3b)$$

where the above operators must be symmetrized in the indices $\mu_1 \dots \mu_N$ and all traces must be subtracted; D^μ is the covariant derivative.

Combining Eqs. (2.11) and (3.2) we have the relations

$$P_{VV} = P_{GG} + P_{\lambda G} = P_{G\lambda} + P_{\lambda\lambda} \quad (3.4a)$$

$$P_{V\phi} = P_{Gq} + P_{\lambda q} = P_{Gs} + P_{\lambda s} \quad (3.4b)$$

$$P_{\phi V} = P_{qG} + P_{sG} = P_{q\lambda} + P_{s\lambda} \quad (3.4c)$$

$$P_{\phi\phi} = P_{qq} + P_{sq} = P_{sq} + P_{ss} \quad (3.4d)$$

It can be seen from Table 1 that these relations are satisfied. The first relation, Eq. (3.4a), was already known from standard QCD (without gluinos or squarks) when the quarks are formally put into the adjoint representations of SU(3), so that the standard QCD Lagrangian looks like a supersymmetric Lagrangian with one vector supermultiplet.

The relations (3.4) are valid up to all orders in perturbation theory provided one uses a renormalization prescription which preserves supersymmetry (e.g. the dimensional reduction scheme, $\overline{DR}$, instead of dimensional renormalization scheme $\overline{MS}$).

We also observe that if $C_F = C_A = T_R$ we have further relations

$$P_{Gi} + P_{\lambda i} + P_{qi} + P_{si} = P_{Gj} + P_{\lambda j} + P_{qj} + P_{sj} \quad (3.5a)$$

for $i, j = G, \lambda, q, s$ and

$$P_{ij} = P_{kl} \quad (3.5b)$$

where i and k or j and l are related by the interchange of λ and q . These relations do not follow from $N = 1$ supersymmetry but from the fact that if the quarks

transform as the adjoint representation of SU(3) (Majorana spinors) then the Lagrangian is symmetric under the interchange $\lambda \leftrightarrow q$ and moreover it is invariant under N = 2 supersymmetry transformations where all the fields transform as members of a single N = 2 supersymmetry vector multiplet, V_2 . The relation (3.5a) now follows from the fact that the combination

$$V_2 = G + \lambda + q_t + S_t$$

is an N = 2 supersymmetric invariant quantity (provided $C_F = C_A = T_R$). This implies that

$$Q^2 \frac{d}{dQ^2} (G + \lambda + q_t + S_t) = \frac{\alpha(a')}{8\pi} P_{V_2 V_2} \otimes (G + \lambda + q + S) \quad (3.6)$$

where $P_{V_2 V_2}$ is the inverse Mellin transform of the N = 2 supersymmetric operator

$$O_N^{V_2} = O_N^V + O_N^\phi \quad (3.7)$$

Relation (3.5a) now follows from Eq. (3.6) and the relation (3.5b) follows from the symmetry of the N = 2 supersymmetric Lagrangian under the interchange $\lambda \leftrightarrow q$. If $i, j = q$ or s , then the relation (3.5a) holds even without the condition $C_F = C_A = T_R$; however, this is an accident of leading order perturbation theory. It follows because the group theory factor for all P_{qi} or P_{si} is C_F , so we can relax the condition $C_F = C_A = T_R$. But in higher orders the kernels will depend on the other independent eigenvalues of the Casimir operators and so this relation will not be valid. In any case the relations (3.5) are unphysical but they are useful for reducing the number of independent kernels P_{ij} .

Since P_{ij} are gauge invariant quantities the relations (3.4) and (3.5) are valid in any gauge. Supersymmetry suggests that the anomalous dimensions of fields which are members of the same supermultiplet must be the same

$$\gamma_V \equiv \gamma_\lambda = \gamma_G \quad (3.8a)$$

and

$$\gamma_\phi \equiv \gamma_q = \gamma_s \quad (3.8b)$$

However, these are not gauge invariant quantities and in covariant gauges, for which the gauge fixing and Faddeev-Popov ghost terms explicitly break the supersymmetry invariance, they are not valid. Nevertheless we find that in the light cone axial gauge these relations are indeed valid

$$-2\gamma_\lambda = 3C_A - T_R - 4C_A \int_0^1 \frac{dx}{(1-x)^{1-\varepsilon}} = 3C_A - T_R - \frac{4C_A}{\varepsilon} \quad (3.9a)$$

$$-2\gamma_G = 3C_A - T_R - 4C_A \int_0^1 \frac{dx}{(1-x)^{1-\varepsilon}} = 3C_A - T_R - \frac{4C_A}{\varepsilon} \quad (3.9b)$$

$$-2\gamma_q = 2C_F - 4C_F \int_0^1 \frac{dx}{(1-x)^{1-\varepsilon}} = 2C_F - \frac{4C_F}{\varepsilon} \quad (3.9c)$$

$$-2\gamma_S = 2C_F - 4C_F \int_0^1 \frac{dx}{(1-x)^{1-\varepsilon}} = 2C_F - \frac{4C_F}{\varepsilon} \quad (3.9d)$$

so that the relations (3.8) are satisfied. Moreover, if the quarks and squarks are in the adjoint representation of SU(3) we have the further relation from N = 2 supersymmetry

$$\gamma_G = \gamma_\lambda = \gamma_q = \gamma_S \Big|_{C_A = T_R = C_F} \quad (3.10)$$

