REF.TH.3461-CERN

CONSTRAINTS ON THE STABILITY
OF MASS HIERARCHIES IN SUPERGRAVITY

H.P. Nilles, M. Srednicki and D. Wyler
CERN -- Geneva

ABSTRACT

We study supersymmetric grand unified models coupled to $N = 1$ supergravity with a gauge hierarchy induced by the breakdown of local supersymmetry, and a low energy scale given by the gravitino mass. Certain conditions must be fulfilled for this hierarchy to remain stable in perturbation theory. We discuss these constraints and show that most presently available models do not have a stable hierarchy.

REF.TH.3461-CERN
12 November 1982

Spontaneously broken supergravity^{1),2)} can lead naturally to a small mass scale given by the gravitino mass, $m_{3/2}$. A grand unified theory with spontaneously broken local supersymmetry is characterized by three scales: the Planck mass $M \approx 10^{19}$ GeV, the grand unification mass $\mu \approx 10^{14} - 10^{16}$ GeV, and the gravitino mass $m_{3/2} \approx 10^1 - 10^3$ GeV. The supersymmetry breaking scale M_{SS} is given by $(m_{3/2} M)^{1/2}$. There might be relations^{3),4)} among M , μ , and $m_{3/2}$ but we will not discuss them here. Rather, we will concentrate our discussion on the relation between $m_{3/2}$ and the scale of weak interaction breakdown, $M_W \approx 10^2$ GeV.

$N = 1$ supergravity is a non-renormalizable theory and is used here as an approximation to the correct theory of gravity, which we assume will incorporate supersymmetry. In order to avoid the problems coming from non-renormalizability in perturbation theory we make use of the fact that both $m_{3/2}$ and μ are small compared to M , and take the limit $M \rightarrow \infty$ with μ and $m_{3/2}$ fixed⁵⁾. In this limit we arrive at a softly broken globally supersymmetric theory and perturbation theory is well defined. The soft breaking terms are of scale $m_{3/2}$ and include in general scalar masses, gauge fermion masses, and trilinear scalar couplings. We want these terms to induce the breakdown of weak interactions; hence we expect $m_{3/2}$ will be order M_W .

A grand unified model contains now two classes of particles: heavy particles with masses of order μ , and light particles, with masses of order $m_{3/2}$ or less. There are, of course, couplings between these two sectors. However, we want the light sector to remain light once higher order corrections are added to the tree level Lagrangian. It is possible that the division between light and heavy sectors is not maintained by these corrections, and that the gauge hierarchy is spoiled. We want to avoid this situation in realistic models.

In the following we will discuss the feed-down of supersymmetry breaking terms from the heavy sector to the light sector, following the analysis done by Polchinski and Susskind⁷⁾ for spontaneously broken globally supersymmetric theories. We will use first a simple toy model to establish the results and will then discuss their relevance for realistic locally supersymmetric grand unified theories⁸⁾⁻¹¹⁾.

The toy model we consider has the following superpotential:

$$g = \mu B^2 + m L^2 + \lambda_1 B^3 + \lambda_2 B^2 L + \lambda_3 L^3 + \lambda_4 B L^2 \quad (1)$$

where B and L are the complex scalar components of chiral superfields. B represents the heavy sector (μ is supposed to be of order of the grand unification mass) and L represents the light sector. To keep our analysis as general as possible, we include a mass term for L in the superpotential, even though we believe that this is unnatural⁶⁾. The parameter m should not be chosen larger than order $m_{3/2}$.

Coupled to spontaneously broken supergravity, this superpotential leads to the following scalar potential^{5),6)} in the limit $M \rightarrow \infty$ with μ and $m_{3/2}$ fixed:

$$V = \left| \frac{\partial g}{\partial B} \right|^2 + \left| \frac{\partial g}{\partial L} \right|^2 + m_{3/2}^2 |B|^2 + m_{3/2}^2 |L|^2 + m_{3/2} (A g - \mu B^2 - m L^2 + h.c.) \quad (2)$$

$$\begin{aligned} &= |2\mu B + 3\lambda_1 B^2 + 2\lambda_2 BL + \lambda_4 L^2|^2 \\ &\quad + |2mL + \lambda_2 B^2 + 3\lambda_3 L^2 + 2\lambda_4 BL|^2 \\ &\quad + m_{3/2}^2 |B|^2 + m_{3/2}^2 |L|^2 \\ &\quad + A m_{3/2} (\lambda_1 B^3 + \lambda_2 B^2 L + \lambda_3 L^3 + \lambda_4 BL^2 + h.c.) \\ &\quad + (A-1) m_{3/2} (\mu B^2 + m L^2 + h.c.) \end{aligned} \quad (3)$$

The constant A was defined and discussed in Ref. 6). In a realistic theory, there are additional terms coming from gauge interactions, but they will play no role in the following discussion. The fermion sector is that of a globally supersymmetric theory with superpotential g , plus (possibly) gauge fermion mass terms of order of $m_{3/2}$ or less.

