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COMPENSATION OF ERRORS BY CROSS-CALIBRATION OF INCOMING AND
SCATTERED ENERGIES IN DEEP-INELASTIC LEPTON SCATTERING

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ABSTRACT

The advantages arising from cross-calibration of the incoming beam energy versus the scattered particle spectrometer in deep-inelastic charged lepton experiments are discussed. After cross-calibration, measured quantities of greatest interest for QCD tests are only weakly affected by uncertainties in absolute energy values.

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Deep-inelastic scattering of charged high-energy leptons has yielded fundamental information on the structure of nucleons [1]. Nowadays it still presents one of the most promising avenues for testing the quantum chromodynamic theory of strong interactions (QCD).

In the kinematical region of interest for experiments at energies of several hundred GeV, the inclusive unpolarized deep-inelastic differential cross section may be written in the form (see fig. 1 for the notation)

$$\frac{d^2\sigma}{dQ^2 dv} = \frac{4\pi\alpha^2}{Q^4 v} \left[1-y + \frac{1}{1+R(x,Q^2)} \frac{y^2}{2} \right] F_2(x,Q^2) . \quad (1)$$

The interesting quantities to be extracted from measured cross sections are the nucleon structure function $F_2(x,Q^2)$ and the ratio $R(x,Q^2) = \sigma_L/\sigma_T$, $\sigma_L(\sigma_T)$ being the absorption cross section for longitudinal (transverse) virtual photons. It is expected that with increasing Q^2 , $R(x,Q^2)$ approaches zero while $F_2(x,Q^2)$ becomes largely independent of Q^2 (Bjorken scaling). The small remaining deviations from exact scaling are predicted to depend logarithmically on Q^2 [2]. Thus they are characterized by the quantity $U = \partial \log F_2(x,Q^2) / \partial \log Q^2$ and are usually parametrized by the QCD renormalization parameter Λ which in zero order perturbative QCD is defined as

$$\alpha_s(Q^2) = \frac{12\pi}{33-2N_f} \frac{1}{\log Q^2/\Lambda^2} ,$$

α_s being the running coupling constant and N_f the number of flavours in the theory.

The relation between U and Λ is given in terms of complicated evolution equations which are usually solved numerically. One such solution [3] can be approximately described by the following simple expression

$$U = U_0(\Lambda) [x-2(x-0.45)^3] + U_1(\Lambda)(x-0.121) \log \frac{Q^2}{Q_0^2}$$

with $Q_0^2 = 50 \text{ GeV}^2$, Λ measured in MeV and

$$U_0(\Lambda) = -0.164 - 8.9 \cdot 10^{-4} \Lambda + 1.4 \cdot 10^{-6} \Lambda^2$$

$$U_1(\Lambda) = 0.014 + 2.1 \cdot 10^{-4} \Lambda - 0.22 \cdot 10^{-6} \Lambda^2.$$

This expression is valid to an accuracy of a few percent in the region $0.25 < x < 0.75$, $10 < Q^2 < 300 \text{ GeV}^2$, $50 < \Lambda < 200 \text{ MeV}$. As an example, at $Q^2 = 50 \text{ GeV}^2$, $x = 0.45$, $\Lambda = 100 \text{ MeV}$, we find

$$U = -0.11$$

and

$$\frac{\partial U}{\partial \log \Lambda} = -0.03 \quad \text{or} \quad \frac{\partial \log |U|}{\partial \log \Lambda} = 0.26 ,$$

i.e. typically U is small and not very sensitive to Λ .

The small values of R and U make them difficult to measure with high relative accuracy. In modern high statistics experiments [4], systematic errors dominate. Among the most important sources of systematic error are the uncertainties on the energies of both the incident and the scattered lepton. From expression (1) follows (see Appendix) that a small relative shift dE/E in incident energy E leads to a relative change in cross section

$$\begin{aligned} \frac{d\sigma}{\sigma} &= A_E^{(1)} \frac{dE}{E} \\ \text{with} \quad A_E^{(1)} &= -2 - \frac{1-y}{y} T + U - \frac{1-y}{y} V . \\ V &= \frac{\partial \log F_2}{\partial \log x} \\ T &= 1 - \frac{\partial \log Y}{\partial \log y} = 1 + \frac{Y}{Y} \left(1 - \frac{y}{1+R}\right), \quad Y = 1-y + \frac{y^2}{2(1+R)} . \end{aligned} \tag{2}$$

Here, terms higher than linear in dE have been neglected and R has been assumed to be constant. The cross section σ is the differential cross section at the original E integrated over a small but finite Q^2, x bin as computed from measured E', θ , and the original E .

