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IMPROVED SUPERCONFORMAL GAUGE CONDITIONS IN THE
 $N = 1$ SUPERGRAVITY - YANG - MILLS - MATTER SYSTEM

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ABSTRACT

Superconformal tensor calculus greatly simplifies the derivation of the component form of the Lagrangian and supersymmetry transformation laws for supergravity-matter system generally, if the extraneous gauge freedoms are fixed by a "natural" set of gauge conditions. We show this explicitly for the Yang-Mills-matter supergravity system recently discussed by Cremmer et al.. The present technique would be useful for the further study of a more general matter-supergravity system.

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1. - INTRODUCTION

Supersymmetric gauge theories have received much attention as possible candidates for the grand unified theories in which the hierarchy problems may be solved¹⁾. Recently, many authors²⁾ have proposed possible scenarios in which the mass scale of supersymmetry breaking is taken to be very large, say of order 10^{10-15}GeV , mainly in connection with the mass splitting problem between quarks/leptons and their scalar partners. It has further been recognized that in such cases the effects of gravity (supergravity) become non-negligible and play even crucial roles in such problems as mass splitting^{3),4)}, gauge hierarchy⁵⁾ and others⁶⁾.

To understand the effects of supergravity in detail, Cremmer, Ferrara, Girardello and Van Proeyen³⁾ discussed a general Yang-Mills-matter system coupled to $N = 1$ supergravity and derived the explicit forms of the Lagrangian and transformation laws in terms of component fields. Their paper is a generalization of Ref. 7) discussing the superHiggs effect for the system containing only one chiral matter multiplet. The calculations in Refs. 3), 7) are, however, very complicated and not easy to trace. The laborious steps included in their derivations are:

- i) Weyl rescaling of the vierbein,
- ii) chiral rotations on the fermion fields,
- iii) recombination of the functions ϕ (of the kinetic term) and g (of the superpotential) into $\mathcal{G} \propto \ln(\phi/|g|^{2/3})$ and
- iv) elimination of auxiliary fields.

The first three steps i) - iii) can, however, be simply bypassed if one uses the superconformal tensor calculus (instead of the superpoincaré one) and chooses a suitable set of gauge fixing conditions for extraneous gauge freedoms in the superconformal theory. The simplification achieved by this is in fact double because those steps of calculation are included in both the derivations of Lagrangian and transformation laws. We intend to show this in the present paper. This is rather technical but we believe that such a technique becomes useful for the further study of more general matter-supergravity systems of $N = 1$ and 2. The reader will recognize that the superconformal tensor calculus, as a practical calculational tool, is much more flexible and convenient than the usual Poincaré one.

Following Cremmer et al.³⁾, we consider the coupled Yang-Mills-matter-supergravity system, the Lagrangian of which is given in superspace by

$$\mathcal{L} = \mathcal{L}_S + \mathcal{L}_{YM} ,$$

$$\mathcal{L}_S = \int d^4\theta \ E \left[\tilde{\Phi}(S, \bar{S} e^{2V_G}) + \text{Re}\left(\frac{1}{R} g(S)\right) \right] , \quad (1.a)$$

$$\mathcal{L}_{YM} = \int d^4\theta \ E \ \text{Re}\left(\frac{1}{R} f_{\alpha\beta}(S) W^{\alpha\tau} \mathcal{C} W^\beta\right) . \quad (1.b)$$

[$\mathcal{C}$: charge conjugation matrix.] Here S_i ($\bar{S}^i$) is a set of chiral (anti-chiral) matter multiplets which transform according to some (generally reducible) representation of a compact group G , and $V_G \equiv \bar{g} V^\alpha T^\alpha$ is a Yang-Mills (YM) vector multiplet, W^α being the chiral spinor superfield containing the YM field strength. The G invariance of the Lagrangian requires that the function $\tilde{\Phi}(z, z^*)$ (real) and $g(z)$ satisfy the conditions

$$\tilde{\Phi}^i T_i^{\alpha j} z_j = z^{*i} T_i^{\alpha j} \tilde{\Phi}_j , \quad g^i T_i^{\alpha j} z_j = 0 , \quad (2)$$

and that $f_{\alpha\beta}(z)$ transform as the symmetric product of the two adjoint representations of G . In this paper we denote the derivatives of functions by abbreviations such as $g^i = \partial g / \partial z_i$, $\tilde{\Phi}_j^i = \partial^2 \tilde{\Phi} / \partial z_i \partial z^{*j}$ etc.

In Section 2, we present our "improved" gauge fixing conditions for this system for the the extraneous gauge freedoms in superconformal theory, which enable us to bypass the laborious steps i) - iii) cited above. This is done after explaining some of the conformal tensor calculus formulae. We refer the reader to Refs. 8) and 9) for the details of superconformal tensor calculus. Based on that gauge, we derive the explicit component form of the Lagrangian in Section 3. For clarity, we intend to greatly systematize our calculations. Section 4 is devoted to the derivation of the supersymmetry transformation law.

2. - IMPROVED GAUGE FIXING CONDITIONS

In the superconformal framework the Lagrangian (1) is written as^{*)}

$$\mathcal{L}_S = -\left(\frac{1}{2}\right) \left[\tilde{\Phi}(S, \bar{S} e^{2V_G}) \Sigma_c \bar{\Sigma}_c \right]_D + \left[g(S) \Sigma_c^3 \right]_F , \quad (3.a)$$

$$\mathcal{L}_{YM} = -\left(\frac{1}{4}\right) \left[f_{\alpha\beta}(S) W^{\alpha\tau} \mathcal{C} W^\beta \right]_F , \quad (3.b)$$

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^{*)} Here the coefficients $-(1/2)$ and $-(1/4)$ in (3) are added just for later convenience.

where Σ_C is a chiral superfield which carries conformal (Weyl) weight $\lambda = 1$ and plays the role of a "compensating multiplet"¹⁰⁾ corresponding to the case of $N = 1$ minimal Poincaré supergravity. $[ ]_D$ indicates the superconformal invariant D-type action formula applicable to the (real) vector multiplet $V_{(\lambda=2)} = [C, Z, H, K, B_m, A, D,]$ with conformal weight $\lambda = 2$ ⁸⁾:

$$[V_{(\lambda=2)}]_D = e \left\{ D - \frac{1}{2} \bar{\Psi} \cdot \gamma_i \gamma_5 \Lambda - \bar{\Phi} \cdot \gamma_i \gamma_5 Z + \frac{1}{3} C (R + e^{-1} \bar{\Psi} \cdot R) + \frac{i}{4} \epsilon^{mnkl} (\bar{\Psi}_m \gamma_n \psi_k) (B_l - \frac{1}{2} \bar{\Psi}_l Z - C A_l) \right\}, \quad (4)$$

and $[ ]_F$ is the F-type formula¹⁰⁾ valid for the chiral multiplet $\Sigma_{(\lambda=3)} = [z, \chi_R, h]^*$ with conformal weight $\lambda = 3$:

$$\begin{aligned} [\Sigma_{(\lambda=3)}]_F &= e (h + \bar{\Psi}_L \cdot \gamma \chi_R + \bar{\Psi}_{mL} \sigma^{mn} \psi_{nL} z) + h.c. \\ &= e \left\{ 2F + \bar{\Psi} \cdot \gamma \chi + \bar{\Psi}_m \sigma^{mn} (A - i \gamma_5 B) \psi_n \right\}. \end{aligned} \quad (5)$$

[Here $z \equiv A + iB$, $h \equiv F + iG$ in terms of real fields A, B, F and G .]
Since the matter multiplets $S_i = [z_i, \chi_{Ri}, h_i]$ and the YM vector multiplet V_G carry vanishing conformal weight $\lambda = 0$, $\Phi(S, \bar{S} e^{2VG})$ and $g(S)$ also have $\lambda = 0$. Therefore, in order to apply the D-term and F-term formulae to Φ and g , respectively, one must adjust their conformal weights by multiplying the compensating multiplet Σ_C as shown in (3.a). On the other hand, such a compensation is unnecessary for the YM kinetic term since the superfield strength W^α carries $\lambda = 3/2$ and hence $f_{\alpha\beta} W^\alpha W^\beta$ already has the correct weight $\lambda = 3$ for the F-term. [See Refs.8), 9) and 11) for details.].

