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RADIATIVE FERMION MASSES AND SUPERSYMMETRY

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ABSTRACT

We explore various ways in which the observed hierarchical mass structure for fermions can be accounted for by successive radiative corrections in supersymmetric theories. Soft breaking of supersymmetry which may arise from gravitational effects can be relevant for a gravitino mass between 0.1 and 1 TeV.

The masses of the known quarks and charged leptons obey a hierarchical pattern. They come in families, or generations, of roughly the same masses which behave as $\alpha^2 : \alpha : 1$ with $\alpha \sim 1/20 - 1/50$. This suggests a mass generation programme in which the masses are generated successively by radiative corrections. It embodies the natural fact that all couplings are of the same order as the gauge coupling.

This picture has been advocated by many authors ¹⁾ in the context of grand unified theories. The main point is that there exists a family symmetry under which the various generations transform so that when the symmetry is broken, they obtain successive radiative masses. In fact, the entire mass matrix has elements of order $\alpha^2 M_t$, αM_t , M_t , where M_t is the mass of the heaviest quark which we typically set equal to M_W . The basic idea behind the mechanism is that due to the family symmetry, the fermions of the lower generations can only couple to non-zero vacuum expectation values (vev's) of such combinations of scalars which arise only in higher orders. These higher order graphs may also include soft breaking terms of the family symmetry in the scalar potential [see in particular the last reference of 1)]. In the following, soft breaking terms, both of supersymmetry and family symmetry, will be frequently introduced. This point was already held in Ref. 1). For a low energy effective theory, it is indistinguishable from a spontaneous breaking at high energies. This picture does not shed a deep light on the family symmetry, but it shows how a successful symmetry would function.

In supersymmetric ²⁾ models, the programme is more difficult to implement as pointed out by Ibáñez ^{3),*)}. The reason is that in order to have a radiative mass, we must not only break the family symmetry, but also supersymmetry. The one-loop mass is then typically of order

$$\delta m \sim \alpha \frac{\mu^2}{M^2} M_W \quad (1)$$

(see Fig. 1) where μ^2 characterizes the supersymmetry breaking scale and M the breaking scale of the family symmetry ^{*)}, ^{**)}. In the models of Ref. 1) M is related to the unification scale M_{GUT} while $\mu^2 \sim M_W \cdot M_{GUT}$ in most realistic models. This yields $\mu^2/M_{GUT}^2 \sim 10^{-13}$. Even if we consider a setting with supergravity then $\mu^2/M_{GUT}^2 < 10^{-4}$ ⁵⁾. Thus it appears that radiative mass generation and supersymmetry are incompatible.

*) Radiative mass generation has been considered, e.g., by Nappi and Ovrut ⁴⁾. In their model all fermions obtain a mass by the same radiative mechanism (see Fig. 2b). The family problem remains as in the standard model.

**) Relation (1) does not really assume that the heaviest fermion generation is $\sim M_W$; also if the intermediate fermion is superheavy we expect M_W in (1). [See, e.g., Barbieri and Nanopoulos ¹⁾].

Let us examine why $M \sim M_{\text{GUT}}$. For definiteness we use the framework of the standard electroweak $SU(2) \times U(1)$ theory. In order that quarks and leptons get a mass at all, they must couple to scalars. For the lower generations, these scalars should not develop a vacuum expectation value, and for ordinary Higgs doublets of charge +1 and 0, this is not guaranteed, in general, by any symmetry. Therefore we must require that these scalars are charged and, as it turns out, they are also coloured. Thus they may mediate proton decay and should have large masses of the order of M_{GUT} .

In supersymmetric theories, however, this argument does not hold for two reasons.

- a) Due to the higher symmetry it might be possible to have ordinary doublets without vev's and hence to have diagrams as in Fig. 1 with Higgs ^{*)} scalars in the boson loop.
- b) There are coloured scalars, the superpartners of quarks which cannot mediate proton decay and need not be superheavy. They can also run in boson loops. Diagrams of this type include the supersymmetric partner of the diagram mentioned in a), as well as the diagrams of Fig. 2.

In the following we consider the various ways in which the fermions can acquire radiative masses. We will generally allow for the most general type of supersymmetric breaking in the low energy theory, spontaneous or soft ⁶⁾. In the latter case we include the possibility that soft breaking is induced by $N=1$ supergravity ⁷⁾.

The possible family symmetries are restricted. Take, for instance, a three-family model with horizontal $SU_H(3)$ symmetry. Left- and right-handed quarks are grouped in the three-representation of $SU_H(3)$ and the Higgs fields belong either to an octet or a singlet. Without the singlet, the mass matrix is traceless, and if the $(3,3)$ entry is of order M_w , so is at least one of the other two diagonal elements, and we would have two large eigenvalues. Adding the singlet solves the problem, but at a price of an unwanted fine tuning. Similar arguments can be made for any non-Abelian family symmetry. This suggests considering only Abelian symmetries. Analogous arguments against non-Abelian symmetries can be made from family mixing considerations.

*) We distinguish between matter fields, containing the quarks Q , leptons L and their scalar partners SQ , SL and Higgs fields containing Higgs scalars and their fermionic partners SH . All fields will be taken to be left-handed; Q^c , L^c denote the charge conjugated right-handed fields.