Similarly, incorrect calibration of the spectrometer which measures the energy E' of the scattered lepton will lead to an error in the determination of the cross section σ . We shall limit ourselves to the simplest case of a calibration curve of the type $E'_{\text{real}} = k E'_{\text{meas}}$, E'_{meas} and E'_{real} being the measured and real energy of the scattered lepton, respectively, and k being a proportionality constant which is subject to error. In the case of a magnetic spectrometer, k is proportional to the magnetic field strength while geometric quantities determining E'_{meas} are usually much better

known than the field strength. More complicated calibration curves involving more than one error-prone parameter would have to be investigated separately. A slightly incorrect calibration of the spectrometer such that the measured energy of the scattered lepton is E' , while in reality it should be $E' - dE'$, leads to a change of measured cross section

$$\frac{d\sigma}{\sigma} = A_{E'}^{(1)} \frac{dE'}{E'}$$

with

$$A_{E'}^{(1)} = \frac{1-y}{y} T + U + \frac{1}{y} V \quad (3)$$

Again, terms higher than linear in dE' have been neglected and σ is the differential cross section at the measured E' integrated over a Q^2, x bin as computed from E , θ and the measured E' .

The "error amplification factors" $A_E^{(1)}$ and $A_{E'}^{(1)}$ are scaling functions of x and y in the limit of exact scaling, i.e. for Q^2 -independent structure functions. Contours of constant $A_E^{(1)}$ and $A_{E'}^{(1)}$ in the $x, v = xy = Q^2/Q_{\max}^2$ plane are presented in fig. 2 assuming $R = 0$ and $F_2(x) \sim (1-x)^3$. We see that $A_E^{(1)}$ and $A_{E'}^{(1)}$ become very large at small y and at large x (*). In addition, their variation with Q^2 leads to a fake scaling violation if the incident and/or outgoing energy calibration is in error. Also, since the measurement of R requires comparing cross sections at constant Q^2 and x , but different y , i.e. different v , energy calibration errors may seriously affect the determination of R .

Let us suppose that it is possible to calibrate the outgoing lepton spectrometer with the incident beam. An incident energy that in reality is higher by dE than the assumed nominal energy E will lead to a spectrometer calibration such that the measured outgoing energy E' is underestimated by dE at $E' = E$.

(*) The singularity of $A_E^{(1)}$ and $A_{E'}^{(1)}$ at $x = 1$ is removed by the Fermi motion in nuclear targets but V is still expected to reach values of the order of 10 at large x .

In other words, at $E' = E$ the value $dE' = dE$ has to be inserted into eq. (3) and we obtain

$$\frac{d\sigma}{\sigma} = A_E^{(1)} \frac{dE}{E} + A_{E'}^{(1)} \frac{dE'}{E'} = (A_E^{(1)} + A_{E'}^{(1)}) \frac{dE}{E} \quad (4)$$

Under the above assumption on the form of the spectrometer calibration curve $dE'/E' = \text{const.}$ and eq. (4) is valid for any E and E' .

From expressions (2) and (3) we obtain

$$A_E^{(1)} + A_{E'}^{(1)} = -2 + 2U + V = -2 + 2 \frac{\partial \log F_2}{\partial \log Q^2} + \frac{\partial \log F_2}{\partial \log x} \quad (5)$$

If $F_2(x, Q^2) = F_2(x)$, i.e. if the structure function scales, the sum of the amplification factors $A_E^{(1)} + A_{E'}^{(1)}$ also depends only on x . Lines of constant $A_E^{(1)} + A_{E'}^{(1)}$ in fig. 2 are then parallel to the v -axis i.e. common incident and outgoing energy errors do not cause fake scaling violations. It is intuitively clear and can be easily checked that also in the real case of F_2 slowly varying with Q^2 , the systematic errors of U are greatly reduced by the cross calibration of incident vs. outgoing energy.

The x -dependence of the structure functions is still strongly affected by energy errors even after cross-calibration. However, with the present level of theoretical understanding of the x -dependence of structure functions [5], the requirements on experimental accuracy are not very strict and can be met with feasible absolute energy calibrations.