To go to the Poincaré supergravity theory, one fixes the extraneous gauge freedoms of superconformal theory^{10),12)}, i.e., dilatation ($\underline{D}$), chiral ($\underline{A}$), conformal supersymmetry ($\underline{S}$) and special conformal ($\underline{K}_m$) gauges. The usual gauge choice to lead to the conventional tensor calculus in the minimal Poincaré supergravity is:

$$\begin{aligned} \underline{D} ; \quad Re z_c = \sqrt{3} \quad , \quad \underline{A} ; \quad Im z_c = 0 \quad , \\ \underline{S} ; \quad \chi_c = 0 \quad , \quad \underline{K}_m ; \quad b_\mu = 0 \quad , \end{aligned} \quad (6)$$

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*) For convenience, we have slightly changed the normalization convention of the chiral multiplet from that adopted in Ref.8) in which we denoted it by $\Sigma = [\mathcal{A}, \mathcal{T}_R \chi, \mathcal{F}]$. The relations are $z = 2\mathcal{A}$, $\chi_R = \mathcal{T}_R \chi$, $h = 2\mathcal{F}$, or equivalently, $Z = \mathcal{A}$, $2\chi_R = \mathcal{T}_R \chi$, $h = \mathcal{F}$.

where z_c and χ_c are the components of the compensating multiplet $\Sigma_c = [z_c, \chi_{RC} \equiv (1+\gamma_5)\chi_c/2, h_c]$ and b_μ is the D-gauge field.

In the present context, however, there is a better gauge choice as announced. Before stating the improved gauge conditions, some preparation is needed. First we introduce a rescaling on the compensating multiplet

$$\Sigma_o \equiv g(S)^{\frac{1}{3}} \Sigma_c \quad (7)$$

and rewrite the Lagrangian (3.a) as ^{*) **)}

$$\begin{aligned} \mathcal{L}_S = & -\frac{i}{2} [\phi(S, \bar{S} e^{2V_G}) \Sigma_o \bar{\Sigma}_o]_D + [\Sigma_o^3]_F, \\ \phi \equiv & \tilde{\phi}(S, \bar{S} e^{2V_G}) \cdot |g(S)|^{-2/3}. \end{aligned} \quad (8)$$

This expression clearly shows that the action only depends on a particular combination $\phi(z, z^*)$ of the functions $\tilde{\phi}(z, z^*)$ and $g(z)$, as has already been noticed by many authors^{7), 3), 13)}. Taking the action of the form (8) from the start makes it possible to skip a considerable part of step iii) automatically. Next we write down the component form of $\phi(S, \bar{S} e^{2V_G})$. As in Ref.3), this can be done most easily in the Wess-Zumino gauge in which $V^\alpha = [0, 0, 0, 0, B_m^\alpha, \lambda^\alpha, D^\alpha]$ and

$$\begin{aligned} \phi(S, \bar{S} e^{2V_G}) = & \phi(S, \bar{S}) + 2\tilde{g} \bar{S}^i T_i^{\alpha j} \phi_j V^\alpha \\ & + 2\tilde{g}^2 (\bar{S}^i T_i^{\alpha k} \bar{S}^j T_j^{\beta l} \phi_{kl} + \bar{S}^i T_i^{\alpha k} T_k^{\beta j} \phi_j) V^\alpha V^\beta, \end{aligned} \quad (9)$$

since $V^\alpha V^\beta V^\gamma = 0$. By using the component formula for arbitrary functions $f(S, \bar{S})$ and the multiplication law in conformal tensor calculus⁸⁾, we find the components of the vector multiplet ϕ :

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*) The expression (8) is valid as far as $g(S) \neq 0$. In perturbation theory, it will make sense if we expand around the point $S = S_o$ such that $g(S_o) \neq 0$. Further, the final expression obtained by this procedure turns out to be valid even when $g = 0$.

**) Note that $g^*(\bar{S} e^{2V_G}) = g^*(\bar{S})$ by the G invariance of $g(S)$.

$$\begin{aligned}
C(\phi) &= \phi(z, z^*) , \quad Z(\phi) = 2i (\phi_i \chi_L^i - \phi^i \chi_{Ri}) , \\
\frac{1}{2} [H(\phi) + iK(\phi)] &= -\phi^i h_i + \phi^{ij} \bar{\chi}_{Ri} \chi_{Rj} , \\
B_m(\phi) &= i (\phi^i \hat{D}_m z_i - \phi_i \hat{D}_m z^{*i}) - 2i \phi_j^i \bar{\chi}_L^j \gamma_m \chi_{Ri} , \\
\frac{1}{2} \Lambda_R(\phi) &= \tilde{g} \phi^\alpha \lambda_R^\alpha + i \{ \phi_j^i (h^{*j} + \hat{\mathcal{D}} z^{*j}) - \phi_{jk}^i (\bar{\chi}_L^j \chi_L^k) \} \chi_{Ri} , \\
\frac{1}{2} D(\phi) &= \tilde{g} \phi^\alpha D^\alpha - 2i \tilde{g} \bar{\lambda}^\alpha (\phi_i^\alpha \chi_L^i - \phi^{\alpha i} \chi_{Ri}) \\
&\quad + \phi_j^i h_i h^{*j} - \{ \phi_{jk}^i h_i (\bar{\chi}_L^j \chi_L^k) + \phi_i^{jk} h^{*i} (\bar{\chi}_{Rj} \chi_{Rk}) \} \\
&\quad + \phi_{kl}^{ij} (\bar{\chi}_{Ri} \chi_{Rj}) (\bar{\chi}_L^k \chi_L^l) \\
&\quad - (\bar{\chi}_L^i \gamma^m \chi_{Rj}) (\phi_i^{jk} \hat{D}_m z_k - \phi_{ik}^j \hat{D}_m z^{*k}) \\
&\quad - \phi_j^i (\hat{D}_m z_i \hat{D}_m z^{*j} + \bar{\chi}_L^j \hat{\mathcal{D}} \chi_{Ri} + \bar{\chi}_{Ri} \hat{\mathcal{D}} \chi_L^j) .
\end{aligned} \tag{10}$$