From Eq. (3.9) we can see that these anomalous dimensions are infrared divergent and we regularize the infrared divergences using dimensional regularization. These divergences always cancel for gauge invariant quantities, such as the anomalous dimensions of gauge invariant operators, which are the sum of the pole part of an amputated Green's function $\gamma_{\Gamma_i}^N$ and a field anomalous dimension $-2\gamma_i$

$$\gamma_{GG}^N = \gamma_{\Gamma_G}^N - 2\gamma_G \quad (3.11a)$$

$$\gamma_{\lambda\lambda}^N = \gamma_{\Gamma_\lambda}^N - 2\gamma_\lambda \quad (3.11b)$$

$$\gamma_{qq}^N = \gamma_{\Gamma_q}^N - 2\gamma_q \quad (3.11c)$$

$$\gamma_{SS}^N = \gamma_{\Gamma_S}^N - 2\gamma_S \quad (3.11d)$$

Thus, for example

$$\gamma_{\Gamma_\lambda}^N = 2C_A \int_0^1 dx x^{N-1} \left[\frac{1+x^2}{(1-x)^{1-\epsilon}} \right] = \frac{4C_A}{\epsilon} + 2C_A \left[-2S(N) + \frac{1}{N(N+1)} \right] \quad (3.12)$$

so we see that for $\gamma_{\lambda\lambda}^N$ this infrared divergence cancels and one obtains a finite gauge invariant anomalous dimension

$$\gamma_{\lambda\lambda}^N = 2C_A \left[-2S(N) + \frac{1}{N(N+1)} \right] + 3C_A - T_R \quad (3.13)$$

where

$$S(N) = \sum_{i=1}^N \frac{1}{i}$$

The reason why the naive supersymmetry relations are valid in the light cone axial gauge ($n \cdot G_a = 0$, $n^2 = 0$) is that the unphysical degrees of freedom $n \cdot G_a$, which explicitly break supersymmetry, do not propagate so that they always decouple from all Green's functions.

(b) LONGITUDINAL MOMENTUM CONSERVATION

$P_{ij}(x)$ represents the probability to generate a parton i from a parton j with a function of longitudinal momentum x . In leading order this happens by the emission of a parton k whose longitudinal momentum fraction is $1-x$, so in this order we have

$$P_{ij}(x) = P_{kj}(1-x) \quad (3.14)$$

In higher order these relations become much more complicated and consequently lose their usefulness. From Eq. (3.14) we have

$$P_{GG}(x) = P_{GG}(1-x) \quad (3.15a)$$

$$P_{\lambda G}(x) = P_{\lambda G}(1-x) \quad (3.15b)$$

$$P_{qG}(x) = P_{qG}(1-x) \quad (3.15c)$$

$$P_{SG}(x) = P_{SG}(1-x) \quad (3.15d)$$

$$P_{G\lambda}(x) = P_{\lambda\lambda}(1-x) \quad (3.15e)$$

$$P_{q\lambda}(x) = P_{S\lambda}(1-x) \quad (3.15f)$$

$$P_{Gq}(x) = P_{qg}(1-x) \quad (3.15g)$$

$$P_{\lambda q}(x) = P_{sq}(1-x) \quad (3.15h)$$

$$P_{Gs}(x) = P_{ss}(1-x) \quad (3.15i)$$

$$P_{\lambda s}(x) = P_{qs}(1-x) \quad (3.15j)$$

(c) MOMENTUM SUM RULE

The total fraction of longitudinal momentum carried by all the constituents of a hadron must sum to one

$$\int_0^1 dx \times [G(x, \alpha^2) + \lambda(x, \alpha^2) + q_t(x, \alpha^2) + S_t(x, \alpha^2)] = 1 \quad (3.16a)$$

Taking the derivative of this with respect to Q^2 we have

$$\int_0^1 dx \times [P_{Gi} + P_{\lambda i} + P_{qi} + P_{si}] = 0, \quad i = G, \lambda, q, s. \quad (3.16b)$$

This relation expresses the fact that the anomalous dimension of the energy momentum tensor is zero (i.e. energy and momentum are conserved).

(d) BARYON NUMBER CONSERVATIONS

The fact that the quarks and squarks carry baryon number $1/3$ and that baryon number is conserved implies that

$$\frac{1}{3} \int_0^1 dx \sum_{i=1}^{n_f} (q_i + s_i - \bar{q}_i - \bar{s}_i) = 1 \quad (\text{for nucleons}) \quad (3.17a)$$

Taking the derivative of this with respect to Q^2 gives us

$$\int_0^1 dx (P_{qq} + P_{sq}) = \int_0^1 dx (P_{qs} + P_{ss}) = 0 \quad (3.17b)$$

This is the statement that baryonic current is conserved so that its anomalous dimension must vanish. Here we observe a fundamental difference between this supersymmetric model and standard QCD. In standard QCD only the quarks carry baryon number, whereas in the supersymmetric version the squarks can also carry baryon number so that only a fraction of the baryon number is now carried by the quarks. It will be shown in the next section that in the asymptotic limit ($Q^2 \rightarrow \infty$) the quarks carry $2/3$ of the total baryon number and the squarks carry $1/3$.

4. VALENCE PARTON DISTRIBUTIONS

Having discussed the properties of the Q^2 evolution kernels we now turn to the discussion of how the constituents are distributed inside hadrons. We begin with the valence distributions $q_v(x, Q^2)$ and $s_v(x, Q^2)$. These quantities do not mix with gluons or gluinos and their Q^2 evolution is given by Eq. (2.10). We denote by $\langle A \rangle_N$ the N^{th} moment of $A(x)$

$$\langle A \rangle_N = \int_0^1 dx x^{N-1} A(x) \quad (4.1)$$

Taking the first and second moments of Eq. (2.10) we obtain the following sets of differential equations.

$$Q^2 \frac{d}{dQ^2} \begin{pmatrix} \langle q_v \rangle_1 \\ \langle s_v \rangle_1 \end{pmatrix} = \frac{\alpha(Q^2)}{8\pi} C_F \begin{pmatrix} -1 & 1 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} \langle q_v \rangle_1 \\ \langle s_v \rangle_1 \end{pmatrix} \quad (4.2)$$

and

$$Q^2 \frac{d}{dQ^2} \begin{pmatrix} \langle q_v \rangle_2 \\ \langle s_v \rangle_2 \end{pmatrix} = \frac{\alpha(Q^2)}{8\pi} C_F \begin{pmatrix} -\frac{11}{3} & 1 \\ \frac{2}{3} & -4 \end{pmatrix} \begin{pmatrix} \langle q_v \rangle_2 \\ \langle s_v \rangle_2 \end{pmatrix} \quad (4.3)$$

