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THE DEUTERON D/S RATIO AND QUADRUPOLE MOMENT :

A QUANTITATIVE EXAMPLE OF PION PHYSICS

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A B S T R A C T

It is demonstrated that the deuteron D/S ratio and quadrupole moment are strongly OPEP dominated. Corrections to this picture are nearly model-independent. The analysis indicates strongly a pointlike πNN coupling. The investigation provides the most quantitative demonstration of pionic effects in nuclear physics.

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Frequent questions from particle physicists in the last few years have been : is there any real evidence that pions exist inside nuclei ? Is the "evidence" not simply evidence for quark-antiquark pairs or gluon exchanges ? Even if there are pions will not their properties be violently changed from those of free pions ?

These questions are very heretical with the present participants. All of us know that there is plenty of evidence for pionic effects in nuclei. If you try to answer the particle physicists, you find that there is a point to their heresy, however. Nearly all the evidence we have depends on specific nuclear physics in some way, and is rather involved to explain or deals with 15-20% effects. In order to refute the heresy we must be able to answer the devil's advocates by pointing to dominant, clear-cut and nearly model-independent evidence.

M. Rosa-Clot and myself have just finished a detailed investigation of the deuteron D/S ratio and quadrupole moment ¹⁾. We find that these provide an excellent, very quantitative answer to the question. Consider the S and D wave deuteron wave function with the asymptotic properties

$$\left. \begin{aligned} u(r) &\rightarrow N e^{-\alpha r} \\ w(r) &\rightarrow \eta N e^{-\alpha r} \end{aligned} \right\} \quad (1)$$

Here N is the asymptotic S wave normalization known to high accuracy in terms of the effective range parameter $\rho(-\epsilon, -\epsilon)$. The D wave asymptotic strength is given by the D/S ratio η . This quantity has recently been determined experimentally to a new level of precision by three independent methods ²⁾⁻⁵⁾

$$\eta = 0.0271 (\pm \text{few } \%). \quad (2)$$

The existence of the deuteron D state is due to the tensor interaction, which has a very strong pionic component.

To explore the effects of the interaction let us assume the usual potential approach and write the deuteron coupled equations

$$u'' = (\alpha^2 + U_{00})u + \sqrt{8} U_T w \quad (3a)$$

$$w'' = (\alpha^2 + 6/r^2 + U_{22})w + \sqrt{8} U_T u. \quad (3b)$$

Here $U_{22} = MV_{22}$ is the diagonal $L = 2$ interaction while $U_T = MV_T$ is the tensor potential. For us the crucial equation is Eq. (3b), which has a driving source $\sqrt{8} U_T u$ generating D state by the tensor potential and out of the S wave function u . In the D state a crucial property of the rescattering potential $(6/r^2 + U_{22})$ is that it is repulsive both owing to the centrifugal barrier as well as to the strong pionic repulsion (Fig. 1). The repulsion has the nice property of making the problem insensitive to unknown short-range effects.

The central step in our new approach to this problem is the use of a source function technique. In fact, the exact formal solution to Eq. (3b) is

$$W(r) = \int_0^\infty G_2(r, r') \sqrt{8} U_T(r') u(r') dr', \quad (4)$$

where $G_2(r, r')$ is the exact Green function for the D wave interaction. As usual the Green function is the product of $r_<$ and $r_>$ exact solutions to the homogeneous problem with correct boundary conditions. It is then immediately possible to write an exact relation for η in the integral form :

$$\eta \equiv 2M\sqrt{8} \int_0^\infty r J_2(r) V_T(r) \tilde{u}(r) dr \equiv \int_0^\infty \eta(r) dr. \quad (5)$$

Here $J_2(r) \xrightarrow{r \rightarrow \infty} \frac{1}{2}e^{+ix}$ is the exact regular solution to the homogeneous problem while $\tilde{u}(r) \xrightarrow{r \rightarrow \infty} e^{-\alpha r}$ is the exact S' wave function in a more convenient normalization.

There is nothing mysterious about Eq. (5). In essence it is the exact analogy to what is written down in multipole theory of electromagnetic sources. It is now straightforward to evaluate $J_2(r)$. This solution corresponds to the tunnelling solution going inwards through the repulsive barrier of Fig. 1.

Let us now look at the pion potential. The crucial part of it is the tensor potential, while the central potential is weak. The tensor potential is of the characteristic form of a dipole-dipole interaction between the axial dipoles g_1 and g_2 . For the deuteron it has the form

$$V_T(x) = -f^2 \left(1 + 3/\mu x + 3/(\mu x)^2 \right) \frac{e^{-\mu x}}{x} S_{12}(\hat{x}). \quad (6)$$

For small x the behaviour is like x^{-3} resembling exactly the interaction between two magnetic moments. To be quite exact : the potential (6) is an average of the charged pion contributions (weight $\frac{2}{3}$) and the neutral pion contributions (weight $\frac{1}{3}$) each with mass and coupling constants. Take $f_c^2 = f_o^2 = 0.0790$ and evaluate Eq. (5) with OPEP and a good S wave function. The shape of the integrand is shown in Fig. 2. The pure OPEP value f_π is

$$\eta_\pi = 0.0276$$

$$[\eta_{exp} = 0.0271]. \quad (7)$$

This is a fantastically good result, since we have hardly started the discussion yet. Indeed, it is so good that corrections can work only to our disadvantage.

