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QUARK MIXING : PHENOMENOLOGY *)

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The low q^2 effective charged current interaction in the standard model is $\mathcal{L}_{cc} = 4G_F/\sqrt{2} J_\mu^+ J_\mu^-$ where

$$J_\mu = (\overline{ucl})_L \gamma_\mu U_{KM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L + (\overline{\nu_1 \nu_2 \nu_3})_L \gamma_\mu U \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_L. \quad (1)$$

To extract information about G_F and U_{KM} it has to be assumed¹ that a) all $m_{\nu_i} = 0$ or b) $U = 1$ or c) all $m_{\nu_i} \ll Q$ in μ -decay and other semi-leptonic decays. Then G_F is fixed by the μ -lifetime and U_{ud} can be measured via ft-values of allowed nuclear β -decays. Radiative corrections play an important role² and the final result for U_{ud} can be written as²⁻⁴

$$|U_{ud}|^2 = \frac{192(\ln 2)\tau_\mu (m_\mu/m_e)^5 f(m_e/m_\mu) [1 - \alpha/8\pi(\pi^2 - 25)]}{ft(1 + \delta_R)(1 - \delta_c)(1 + \Delta_R - \delta_\mu)} \quad (2)$$

where δ_c and δ_R are Coulomb and internal radiative nuclear corrections, Δ_R and δ_μ are electro-weak corrections to β -decay and μ -decay. The most accurate ft values are those of 0^{14} and $A1^{21}$. Combining the recent analyses, we conclude:

$$|U_{ud}| = 0.9734 \pm 0.0025. \quad (3)$$

The largest uncertainties are in δ_c and in evaluating hadronic matrix elements of quark operators.

U_{us} can be extracted in a similar way from $K \rightarrow \pi e^- \nu_e$ and $Y \rightarrow N e^- \nu_e$, albeit with less accuracy. In case of Ke_3 , apart from U_{us} and G_F there are two form factors $f_-(q^2)$ and $f_+(q^2)$, whose ratio and q^2 dependence is measured. $|f_+(0)|^2$ is determined theoretically⁵ (broken chiral $SU(3) \times SU(3)$) to be about 0.98. Then one finds^{4,6} $|U_{us}| = 0.219 \pm 0.003$. The hyperon semileptonic decays⁷ are slightly problematic. One potential problem is the rate for $\Gamma(\Sigma^- \rightarrow \Lambda e^- \nu_e)$ which is lower than $SU(3)$ fits would like and the other is the electron asymmetry in polarized Σ^- -decay ($\Sigma^- \rightarrow n e^- \nu_e$) which may have the sign opposite to the expected value. Now the first can be accommodated with $SU(3)$ breaking in the axial-vector current matrix element without affecting the overall fit drastically whereas the second, if correct, would require a profound change in our understanding of charged weak currents. Ignoring this second problem, recent fits^{3,8} to hyperon decays yield values for $|U_{us}|$ ranging from 0.225 ± 0.002 , to 0.227 ± 0.003 or even 0.228, all of which are somewhat higher than values obtained earlier⁴; viz. 0.222 ± 0.003 . For the present purposes, we adopt a crude average from Ke_3 and hyperon decays of

$$|U_{us}| = 0.224 \pm 0.006. \quad (4)$$

A better way to use the hyperon data is to make a two angle fit (θ_V and θ_A) and use only θ_V to determine $|U_{us}|$, thus minimizing dependence on the uncertain axial vector matrix elements.

From the bounds (at 1σ level) on $|U_{ud}|$ and $|U_{us}|$ in Eq. (3) and (4) and the unitarity of U_{KM} we find

$$|U_{ub}| < 0.1 \quad (5)$$

in agreement with the earliest bound⁹ obtained on $|U_{ub}|$. In principle, $|U_{ub}|$ can also be bounded by the same-sign dimuon rate in $\bar{\nu}_\mu N$. This has a contribution from $\nu_\mu u \rightarrow \mu^+ b \rightarrow \mu^+ c \rightarrow \mu^+ \mu^+ x$ which can be bounded by the unknown rate. However, due to the high $\sigma(\bar{\nu}_\mu \rightarrow \mu^+ \mu^+)$, the threshold suppression in b-production and the unfavorable y-dependence, the bound obtained on $|U_{ub}|$ is weaker than (5).¹⁰ A more promising way of constraining $|U_{ub}|$ may be to use the "unexplained" part of $\sigma(\bar{\nu}_\mu \rightarrow \mu^+ \mu^- \mu^+)$ i.e., the observed cross-section after subtracting the cross-section calculated from all conventional sources.