The general solution of Eq. (4.2) is

$$\langle q_v(Q^2) \rangle_1 = 2 + (\langle q_v(Q_0^2) \rangle_1 - 2) \exp[-3C_F S(Q^2, Q_0^2)] \quad (4.4a)$$

$$\langle s_v(Q^2) \rangle_1 = 1 - (\langle q_v(Q_0^2) \rangle_1 - 2) \exp[-3C_F S(Q^2, Q_0^2)] \quad (4.4b)$$

where we have used the fact that the total number of valence partons in a number is 3 and

$$S(Q^2, Q_0^2) = \frac{1}{b_0} \log \left(1 + \frac{\alpha(Q_0^2)}{4\pi} b_0 \log \frac{Q^2}{Q_0^2} \right) \quad (4.5a)$$

with

$$b_0 = 3C_A - T_R \quad (= 3 \text{ for } C_A = 3 \text{ and } T_R = n_f = 6)^* \quad (4.5b)$$

The solution to Eq. (4.3) is

*) We define T_R to be $\frac{1}{2} \times (\text{no. of left-handed flavours} + \text{no. of right-handed flavours})$.

$$\begin{aligned} \langle q_v(q^2) \rangle_2 = \langle q_v(q_0^2) \rangle_2 & \left[\frac{3}{5} \exp[-3C_F S(q^2, q_0^2)] + \frac{2}{5} \exp\left[-\frac{14}{3} C_F S(q^2, q_0^2)\right] \right] \\ & + \langle S_v(q_0^2) \rangle_2 \left[\frac{3}{5} \exp[-3C_F S(q^2, q_0^2)] - \frac{3}{5} \exp\left[-\frac{14}{3} C_F S(q^2, q_0^2)\right] \right]. \end{aligned} \quad (4.6a)$$

$$\begin{aligned} \langle S_v(q^2) \rangle_2 = \langle q_v(q_0^2) \rangle_2 & \left[\frac{3}{5} \exp[-3C_F S(q^2, q_0^2)] - \frac{3}{5} \exp\left[-\frac{14}{3} C_F S(q^2, q_0^2)\right] \right] \\ & + \langle S_v(q_0^2) \rangle_2 \left[\frac{2}{5} \exp[-3C_F S(q^2, q_0^2)] + \frac{3}{5} \exp\left[-\frac{14}{3} C_F S(q^2, q_0^2)\right] \right]. \end{aligned} \quad (4.6b)$$

Asymptotically (as $Q^2 \rightarrow \infty$) the quarks carry $2/3$ of the baryon number and the squarks $1/3$. Furthermore the fraction of momentum carried by the valence partons goes to zero; this means that asymptotically all the momentum is transferred to the gluons, gluinos and "sea" quarks and squarks (and antiquarks and antisquarks). This means that asymptotically the valence parton distributions become a delta function at $x = 0$

$$q_v(x, Q^2) \xrightarrow{Q^2 \rightarrow \infty} 2 \delta(x) \quad (4.7a)$$

$$S_v(x, Q^2) \xrightarrow{Q^2 \rightarrow \infty} \delta(x) \quad (4.7b)$$

For the large moments ($N \rightarrow \infty$) we have

$$Q^2 \frac{d}{dQ^2} \begin{pmatrix} \langle q_v \rangle_N \\ \langle S_v \rangle_N \end{pmatrix} = \frac{\alpha(Q^2)}{8\pi} (-4C_F \log N) \begin{pmatrix} \langle q_v \rangle_N \\ \langle S_v \rangle_N \end{pmatrix} \quad (4.8)$$

which implies that the behaviour of the distributions near $x = 1$ is given by

$$q_v(x, Q^2) \xrightarrow{x \rightarrow 1} q_v(x, Q_0^2) \left[\log \frac{1}{x} \right]^{4C_F S(Q^2, Q_0^2)} \quad (4.9a)$$

$$S_v(x, Q^2) \xrightarrow{x \rightarrow 1} S_v(x, Q_0^2) \left[\log \frac{1}{x} \right]^{4C_F S(Q^2, Q_0^2)} \quad (4.9b)$$

If the threshold for the productions of squarks in deep inelastic scattering is $Q^2 = m_t^2$, then at this value of Q^2 we have

$$\langle q_v(M_E^2) \rangle_1 = 3, \quad \langle S_v(M_E^2) \rangle_1 = 0 \quad (4.10a)$$

$$\langle q_v(M_E^2) \rangle_2 \approx 0.2, \quad \langle S_v(M_E^2) \rangle_2 = 0 \quad (4.10b)$$

The first condition follows from the fact that no squarks are observed below threshold and the second from previously fitted structure functions below m_t^2 , with m_t taken to be 34 GeV (i.e. assuming that the squark mass is 17 GeV, which is its current experimental lower limit). In order to be consistent with Eqs. (4.5), (4.6), and (4.9) we parameterize the valence parton distributions as

$$q_v(x, q^2) = C_{q_v}(q^2) \times A_{q_v}(q^2) \left[\log \frac{1}{x} \right]^{B_{q_v}(q^2)} \quad (4.11a)$$

$$S_v(x, q^2) = C_{S_v}(q^2) \times A_{S_v}(q^2) \left[\log \frac{1}{x} \right]^{B_{S_v}(q^2)} \quad (4.11b)$$

where

$$B_{q_v}(q^2) = B_{q_v}(q_0^2) + 4C_F S(q^2, q_0^2) \quad (4.12a)$$

$$B_{S_v}(q^2) = B_{S_v}(q_0^2) + 4C_F S(q^2, q_0^2) \quad (4.12b)$$

$$A_{q_v}(q^2) = \frac{R_{q_v}(q^2) - 2}{1 - R_{q_v}(q^2)} \quad \text{with} \quad R_{q_v}(q^2) = \left(\frac{\langle q_v(q^2) \rangle_1}{\langle q_v(q^2) \rangle_2} \right)^{\frac{1}{1 + B_{q_v}(q^2)}} \quad (4.12c)$$

$$A_{S_v}(q^2) = \frac{R_{S_v}(q^2) - 2}{1 - R_{S_v}(q^2)} \quad \text{with} \quad R_{S_v}(q^2) = \left(\frac{\langle S_v(q^2) \rangle_1}{\langle S_v(q^2) \rangle_2} \right)^{\frac{1}{1 + B_{S_v}(q^2)}} \quad (4.12d)$$

$$C_{q_v}(q^2) = \langle q_v(q^2) \rangle_1 \frac{[1 + A_{q_v}(q^2)]^{\frac{1}{1 + B_{q_v}(q^2)}}}{\Gamma(1 + B_{q_v}(q^2))} \quad (4.12e)$$

$$C_{S_v}(q^2) = \langle S_v(q^2) \rangle_1 \frac{[1 + A_{S_v}(q^2)]^{\frac{1}{1 + B_{S_v}(q^2)}}}{\Gamma(1 + B_{S_v}(q^2))} \quad (4.12f)$$

where $\langle q_V(Q^2) \rangle_{1,2}$ and $\langle s_V(Q^2) \rangle_{1,2}$ are obtained from Eqs. (4.5) and (4.6). From this parameterization we can calculate $q_V(x, Q^2)$ or $s_V(x, Q^2)$ for any x and Q^2 once they are known at $Q^2 = m_t^2$.