Here

$$\phi^\alpha \equiv z^{*i} T_i^{\alpha j} \phi_j = \phi^i T_i^{\alpha j} z_j \tag{11}$$

and the derivatives $\hat{D}_\mu$ are fully conformal and YM covariant:

$$\begin{aligned}
\hat{D}_\mu z_i &= D_\mu^c z_i - i \tilde{g} T_i^{\alpha j} B_\mu^\alpha z_j ; \quad D_\mu^c z_i = \partial_\mu z_i - \bar{\psi}_{\mu R} \chi_{Ri} - \frac{1}{2} i \lambda A_\mu z_i , \\
\hat{D}_\mu \chi_{Ri} &= D_\mu^\omega \chi_{Ri} - i \tilde{g} T_i^{\alpha j} B_\mu^\alpha \chi_{Rj} - \frac{1}{2} (\hat{\mathcal{D}} z_i \psi_{L\mu} + h_i \psi_{R\mu}) \\
&\quad + i \left(\frac{3}{4} - \frac{\lambda}{2} \right) A_\mu \chi_{Ri} - \lambda z_i \varphi_{R\mu} ,
\end{aligned}$$

where $D_{\mu}^C(D_{\mu}^{\omega})$ is the conformal (local Lorentz) covariant derivative and the conformal weight λ is kept for later convenience although $\lambda = 0$ for this multiplet S_i . The component formula (10) is general enough to hold for any functions of the form $\phi(S, \bar{S}e^{2V_G})$, and applicable even directly to the function

$$\Phi(S, \bar{S}e^{2V_G}, \Sigma_0, \bar{\Sigma}_0) \equiv \phi(S, \bar{S}e^{2V_G}) \Sigma_0 \bar{\Sigma}_0 \quad (13)$$

that appears in the D-type term of $\mathcal{L}_S$ (8). Regarding the compensating field Σ_0 as a G-singlet "matter" field $S_0 \equiv [z_0, \chi_{R0}, h_0]$ with index 0, we treat Σ_0 and the matter fields S_i on the same footing and denote them inclusively by $S_I = (S_0, S_i)$ with capital index I ^{*)}. We can then read the formula (10) for the multiplet (13) by replacing ϕ by Φ and the lower case indices i by the capital one I ; [notice that we should take $\lambda = 1$ for the $I = 0$ case in covariant derivatives (12)]; for instance,

$$\begin{aligned} C(\Phi) &= \Phi(z, z^*) = \phi(z, z^*) / |z_0|^2, \\ \frac{1}{2} Z(\Phi) &= i (\Phi_I \chi_L^I - \Phi^I \chi_{RI}) \\ &= i |z_0|^2 (\phi_i \chi_L^i - \phi^i \chi_{Ri}) + i \phi (z_0 \chi_L^0 - z^{*0} \chi_{R0}). \end{aligned} \quad (14)$$

We are now ready to state our gauge conditions. In view of (14) and the D-term action formula (4), it would be clear that the most convenient gauge in the present case is such that one realizes

$$C(\Phi) = 3 \quad \text{and} \quad Z(\Phi) = 0. \quad (15)$$

So we choose the following gauge fixing conditions:

$$\begin{aligned} \underline{D}; \quad \text{Re } z_0 &= \sqrt{3} \phi^{-\frac{1}{2}}(z, z^*), \quad \underline{A}; \quad \text{Im } z_0 = 0, \\ \underline{S}; \quad \chi_{R0} &= -z_0 \phi^{-1} \phi^i \chi_{Ri}, \quad \underline{K}_m; \quad b_{\mu} = 0. \end{aligned} \quad (16)$$

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^{*)} In order to avoid confusion with the chirality index L of χ_L we use only I, J, K, M for this capital index.

This D-gauge choice guarantees the appearance of the Einstein and Rarita-Schwinger terms in canonical form $-(1/2)(R + e^{-1}\bar{\psi}\cdot R) \equiv \mathcal{L}_{SG}$ in $\mathcal{L}_S$, thus making the Weyl rescaling process i) stated in the Introduction unnecessary. The chiral rotation step ii) is also skipped by virtue of the fact that the A-gauge condition has been imposed on the rescaled compensating field $S_0(=\Sigma_0) = g^{1/3}(S)\Sigma_C$. Further, the S-gauge choice (16) not only enables us to avoid the shift of the Rarita-Schwinger field⁷⁾, $\psi_{\mu R} \rightarrow \psi_{\mu R} - \gamma_\mu \phi^{-1} \phi_i \chi_L^i$, but also greatly simplifies the algebraic calculations below.

3. - DERIVATION OF THE COMPONENT FORM OF THE LAGRANGIAN

Settling the gauge fixing conditions, we can write down the D-term Lagrangian $[\Phi]_D$ by applying the formulae (4) and (10) to Φ . Then we simultaneously rewrite the full covariant derivatives $\hat{D}_\mu$ with the following ones ∇_μ which are covariant only under YM and local-Lorentz gauge transformations:

$$\begin{aligned} \nabla_\mu z_i &\equiv \partial_\mu z_i - i\tilde{g} T_i^{\alpha j} B_\mu^\alpha z_j, \\ \nabla_\mu \chi_{Ri} &\equiv (\partial_\mu + \frac{1}{2}\omega_\mu^{mn}(e,\psi)\sigma_{mn})\chi_{Ri} - i\tilde{g} T_i^{\alpha j} B_\mu^\alpha \chi_{Rj}. \end{aligned} \quad (17)$$

Interestingly enough the result written with ∇_μ is no more complicated than that with $\hat{D}_\mu$ since many cancellations of superconvariantization terms (i.e. containing ψ_μ) of $\hat{D}_\mu$ occur between $D(\Phi)$ and $-(1/2)\bar{\psi}\cdot\gamma_5\Lambda(\Phi)$ in $[\Phi]_D$. We thus obtain:

$$\begin{aligned} -\frac{1}{2}e^{-1}[\Phi]_D &= \Phi_J^I \nabla_m z_I \nabla_m z^{*J} + \{ \Phi_J^I \bar{\chi}_L^J \not{D} \chi_{RI} + h.c. \} + \mathcal{L}_{SG} \\ &+ \{ \Phi_J^{IK} (\bar{\chi}_L^J \gamma^m \chi_{RI}) \nabla_m z_K - \Phi_J^I (\bar{\psi}_{mL} \not{D} z_I \gamma_m \chi_L^J) + h.c. \} \\ &- \Phi_{KM}^{IJ} (\bar{\chi}_{RI} \chi_{RJ}) (\bar{\chi}_L^K \chi_L^M) - \frac{1}{2} \Phi_J^I (\bar{\chi}_L^J \gamma_m \chi_{RI}) (\bar{\psi}_{nR} \gamma_m \psi_{nL}) \\ &+ \frac{1}{8} \epsilon^{mnkl} (\bar{\psi}_m \gamma_n \psi_k) \{ (\Phi^I \nabla_l z_I - \Phi_I \nabla_l z^{*I}) + 2 \Phi_J^I (\bar{\chi}_L^J \gamma_l \chi_{RI}) \} \\ &- i\tilde{g} \{ 2\Phi^{I1} \bar{\chi}_{RI} \lambda_R^\alpha - \frac{1}{2} \Phi^\alpha \bar{\psi}_L \cdot \gamma \lambda_R^\alpha - h.c. \} - \tilde{g} \Phi^\alpha D^\alpha \\ &- \Phi_J^I h_I h^{*J} + \{ \Phi_{JK}^I (\bar{\chi}_L^J \chi_L^K) h_I + h.c. \} + \frac{3}{4} A_m^2 \\ &- \frac{1}{2} A_m \{ i z_0 \Phi_J^I \nabla_m z^{*J} + i (z_0 \Phi_J^{I0} - \frac{3}{2} \Phi_J^I) (\bar{\chi}_L^J \gamma_m \chi_{RI}) + h.c. \}, \end{aligned} \quad (18)$$

where $\phi^\alpha = \phi_{\mathbf{I}}^{\mathbf{I}\alpha\mathbf{J}} z_{\mathbf{J}} = \phi_{\mathbf{I}}^{\mathbf{I}\alpha\mathbf{j}} z_{\mathbf{j}}$. Equation (18) may be regarded as a D-type action formula for general function $\Phi = \phi(S, \bar{S} e^{2VG}) S_{\mathbf{O}} \bar{S}_{\mathbf{O}}$ in the gauge $C(\Phi) = 3$ and $Z(\Phi) = 0$.