In the language of superfields the terms in V that explicitly break supersymmetry can be represented by two spurion superfields¹²⁾

$$\hat{U} = \theta^2 \bar{\theta}^2 m_{3/2}^2, \quad \hat{V} = \theta^2 m_{3/2} \quad (4)$$

In this case, however, the use of supergraphs does not lead to very transparent results; we will discuss perturbation theory in terms of component graphs.

Before beginning this discussion we want to identify the magnitudes of the tree level mass splittings in the different sectors. The mass splittings among superpartners in the light sector are not larger than $m_{3/2}$. The splitting between the partners of the absorbed goldstino is also of the order $m_{3/2}$. (This sector has not been displayed here since it decouples in the limit $M \rightarrow \infty$.) In the heavy sector, however, the situation is different. There is a $B^* B$ term with coefficient $\mu^2 + m_{3/2}^2$ and a $(B^2 + B^{*2})$ term with coefficient $\mu m_{3/2}$. With $\mu \gg m_{3/2}$ the dominant splittings between the heavy scalars are thus of the order $\mu m_{3/2}$. This should be compared to the case of spontaneously broken global supersymmetry, where the splittings are of the order M_{SS}^2 . In our case M_{SS}^2 is $M m_{3/2}$ and so the splittings $\mu m_{3/2}$ are smaller. They are still large enough, however, to be dangerous if they enter the light sector through higher order corrections.

In our investigation of the feed-down from the heavy sector to the light sector, we concentrate on the generation of an $(L^2 + L^{*2})$ mass term. This term is among the most dangerous and it illustrates the general mechanism. In principle, the hierarchy could be destroyed by the appearance of quadratic divergences for the $(L^2 + L^{*2})$ term. However, they do not appear (as can be explicitly checked) since the global supersymmetry is only softly broken¹²⁾.

The coupling constants that are responsible for the feed-down from the heavy to the light sector are λ_2 and λ_4 (see Eq. 1). Let us first consider contributions proportional to λ_4 . The graphs that could induce large masses for L are shown in Fig. 1. Each of the graphs individually induces a mass term of the order $\lambda_1 \lambda_4 \mu m_{3/2} (L^2 + L^{*2})$, but the sum cancels to the leading order in $m_{3/2}/\mu$, leaving a coefficient of the order $\lambda_1 \lambda_4 m_{3/2}^2$. There are also graphs like Fig. 1a in which the couplings $\mu \lambda_1$ and/or $\mu \lambda_4$ are replaced by $m_{3/2} \lambda_1$ and/or $m_{3/2} \lambda_4$, respectively. Hence these contributions do not affect the stability of the hierarchy. This is true of all contributions proportional to λ_4 .

One remark should be added here. We have seen that supersymmetry breaking in the heavy sector induces mass splittings of the order $m_{3/2}$ in the light sector. Hence, even if the light scalars were massless at the tree level we

* All the graphs we discuss are logarithmically divergent; the divergences can be treated via the usual renormalization procedure.

would expect them to receive a mass of the order $m_{3/2}$ through radiative corrections. This implies that $m_{3/2}$ cannot be much larger than M_W even in the absence of explicit breaking terms in the light sector.

Next we discuss radiative corrections due to the $\lambda_2 B^2 L$ coupling. The "dangerous" contributions are shown in Fig. 2. If m is no larger than order $m_{3/2}$, these graphs give rise to a term of the order $\lambda_2 \lambda_3 \mu m_{3/2} (L^2 + L^{*2})$ in the low energy effective potential and spoil the hierarchy that existed at the tree level*. It is interesting to note that the soft breaking term $\mu m_{3/2} (B^2 + B^{*2})$ alone is not enough to spoil the hierarchy. It is the combination of this term and the terms $m_{3/2} \lambda_2 B^2 L$ and $m_{3/2} \lambda_3 L^3$ that leads to the large coefficient for $(L^2 + L^{*2})$, and introduces mass splittings in the light sector of the order $\lambda_2 \lambda_3 \mu m_{3/2} \gg m_{3/2}^2$.

Locally supersymmetric grand unified theories should avoid this disease of our toy model since the stability of the hierarchy was the only motivation for considering this type of model. There are two obvious ways to avoid the problem:

- i) absence of light $SU(3) \times SU(2) \times U(1)$ singlets, which are the only light fields that can have couplings analogous to $\lambda_2 B^2 L$, and
- ii) absence of couplings analogous to $\lambda_2 B^2 L$ even though there are one or more light singlets.

Both possibilities lead to other difficulties.