Before continuing, we should say that not only must we reproduce the right mass scale pattern, but also account for Cabibbo mixing. A sizeable Cabibbo angle (greater than m_d/m_s) requires some off-diagonal entries of the quark mass matrix to be of order M_W , rather than just αM_W ⁸⁾. This will prove to be a severe constraint.

We first consider the mechanism of Fig. 3a. The superpotential is

$$W = Q_i Q_j^c H_{ij} + \dots \quad (2)$$

where i, j are the generation indices and W is assumed to have a certain symmetry, the family symmetry, which allows H_{ij} to couple only to Q_i and Q_j^c (*). In order to obtain the right form for the mass matrix [e.g., case 2 of Ref. 8] we require

$$\langle H_{32} \rangle \neq 0, \quad \langle H_{33} \rangle \neq 0$$

and $\langle H_{ij} \rangle = 0$ ***) otherwise.

The crucial point is that $\langle H_{32} \rangle \neq 0$ in order to have a sizeable Cabibbo angle. To obtain one loop masses in the entries (2,2) and (2,3) we must have the diagram of Fig. 3a. However, this implies that of Fig. 4, leading to a tree mass for the second generation. As to the graphs of Fig. 3b, no such conflict appears. In fact, we shall see that this mechanism can be attributed to supergravity where the soft breaking term $(SQ_1 SQ_j^c)$ appearing in the diagram of Fig. 3b is connected to the gravitino mass.

Consider for instance the model of Barbieri, Ferrara and Savoy⁷⁾ in which supersymmetry is broken by supergravity. For simplicity we take only two generations (Q_1 and Q_2) and write a superpotential

$$W = \lambda_1 Q_1 Q_2^c H + \lambda_2 Q_2 Q_1^c H + \lambda_3 Q_2 Q_2^c H' + \beta H^2 T + \dots \quad (3)$$

The Yukawa couplings are constructed in the usual way, whereas the scalar potential is^{7),9)}

$$V = \sum_i \left| \frac{\partial W}{\partial \varphi_i} \right|^2 + m_{3/2}^2 \sum_i |\varphi_i|^2 + A m_{3/2} (W + W^*) + \frac{1}{2} \sum_a (D^a)^2 \quad (4)$$

*) Actually H_{ij} can be linear combination of fields $\phi_{ij}^{(a)}$ which couple to $Q_i Q_j^c$ but this does not change the argument.

**) We will denote by $\langle \phi \rangle$ the vev of the scalar component of the superfield ϕ .

where A is a number determined by supergravity and $m_{\frac{3}{2}}$ the mass of the spin $\frac{3}{2}$ gravitino. The complete superpotential is assumed to be such that $\langle H' \rangle, \langle T \rangle \neq 0$; $\langle H \rangle = 0$ ^{*}). Then, Figs. 3a and 3b yield ^{**})

$$\begin{aligned} \delta m_a &= \frac{A \lambda_1 \lambda_2 m m_F m_{\frac{3}{2}}}{16\pi^2} \int_0^1 \frac{x dx}{m^2(1-x) + m_{\frac{3}{2}}^2 x} \\ \delta m_b &= \frac{A \lambda_1 \lambda_2 m m_F m_{\frac{3}{2}}}{16\pi^2} \int_0^1 \frac{x dx}{m_F^2(1-x) + m_{\frac{3}{2}}^2 x} \end{aligned} \quad (5)$$

where $m = \beta \langle T \rangle$ and $m_F = \lambda_3 \langle H' \rangle$. Notice that the couplings in both terms are equal due to the universal form of the soft breaking. Though, as it stands the model does not allow Cabibbo mixing ^{***}), it is interesting to note that if $m \sim m_{\frac{3}{2}}$, one gets, setting $m \sim m_{\frac{3}{2}}$, $m_F < m_{\frac{3}{2}}$

$$\delta m \sim O(1) \frac{\alpha}{\pi} m_F \quad (6)$$

where we have set $\lambda_1 \lambda_2 / 4\pi = \alpha$, since the λ_1, λ_2 can be naturally of the order of the gauge coupling. Thus, if $m \sim m_{\frac{3}{2}}$, we get just the right ratios and values for the fermion masses. m is related to the weak mass scale m_W , thus $m_{\frac{3}{2}}$ lies between 0.1 and 1 TeV.

We next turn to case b). Figure 2 shows the possible diagrams. We treat them in turn, considering also their extension to two loops.

- i) In the diagram 2a ¹⁰⁾, the mixing of left and right-handed fields (Q and Q^c , respectively) occurs at the tree level only for the scalar partners of the fermions. The soft Majorana gaugino mass term may be due to supergravity ¹¹⁾. The diagram occurs, because an R symmetry might forbid QQ^c mass terms, but not $SQ SQ^c$ terms. The gaugino mass breaks this symmetry, giving rise to the radiative QQ^c mass. For the quark masses, both the $SU(3)_c$ and $SU(2) \times U(1)$ gauginos are effective; as to the lepton masses, only the latter can contribute. This might explain why quarks are heavier than corresponding leptons. The order of magnitude estimate for a one-loop radiative mass is $\sim \alpha / \pi m_{\tilde{g}}$, if all masses in the scalar sector are similar, as is the case in models where supergravity induces supersymmetry breaking ⁷⁾.