In Figs. 1 to 4 we plot $q_V(x, Q^2)$ and $s_V(x, Q^2)$ for three different values of Q^2 . The quantity $q_V(x, m_t^2)$ is found from the evolution equations of QCD without squarks. In principle this is affected by the possible existence of gluinos and it is for this reason that we plot the distributions for a small gluino mass of 1 GeV (Figs. 1 and 2) and for the gluino mass equal to the squark mass (Figs. 3 and 4). In practice, it is found that this makes very little difference and that $\langle q_V(m_t^2) \rangle_2 \approx 0.21$ independent of the gluino mass. We also find that taking $\Lambda \approx 0.2$ GeV for Q^2 between the c-quark and b-quark thresholds

$$\frac{\alpha(M_t^2)}{4\pi} \approx 0.012 \quad \text{for} \quad m_\lambda = \frac{m_t}{2} \quad (4.13a)$$

and

$$\frac{\alpha(M_t^2)}{4\pi} \approx 0.014 \quad \text{for} \quad m_\lambda \approx 1 \text{ GeV} \quad (4.13b)$$

We also find

$$B_q(M_t^2) = B_{s_V}(M_t^2) \approx 2.94 \quad (4.14)$$

In the case of standard QCD without squarks it was found that $q_V(x, Q^2)$ was Q^2 independent at $x_0 \approx 0.15$. Above $Q^2 = m_t^2$ this is no longer the case because the integral $\langle q_V(Q^2) \rangle_1$ is no longer a constant due to the presence of squarks. It is now the sum $q_V(x, Q^2) + s_V(x, Q^2)$ whose integral is Q^2 independent and there exists a value of x_0 for which this sum is constant, although as can be seen from Figs. 1 to 4 this x_0 has the lower value of $x_0 = 0.05$.

5. SINGLET PARTON DISTRIBUTIONS (q_t , s_t , G , λ)

Equation (2.11) is a system of coupled differential equations describing the Q^2 evolution of $q_t(x, Q^2)$, $s_t(x, Q^2)$, $G(x, Q^2)$ and $\lambda(x, Q^2)$. Taking the second moment of these equations and using the second moments of P_{ij} shown in Table 2 we have

$$Q^2 \frac{d}{dQ^2} \begin{pmatrix} \langle G \rangle_2 \\ \langle \lambda \rangle_2 \\ \langle q_t \rangle_2 \\ \langle S_+ \rangle_2 \end{pmatrix} = \frac{\alpha(Q^2)}{8\pi} \begin{pmatrix} -\frac{2}{3}C_A - T_R & \frac{8}{3}C_A & \frac{8}{3}C_F & 2C_F \\ \frac{8}{3}C_A & -\frac{8}{3}C_A - T_R & \frac{1}{3}C_F & C_F \\ \frac{8}{3}T_R & \frac{1}{3}T_R & -\frac{11}{3}C_F & C_F \\ \frac{1}{3}T_R & \frac{2}{3}T_R & \frac{8}{3}C_F & -4C_F \end{pmatrix} \begin{pmatrix} \langle G \rangle_2 \\ \langle \lambda \rangle_2 \\ \langle q_t \rangle_2 \\ \langle S_+ \rangle_2 \end{pmatrix} \quad (5.1)$$

The above 4×4 matrix has the eigenvalues

$$\lambda_0 = 0 \quad (5.2a)$$

$$\lambda_1 = -(T_R + 3C_F) \quad (5.2b)$$

$$\lambda_{2,3} = -\frac{1}{3} \left[(7C_F + 5C_A + \frac{3}{2}T_R) \pm \sqrt{(7C_F + 5C_A + \frac{3}{2}T_R)^2 - 20C_F(7C_A + 2T_R)} \right] \quad (5.2c)$$

with corresponding eigenvectors

$$V_0 = \langle G \rangle_2 + \langle \lambda \rangle_2 + \langle q_t \rangle_2 + \langle S_+ \rangle_2 \quad (5.3a)$$

$$V_1 = \langle G \rangle_2 + \langle \lambda \rangle_2 - \frac{3C_F}{T_R} (\langle q_t \rangle_2 + \langle S_+ \rangle_2) \quad (5.3b)$$

$$V_{2,3} = (\lambda_{2,3} - \frac{14}{3}C_F) \left[\frac{2}{5} \langle G \rangle_2 - \frac{8}{5} \langle \lambda \rangle_2 \right] + \frac{2}{3}C_F \left[\frac{4}{5} \langle q_t \rangle_2 - \frac{6}{5} \langle S_+ \rangle_2 \right] \quad (5.3c)$$