The unitarity limit on U_{cd} from U_{ud} in (3) is $|U_{cd}| < 0.24$. To get a non-zero lower bound it was suggested¹¹ that one use the valence contribution to $\sigma(\nu_\mu \rightarrow \mu^- l^+)$ which comes from $\nu_\mu d \rightarrow \mu^- c \rightarrow \mu^- l^+ x$. Using the recent CDHS data for the fraction of valence in $\sigma(\nu_\mu \rightarrow \mu^- \mu^+)$ viz. 52%, one finds $|U_{cd}| > 0.2$. A more complete analysis by CDHS¹² using the identity

$$|U_{cd}|^2 = \frac{2}{3\langle B \rangle} \frac{\left[\frac{\sigma(\nu \rightarrow \mu^- \mu^+)}{\sigma(\nu \rightarrow \mu^-)} - R \frac{\sigma(\bar{\nu} \rightarrow \mu^+ \mu^-)}{\sigma(\bar{\nu} \rightarrow \mu^+)} \right]}{1 - R} \quad (6)$$

where $R = \sigma(\bar{\nu} \rightarrow \mu^+)/\sigma(\nu \rightarrow \mu^-)$ and using the fragmentation function they measured finds $|U_{cd}| = 0.24 \pm 0.03$. So summarizing

$$0.2 < |U_{cd}| < 0.24. \quad (7)$$

In the same analysis CDHS¹² also find that $(|U_{cs}|^2/|U_{cd}|^2)(2S/U + D) = 0.92 \pm 0.06$ and from their charged current data $(\bar{U} + \bar{D})/(U + D) = 0.13 \pm 0.02$. So if $2S < \bar{U} + \bar{D}$ they deduce the most conservative bound $|U_{cs}| \geq 0.59$. But from the CHARM analysis¹³ of their neutral current data (assuming s and d have identical neutral current couplings) we know that $2S/(\bar{U} + \bar{D})$ is close to 0.5. From this, it is possible to use CDHS data to deduce a stronger bound¹⁴ on $|U_{cs}|$ viz. $|U_{cs}| > 0.7$ to 0.74. A similar bound on $|U_{cs}|$ can be obtained¹¹ from $\Gamma(D^+ \rightarrow \bar{K}^0 e^+ \nu_e)$. Using an F^* -dominance model for the form factors and a calculated value¹⁵ for $|f_+(0)|^2 \approx \frac{1}{2}$, one finds $|U_{cs}| > 0.8$. The main uncertainties are the values for $B(D^+ \rightarrow \bar{K}^0 e^+ \nu_e)$ and $\tau(D^+)$. Including the unitarity limit from $|U_{cs}|$, then³

$$0.8 < |U_{cs}| < 0.98. \quad (8)$$

It would be nice to check that indeed $|U_{cs}| < 1$ and that unitarity is satisfied.

There are several ways of measuring the ratio $|U_{ub}/U_{cb}|$ as reviewed¹⁶ in Madison. CLEO has reported¹⁷ on a new measurement of $\langle N_K \rangle_T / \langle N_K \rangle_C = 1.88 \pm 0.28$ from which a limit $|U_{ub}/U_{cb}| < 0.4$ can be deduced. A stronger result from a study of the shape and the end-point of the electron spectrum in $B \rightarrow e \nu x$ is reported to this conference by CUSB¹⁸:

$$|U_{ub}/U_{cb}| < 0.2. \quad (9)$$

If B-decay is described well by quark decay, as seems likely, the lifetime τ_B is given approximately by

$$\tau_B \approx \frac{1}{3} \tau_\mu \left(\frac{m_\mu}{m_B} \right)^5 \frac{1}{(|U_{cb}|^2 + 2.5|U_{ub}|^2)}. \quad (10)$$

With the new limit¹⁹ on τ_B from JADE $\tau_B < 1.4 \times 10^{-12}$ s, and the smallness of $|U_{ub}|$, $|U_{cb}|$ should satisfy

$$0.57 > |U_{cb}| > 0.05. \quad (11)$$

where the upper bound comes from unitarity.