^{*}) Conditions for this can be constructed along the lines of Ref. 9).

^{**}) For simplicity, we have set all scalar masses equal to $m_{\frac{3}{2}}$ inside the Feynman integral.

^{***}) It is impossible to generate a (fermionic) generational mixing by radiative corrections because of the special form of the supersymmetric breaking in V , which is "proportional" to the Yukawa couplings.

The gaugino mass term may be generated perturbatively through the one loop graphs of Fig. 3c. They are typically of order

$$\delta m \sim \alpha(1) \frac{\alpha}{\pi} \frac{m_{H(Q)}^2}{m_{3/2}} \quad (7)$$

where $m_{H(Q)}$ is the mass of H or of the heaviest quark, and α the fine structure constant in question. For the partners of W and Z this gives a mass between 1 and 10 GeV, if $m_H \sim m_{\frac{3}{2}}$ ^{*)}; for gluinos it is ~ 1 GeV ^{**)} if $\alpha_s \sim 0.1$ and $m_Q \sim 30$ GeV. Then, the graph 2a occurs only at the two-loop level; it would give the lowest generation the desired radiative mass. It is remarkable that there is a natural way to obtain two-loop masses. If we take all Majorana gaugino masses to be roughly equal, and set $\alpha_{em} \sim 0.1 \alpha_{strong}$ then the electron is roughly ten times lighter than the up and down quarks. In this programme the middle generation might get a one-loop mass through some of the mechanisms described below.

- ii) The diagram 2b implies a direct mass term between SQ_k and SQ_j . We see that only the case 2) [Eq. (7)] of Ref. 8) is applicable. To reproduce this matrix, breaking terms $SQ_i^* SQ_j$ are needed to obtain the one-loop elements of the mass matrix. As to the lowest generation, it could get a two-loop mass as outlined under a). Alternatively, we note that Q_1^C can couple only to Higgs fields with no vev in order to avoid one-loop masses. This leads then immediately to the two-loop graph of Fig. 5, which requires soft terms of the form $SQ_i SQ_j^C H$. Again, there exists a natural two-loop mechanism for the smallest masses.

We can also replace Q by Q^C and vice versa. This gives an analogous contribution; notice that only the $U(1)$ gaugino can contribute in this case.

To summarize, we have shown in this note that supersymmetric theories allow for a variety of ways in which fermion masses arise radiatively. All involve soft breaking terms (of supersymmetry and family symmetry) for the scalar partners of the fermions ^{***)}. In $N=1$ local supersymmetry soft breaking terms are automatically present, and are related to the gravitino

*) If there are coloured Higgs fields, this mechanism would give the gluinos a mass bigger than 10 GeV.

**) Such gluino masses might be soon testable in experiments.

***) This essentially excludes spontaneously broken $N=1$ rigid supersymmetry and horizontal symmetries as source of radiative masses.

mass, $m_{\frac{3}{2}}$ ⁷⁾. For a typical value, $m_{\frac{3}{2}} \gtrsim m_W$, it is indeed possible to obtain the desired generational hierarchy $\alpha^2 M_W, \alpha M_W, M_W$ for the fermion masses. It should be noted that because the soft breaking terms of the scalar quarks and leptons are of the same order of magnitude, quarks and leptons of one family get roughly the same mass. This important fact is usually only accounted for by grand unification. Contrary to some of the examples in Ref. 1), the neutrinos need not acquire a Majorana mass. Unlike in ordinary theories ¹²⁾, it might be possible to find models in which there are no (or heavily suppressed) flavour changing neutral Higgs exchange; due to the many scalars coupling to the lower generations, one ordinary Higgs might be sufficient to give radiative masses.

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FIGURE CAPTIONS

- Figure 1 : Typical one-loop mass in a supersymmetric theory. f, f', ψ, ψ' are two-component Weyl spinors, A and A' are scalars and F is the auxiliary field of a chiral multiplet.
- Figure 2a : One-loop radiative mass with squarks (SQ_i, SQ_j^C) and gauginos $(\tilde{g})$ in the internal lines.
- Figure 2b : One-loop radiative mass. SQ_i are squarks and SU_{kj} Higgs fermions.
- Figure 3a : One-loop diagram giving rise to a mass of the second generation. The family symmetry is broken at the $H_{33}H_{32}(3)^F$ vertex or spontaneously [see Ref. 6].
- Figure 3b : Radiative quark mass. The dashed lines are squarks SQ or SQ^C while the solid internal line is the internal line Higgs fermion SH, SH' .
- Figure 3c : Radiative gaugino mass. H denotes Higgses and SH Higgs fermions.
- Figure 4 : $Q_2 Q_2^C$ mass term.
- Figure 5 : Two-loop radiative mass for the lowest generation.

Figure 1

Figure 2a

Figure 2b

Figure 3a

Figure 3b

Figure 3c

Figure 4

Figure 5