The fact that the combination V_0 has eigenvector zero is a statement of the fact that the energy momentum tensor is a conserved quantity ($V_0 \equiv 1$ for all Q^2). The other eigenvalues are all negative so that the other combinations V_1 , V_2 and V_3 vanish asymptotically. From this we can determine the fraction of momentum caused by each type of constituent in the limit $Q^2 \rightarrow \infty$

$$\langle G(Q^2) \rangle_2 \xrightarrow{Q^2 \rightarrow \infty} \frac{4}{5} \frac{3C_F}{T_R + 3C_F} = 0.32 \quad (5.4a)$$

$$\langle \lambda(Q^2) \rangle_2 \xrightarrow{Q^2 \rightarrow \infty} \frac{1}{5} \frac{3C_F}{T_R + 3C_F} = 0.08 \quad (5.4b)$$

$$\langle q_t(Q^2) \rangle_2 \xrightarrow{Q^2 \rightarrow \infty} \frac{3}{5} \frac{T_R}{T_R + 3C_F} = 0.36 \quad (5.4c)$$

$$\langle s_t(Q^2) \rangle_2 \xrightarrow{Q^2 \rightarrow \infty} \frac{2}{5} \frac{T_R}{T_R + 3C_F} = 0.24 \quad (5.4d)$$

where we have assumed six flavours ($T_R = 6$) and ($C_A = 3$, $C_F = 4/3$). The above asymptotic results for the second moment agree with those presented at the CERN Susy workshop (21-23 April) by C.H. Llewellyn Smith [17]. These asymptotic results should be compared with standard QCD in the absence of gluinos and squarks in which case 0.53 of the momentum is carried by gluons and 0.47 by quarks in the asymptotic limit. Thus we see that for the supersymmetric model a larger fraction of the longitudinal momentum of the hadron is carried by charged particles (0.60 instead of 0.47). This increase may affect the $p\bar{p} \rightarrow \mu^+\mu^- + X$ cross-section by 10-15%.

Putting in numbers into Eqs. (5.1), (5.2) and (5.3) (for the case of six flavours) the second moments of the distributions $G(x, Q^2)$, $\lambda(x, Q^2)$, $q_t(x, Q^2)$ and $s_t(x, Q^2)$ have Q^2 evolutions given by

$$\begin{aligned} \langle G(Q^2) \rangle_2 &= 0.32 + \frac{4}{5} (G_0 + \lambda_0 - \frac{2}{5}) e^{-10S} \\ &+ \frac{1}{5} [0.982 (G_0 - 4\lambda_0) - 0.088 (2q_0 - 3s_0)] e^{-16.18S} \\ &+ \frac{1}{5} [0.018 (G_0 - 4\lambda_0) + 0.088 (2q_0 - 3s_0)] e^{-6.04S} \end{aligned} \quad (5.5a)$$

$$\begin{aligned} \langle \lambda(Q^2) \rangle_2 &= 0.08 + \frac{1}{5} (G_0 + \lambda_0 - \frac{2}{5}) e^{-10S} \\ &- \frac{1}{5} [0.982 (G_0 - 4\lambda_0) - 0.088 (2q_0 - 3s_0)] e^{-16.18S} \\ &- \frac{1}{5} [0.018 (G_0 - 4\lambda_0) + 0.088 (2q_0 - 3s_0)] e^{-6.04S} \end{aligned} \quad (5.5b)$$

$$\begin{aligned}
 \langle q_+(Q^2) \rangle_2 &= 0.36 + \frac{3}{5} (q_0 + s_0 - \frac{3}{5}) e^{-10S} \\
 &+ \frac{1}{5} \left[-0.197 (G_0 - 4\lambda_0) + 0.018 (2q_0 - 3s_0) \right] e^{-16.18S} \quad (5.5c) \\
 &+ \frac{1}{5} \left[0.197 (G_0 - 4\lambda_0) + 0.982 (2q_0 - 3s_0) \right] e^{-6.04S}
 \end{aligned}$$

$$\begin{aligned}
 \langle S_+(Q^2) \rangle_2 &= 0.24 + \frac{2}{5} (q_0 + s_0 - \frac{3}{5}) e^{-10S} \quad (5.5d) \\
 &- \frac{1}{5} \left[-0.197 (G_0 - 4\lambda_0) + 0.018 (2q_0 - 3s_0) \right] e^{-16.18S} \\
 &- \frac{1}{5} \left[0.197 (G_0 - 4\lambda_0) + 0.982 (2q_0 - 3s_0) \right] e^{-6.04S}
 \end{aligned}$$

where

$$G_0 = \langle G(Q_0^2) \rangle_2, \quad \lambda_0 = \langle \lambda(Q_0^2) \rangle_2, \quad q_0 = \langle q_+(Q_0^2) \rangle_2, \quad s_0 = \langle S_+(Q_0^2) \rangle_2$$

and

$$S = S(Q^2, Q_0^2) = \frac{1}{3} \log \left(1 + 3 \frac{\alpha(Q_0^2)}{4\pi} \log \frac{Q^2}{Q_0^2} \right)$$

From the initial values of G_0 , λ_0 , q_0 and s_0 at the squark threshold $Q_0^2 = m_t^2$ we can determine the fraction of longitudinal momentum carried by each constituent at any Q^2 . For $m_t = 34$ GeV the results of the low energy phenomenology give [12]

$$G_0 = 0.53, \quad q_0 = 0.47, \quad \lambda_0 = s_0 = 0 \quad \text{for } m_\lambda = \frac{m_t}{2} = 17 \text{ GeV} \quad (5.6a)$$

and

$$G_0 = 0.45, \quad q_0 = 0.47, \quad \lambda_0 = 0.08, \quad s_0 = 0 \quad \text{for } m_\lambda = 1 \text{ GeV} \quad (5.6b)$$

We now look at the large x distributions for the partons and their Q^2 development. From the large N limits of the moments of P_{ij} shown in Table 2 the Q^2 evolution for the large moments is given by the uncoupled differential equations

$$Q^2 \frac{d}{dQ^2} \begin{pmatrix} \langle G \rangle_N \\ \langle \lambda \rangle_N \\ \langle q_t \rangle_N \\ \langle S_+ \rangle_N \end{pmatrix} \xrightarrow{N \rightarrow \infty} \frac{\alpha(Q^2)}{8\pi} \log N \begin{pmatrix} -4C_A & & & \\ & -4C_A & 0 & \\ & 0 & -4C_F & \\ & & & -4C_F \end{pmatrix} \begin{pmatrix} \langle G \rangle_N \\ \langle \lambda \rangle_N \\ \langle q_t \rangle_N \\ \langle S_+ \rangle_N \end{pmatrix} \quad (5.7)$$

which implies for the large x behaviour of the distributions

$$G(x, Q^2) \xrightarrow{x \rightarrow 1} G(x, Q_0^2) \left[\log \frac{1}{x} \right]^{4C_A S(Q^2, Q_0^2)} \quad (5.8a)$$

$$\lambda(x, Q^2) \xrightarrow{x \rightarrow 1} \lambda(x, Q_0^2) \left[\log \frac{1}{x} \right]^{4C_A S(Q^2, Q_0^2)} \quad (5.8b)$$

$$q_t(x, Q^2) \xrightarrow{x \rightarrow 1} q_t(x, Q_0^2) \left[\log \frac{1}{x} \right]^{4C_F S(Q^2, Q_0^2)} \quad (5.8c)$$

$$S_+(x, Q^2) \xrightarrow{x \rightarrow 1} S_+(x, Q_0^2) \left[\log \frac{1}{x} \right]^{4C_F S(Q^2, Q_0^2)}. \quad (5.8d)$$