In writing (18), we have already used the particular properties

$$\Phi^{\mathbf{I}} \chi_{\mathbf{RI}} = \Phi^i \chi_{\mathbf{R}i} + \Phi^0 \chi_{\mathbf{R}0} = 0, \quad \Phi_{\mathbf{O}}^{\mathbf{I}} \chi_{\mathbf{RI}} = 0, \quad (19)$$

which hold owing to our $\mathbb{S}$ -gauge condition (16). Consider more generally such quantities $A_{\mathbf{I}} = (A_{\mathbf{O}}, A_{\mathbf{i}})$ like $\chi_{\mathbf{RI}}$ in which the zeroth components are given by the rest as

$$A_{\mathbf{O}} = -z_{\mathbf{O}} \phi^{-1} \phi^i A_i = -(\Phi^0)^{-1} \Phi^i A_i, \quad (20)$$

and their Hermitean conjugates denoted by $A^{\mathbf{I}}$. They then enjoy the following interesting properties:

$$\begin{aligned} \Phi^{\mathbf{I}} A_{\mathbf{I}} = \Phi_{\mathbf{O}}^{\mathbf{I}} A_{\mathbf{I}} = 0, \quad z_{\mathbf{O}} \Phi_{\mathbf{J}}^{\mathbf{I}\mathbf{O}} A_{\mathbf{I}} B^{\mathbf{J}} = \Phi_{\mathbf{J}}^{\mathbf{I}} A_{\mathbf{I}} B^{\mathbf{J}} = \mathcal{G}_j^i A_i B^j, \\ \Phi_{\mathbf{J}\mathbf{K}}^{\mathbf{I}} A_{\mathbf{I}} B^{\mathbf{J}} C^{\mathbf{K}} = \mathcal{G}_{\mathbf{j}\mathbf{k}}^i A_i B^{\mathbf{J}} C^{\mathbf{K}}, \end{aligned} \quad (21)$$

where the function $\mathcal{G}(z, z^*)$ is defined by

$$\mathcal{G} \equiv 3 \ln(\phi/3). \quad (22)$$

These relations (21) are very helpful in rewriting the Lagrangian (18) with $\mathcal{G}$ instead of Φ . Motivated by this we also write $\nabla_{\mathbf{m}} z_{\mathbf{O}}$ and $h_{\mathbf{O}}$ as

$$\nabla_{\mathbf{m}} z_{\mathbf{O}} = -z_{\mathbf{O}} \phi^{-1} \phi^i \nabla_{\mathbf{m}} z_i + \frac{z_{\mathbf{O}}}{6} (\mathcal{G}^i \nabla_{\mathbf{m}} z_i - \mathcal{G}_i \nabla_{\mathbf{m}} z^{*i}), \quad (23.a)$$

$$h_{\mathbf{O}} = -z_{\mathbf{O}} \phi^{-1} \phi^i h_i + \tilde{h}_{\mathbf{O}}, \quad (23.b)$$

by extracting the terms of the form (20) into the first terms. [The former (23.a) is an identity while the latter (23.b) defines a change of variable $h_{\mathbf{O}} \rightarrow \tilde{h}_{\mathbf{O}}$]. The conversion of Φ into $\mathcal{G}$ is then performed quite easily as follows. First, forgetting about the second terms of the right-hand side of (23), we regard both of $\nabla_{\mathbf{m}} z_{\mathbf{I}}$ and $h_{\mathbf{I}}$ as quantities satisfying (20) as does $\chi_{\mathbf{RI}}$ and fully use the identities (21) as well as

$$\begin{aligned}\Phi_I^\alpha A^I &= \mathcal{G}_i^j T_j^{\alpha k} z_k A^i, & z_o \Phi_{IJ}^o A^I B^J &= (\mathcal{G}_{ij} - \frac{1}{3} \mathcal{G}_i \mathcal{G}_j) A^i B^j, \\ (\Phi_{km}^{IJ} - \phi^{-1} \Phi_{km}^o \Phi_o^{IJ}) A_I A_J B^k B^m &= (\mathcal{G}_{kl}^{ij} + \frac{2}{3} \mathcal{G}_k^i \mathcal{G}_l^j) A_i A_j B^k B^l. \quad (24)\end{aligned}$$

After that we calculate separately the contributions of the residual second terms by picking up only the terms containing $\nabla_m z_o$ or h_o in (18). The result is:

$$\begin{aligned}-\frac{1}{2} e^{-1} [\Phi]_D &= \mathcal{G}_j^i \nabla_m z_i \nabla_m z^{*j} + \{ \mathcal{G}_j^i \bar{\chi}_L^j \not{\nabla} \chi_{Ri} + h.c. \} + \mathcal{L}_{SG} \\ &+ \{ (\mathcal{G}_j^{ik} + \frac{1}{2} \mathcal{G}_j^i \mathcal{G}^k) (\bar{\chi}_L^j \not{\nabla} z_k \chi_{Ri}) - \mathcal{G}_j^i (\bar{\psi}_{mL} \not{\nabla} z_i \gamma_m \chi_L^j) + h.c. \} \\ &- [\mathcal{G}_{kl}^{ij} + \frac{1}{2} \mathcal{G}_k^i \mathcal{G}_l^j - \mathcal{G}_m^{ij} (\mathcal{G}^{-1})_n^m \mathcal{G}_{kl}^n] (\bar{\chi}_{Ri} \chi_{Rj}) (\bar{\chi}_L^k \chi_L^l) \\ &- \frac{1}{2} \mathcal{G}_j^i (\bar{\chi}_L^j \gamma_m \chi_{Ri}) (\bar{\psi}_{nR} \gamma_m \psi_{nL}) \\ &+ \frac{1}{8} \epsilon^{mnkl} (\bar{\psi}_m \gamma_n \psi_k) \{ (\mathcal{G}^i \nabla_l z_i - \mathcal{G}_i \nabla_l z^{*i}) + 2 \mathcal{G}_j^i (\bar{\chi}_L^j \gamma_l \chi_{Ri}) \} \\ &+ i \oint \{ 2 \mathcal{G}_j^i T_i^{\alpha k} z_k (\bar{\chi}_L^j \lambda_L^\alpha) + \frac{1}{2} \mathcal{G}^i T_i^{\alpha j} z_j (\bar{\psi}_L \gamma \lambda_R^\alpha) - h.c. \} \quad (25) \\ &+ e^{-1} \mathcal{L}_{aux.}^{[\Phi]},\end{aligned}$$

where $(\mathcal{G}^{-1})_j^i$ denotes the inverse matrix of $\mathcal{G}_j^i$ and

$$\begin{aligned}e^{-1} \mathcal{L}_{aux.}^{[\Phi]} &= - \oint \mathcal{G}^i T_i^{\alpha j} z_j D^\alpha - \mathcal{G}_j^i \tilde{h}_i \tilde{h}^{*j} \\ &- 3 \left| \tilde{h}_o z_o^{-1} - \frac{1}{3} (\mathcal{G}_{ij} - \frac{1}{3} \mathcal{G}_i \mathcal{G}_j) (\bar{\chi}_L^i \chi_L^j) \right|^2 \\ &+ \frac{3}{4} \left[A_m + \frac{i}{3} (\mathcal{G}^i \nabla_m z_i - \mathcal{G}_i \nabla_m z^{*i} + \mathcal{G}_j^i \bar{\chi}_L^j \gamma_m \chi_{Ri}) \right]^2, \quad (26) \\ \tilde{h}_i &\equiv h_i - (\mathcal{G}^{-1})_i^j \mathcal{G}_j^{kl} \bar{\chi}_{Rk} \chi_{Rl}.\end{aligned}$$

