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GENERALIZED O'RAIFEARTAIGH MODELS COUPLED TO $N = 1$ SUPERGRAVITY

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A B S T R A C T

Generalized O'Raifeartaigh models coupled to $N = 1$ supergravity are studied in the limit of infinite Planck mass. This limit does not reproduce the usual, globally supersymmetric theory. Relevance of the results to realistic models of particle physics is discussed.

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There is now much interest ¹⁾⁻³⁾ in supersymmetric models of particle physics coupled to $N = 1$ supergravity ^{4),5)}. Local supersymmetry seems to cure many of the phenomenological difficulties which plagued simple, globally supersymmetric models. Indeed, even the simplest locally supersymmetric theories appear to have no phenomenological difficulties ! This astounding situation may indicate that these ideas are basically correct.

It is therefore important to study different possible mechanisms of spontaneous supersymmetry breaking, as any one choice might not include the correct theory. To date, all specific, realistic models ^{*)} have broken supersymmetry via gravitational couplings to a chiral field X with superpotential $W = \mu^2(X + \beta M)$, $\beta = 2 - \sqrt{3}$, $M = M_{\text{Planck}}/\sqrt{8\pi}$, μ arbitrary. This leads to a theory with zero cosmological constant and broken supersymmetry ; the gravitino mass is $m_{\frac{3}{2}} = \exp(2 - \sqrt{3})\mu^2/M$ ^{1),4)}. All the remaining physics comes from the choice of the other fields and their superpotential.

In this paper we study generalized O'Raifeartaigh models ⁶⁾ coupled to $N = 1$ supergravity. The O'Raifeartaigh mechanism of spontaneous supersymmetry breaking has often been used in attempts to construct realistic, globally supersymmetric models.

We begin with two sets of chiral superfields with complex scalar components X_i , $i = 1, \dots, n$ and Z_α , $\alpha = 1, \dots, p$. The superpotential is chosen to be

$$W = X_i F_i(Z) + F_0(Z) + \Lambda \quad (1)$$

where the F 's are analytic functions of the Z_α . A possible constant term in F_0 has been written explicitly as Λ ; $F_0(0) = 0$. In global supersymmetry, the scalar potential is

$$V_{\text{global}} = |F_i|^2 + |X_i F_{i,\alpha} + F_{0,\alpha}|^2 \quad (2)$$

with understood sums over i and α . The F_i are chosen so that the system of n equations $F_i = 0$ in the p unknowns Z_α has no solution. Then global supersymmetry is spontaneously broken. The VEV's of the Z_α are those which minimize $|F_i|^2$. The VEV's of the X_i satisfy the p equations $X_i F_{i,\alpha} + F_{0,\alpha} = 0$ (with the F 's evaluated at the VEV's of the Z_α). All other linear combinations of the X_i are undetermined at the tree level.

*) By specific, we mean that the complete Lagrangian was given. By realistic, we mean that it reproduces the phenomenology of the standard model at energies $\lesssim 20$ GeV.

Now let us couple this model to $N = 1$ supergravity ^{4),5)}, with "minimal" kinetic terms ⁵⁾. The scalar potential becomes ^{5),7)}

$$V = \exp(|X_i|^2/M^2 + |Z_\alpha|^2/M^2) \tilde{V} \quad (3)$$

$$\begin{aligned} \tilde{V} = & |F_i + X_i^* W/M^2|^2 - 3|W|^2/M^2 \\ & + |X_i F_{i,\alpha} + F_{0,\alpha} + Z_\alpha^* W/M^2|^2 \end{aligned} \quad (4)$$

with $M = M_{\text{Planck}}/\sqrt{8\pi}$.

In general, this is a horrible mess. Therefore, let us make some very mild assumptions which will simplify V considerably.

Consider expanding V in powers of the fields X_i and Z_α . Each term with d fields must be multiplied by some mass parameter to the power $4-d$. We assume that terms with $d > 4$ have coefficients no larger than $O(M^{4-d})$, and terms with $0 < d < 4$ have coefficients no larger than $O(\mu^{4-d})$, with $\mu \ll M$. Thus the renormalizable part of V is characterized by the small mass scale μ , and the violations of renormalizability are characterized by the large mass scale M .

Our assumptions imply that Λ is no larger than $O(\mu M^2)$. In practice, it will be possible to have a minimum of V with zero cosmological constant only if Λ is $O(\mu^2 M)$. It is always necessary to fine tune parameters in $N = 1$ supergravity in order to have zero cosmological constant. For our generalized O'Raifeartaigh models, we will treat Λ as the parameter to be adjusted.