R is determined by ratios of cross sections and functions of y , for different incident energies but constant Q^2 and x . After cross-calibration (at all incident energies used in the R determination), $y = 1 - E'/E$ is not affected by energy errors and the measured cross sections differ from the true ones by a constant factor f , namely

$$f = 1 + (A_E^{(1)} + A_{E'}^{(1)}) \frac{dE}{E} \quad .$$

The factor f is a constant provided $\frac{dE}{E} = \frac{dE'}{E'}$ is constant for all incident and outgoing energies used in the R determination, i.e. if our assumption about the form of the spectrometer calibration curve is valid. Then R is not affected by a potential common error in absolute energy calibration even if $A_E^{(1)} + A_{E'}^{(1)}$ had an arbitrary Q^2, x dependence.

We see that the cross-calibration has a somewhat different role in the determination of U and R . The quantity U can be measured, in principle, at a single incident energy. An outgoing energy spectrometer with linear response calibrated with the incident beam dramatically reduces the effect on U of uncertainties in absolute energy calibration, because structure functions happen to obey Bjorken scaling to a very good approximation. In the determination of R the essential advantage of cross calibration is the use of a single outgoing energy spectrometer with linear response for the relative calibration of different incident energies. In the case of R , the error compensating mechanism does not depend on any particular behaviour of the structure functions.

The method of cross-calibrating incoming and scattered energies is not entirely new [6,7] though no explicit discussion is given by previous authors about its benefits in compensating systematic errors. In electron experiments the absolute accuracy in the calibration of E was $\pm 0.1\%$, and the measurement of elastic ep scattering allowed to calibrate the spectrometer value of E' versus E to 0.05% [6]. In the Fermilab measurement of deep inelastic muon scattering on hydrogen and deuterium the relative errors in both beam momentum tagging and spectrometer magnet were about $\pm 0.1\%$, whilst the δB were known for both magnets to better than 1% : the cross calibration of E vs E' was obtained by using a large number of μe elastic scatters and beam events [7]. The EMC group at CERN have measured beam tracks at several energies in their air-gap magnetic spectrometer [8] in order to correlate it with the beam momentum station [9]. This latter method of shooting the beam into the spectrometer is also the most promising one for a magnetized iron spectrometer such as is used by the BCDMS group [10].

With respect to the idealized case of homogeneous magnetic field and monochromatic beam, as treated above, a magnetized iron spectrometer presents several problems. In fact, the field is not constant but shows a mild radial dependence [11] and the energy loss of muons in iron is far from being negligible, especially at the lower energies. Knowing the stopping power and being able to measure the path length in iron as well as the local radius of curvature, it should be possible to calibrate the local B with respect to the incident energy E . However, to reach accuracies of the order of 10^{-3} one needs a proper parametrization of the magnetic field shape as well as a large sample (10^5 - 10^6) of beam tracks.

Any cross-calibrating method that is not using a calibrating monochromatic energy line embedded in all data depends on the time stability of the beam line for the incident leptons and of the spectrometer for the outgoing leptons. However, with proper care, it is possible to ensure this stability in time to a much better accuracy than it is generally feasible to carry out absolute energy calibrations of spectrometers and beam transport lines.

A comment is due about the alternative to the measurement of the energies of incident and scattered lepton: using a calorimeter, one can measure the energy E'' of the hadronic shower in the final state. Since $E + M = E' + E''$ any two of the energies E , E' and E'' determine the third one. In case the energies of the scattered lepton and the shower are measured but not the incident lepton energy, we call the corresponding amplification factors $A_{E'}^{(2)}$ and $A_{E''}^{(2)}$. By the method sketched in the Appendix one derives

$$A_{E''}^{(2)} = y A_E^{(1)}$$

$$A_{E'}^{(2)} + A_{E''}^{(2)} = -2 + 2U + V = A_E^{(1)} + A_{E'}^{(1)}. \quad (6)$$

The alternative measurement is advantageous at small y : if only E and E' are measured one has to calculate E'' , i.e. v and y , by taking differences of nearly equally large quantities. However, the alternative measurement technique requires an accurately calibrated hadron calorimeter. Fig. 3 shows contours of constant $A_{E'}^{(2)}$ and $A_{E''}^{(2)}$.