Next consider the $[\Sigma_o^3]_F$ term in $\mathcal{L}_S$. Recalling the general component formula^{3),8)}.

$$f(\Sigma) = [f(z_o), f'(z_o) \chi_{Ro}, f'(z_o) h_o - f''(z_o) \bar{\chi}_{Ro} \chi_{Ro}] \quad (27)$$

valid for any function f of chiral multiplet $\Sigma = [z_o, \chi_{Ro}, h_o]$, and applying the F-term formula (5), we obtain

$$\begin{aligned}e^{-1} [\Sigma_o^3]_F &= \left\{ 3 z_o^2 \tilde{h}_o - z_o^3 \mathcal{G}^i h_i - 6 \bar{\chi}_{Ro} \chi_{Ro} z_o \right. \\ &\quad \left. + 3 \bar{\psi}_L \gamma \chi_{Ro} z_o^2 + \bar{\psi}_{mL} \sigma^{mn} \psi_{nL} z_o^3 + h.c. \right\} \quad (28)\end{aligned}$$

Finally we proceed to the calculation of the YM kinetic term $\mathcal{L}_{YM}$ (3.b). The superfield strength $W_a \equiv W_a^{\alpha T \alpha}$ is a spinor chiral multiplet corresponding to the following quantity in the rigid supersymmetry case:

$$\begin{aligned} W_a^{rigid} &= (2\tilde{g})^{-1} \bar{D}\bar{D} (e^{-2V_G} D_a e^{2V_G}) \\ &= \tilde{g}^{-1} \bar{D}\bar{D} \{ D_a V_G + [D_a V_G, V_G] + O(V_G^3) \} \end{aligned} \quad (29)$$

The analogue to this flat space quantity $D_a V_G$ is given in the local superconformal case by ^{*}) (suppressing the external spinor index by multiplying a constant spinor η)

$$\begin{aligned} \bar{\eta}^a D_a V_G|_{local} &= \bar{\eta}_R (DV_G)_R|_{local} \\ &= \left[\bar{\eta}_R Z_R, (H+iK+i\mathcal{B}-\mathcal{D}^c C)\eta_R, \bar{\eta}_R \lambda_R, i\bar{\eta}_R \lambda_L, \right. \\ &\quad \bar{\eta}_R (2i\sigma_{mn} D_n^c Z_R + i\gamma_m \lambda_L), \\ &\quad \left\{ \square^c C + \frac{i}{2} (R_{ab}(Q) Z_R) \sigma_{ab} - i D_m^c B_m + D - \mathcal{D}^c (H+iK) \right\} \eta_R, \\ &\quad \left. - \bar{\eta}_R \left\{ \square^c Z_R - \frac{i}{4} \sigma_{ab} Z_R R_{ab}(A) + \mathcal{D}^c \lambda_L \right\} \right] \end{aligned} \quad (30)$$

Then, using the multiplication rule and chiral embedding formula ^{*} (correspondent of $\bar{D}\bar{D}$ operation) in conformal tensor calculus, we obtain the superfield strength multiplet in the Wess-Zumino gauge:

$$\bar{\eta}_R W_R^\alpha = \left[\bar{\eta}_R \lambda_R^\alpha, \frac{1}{2} (-\sigma \cdot \hat{F}^\alpha + i D^\alpha) \eta_R, \bar{\eta}_R \hat{\mathcal{B}} \lambda_L^\alpha \right], \quad (31)$$

where $\hat{F}^\alpha$ and $\hat{D}_m \lambda_L$ are fully covariant quantities under superconformal as well as YM gauge transformations:

$$\begin{aligned} \hat{F}_{\mu\nu}^\alpha &= F_{\mu\nu}^\alpha + \frac{1}{2} (\bar{\psi}_\mu \gamma_\nu \lambda^\alpha - \bar{\psi}_\nu \gamma_\mu \lambda^\alpha), \\ \hat{D}_\mu \lambda_L^\alpha &= \nabla_\mu \lambda_L^\alpha - \frac{1}{2} (\sigma \cdot \hat{F}^\alpha - i D^\alpha) \psi_{\mu L} + \frac{3}{4} i A_\mu \lambda_L^\alpha. \end{aligned} \quad (32)$$

---

^{*}) Notice that the transformation laws and chiral embedding formula for the vector multiplet with external Lorentz index are slightly different from the usual ones for that without external index. Eqs. (30) and (31) are calculated by using those correct formulae, which will be reported elsewhere. Fortunately, however, this is irrelevant here for obtaining the result (31) since the differences appear only in such terms as vanish in the Wess-Zumino gauge.

[We have used the usual YM field strength $F_{\mu\nu}^\alpha$ and the local-Lorentz and YM covariant derivative ∇_μ .]

Now, with (31) it is straightforward to write down the component form of $\mathcal{L}_{YM} = -4^{-1} [fW^2]_F$ by applying the general formulae (27). Here also, similar to the previous case of $[\Phi]_D$, many contributions of supercovariantization terms contained in $\hat{F}_{\mu\nu}^\alpha$ and $\hat{D}_\mu \lambda_L^\alpha$ cancel among the terms in $[fW^2]_F$:

$$\begin{aligned} -\frac{1}{4} e^{-1} [fW^2]_F = & \operatorname{Re} f_{\alpha\beta} \left(-\frac{1}{4} F_{\mu\nu}^\alpha F_{\mu\nu}^\beta - \frac{1}{2} \bar{\lambda}^\alpha \not{D} \lambda^\beta \right) \\ & + \frac{1}{4} \operatorname{Im} f_{\alpha\beta} \left[i F_{\mu\nu}^\alpha \tilde{F}_{\mu\nu}^\beta - e^{-1} \nabla_\mu (e \bar{\lambda}^\alpha i \gamma_5 \gamma^\mu \lambda^\beta) \right] \\ & + \left\{ \frac{1}{4} \operatorname{Re} f_{\alpha\beta} (\bar{\Psi}_m \sigma^{\mu\nu} \gamma^m \lambda^\alpha) (F_{\mu\nu}^\beta + \frac{1}{2} \bar{\Psi}_\mu \gamma_\nu \lambda^\beta) \right. \\ & - \frac{1}{2} f_{\alpha\beta}^i (\bar{\chi}_{Ri} \sigma^{\mu\nu} \lambda_R^\alpha) (F_{\mu\nu}^\beta + \bar{\Psi}_{\mu R} \gamma_\nu \lambda_L^\beta) + \frac{1}{8} f_{\alpha\beta}^i (\bar{\Psi}_L \gamma \chi_{Ri}) \bar{\lambda}_R^\alpha \lambda_R^\beta \\ & + \left[\frac{1}{4} f_{\alpha\beta}^{ij} - \frac{1}{16} f_{\alpha\gamma}^i (\operatorname{Re} f)_{\gamma\delta}^{-1} f_{\delta\beta}^j \right] (\bar{\chi}_{Ri} \chi_{Rj}) \bar{\lambda}_R^\alpha \lambda_R^\beta \\ & - \frac{1}{16} f_{\alpha\gamma}^i (\operatorname{Re} f)_{\gamma\delta}^{-1} f_{\delta\beta}^j (\bar{\chi}_{Ri} \sigma_{mn} \chi_{Rj}) (\bar{\lambda}_R^\alpha \sigma_{mn} \lambda_R^\beta) \\ & \left. + \frac{1}{16} f_{\alpha\gamma}^i (\operatorname{Re} f)_{\gamma\delta}^{-1} f_{\delta\beta}^{*j} (\bar{\chi}_{Lj} \gamma_m \chi_{Ri}) (\bar{\lambda}_R^\alpha \gamma_m \lambda_L^\beta) + h.c. \right\} \\ & + e^{-1} \mathcal{L}_{aux.}^{[fW^2]}, \end{aligned} \quad (33)$$