To derive the behaviour of the distributions for small x we must examine the leading poles in the Mellin transform, which (from Table 2) is at $N = 1$. In this case we have

$$Q^2 \frac{d}{dQ^2} \langle G \rangle_N \xrightarrow{N \rightarrow 1} \frac{\alpha(Q^2)}{8\pi} \frac{4C_A}{N-1} \left[\langle G \rangle_N + \langle \lambda \rangle_N + \frac{C_F}{C_A} (\langle q_t \rangle_N + \langle S_+ \rangle_N) \right] \quad (5.9)$$

whereas the Q^2 derivative of the Mellin transforms of all other distributions are regular at $N = 1$.

We seek a parameterization of the parton distribution which is consistent with Eqs. (5.5), (5.8) and (5.9) so that they correctly describe the Q^2 evolution of the low x limit, the large x limit and the fraction of momentum as carried by each constituent. We begin by decomposing the quark and squark distributions into "valence" and "sea" parts.

$$q_t(x, Q^2) = q_v(x, Q^2) + q_s(x, Q^2) \quad (5.10a)$$

$$S_+(x, Q^2) = S_v(x, Q^2) + S_s(x, Q^2) \quad (5.10b)$$

A parameterization of the parton distribution consistent with Eqs. (5.5), (5.8) and (5.9) is given by

$$\lambda(x, Q^2) = \frac{C_\lambda(Q^2)}{x} \left[\log \frac{1}{x} \right]^{B_\lambda(Q^2)} \quad (5.11a)$$

$$q_s(x, Q^2) = \frac{C_{q_s}(Q^2)}{x} \left[\log \frac{1}{x} \right]^{B_{q_s}(Q^2)} \quad (5.11b)$$

$$S_s(x, Q^2) = \frac{C_{S_s}(Q^2)}{x} \left[\log \frac{1}{x} \right]^{B_{S_s}(Q^2)} \quad (5.11c)$$

$$q_v(x, Q^2) = C_{q_v}(Q^2) x^{A_{q_v}(Q^2)} \left[\log \frac{1}{x} \right]^{B_{q_v}(Q^2)} \quad (5.11d)$$

$$S_v(x, Q^2) = C_{S_v}(Q^2) x^{A_{S_v}(Q^2)} \left[\log \frac{1}{x} \right]^{B_{S_v}(Q^2)} \quad (5.11e)$$

$$G(x, Q^2) = \frac{C_G(Q^2)}{x} R_G(x, Q^2) + \frac{C_\lambda(Q^2)}{x} R_\lambda(x, Q^2) - \lambda(x, Q^2) + \frac{C_F}{C_A} \left\{ \frac{C_{q_s}(Q^2)}{x} R_{q_s}(x, Q^2) + \frac{C_{S_s}(Q^2)}{x} R_{S_s}(x, Q^2) - q_s(x, Q^2) - S_s(x, Q^2) \right\} \quad (5.11f)$$

where

$$R_i(x, Q^2) = \Gamma(1 + B_i(Q^2)) \left[\frac{\log \frac{1}{x}}{a_i(Q^2)} \right]^{\frac{B_i(Q^2)}{2}} \cdot I_{B_i(Q^2)} \left(2 \sqrt{a_i(Q^2) \log \frac{1}{x}} \right) \quad (5.12)$$

where $I_\nu(x)$ is the modified Bessel function of order ν . The quantities $a_i(Q^2)$ are given by

$$a_G(Q^2) = a_\lambda(Q^2) = 4 C_A S(Q^2, Q_0^2) \quad (5.13a)$$

$$a_{q_s}(Q^2) = a_{S_s}(Q^2) = 4 C_F S(Q^2, Q_0^2) \quad (5.13b)$$

These quantities vanish as $Q^2 \rightarrow Q_0^2$ so that $G(x, Q_0^2)$ is given by

$$G(x, Q_0^2) = \frac{C_G(Q_0^2)}{x} \left[\log \frac{1}{x} \right] B_G(Q_0^2). \quad (5.14)$$

The quantities $A_{q_V}, B_{q_V}, C_{q_V}, A_{S_V}, B_{S_V}, C_{S_V}$ have been discussed in the previous section. B_{q_S}, B_{S_S}, B_G and B_λ are given by

$$B_{q_S}(Q^2) = B_{q_S}(Q_0^2) + 4C_F S(Q^2, Q_0^2) \quad (5.15a)$$

$$B_{S_S}(Q^2) = B_{S_S}(Q_0^2) + 4C_F S(Q^2, Q_0^2) \quad (5.15b)$$

$$B_G(Q^2) = B_G(Q_0^2) + 4C_A S(Q^2, Q_0^2) \quad (5.15c)$$

$$B_\lambda(Q^2) = B_\lambda(Q_0^2) + 4C_A S(Q^2, Q_0^2) \quad (5.15d)$$

where the quantities $B_{q_S}(Q_0^2), B_{S_S}(Q_0^2), B_G(Q_0^2), B_\lambda(Q_0^2)$ are obtained from the large x behaviour of the distributions below $Q^2 = m_t^2 (\equiv Q_0^2)$. We find from low energy phenomenology

$$B_{q_S}(Q_0^2) = B_{S_S}(Q_0^2) = 2.94 \quad (5.16a)$$

$$B_G(Q_0^2) = 3.6 \quad (5.16b)$$

$$B_\lambda(Q_0^2) = 3.3 \quad (5.16c)$$

The quantities $C_{q_S}(Q^2), C_{S_S}(Q^2)$ and $C_\lambda(Q^2)$ are related to the second moments of the distributions and $B_{q_S}(Q^2), B_{S_S}(Q^2), B_\lambda(Q^2)$ via the relations