We can now use unitarity to limit U_{ti} , e.g., we find $|U_{td}| < 0.13$, $|U_{ts}| < 0.56$ and $0.99 > |U_{tb}| > 0.82$. Hence $|U_{ts} U_{td}| < 0.073$. However, a better bound on $|U_{ts} U_{td}|$ is obtained from saturating the dispersive part of $K_L \rightarrow \mu^+ \mu^-$ amplitude²⁰ by one loop graphs which are dominated by t . If the fourth generation heavy quark contribution is small compared to t , then for $m_t > 20$ GeV, it can be shown that¹ $|U_{ts} U_{td}| < 0.035$. $|U_{tb}|^2 + |U_{ts}|^2 + |U_{td}|^2$ which should be ≤ 1 can be measured in principle in weak decays of toponium: $(t\bar{t}) \rightarrow \bar{t} + b + \nu$ etc. The rate goes as $G_F^2 m_t^5 \sum_i |U_{ti}|^2$ and the branching ratio can be as high as 5 to 6 % for $M_{t\bar{t}} \sim 50$ GeV. A possible signature is prompt e/μ accompanied by hadrons.²¹ Since $\tau_T < 10^{-17}$ s, this may be the only way to measure the absolute value of t -couplings. Relative strengths such as $|U_{ts}/U_{tb}|$ can only be measured in $t \rightarrow b\nu$, $t \rightarrow s\nu$ etc.

A summary of our present knowledge of U_{KM} elements is:

$$U_{KM} = \begin{pmatrix} |U_{ud}| = 0.9734 \pm 0.024 & |U_{us}| = 0.224 \pm 0.006 & 0 < |U_{ub}| < 0.1 \\ 0.2 < |U_{cd}| < 0.24 & 0.8 < |U_{cs}| < 0.98 & 0.05 \leq |U_{cb}| < 0.57 \\ 0 < |U_{td}| < 0.13 & 0 \leq |U_{ts}| < 0.56 & 0.82 \leq |U_{tb}| \leq 0.99 \end{pmatrix}$$

If and only if there are three families, U_{KM} can be parameterized à la Kobayashi-Maskawa²²

$$U_{KM} = \begin{pmatrix} C_1 & S_1 C_3 & S_1 S_3 \\ -S_1 C_2 & C_1 C_2 S_3 - S_2 S_3 e^{i\delta} & C_1 C_2 S_3 + C_3 S_2 e^{i\delta} \\ S_1 S_2 & -C_1 S_2 C_3 - C_2 S_3 e^{i\delta} & -C_1 S_2 S_3 + C_2 C_3 e^{i\delta} \end{pmatrix}$$

where C_i and S_i are $\cos\theta_i$ and $\sin\theta_i$ with $0 \leq \theta \leq \pi/2$ and $0 \leq \delta \leq 2\pi$. The value of $|U_{ud}|$ fixes $S_1 = 0.229$. The other constraints on U_{ij} restrict S_2 and S_3 as shown in Figs. 1 to 3. In Fig. 1 and Fig. 2, $\sin\delta$ is small and δ is near 0 and π respectively. In Fig. 3, $\sin\delta$ is large (>0.1). These constraints can be converted into other parameterizations²³ for U_{ij} .

Bounds on θ_i and δ can also be derived from evaluating δM_{L-S} and $\text{Re } \epsilon$ for the $K_L - K_S$ system.²⁴ The principle uncertainty is the evaluation of the hadronic matrix element. The bag-model evaluation is too sensitive to the parameters chosen and even the sign is unstable.²⁵ There is also the factor of 2 to 3 uncertainty due to possible long-range effects²⁶ (e.g., π^0 , η , η' intermediate states). Two recent attempts to estimate this matrix element are very encouraging. One²⁷ relates the $\Delta S = 2$, $\Delta T = 1$ matrix element for $K^0 \leftrightarrow \bar{K}^0$ to the $\Delta S = 1$, $\Delta T = 3/2$ matrix element $\bar{K}^0 \rightarrow \pi^0$ in $K^+ \rightarrow 2\pi$ and obtains a suppression factor of 0.33 compared to vacuum saturation. The other²⁸ uses QCD sum rules to place a bound on the form factor for vacuum $\rightarrow KK$ yielding a limit of the suppression factor <0.6 . Allowing for this uncertainty and allowing m_t to range between 20 and 60 GeV, the range for S_2 and S_3 preferred^{24,29} by the $K_L - K_S$ parameters is shown on the figures. Similar constraints^{20,30} can be obtained from $K_L \rightarrow \mu^+ \mu^-$.

Meanwhile, it seems to me that the real problem, viz. that of predicting quark and lepton masses and mixings, remains open. As a beginning, we need to see the regularities, i.e., find the "Balmer formula" for fermion spectrum. One speculation on the t -quark mass not yet ruled out is $m_t/m_c \sim m_\tau/m_\mu$. This leads to the expectation³¹ that the threshold for naked t i.e., $2M_t$ is between 46 and 52 GeV. Are there simple relations between mixings and masses? Tune in next year.

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Fig.1

Fig.2

Fig.3