The alternative technique is the only one accessible to experiments on charged currents with incident neutrino beams. In this case, an added advantage is the absence of the Q^{-4} factor in the deep-inelastic cross section. For neutrinos we derive (considering only the sum of cross sections for neutrino and anti-neutrino deep inelastic scattering on isoscalar targets)

$$A_{E''}^{(3)} = y(A_E^{(1)} + 2)$$

$$A_{E'}^{(3)} + A_{E''}^{(3)} = 2 + 2U + V = A_E^{(1)} + A_{E'}^{(1)} + 4 \quad (7)$$

Fig. 4 shows contours of constant $A_{E'}^{(3)}$ and $A_{E''}^{(3)}$. An interesting and often debated question is whether neutrinos or charged leptons are more likely to yield accurate F_2 functions. While the answer requires careful consideration of many factors, from the present point of view neutrinos seem superior at least with respect to the Q^2 -dependence of F_2 .

A quantitative measure of the fake scaling violations induced by energy errors are the partial derivatives of the amplification factors with respect to $\log Q^2$ or $\log y$ at constant x and E . In the approximation of a scaling F_2 -function we derive from eqs. (2), (3), (6), (7)

$$\frac{\partial A_E^{(1)}}{\partial \log y} = - \frac{\partial A_{E'}^{(1)}}{\partial \log y} = \frac{1}{y} [f_1(y) + V]$$

$$\frac{\partial A_{E''}^{(3)}}{\partial \log y} = - \frac{\partial A_{E'}^{(3)}}{\partial \log y} = y [f_2(y) + V]$$

$$\frac{\partial A_{E''}^{(2)}}{\partial \log y} = - \frac{\partial A_{E'}^{(2)}}{\partial \log y} = y [f_2(y) - 2 + V]$$

with

$$f_1(y) = 1 + (1-y) \left(\frac{y}{Y} \right)^2$$

and

$$f_2(y) = \frac{y}{Y^2} \left(2 - \frac{5}{2} y + y^2 - \frac{y^3}{4} \right)$$

The functions $f_1(y)$ and $f_2(y)$ are shown in fig. 5. We see that at sufficiently small y , neutrinos are certainly superior to charged leptons, except perhaps in a narrow x -region around $x \approx 0.25$ where $f_1(y) + V \approx 0$. On the other hand, at sufficiently large y we have $f_1(y) \approx f_2(y)$ and

$$\frac{\partial A_E^{(1)}}{\partial \log y} \approx \frac{1}{y^2} \frac{\partial A_{E''}^{(3)}}{\partial \log y}.$$

If the accuracy of the hadron calorimeter and muon spectrometer calibration in neutrino experiments were comparable to the accuracy of the beam-spectrometer intercalibration in charged lepton experiments, neutrinos would be superior to charged leptons also at large y -values. However, the beam-spectrometer intercalibration is capable of much higher accuracy than the energy calibrations in neutrino experiments^(*). Assuming e.g. that the former has been performed ten times more accurately than the latter, fake scaling violations should be less probable in the charged lepton data

* In a certain sense, neutrino experiments also intercalibrate the hadron calorimeter and the muon spectrometer by requiring that on the average E' and E'' add to the known average energy of a narrow band neutrino beam [12]. However the accuracies reached with this procedure are only of the order of 1%.

than in the neutrino data for $y \gtrsim 0.3$. We see that, in practice, charged leptons and neutrinos complement each other in their relative immunity to fake scaling violations induced by systematic energy errors.

Another comment concerns future experiments with e^-p colliders. They will study both neutral current interactions of the type $e^-p \rightarrow e^-X$ and charged current interactions of the type $e^-p \rightarrow \nu X$, X being the hadronic system in the final state. To detect the latter, a hadron calorimeter (or any equivalent apparatus aiming at determining the four-momentum of X) is necessary. In all probability, it will have to be a sophisticated device, not easy to calibrate. Furthermore, it is expected that X consists of two components: a parton jet and a target fragmentation jet. The latter will disappear nearly undetected down the beam pipe, necessitating model dependent assumptions in the data analysis. All these problems in the study of charged currents can be greatly alleviated by the comparison with neutral current events where the electron in the final state can be accurately measured. Therefore, accurate determination of the energies and angles of the incident proton and the incident and scattered electron will presumably represent the basic experimental method at e^-p colliders. This method is essentially the first method of our discussion and the amplification factors $A_E^{(1)}$ and $A_{E'}^{(1)}$, as given in Lorentz invariant form by expressions (2) and (3), turn out to be applicable also in the case of collider experiments. This means that it is again nearly mandatory to intercalibrate the primary electron energy and the spectrometer for the scattered electron. Bhabha scattering from an auxiliary positron beam seems to be the method of choice for this purpose. In addition, one may worry about the uncertainty in the primary proton energy E_p . We derive