We will now systematically establish corrections and uncertainties of this result. To give a first idea of the magnitude of corrections, Fig. 2 also shows the same calculation based on the full Paris potential ⁶⁾ with the same f^2 which gives

$$\eta_{Paris} = 0.0263, \quad (8)$$

i.e., all the additional sophistication is only a -4.3% correction coming mainly from the region 1.0-1.5 fm.

Let us first determine the sensitivity to the S wave function. Figure 3 shows the result to be extremely insensitive. Even the exponential wave function with a normalization integral of 1.6 (!) badly violating unitarity gives a result only 20% too high, while Reid SC and Paris wave functions agree to 0.5%. This last agreement is very significant, since the Reid wave is inconsistent with data because of its somewhat wrong effective range. When renormalized to the correct value the S state probability differs from the Paris one by about 1.5% and this is the origin of the difference. Any S wave function with correct $\rho(-\epsilon, -\epsilon)$ and normalized to the range 0.98-0.97 taken within reason gives only $\frac{1}{2}\%$ or less differences in η .

Let us next study the effect of the 2π exchange potential. We saw in Fig. 2 that in the Paris potential all higher corrections were only -4%. The Paris potential has a theoretically well-defined 2π potential, which is sound up to mass transfers of the order of 7π or so and then becomes debatable. The potential is then completed by a phenomenological short-range part regularizing the

interaction at the origin. The tensor potential is dominated by exchange of P wave $\pi\pi$ systems, the properties of which are well understood. In Fig. 4 we display the 2π potential contributions to η as a function of a cut-off mass μ_0 , so that all contributions $\mu \leq \mu_0$ are included. One notes that the region $\mu < 5$ which does not yet include the ρ resonance gives 50% of the contribution. Once the ρ resonance is included, the contributions saturate. The full Paris potential including its phenomenology gives essentially the same result as the theoretical 2π potential. Figure 4 also demonstrates that the contributions from the region 0 to 1 fm are quite model-independent. Once the minimum around 1 fm is reached the contribution from the inside region is determined by the requirements $\eta(r=0) = 0$ and continuity. We therefore conclude that there are very small uncertainties associated with the 2π exchange potential contributions.

Another way of visualizing this information is to fix r and investigate which masses contribute most importantly to the 2π potential at that r . This is shown for the 2π tensor potential in Fig. 5, where we display everything relatively to the OPEP potential. There are two outstanding features. First that the ρ is nearly irrelevant for $r > 1$ fm, with the contributions coming from the non-resonant region below the ρ mass. The second one is that even at 0.5 fm there are only 20% contributions for masses $> 6\mu$, the reason being the very strong exponential damping of the high mass region. Figure 5 therefore emphasizes that the physics of 2π exchange down to $\frac{1}{2}$ fm is not a high mass phenomenon. We therefore conclude that 2π exchange is a correction but not a major source of uncertainty.

A more troublesome uncertainty is that the πNN coupling has a form factor. Very little is experimentally (and even theoretically) known about the πNN source function. Nucleon-nucleon descriptions nearly invariably use a point-like coupling, since serious problems occur with a spread-out source. Bag models have the source size associated with the bag radius. Let us take a purely phenomenological position and find out how η varies with the form factor, Fig. 6. Using a parametrization $K(t) \propto (\Lambda^2 - \mu^2/t + \Lambda^2)$, which has an equivalent uniform source radius $R = \sqrt{10}/\Lambda$ ("bag radius") we find for $R = 1.2$ fm a 25% decrease and for $R = 0.85$ fm an 11% decrease. These are very large effects indeed. (They are 1.6 times larger in the quadrupole moment.) It is not reasonable to envisage any phenomenological radius $R \gtrsim 0.7$ fm. (One should note that this is strictly speaking an "effective form factor", since it describes empirically the joint effect of the form factor and heavy mass exchanges.) This causes a serious challenge to bag models.

Another uncertainty comes from the πNN coupling constant. The charged coupling is known up to 1.3%, with $g^2 = 14.26$, while the neutral one is more disputed, with the best value given as $g^2 = 15.3$ with 2% accuracy. The difference between pseudoscalar and pseudovector couplings is quite important. The choice of one or the other changes η by 2%. Present experiments favour pseudovector couplings.

For pointlike coupling we therefore find

$$\begin{aligned}\eta_{th} &= 0.0263 (\pm 1.5\%) \\ \eta_{exp} &= 0.0271 (\pm \text{few}\%).\end{aligned}\quad (9)$$

This result restricts the πNN form factor to an approximately pointlike shape.