$$C_{q_S}(Q^2) = \frac{\langle q_L^2(Q^2) \rangle_2 - \langle q_V^2(Q^2) \rangle_2}{\Gamma(1 + B_{q_S}(Q^2))} \quad (5.17a)$$

$$C_{S_S}(Q^2) = \frac{\langle S_+^2(Q^2) \rangle_2 - \langle S_V^2(Q^2) \rangle_2}{\Gamma(1 + B_{S_S}(Q^2))} \quad (5.17b)$$

$$C_\lambda(Q^2) = \frac{\langle \lambda^2(Q^2) \rangle_2}{\Gamma(1 + B_\lambda(Q^2))} \quad (5.17c)$$

where the second moments satisfy Eqs. (4.6) and (5.5). For the gluon distribution the evolution is more complicated because of the pole at $N = 1$ in the Mellin transforms of the kernels which govern the Q^2 evolution of the gluon distribution and give it a more complicated behaviour near $x = 0$. Equation (5.11f) is consistent with that Q^2 evolution near $x = 0$ and near $x = 1$ provided

$$G(Q^2) = \frac{\langle G(Q^2) \rangle_2 + \langle \lambda(Q^2) \rangle_2 (1 - e^{4C_A S}) + \frac{C_F}{C_A} (\langle q_s(Q^2) \rangle_2 + \langle S_s(Q^2) \rangle_2)}{\Gamma(1 + B_g(Q^2)) e^{4C_A S}} \quad (5.18)$$

From Eqs. (5.11) to (5.18) and the initial conditions at the squark threshold, Eqs. (5.6) the singlet quark, singlet squark, gluon and gluino distributions at any Q^2 can be calculated. We plot the singlet quark, total quark, singlet squark, total squark, gluon and gluino distributions for $\sqrt{Q^2} = 50, 200$, and 1000 GeV, in Figs. 5 to 10 respectively for the case of the gluino mass being taken to be 1 GeV and again in Figs. 11 to 16 respectively for a gluino mass of 17 GeV.

In the asymptotic limit, $Q^2 \rightarrow \infty$, all distributions are concentrated into small x , so that as well as Eq. (4.7) we have

$$x q_s(x, Q^2) \xrightarrow{Q^2 \rightarrow \infty} \frac{9}{25} \delta(x) \quad (5.19a)$$

$$x S_s(x, Q^2) \xrightarrow{Q^2 \rightarrow \infty} \frac{6}{25} \delta(x) \quad (5.19b)$$

$$x G(x, Q^2) \xrightarrow{Q^2 \rightarrow \infty} \frac{8}{25} \delta(x) \quad (5.19c)$$

$$x \lambda(x, Q^2) \xrightarrow{Q^2 \rightarrow \infty} \frac{2}{25} \delta(x) \quad (5.19d)$$

6. CONCLUSION

In this paper we have studied the short distance internal structure of nucleons (i.e. the structure functions at large Q^2) where supersymmetry may be considered to be exact.

In order to analyse hadron structure it was necessary to know how the various types of constituents are distributed at any Q^2 . We have been able to do this

using the well known low Q^2 quark and gluon distribution and calculating all Q^2 evolution kernels P_{ij} . In the supersymmetric limit we have derived several relations between these kernels which follow from $N = 1$ supersymmetry. There are further relations which follow from $N = 2$ supersymmetry when quarks and squarks are (formally) put into the adjoint representation. We have also shown that, in contrast to covariant gauges, the light cone axial gauge preserves the supersymmetry relations even for non gauge invariant quantities.

Finally, we have given a simple parameterization for quarks, squark, gluon and gluino distributions, which are consistent with the Q^2 evolution of the distributions at low x and large x ($x \approx 1$) and the Q^2 evolution of the second moments of the distributions (the fraction of the longitudinal momentum carried by each type of constituent). These distributions depend on the distributions at the threshold for squark production, which is taken as the reference momentum. These distributions may play an important role in future experiments involving hard scattering processes above the threshold for squark production. In particular the presence of squarks may radically affect the measurement of the ratio σ_L/σ_T in high Q^2 deep inelastic scattering, since for scalar particles this quantity does not vanish in leading order.

Above the threshold for the production of squarks some fraction of the nucleon's baryon number is carried by them. In the asymptotic limit there are two valence quarks and one valence squark rather than three valence quarks at thresholds. Also in the asymptotic limit the gluons carry 0.32 of the longitudinal momentum of the hadrons, gluinos 0.08, quarks 0.36 and squarks 0.24.

Since finishing this work we learned that S.K. Jones and C.H. Llewellyn Smith [18] have calculated the evolution kernels and studied the non-singlet and singlet solutions. Our results agree completely with them.

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Table 1

The analytic expression of the $P_{ij}(x)$ kernels of Eqs. (2.10) and (2.11). The $N = 1$ and $N = 2$ supersymmetry relations [Eqs. (3.4) and (3.5)] are automatically valid. The same is true for the relations (3.5). The group factors C_A , T_R and C_F for the $SU(N)$ colour with n_F^R right-handed and n_F^L left-handed flavours in the fundamental representation are given as $C_A = n$, $T_R = 1/2(n_F^R + n_F^L)$, $C_F = (N^2 - 1)/2N$.