$$A_{E_p}^{(1)} = -V = -\frac{\partial \log F_2}{\partial \log x}$$

which has a benign y -dependence, if any, in contrast to $A_E^{(1)}$ and $A_{E'}^{(1)}$.

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Figures Caption

Fig. 1 Kinematics of deep-inelastic charged lepton scattering on a nucleon.

Fig. 2 Contours of constant error amplification factors in the x, v plane:

- a) concerning the incoming energy, $A_E^{(1)}$;
 - b) concerning the scattered lepton energy, $A_{E'}^{(1)}$;
 - c) sum of $A_E^{(1)}$ and $A_{E'}^{(1)}$ showing scaling behaviour with v .
- $F_2 \approx (1-x)^3$ and $R = 0$ have been assumed in the calculation.

Fig. 3 Contours of constant error amplification factors in the x, v plane:

- a) concerning the hadron energy, $A_{E''}^{(2)}$;
 - b) concerning the scattered lepton energy, $A_{E'}^{(2)}$.
- $F_2 \approx (1-x)^3$ and $R = 0$ have been assumed in the calculation.
 $A_{E'}^{(2)}$ reaches its maximum value $A_{E'}^{(2)} = 0$ at $x=0, y=1$
while $A_{E''}^{(2)}$ reaches its minimum value $A_{E''}^{(2)} = -2$ at $y=1$.

Fig. 4 Contours of constant error amplification factors in the x, v plane:

- a) concerning the hadron energy, $A_{E''}^{(3)}$;
 - b) concerning the scattered lepton energy, $A_{E'}^{(3)}$.
- $F_2 \approx (1-x)^3$ and $R = 0$ have been assumed in the calculation.
At $x = y = 0$, $A_{E'}^{(3)}$ reaches its maximum value $A_{E'}^{(3)} = 3$
while $A_{E''}^{(3)}$ reaches its minimum value $A_{E''}^{(3)} = -1$.

Fig. 5 Functions of y entering in the partial derivatives of the error amplification factors.

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APPENDIX: Derivation of $A_E^{(1)}$ and $A_{E'}^{(1)}$.

Assuming $R(x, Q^2) = \text{const.}$, the deep inelastic cross section (1) may be written in the form

$$d^2\sigma = \text{const.} \frac{dQ^2}{Q^4} \frac{dv}{v} Y(y) F_2(x, Q^2) \quad (\text{A1})$$

with

$$Y(y) = 1 - y + \frac{y^2}{2(1+R)}$$

We assume an error-free measurement of the scattering angle and a calibration of the spectrometer for scattered leptons of the form

$$E' = k E'_m \quad (\text{A2})$$

where k is a proportionality factor such that the scattered energy E' is obtained by multiplying a measured quantity E'_m by k . The factor k may be a dimensioned quantity, e.g. the magnetic field B , and it may be subject to error. On the other hand, we assume that E'_m is error-free. We have

$$\left| \frac{dQ^2 dv}{Q^4 v} \right| = \left| \frac{d\cos\theta}{2EE'(1-\cos\theta)^2(E-E')} \frac{dE'}{dE'_m} \right| = \left| \frac{d\cos\theta}{2(1-\cos\theta)^2 E'^2 y E'_m} \frac{dE'_m}{dE'_m} \right| \sim \frac{1}{E'^2 y} \quad (\text{A3})$$

where error-free factors have been omitted from the last expression.