Concerning possibility (i), we note that it is impossible to construct a low energy theory consisting of Higgs doublet and quark and lepton superfields alone which has an acceptable pattern of $SU(2) \times U(1)$ breakdown at the tree level**. It is possible (though by no means certain) that the breakdown can be induced radiatively if there are very heavy quarks***. It is thus tempting to introduce additional fields. The simplest choice is an $SU(3) \times SU(2) \times U(1)$ singlet leading to superpotentials like

* The graphs all vanish if $A = 1$ exactly. Even if we arrange $A = 1$ at the tree level, however, it is unlikely that the hierarchy would survive in higher orders of perturbation theory.

** This is because the only terms in the low energy effective potential which are not minimized by vanishing VEV's for all fields are of the form $(H Q \bar{Q} + \text{h.c.})$, where H is a Higgs doublet and Q and $\bar{Q}$ are quark or lepton scalars. We must have $Q \neq 0 \neq \bar{Q}$ for such a term to produce a local minimum.

*** Note that in Ref. 10), no attempt is made to compute the full, one loop effective potential, and hence no firm conclusions can be drawn concerning the pattern of spontaneous symmetry breaking.

$$g = \lambda_3 Y H \bar{H} + \lambda'_3 Y^3 \quad (5)$$

A more complicated way (the next simplest) is to use an $SU(2)$ triplet instead of the singlet. This triplet will get a VEV along with the Higgs doublets, and one would have to arrange parameters carefully in order to preserve $M_W \cong M_Z \cos \theta_W$. The introduction of $SU(2)$ triplets (possibly with non-zero hypercharge and coupling like $T_1 H H + T_0 H \bar{H} + T_{-1} \bar{H} \bar{H}$) is also somewhat problematic in a grand unified theory.

On the other hand, one could try to live with the singlet and try possibility (ii), i.e., not couple it to any heavy fields. At low energies we would have the superpotential of Eq. 5. The simplest way to embed this in a grand unified theory such as $SU(5)$ would be to make Y an $SU(5)$ singlet. Then one is forced to have a coupling $\lambda_3 Y H_3 \bar{H}_3$ to the necessarily heavy Higgs colour triplets, which have mass splittings of the order $\mu m_{3/2}$. Higgs doublet masses of the order $(\mu m_{3/2})^{1/2}$ would be induced by the graph in Fig. 3 and the hierarchy would be destroyed. Thus Y cannot be an $SU(5)$ singlet. Any resolution of this problem would require a highly complicated $SU(5)$ Higgs sector.

To conclude, we have shown that the supergravity hierarchy is potentially unstable in the presence of light $SU(3) \times SU(2) \times U(1)$ singlets. This gives strong constraints on the construction of locally supersymmetric grand unified theory. A model that fulfils these constraints and is definitely phenomenologically viable has yet to be constructed.

ACKNOWLEDGEMENTS

We would like to thank S. Ferrara, D. Nanopoulos and C. Savoy for discussions.

REFERENCES

- 1) E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello and P. van Nieuwenhuizen, Nucl. Phys. B147 (1979) 105.
- 2) E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, Phys. Lett. 116B (1982) 219 and CERN preprint TH.3348 (1982).
- 3) H.P. Nilles, Phys. Lett. 115B (1982) 193, and Proceedings of the Johns Hopkins Workshop on Current Problems in Particle Theory 6, ed. G. Domokos et al., Baltimore (1982).
- 4) H.P. Nilles, CERN preprint TH.3398 (1982).
- 5) R. Barbieri, S. Ferrara and C. Savoy, CERN preprint TH.3365 (1982).
- 6) H.P. Nilles, M. Srednicki and D. Wyler, CERN preprint TH.3432 (1982).
- 7) J. Polchinski and L. Susskind, SLAC preprint PUB 2924 (1982).
- 8) R. Arnowitt, P. Nath and A.H. Chamseddine, Phys. Rev. Letters 49 (1982) 970 and Northeastern University preprint 2565 (1982).
- 9) L. Ibáñez, CERN preprint TH.3374 (1982) and University of Madrid preprint FTUAM/82-8 (1982).
- 10) J. Ellis, D.V. Nanopoulos and K. Tamvakis, CERN preprint TH.3418 (1982).
- 11) N. Ohta, University of Tokyo preprint UT-388 (1982).
- 12) L. Girardello and M. Grisaru, Nucl. Phys. B.194 (1982) 65.

FIGURE CAPTIONS

Fig. 1 Graphs that contribute to $(L^2 + L^{*2})$ mass terms.

Fig. 2 Graphs that give large masses to light fields.

Fig. 3 A graph in which a light $SU(5)$ singlet spoils the hierarchy.

Fig. 1

Fig. 2

Fig. 3