$$P_{gg} = 2C_A \left[\frac{1+x^2}{(1-x)_+} + \frac{1+(1-x)^2}{x} - (x^2 + (1-x)^2) \right] + [3C_A - T_R] \delta(1-x)$$

$$P_{\lambda g} = 2C_A [x^2 + (1-x)^2]$$

$$P_{qg} = 2T_R [x^2 + (1-x)^2]$$

$$P_{sg} = 2T_R [1 - [x^2 + (1-x)^2]]$$

$$P_{g\lambda} = 2C_A \left[\frac{1+(1-x)^2}{x} \right]$$

$$P_{\lambda\lambda} = 2C_A \left[\frac{1+x^2}{(1-x)_+} \right] + (3C_A - T_R) \delta(1-x)$$

$$P_{q\lambda} = 2T_R [1-x]$$

$$P_{s\lambda} = 2T_R [x]$$

$$P_{gq} = 2C_F \left[\frac{1+(1-x)^2}{x} \right]$$

$$P_{\lambda q} = 2C_F (1-x)$$

$$P_{qq} = 2C_F \left[\frac{1+x^2}{(1-x)_+} \right] + 2C_F \delta(1-x)$$

$$P_{sq} = 2C_F [x]$$

$$P_{gs} = 2C_F \left[\frac{1+(1-x)^2}{x} - x \right]$$

$$P_{\lambda s} = 2C_F [1]$$

$$P_{qs} = 2C_F [1]$$

$$P_{ss} = 2C_F \left[\frac{1+x^2}{(1-x)_+} - (1-x) \right] + 2C_F \delta(1-x)$$

Table 2

The analytic expression of the anomalous dimensions γ_{ij}^n and γ_{ij}^2 .
 $\gamma_{ij}^n = \int_0^1 dx x^{n-1} P_{ij}(x)$. $s(n)$ is defined as $s(n) = \sum_{j=1}^n 1/j$.

	$\frac{n=2}{}$
$P_{gg}^n = \left(-2s(n) + \frac{2}{n-1} - \frac{2}{n} + \frac{2}{n+1} - \frac{2}{n+2} + \frac{3}{2} \right) 2C_A - \frac{1}{2} 2T_R$	$-\frac{1}{3} 2C_A - \frac{1}{2} 2T_R$
$P_{\lambda g}^n = \left(\frac{1}{n} - \frac{2}{n+1} + \frac{2}{n+2} \right) 2C_A$	$\frac{1}{3} 2C_A$
$P_{qg}^n = \left(\frac{1}{n} - \frac{2}{n+1} + \frac{2}{n+2} \right) 2T_R$	$\frac{1}{3} 2T_R$
$P_{sg} = \left(\frac{2}{n+1} - \frac{2}{n+2} \right) 2T_R$	$\frac{1}{6} 2T_R$
$P_{g\lambda} = \left(\frac{2}{n-1} - \frac{2}{n} + \frac{1}{n+1} \right) 2C_A$	$\frac{4}{3} 2C_A$
$P_{\lambda\lambda} = \left(-2s(n) + \frac{1}{n} - \frac{1}{n+1} + \frac{3}{2} \right) 2C_A - \frac{1}{2} 2T_R$	$-\frac{4}{3} 2C_A - \frac{1}{2} 2T_R$
$P_{q\lambda} = \left(\frac{1}{n} - \frac{1}{n+1} \right) 2T_R$	$\frac{1}{6} 2T_R$
$P_{s\lambda} = \left(\frac{1}{n+1} \right) 2T_R$	$\frac{1}{3} 2T_R$
$P_{gq} = \left[\frac{2}{n-1} - \frac{2}{n} + \frac{1}{n+1} \right] 2C_f$	$\frac{4}{3} 2C_f$
$P_{\lambda q} = \left[\frac{1}{n} - \frac{1}{n+1} \right] 2C_f$	$\frac{1}{6} 2C_f$
$P_{qq} = \left[-2s(n) + \frac{1}{n} - \frac{1}{n+1} + 1 \right] 2C_f$	$-\frac{11}{6} 2C_f$
$P_{sq} = \left[\frac{1}{n+1} \right] 2C_f$	$\frac{1}{3} 2C_f$
$P_{gs} = \left[\frac{2}{n-1} - \frac{2}{n} \right] 2C_f$	$1 2C_f$
$P_{\lambda s} = \left[\frac{1}{n} \right] 2C_f$	$\frac{1}{2} 2C_f$
$P_{qs} = \left[\frac{1}{n} \right] 2C_f$	$\frac{1}{2} 2C_f$
$P_{ss} = \left[-2s(n) + 1 \right] 2C_f$	$-2 \cdot 2C_f$

Figure captions

- Fig. 1 : $x q_v(x, Q^2)$ against x for different values of Q^2 , in the case where the gluino mass is taken to be 1 GeV.
- Fig. 2 : $x s_v(x, Q^2)$ against x for different values of Q^2 , in the case where the gluino mass is taken to be 1 GeV.
- Fig. 3 : $x q_v(x, Q^2)$ against x for different values of Q^2 , in the case where the gluino mass is taken to be 17 GeV.
- Fig. 4 : $x s_v(x, Q^2)$ against x for different values of Q^2 , in the case where the gluino mass is taken to be 17 GeV.
- Fig. 5 : $x q_s(x, Q^2)$ against x for different values of Q^2 , in the case where the gluino mass is taken to be 1 GeV.
- Fig. 6 : $x q_t(x, Q^2)$ against x for different values of Q^2 , in the case where the gluino mass is taken to be 1 GeV.
- Fig. 7 : $x s_s(x, Q^2)$ against x for different values of Q^2 , in the case where the gluino mass is taken to be 1 GeV.
- Fig. 8 : $x s_t(x, Q^2)$ against x for different values of Q^2 , in the case where the gluino mass is taken to be 1 GeV.
- Fig. 9 : $x G(x, Q^2)$ against x for different values of Q^2 , in the case where the gluino mass is taken to be 1 GeV.
- Fig. 10 : $x \lambda(x, Q^2)$ against x for different values of Q^2 , in the case where the gluino mass is taken to be 1 GeV.
- Fig. 11 : $x q_s(x, Q^2)$ against x for different values of Q^2 , in the case where the gluino mass is taken to be 17 GeV.
- Fig. 12 : $x q_t(x, Q^2)$ against x for different values of Q^2 , in the case where the gluino mass is taken to be 17 GeV.
- Fig. 13 : $x s_s(x, Q^2)$ against x for different values of Q^2 , in the case where the gluino mass is taken to be 17 GeV.

Fig. 14 : $x s_t(x, Q^2)$ against x for different values of Q^2 , in the case where the gluino mass is taken to be 17 GeV.

Fig. 15 : $x G(x, Q^2)$ against x for different values of Q^2 , in the case where the gluino mass is taken to be 17 GeV.

Fig. 16 : $x \lambda(x, Q^2)$ against x for different values of Q^2 , in the case where the gluino mass is taken to be 17 GeV.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12

Fig. 13

Fig. 14

Fig. 15

Fig. 16