Indicating the correct value of any quantity by the subscript o and the difference between the incorrect and the correct value with the symbol δ , we obtain from (A1) and (A3) to first order in the errors $\delta E = E - E_o$,

$$\delta k = k - k_o$$

$$\frac{\delta(d^2\sigma)}{d^2\sigma} = \frac{d^2\sigma - d^2\sigma_o}{d^2\sigma_o} = -2 \frac{\delta E}{E} - \frac{\delta y}{y} + \frac{\delta Y}{Y} + \frac{\delta F_2}{F_2} = A_E^{(1)} \frac{\delta E}{E} + A_{E'}^{(1)} \frac{\delta k}{k} \quad (\text{A4})$$

From (A2) follows

$$\frac{\delta E'}{E'} = \frac{\delta k}{k} \quad (\text{A5})$$

Because of its direct physical meaning we shall use $\delta E'/E'$ in the subsequent formulae but it should be kept in mind that we obtain the value of $\delta E'/E'$ from expression (A5) and a known (or estimated) calibration error $\delta k/k$.

Considering that

$$\frac{\delta Q^2}{Q^2} = \frac{\delta E}{E} + \frac{\delta E'}{E'}$$

$$\frac{\delta y}{y} = \frac{1-y}{y} \left(\frac{\delta E}{E} - \frac{\delta E'}{E'} \right)$$

$$\frac{\delta x}{x} = \frac{\delta Q^2}{Q^2} - \frac{\delta v}{v} = - \frac{1-y}{y} \frac{\delta E}{E} + \frac{1}{y} \frac{\delta E'}{E'}$$

$$\frac{\delta Y}{Y} = (1-T) \frac{\delta y}{y}$$

$$\frac{\delta F_2}{F_2} = U \frac{\delta Q^2}{Q^2} + V \frac{\delta x}{x}$$

with

$$T = 1 - \frac{\partial \log Y}{\partial \log y} = 1 + \frac{y}{Y} \left(1 - \frac{Y}{1+R} \right)$$

$$U = \frac{\partial \log F_2}{\partial \log Q^2}$$

$$V = \frac{\partial \log F_2}{\partial \log x}$$

we obtain from (A4)

$$\frac{\delta(d^2\sigma)}{d^2\sigma} = A_E^{(1)} \frac{\delta E}{E} + A_{E'}^{(1)} \frac{\delta E'}{E'} \quad (A6)$$

with

$$A_E^{(1)} = -2 - \frac{1-y}{y} T + U - \frac{1-y}{y} V \quad (A7)$$

$$A_{E'}^{(1)} = \frac{1-y}{y} T + U + \frac{1}{y} V \quad (A8)$$

The assumption (A2) could be replaced by a more complicated calibration relation, e.g.

$$E' = a + bE_m' + cE_m'^2 \quad (A9)$$

with the coefficients a , b , and c subject to error. The subsequent analysis would be as simple as before but the relation (A5) would not anymore be applicable, of course. Therefore, the relative error in $d^2\sigma$ (expression (A6)) would have to be expressed in terms of $\delta E/E$, $\delta a/a$, $\delta b/b$ and $\delta c/c$, by means of "error amplification coefficients" A_E , A_a , A_b , and A_c respectively.

The quantities Q^2 , v , y and x may be expressed in their invariant form:

$$Q^2 = - (p_1 - p_3)^2$$

$$v = p_2(p_1 - p_3)/M_2$$

$$y = 1 - \frac{p_2 p_3}{p_1 p_2}$$

$$x = \frac{Q^2}{2M_2 v}$$

p_i being the four-momentum of the particle i ($i = 1, 2, 3, 4$) participating in the scattering $1 + 2 \rightarrow 3 + 4$. Here it is assumed that the lepton is designated by 1 or 3 depending on whether it is the incident or scattered one. M_2 is the mass of the target.

Using the method of this Appendix, it is then a simple matter to derive error amplification factors for any method of measuring energies and any experiment, either with a fixed target or in a collider. The essential steps are the choice of the two or three independent energies to be measured and the proper calibration relation (e.g. expression (A9)) for the energy entering into expression (A1) as a differential. In principle, instead of one angle differential and one energy differential, two energy differentials dE_3 and dE_4 could be used, requiring two calibration relations of the type (A9).

$$Q^2 \approx EE' \quad \theta^2 \approx E p_T^2 / E'$$

$$\nu = E_{\text{had}} = E - E'$$

$$x = Q^2 / 2M_N \nu$$

$$y = \nu / E$$

FIG. 1

FIG. 2

$V = x, y$

FIG. 3

$V=X, y$

FIG. 4

FIG. 5