Now we are ready to expand V in powers of μ/M . One might expect that the first term in this expansion is V_{global} , but this is not so. The reason is that the X_i , whose over-all scale is not determined by V_{global} , will get VEV's of order M once corrections of order μ^2/M^2 are included. The VEV's will cancel some inverse powers of M which appear explicitly in V . We can anticipate this by working with appropriately rescaled, dimensionless fields :

$$\begin{aligned} x_i &= X_i/M & \lambda &= \Lambda/\mu^2 M \\ z_\alpha &= Z_\alpha/\mu & \omega &= W/\mu^2 M \\ f_i &= F_i/\mu^2 & &= x_i f_i + \lambda + (\mu/M) f_0 \\ f_0 &= F_0/\mu^3 & & \end{aligned} \quad (5)$$

In terms of these new fields and functions, the scalar potential is

$$V = \mu^4 \exp [|\kappa_i|^2 + (\mu/M)^2 |z_\alpha|^2] \tilde{V} \quad (6)$$

$$\begin{aligned} \tilde{V} = & |f_i + \kappa_i^* \omega|^2 - 3|\omega|^2 \\ & + (M/\mu)^2 |\kappa_i f_{i,\alpha} + (\mu/M) f_{0,\alpha} + (\mu/M)^2 z_\alpha^* \omega|^2. \end{aligned} \quad (7)$$

We see that minimizing V requires

$$\kappa_i f_{i,\alpha} = O(\mu/M) \quad (8)$$

which is the same as in the global case (for X_1 of order M). To lowest order in μ/M we have

$$\omega = \kappa_i f_i + \lambda \quad (9)$$

$$V = \mu^4 \exp (|\kappa_i|^2) \tilde{V} \quad (10)$$

$$\tilde{V} = |f_i + \kappa_i^* \omega|^2 - 3|\omega|^2 \quad (11)$$

with the x_1 subject to the constraints

$$\kappa_i f_{i,\alpha} = \omega_{,\alpha} = 0. \quad (12)$$

There are corrections to Eqs. (9)-(12) of order μ/M .

Now we proceed to the analysis of the lowest order V as given by Eqs. (9)-(12).

We want to find minima of V at which $V = 0$. (As already mentioned, this means adjusting $\lambda = \Lambda/\mu^2 M$.) In addition to Eq. (12), the equations we must solve are

$$\tilde{V} = |f_i + \kappa_i^* \omega|^2 - 3|\omega|^2 = 0 \quad (13)$$

$$\begin{aligned} \frac{\partial \tilde{V}}{\partial \kappa_i} = & \kappa_i^* |\omega|^2 + f_i (\kappa_j f_j)^* + (|\kappa_j|^2 - 2) f_i \omega^* \\ & + c_\alpha f_{i,\alpha} = 0 \end{aligned} \quad (14)$$

$$\frac{\partial \tilde{\mathcal{N}}}{\partial z_\alpha} = f_i^* f_{i,\alpha} + c_\beta \omega_{\beta\alpha} = 0. \quad (15)$$

where the c_α are Lagrange multipliers enforcing Eq. (12). The parameter λ should be considered a variable along with x_i , z_α , and c_α . Supersymmetry is spontaneously broken if the gravitino mass ⁵⁾,

$$m_{3/2} = |\omega| \exp\left(\frac{1}{2}|\kappa_i|^2\right) \mu^2/M \quad (16)$$

does not vanish.

Two useful equations can be derived by multiplying Eq. (14) by either x_i or f_i^* and summing over i :

$$\kappa_i \frac{\partial \tilde{\mathcal{N}}}{\partial \kappa_i} = |\kappa_i|^2 |\omega|^2 + |\kappa_i f_i|^2 + (|\kappa_i|^2 - 2)(\kappa_j f_j) \omega^* = 0, \quad (17)$$

$$f_i^* \frac{\partial \tilde{\mathcal{N}}}{\partial \kappa_i} = (|f_i|^2 + |\omega|^2)(\kappa_j f_j)^* + (|\kappa_i|^2 - 2)|f_i|^2 \omega^* + c_\alpha f_i^* f_{i,\alpha} = 0. \quad (18)$$

Without loss of generality, we can choose λ to be real. Then Eqs. (9) and (17) imply

$$\text{Im}(\kappa_i f_i) = \text{Im}(\omega) = 0. \quad (19)$$

From now on, we will take λ real, and make use of Eq. (19).

Let us first try setting $c_\alpha = 0$, and see if there are any solutions. Equation (15) now reads $f_i^* f_{i,\alpha} = 0$, which is the set of equations we must solve to minimize V_{global} . Hence the z have the VEV's that they would in the absence of gravitational couplings ^{*)}.