where

$$\begin{aligned} e^{-1} \mathcal{L}_{aux.}^{[fW^2]} = & \frac{1}{2} \operatorname{Re} f_{\alpha\beta} \tilde{D}^\alpha \tilde{D}^\beta - \frac{3}{4} i \operatorname{Re} f_{\alpha\beta} A_m \bar{\lambda}_R^\alpha \gamma_m \lambda_L^\beta \\ & - \frac{1}{4} \{ h_i f_{\alpha\beta}^i (\bar{\lambda}_R^\alpha \lambda_R^\beta) + h.c. \}, \end{aligned} \quad (34)$$

$$\tilde{D}^\alpha \equiv D^\alpha + \frac{1}{2} (\operatorname{Re} f)_{\alpha\beta}^{-1} (i f_{\beta\gamma}^i \bar{\chi}_{Ri} \lambda_R^\gamma + h.c.).$$

In deriving the term $\nabla_\mu (e \bar{\lambda}^\alpha i \gamma_5 \gamma^\mu \lambda^\beta)$ in (33), we have used a useful identity

$$e^{-1} \mathcal{D}_\mu^\omega (e e_m^\mu) + \frac{1}{2} \bar{\Psi} \cdot \gamma \Psi_m = 0. \quad (35)$$

The total Lagrangian has been given explicitly by the sum of (25), (28) and (33). The auxiliary fields h_o, h_i, A_m and D^α are contained only in $[\Phi]_{aux.}$, $[fW^2]_{aux.}$, and $[\Sigma_o^3]_F$. Elimination of those auxiliary fields by the use of the equations of motion

$$z_o^{-1} \tilde{h}_o = \frac{1}{3} (\mathcal{G}_{ij} - \frac{1}{3} \mathcal{G}_i \mathcal{G}_j) \bar{\chi}_L^i \chi_L^j + e^{-\frac{\mathcal{G}}{2}},$$

$$\begin{aligned}
 h_i &= (\mathcal{G}^{-1})^j_i \left[\mathcal{G}^k_l \bar{\chi}_{Rk} \chi_{Rl} - \frac{1}{4} f_{\alpha\beta}^* \bar{\lambda}_L^\alpha \lambda_L^\beta - e^{-\frac{g}{2}} \mathcal{G}_j \right], \\
 A_m &= -\frac{i}{3} \left[\mathcal{G}^i \nabla_m z_i - \mathcal{G}_i \nabla_m z^{*i} + \mathcal{G}^j_i \bar{\chi}_L^j \gamma_m \chi_{Ri} - \frac{3}{2} \text{Re} f_{\alpha\beta} \bar{\lambda}_R^\alpha \gamma_m \lambda_L^\beta \right], \\
 D^\alpha &= \frac{1}{2} (\text{Re} f)_{\alpha\beta}^{-1} \left[\tilde{\mathcal{F}} \mathcal{G}^i T_i^{\beta j} z_j - i f_{\rho\gamma}^i \bar{\chi}_{Ri} \lambda_R^\gamma + h.c. \right], \quad (36)
 \end{aligned}$$

effects the following replacement in the Lagrangian:

$$\begin{aligned}
 &e^{-1} \{ \mathcal{L}_{aux.}^{[\Phi]} + [\Sigma_\sigma^3]_F + \mathcal{L}_{aux.}^{[fW^2]} \} \\
 &\Rightarrow e^{-g} \left[3 + \mathcal{G}_i (\mathcal{G}^{-1})^i_j \mathcal{G}^j \right] - \frac{1}{2} \tilde{\mathcal{F}}^2 (\text{Re} f)_{\alpha\beta}^{-1} (\mathcal{G}^i T_i^{\alpha j} z_j) (\mathcal{G}^k T_k^{\beta l} z_l) \\
 &+ e^{-g/2} \left\{ \bar{\psi}_{mL} \sigma^{mn} \psi_{nL} - \mathcal{G}^i \bar{\psi}_L \gamma \chi_{Ri} + \frac{1}{4} \mathcal{G}^i (\mathcal{G}^{-1})^j_i f_{\alpha\beta}^* \bar{\lambda}_L^\alpha \lambda_L^\beta \right. \\
 &\quad \left. + (\mathcal{G}^{ij} - \mathcal{G}^i \mathcal{G}^j - \mathcal{G}^k (\mathcal{G}^{-1})^l_k \mathcal{G}^{ij}) \bar{\chi}_{Ri} \chi_{Rj} + h.c. \right\} \\
 &+ \left\{ \frac{i}{2} \tilde{\mathcal{F}} \mathcal{G}^i T_i^{\alpha j} z_j (\text{Re} f)_{\alpha\beta}^{-1} f_{\beta\gamma}^k \bar{\chi}_{Rk} \lambda_R^\gamma - \frac{1}{4} (\mathcal{G}^i \nabla_m z_i + \frac{1}{2} \mathcal{G}^j_i \bar{\chi}_L^j \gamma_m \chi_{Ri}) \text{Re} f_{\alpha\beta} \bar{\lambda}_R^\alpha \gamma_m \lambda_L^\beta \right. \\
 &\quad \left. + \frac{1}{32} f_{\alpha\beta}^i (\mathcal{G}^{-1})^j_i f_{\gamma\delta}^* (\bar{\lambda}_R^\alpha \lambda_R^\beta) (\bar{\lambda}_L^\gamma \lambda_L^\delta) + \frac{3}{32} (\text{Re} f_{\alpha\beta} \bar{\lambda}_R^\alpha \gamma_m \lambda_L^\beta)^2 \right. \\
 &\quad \left. - \frac{1}{4} \mathcal{G}^{ij} (\mathcal{G}^{-1})^k_l f_{\alpha\beta}^l (\bar{\chi}_{Ri} \chi_{Rj}) (\bar{\lambda}_R^\alpha \lambda_R^\beta) + h.c. \right\} \quad (37)
 \end{aligned}$$

This finishes the derivation of the final form of the Lagrangian $\mathcal{L}$:
 $e^{-1} \mathcal{L}$ is given by (25) + (33) + (37) with $\mathcal{L}_{aux.}^{[\Phi]}$ and $\mathcal{L}_{aux.}^{[fW^2]}$ discarded and exactly coincides term by term with the result of Cremmer et al.³⁾.