A simple analysis of Eqs. (13), (17) and (18) with $c_\alpha = 0$ shows that

) Of course, solving $f_i^ f_{i,\alpha} = 0$ only finds extrema of V_{global} ; at this point, all we know is that the z_α have VEV's corresponding to some extremum of V_{global} . Eventually we must check to see whether any solution of Eqs. (12)-(16) corresponds to (at least) a local minimum of V as given by Eqs. (9)-(11).

$$|\kappa_i f_i|^2 = |\kappa_i|^2 |f_i|^2 \quad (20)$$

Thus, Eqs. (13), (17) and (18) depend only on $x \equiv (|x_i|^2)^{\frac{1}{2}}$ and the value of $f \equiv (|f_i|^2)^{\frac{1}{2}}$ corresponding to a solution of $f_i^* f_{i,\alpha} = 0$. Furthermore, these are just the equations we would get if we started with a superpotential $\omega = xf + \lambda$, where $f = \text{constant}$. This is identical to the superpotential already used to break local supersymmetry in all realistic models to date ! To see what this means in practice, consider enlarging the original superpotential by adding a scale invariant function of new fields ^{*)}, so that

$$W = X_i F_i(z) + F_0(z) + \Lambda + G(Y) . \quad (21)$$

Then the low energy theory (i.e., at scales much less than μ) depends only on the function G , the gravitino mass $m_{\frac{3}{2}}$ [see Eq. (16)], and the following function of the VEV's of the X_i and $\hat{Z}_\alpha$ ³⁾ :

$$A \equiv |\kappa_i|^2 + (\kappa_i f_i)/\omega \quad (22)$$

$$= 3\kappa_i f_i / (2\kappa_j f_j + \lambda) \quad (23)$$

In Ref. 3), it was shown that we must have $|A| > 3$ in order to have weak interaction breakdown at the tree level (assuming G is scale invariant). The generalized O'Raifeartaigh models with $c_\alpha = 0$ mimic the $\mu^2(X + \beta M)$ superpotential, which yields $A = 3 - \sqrt{3}$ ^{1),3)}.

What about solutions with $c_\alpha \neq 0$? Alas, if $c_\alpha \neq 0$, Eqs. (12)-(16) cannot be solved analytically. A computer was engaged to solve them numerically for $n = 2$, $p = 1$, and various choices of $f_1(z)$ and $f_2(z)$. The computer did find solutions with $c_\alpha \neq 0$, but they always turned out to be local maxima or saddle points. These numerical results are disappointing, but it is possible that a more exhaustive search for solutions would yield more encouraging results.

Of course, we could also relax our assumptions by allowing operators of dimension less than four to have coefficients with factors of M as well as μ . Investigation of this larger class of models is underway.

^{*)} Scale invariance of G is a "naturalness" condition ; see Ref. 3).

Another way to try to use these results in realistic models is to let one or more of the X_i transform non-trivially under some unified gauge group. For example, we could let one of the X_i be a 24 of $SU(5)$. Then this field would get a VEV of order M , breaking $SU(5)$ to (one hopes) $SU(3) \times SU(2) \times U(1)$. This is just Witten's inverted hierarchy mechanism⁸⁾, except that now it is gravitational interactions, rather than radiative corrections, which give the 24 its large expectation value. Note that for this scheme to be viable, the "bare" Planck mass must be large, in contrast to Witten's idea of having the large value of M generated dynamically⁸⁾.

It is important to note that one has some control over the magnitude of the VEV of the 24. To see this, consider the superpotential⁹⁾

$$W = \lambda_1 \text{Tr}(B Z^2) + \lambda_2 X [\text{Tr}(Z^2) - \mu^2] + \Lambda \quad (24)$$

where X is an $SU(5)$ singlet and B and Z are 24's. Then a solution of Eqs. (12)-(15) which yields $V \geq 0$ everywhere is

$$\begin{aligned} Z &= a^{-1} \lambda_2 \mu Y \\ B &= a^{-1} \lambda_2 (\sqrt{3} - 1) M Y \\ X &= a^{-1} \lambda_1 (\sqrt{3} - 1) M \\ \Lambda &= a^{-1} \lambda_1 \lambda_2 (2 - \sqrt{3}) \mu^2 M \end{aligned} \quad (25)$$

where $Y = \text{diag}(2, 2, 2, -3, -3)$ is the hypercharge generator, and $a = (\lambda_1^2 + 30\lambda_2^2)^{\frac{1}{2}}$. We see that for $\lambda_2 \lesssim 10^{-3} \lambda_1$, $B \lesssim (2 \times 10^{15} \text{ GeV}) Y$. Thus supergravity is capable of generating a grand unification scale significantly below M at the cost of some modest fine tuning of parameters.

After this manuscript was completed, I learned of a paper by Ibáñez¹⁰⁾ in which he points out that fields whose VEV's are undetermined by V_{global} may get VEV's of order M in $N = 1$ supergravity. All the models considered by Ibáñez have a non-zero cosmological constant. In this case, it is not clear that the mass terms in the Lagrangian correspond to physical particle masses¹¹⁾.

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