4. - SUPERSYMMETRY TRANSFORMATION LAW

The derivation of the supersymmetry transformation law becomes particularly simple in the present conformal framework. We have only to follow exactly the same procedure as was employed in deducing the usual Poincaré tensor calculus from the superconformal one. The Q supersymmetry transformation in conformal theory does not leave the various gauge conditions invariant. Just as for the Lorentz transformation in QED of Coulomb gauge, the Q-transformation combined with suitable gauge transformations (so as to preserve the imposed gauge conditions) gives the desired supersymmetry transformation.

First let us recall the necessary transformation laws of the fields under the Q, A and S transformations in conformal theory^{*)}:

$$\begin{aligned}
 \delta e^\mu_\mu &\equiv [\delta_\alpha(\epsilon) + \delta_A(\theta) + \delta_S(\zeta)] e^\mu_\mu = \frac{1}{2} \bar{\epsilon} \gamma^\mu \psi_\mu, \\
 \delta \psi_{\mu L} &= (D_\mu^\omega + \frac{3}{4} i A_\mu) \epsilon_L - \frac{3}{4} i \theta \psi_{\mu L} - \gamma_\mu \zeta_R, \\
 \delta z_I &= \bar{\epsilon}_R \chi_{RI} + \frac{1}{2} \lambda i \theta z_I, \\
 \delta \chi_{RI} &= \frac{1}{2} \not{D}^\epsilon z_I \epsilon_L + \frac{1}{2} h_I \epsilon_R - (\frac{3}{4} - \frac{\lambda}{2}) i \theta \chi_{RI} + \lambda z_I \zeta_R, \\
 \delta B_\mu^\alpha &= -\frac{1}{2} \bar{\epsilon} \gamma_\mu \lambda^\alpha, \\
 \delta \lambda_L^\alpha &= \frac{1}{2} (\sigma \cdot F_c^\alpha - i D^\alpha) \epsilon_L - \frac{3}{4} i \theta \lambda_L^\alpha,
 \end{aligned} \tag{38}$$

where λ in δz_I and $\delta \chi_{RI}$ is the conformal weight and takes a non-vanishing value $\lambda = 1$ only for $I = 0$. The YM gauge transformation

$$\begin{aligned}
 e^{2V_\epsilon} &\rightarrow e^{-i\bar{\Lambda}} e^{2V_\epsilon} e^{i\Lambda}, \\
 S_i &\rightarrow e^{-i\Lambda} S_i, \quad (\Lambda \equiv \tilde{f} T^\alpha \Lambda^\alpha)
 \end{aligned} \tag{39}$$

is easily calculable in the Wess-Zumino gauge by expanding the exponentials. For the infinitesimal chiral parameter $\Lambda^\alpha = [\epsilon^\alpha \equiv a^\alpha + i b^\alpha, \eta_R^\alpha, \mathcal{F}^\alpha]$, the component fields of the YM multiplet $V^\alpha = [C^\alpha, Z^\alpha, H^\alpha, K^\alpha, B_m^\alpha, \lambda_L^\alpha, D^\alpha]$ and the matter multiplet S_i transform

$$\left. \begin{aligned}
 \delta_{YM}(\Lambda^\alpha) C^\alpha &= -b^\alpha, & \delta_{YM} Z_R^\alpha &= \eta_R^\alpha, \\
 \delta_{YM}(H^\alpha + i K^\alpha) &= -i \mathcal{F}^\alpha,
 \end{aligned} \right\} \tag{40.a}$$

$$\delta_{YM} \lambda_L^\alpha = \tilde{f} f^{\alpha\beta\gamma} (a^\beta \lambda_L^\gamma - B^\gamma \eta_R^\beta), \quad \delta_{YM} B_m^\alpha = -\hat{D}_m a^\alpha, \tag{40.b}$$

$$\delta_{YM} z_i = -i \tilde{f} T_i^{\alpha j} a^\alpha z_j, \quad \delta_{YM} \chi_{Ri} = -i \tilde{f} T_i^{\alpha j} (a^\alpha \chi_{Rj} + z_j \eta_R^\alpha). \tag{40.c}$$

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^{*)} We have already set $b_\mu = 0$ by using the $\tilde{K}_m$ -gauge choice (16).

We are now ready to derive the desired supersymmetry transformation. First notice that the Wess-Zumino gauge condition $C^\alpha = Z^\alpha = H^\alpha = K^\alpha = 0$ is violated by the Q-transformation; indeed in Wess-Zumino gauge

$$\begin{aligned}\delta_Q(\epsilon) C^\alpha &= 0, \\ \delta_Q(\epsilon) Z_R^\alpha &= -\frac{1}{2} \mathcal{B}^\alpha \epsilon_L, \\ \delta_Q(\epsilon) (H^\alpha + i K^\alpha) &= -i \bar{\epsilon}_L \left(\frac{1}{2} \gamma^\mu \mathcal{B}^\alpha \psi_{\mu L} + \lambda_L^\alpha \right).\end{aligned}\quad (41)$$

Hence, taking account of (40.a), we find the following combination of the Q-transformation and the (field-dependent) YM transformation preserves the Wess-Zumino gauge condition:

$$\begin{aligned}\delta'_Q(\epsilon) &= \delta_Q(\epsilon) + \delta_{YM}(\Lambda^\alpha(\epsilon)), \\ \Lambda^\alpha(\epsilon) &= \left[0, \frac{1}{2} \mathcal{B}^\alpha \epsilon, -\bar{\epsilon}_L \left(\frac{1}{2} \gamma^\mu \mathcal{B}^\alpha \psi_{\mu L} + \lambda_L^\alpha \right) \right].\end{aligned}\quad (42)$$

This transition $\delta_Q \rightarrow \delta'_Q$ effects only the changes $D_\mu^C \rightarrow \hat{D}_\mu$ and $F_{C,\mu\nu}^\alpha \rightarrow \hat{F}_{\mu\nu}^\alpha$ in the supersymmetry transformation law (38). Next we have to consider the conformal gauge fixing (16) of the extraneous gauge freedoms $\underline{D}$, $\underline{A}$, $\underline{S}$ and $\underline{K}_m$. The transformation of the $\underline{D}$ - and $\underline{A}$ -gauge conditions under δ'_Q is estimated by using (38):

$$\begin{aligned}\delta'_Q(\epsilon) (z_0 - \sqrt{3} \phi^{-\frac{1}{2}}(z, z^*)) \\ = \bar{\epsilon}_R \chi_{R0} + (z_0/2\phi) (\phi^i \bar{\epsilon}_R \chi_{Ri} + \phi_i \bar{\epsilon}_L \chi_L^i) \\ = -(z_0/6) (\mathcal{G}^i \bar{\epsilon}_R \chi_{Ri} - \mathcal{G}_i \bar{\epsilon}_L \chi_L^i).\end{aligned}\quad (43)$$

The second equality follows by utilizing our $\underline{S}$ -gauge condition (16). This quantity (43) is purely imaginary and can be compensated by the chiral ($\underline{A}$)-transformation, since $\delta_A(\theta) (z_0 - \sqrt{3}\phi^{-1/2}) = (1/2)i\theta z_0$. The combination

$$\begin{aligned}\delta''_Q(\epsilon) &= \delta'_Q(\epsilon) + \delta_A(\theta(\epsilon)), \\ \theta(\epsilon) &= -\frac{i}{3} (\mathcal{G}^i \bar{\epsilon}_R \chi_{Ri} - \mathcal{G}_i \bar{\epsilon}_L \chi_L^i),\end{aligned}\quad (44)$$

therefore, leaves invariant the $\underline{D}$ - and $\underline{A}$ -gauge conditions (as well as the previous Wess-Zumino gauge conditions, of course). Notice that the imaginary property of the quantity (43) comes from our particular $\underline{S}$ -gauge condition (16). Otherwise it would be necessary to add a $\underline{D}$ -gauge transformation term in (44) to compensate the real term, resulting in a bizarre supersymmetry transformation law for the vierbein. This is one of the reasons why we adopt the present $\underline{S}$ -gauge condition (16).

Repeating the same procedures on the $\underline{S}$ -gauge and $\underline{K}_m$ -gauge conditions successively, we find that they require the compensations of $\underline{S}$ -gauge and $\underline{K}_m$ -gauge transformation, respectively^{*}). Thus we obtain finally the desired supersymmetry transformation:

$$\delta_Q^{\mathbb{P}}(\epsilon) = \delta_Q(\epsilon) + \delta_{YM}(\Lambda^\alpha(\epsilon)) + \delta_A(\theta(\epsilon)) + \delta_S(\zeta(\epsilon)) + \delta_K(\xi_m(\epsilon)), \quad (45)$$

where

$$\begin{aligned} \zeta_R(\epsilon) &= -\frac{1}{2} \tilde{h}_0 \tilde{z}_0^{-1} \epsilon_R - \frac{1}{3} \left\{ (\mathcal{G}^{ij} - \frac{1}{3} \mathcal{G}^i \mathcal{G}^j) \bar{\epsilon}_R \chi_{Rj} + \mathcal{G}_j^i \bar{\epsilon}_L \chi_L^j \right\} \chi_{Ri} \\ &\quad - \frac{1}{12} (\mathcal{G}^i \not{z}_i - \mathcal{G}_i \not{z}^{*i}) \epsilon_L + \frac{1}{4} i \not{A} \epsilon_L \\ &= -\frac{1}{2} e^{-\mathcal{G}/2} \epsilon_R + \frac{1}{4} \left\{ \mathcal{G}_j^i (\bar{\chi}_L^j \gamma^m \chi_{Ri}) - \mathcal{R}_\alpha f_{\alpha\beta} (\bar{\lambda}_R^\alpha \gamma^m \lambda_L^\beta) \right\} \gamma_m \epsilon_L, \end{aligned} \quad (46.a)$$

$$\xi_m(\epsilon) = \frac{1}{4} (\bar{\varphi}_m \epsilon - \bar{\psi}_m \epsilon). \quad (46.b)$$

In going to the second equality in (46.a), we have used the equations of motion (36) for the auxiliary fields $\tilde{h}_0$ and A_m . Although we have given the $\underline{K}_m$ -transformation term, it plays no role here because it affects the transformation only on those vector multiplets having non-zero conformal weight.

Now applying the obtained Poincaré supersymmetry transformation (45) with the help of the conformal and YM transformation laws (38) and (40), we straightforwardly get the transformation laws of fields:

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^{*}) The reason why we can find the desired supersymmetry transformation in such successive way is that at each step the newly added gauge transformation keeps invariant the preceding gauge conditions. Therefore the order of considered gauge conditions has a meaning here.

$$\begin{aligned}
\delta_a^P(\epsilon) e_\mu^m &= \frac{1}{2} \bar{\epsilon} \gamma^m \psi_\mu, \\
\delta_a^P \psi_{\mu L} &= D_\mu^\omega \epsilon_L + \frac{1}{2} \sigma_{\mu\nu} \epsilon_L \mathcal{G}_j^i (\bar{\chi}_{Ri} \gamma^\nu \chi_L^j) + \frac{1}{4} (\partial_{\mu\nu} - \sigma_{\mu\nu}) \epsilon_L \text{Ref}_{\alpha\beta} (\bar{\lambda}_L^\alpha \gamma^\nu \lambda_R^\beta) \\
&\quad + \frac{1}{2} \gamma_\mu \epsilon_R e^{-g/2} + \frac{1}{4} (\mathcal{G}^i \nabla_\mu z_i - \mathcal{G}_i \nabla_\mu z^{*i}) - \frac{1}{4} \psi_{\mu L} (\mathcal{G}^i \bar{\chi}_{Ri} \epsilon_R - \mathcal{G}_i \bar{\chi}_L^i \epsilon_L), \\
\delta_a^P z_i &= \bar{\epsilon}_R \chi_{Ri}, \\
\delta_a^P \chi_{Ri} &= \frac{1}{2} \hat{\mathcal{D}} z_i \epsilon_L + \frac{1}{2} (\mathcal{G}^{-1})_i^j [\mathcal{G}_j^{kl} \bar{\chi}_{Rk} \chi_{Rl} - \frac{1}{4} f_{\alpha\beta}^* \bar{\lambda}_L^\alpha \lambda_L^\beta - e^{-g/2} \mathcal{G}_j] \epsilon_R \\
&\quad - \frac{1}{4} \chi_{Ri} (\mathcal{G}^i \bar{\epsilon}_R \chi_{Ri} - \mathcal{G}_i \bar{\epsilon}_L \chi_L^i), \\
\delta_a^P \mathcal{B}_\mu^\alpha &= -\frac{1}{2} \bar{\epsilon} \gamma_\mu \lambda^\alpha, \\
\delta_a^P \lambda_L^\alpha &= \frac{1}{2} \sigma \cdot \hat{F}^\alpha \epsilon_L - \frac{i}{4} (\text{Ref})_{\alpha\beta}^{-1} [2 \hat{\mathcal{D}} \mathcal{G}^i T_i^{\beta j} z_j - i f_{\beta\gamma}^i \bar{\chi}_{Ri} \lambda_R^\gamma + i f_{\beta\gamma}^* \bar{\chi}_L^i \lambda_L^\gamma] \epsilon_L \\
&\quad - \frac{1}{4} \lambda_L^\alpha (\mathcal{G}^i \bar{\epsilon}_R \chi_{Ri} - \mathcal{G}_i \bar{\epsilon}_L \chi_L^i). \tag{47}
\end{aligned}$$

Here we have inserted the equations of motion (36) of auxiliary fields. This result already gives the final form of the transformation law of Cremmer et al.³⁾, who obtained it by repeating the same four steps i) - iv) as performed in obtaining the Lagrangian.

Before concluding this paper we add some comments:

- i) We did not give the derivation of the mass formula since our method provides no improvement to that given in Ref.3). In this connection the superfield method recently given by Grisaru, Roček and Karlhede¹⁴⁾ is remarkably powerful. Nevertheless we notice that in deriving the full non-linear component form of the Lagrangian their method also encounters many complications and requires the same gauge fixing device as ours to simplify the calculation.
- ii) The matter-YM-supergravity coupled system (1) which we have considered stands for a fairly general system in $N = 1$ supergravity but is not yet completely general; indeed it excludes the possibilities of linear multiplets L_i and (non-gauge) vector multiplets V_j as the matter fields. It would be interesting to study the

the nature and effect of such matter multiplets especially in connection with the superHiggs phenomenon. Clearly, similar considerations should also be performed for the $N = 2$ and higher cases if possible.

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