Let us now turn to the deuteron quadrupole moment. Inside the potential description the same approach can be used.

The "experimental" value for Q_D is a theoretically constructed value from highly accurate hyperfine splitting using atomic beams ($\pm 0.02\%$). By variational techniques Reid and Vaidya⁷⁾ find the standard value

$$Q_D = 0.2860(15) \text{ fm}^2 (\pm 1/2\%). \quad (10)$$

We were worried about this precision and found that more accurate values were implicit in a study by Bishop and Cheung⁸⁾. Professor Bishop has allowed us to quote the improved value²⁾

$$Q_D = 0.2859(3) \text{ fm}^2 [\pm 0.1\%]. \quad (11)$$

The quadrupole moment is given by

$$Q_D = Q_1 + Q_2 = \frac{1}{\sqrt{50}} \int_0^\infty u w r^2 dr - \frac{1}{20} \int_0^\infty w^2 r^2 dr. \quad (12)$$

Here Q_2 is a -6% correction, which does not give rise to uncertainties of importance. The main term can be identically rewritten using the Green function technique

$$\begin{aligned}Q_1 &= \frac{N^2}{50} \int_0^\infty \gamma(r) dr \left[r \frac{1}{r} \int_0^\infty G_2(r, r') r'^2 \tilde{u}(r') dr' \right] \\ &\equiv \frac{N^2}{\sqrt{50}} \int_0^\infty \gamma(r) F(r) dr.\end{aligned}\quad (13)$$

The function $F_x(r)$ is extremely stable with respect to assumptions and it is very good to use even the free value for the propagation. This is seen in Fig. 7. The function is practically identical for various assumptions apart from the uninteresting region near the origin where the function $\eta(r)$ vanishes anyway. Because of this, η and the quadrupole moment are in essence the same quantity but for the non-potential effects (i.e., relativistic and meson exchange current effects) and a weak shape dependence on $\eta(r)$.

Therefore, from the experimental effective range (N^2) the experimental η and the rough shape of $\eta(r)$ follows Q_D to a high precision without further theory inside a potential description. One finds that $\Delta Q = (Q_{\text{exp}} - Q(\eta_{\text{exp}})) = -0.0036(54) \text{ fm}^2$ for a pointlike πNN form factor, i.e., 0 to a few per cent. The question is whether relativistic and meson exchange corrections are needed. These have been calculated to yield $\Delta Q_{\text{mec}} \approx +0.010 \text{ fm}^2$ ^{9),10)}. One sees that it is on the borderline to discuss whether there is any evidence whatsoever for isoscalar meson exchange. One must, however, discuss this carefully in conjunction with form factor effects both in η and Q_D . This is now being done. Already, however, we have an excellent description of Q_D anyway.

In conclusion, we have found :

- a) that the pion dominates the D/S ratio to high precision ;
- b) that η (and Q) give strong restrictions on the πNN vertex function and form factor with clear counter-evidence for bag sizes larger than 0.7 fm ;
- c) that the quadrupole moment gives new information mainly on meson exchange currents and on the πNN form factor but it gives no other information on the tensor interaction ;
- d) it is important to obtain independent compelling evidence from other sources on (i) the πNN form factor, (ii) the charge-neutral πNN coupling constants.

If we assume near pointlike πNN coupling, we have even now one of the best determinations of g^2 from η with $g^2 = (14.7 \pm 0.3)$. It is fascinating and a real challenge to us that shortly the best information on the πNN coupling may come from determinations of η by nuclear experiments.

This is the level of accuracy and understanding we have achieved. Indeed we have refuted the heretics and there is no grounds to doubt that pions can be used quantitatively in nuclear physics.

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FIGURE CAPTIONS

- Fig. 1 Centrifugal and OPEP repulsive potentials in the deuteron D state.
- Fig. 2 Shape of $\eta(r)$ for pure OPEP and Paris potential using the same $u(r)$.
- Fig. 3 Wave function dependence of $\eta(r)$ constructed using OPEP with :
(a) $u(r) = \exp(-\alpha r)$; (b) Reid soft case $\tilde{u}_{RSC}$; (c) Paris potential $u_{Paris}(r)$.
- Fig. 4 Contributions of the 2π potential to $\eta(r)$ as a function of maximal 2π mass μ_0 .
- Fig. 5 Contribution of different squared 2π mass μ^2 to the 2π tensor potential for fixed r in units of the OPE tensor potential.
- Fig. 6 Dependence of $\eta(r)$ on the πNN form factor, assumed to have the form $K(t) = (\Lambda^2 - \mu^2)/(\Lambda^2 + t)$ where the equivalent uniform radius $R = \sqrt{10}/\Lambda$.
- Fig. 7 The quadrupole weight function $F(r)$ for purely asymptotic wave function as well as good wave function $\tilde{u}(r)$ for free and realistic interactions. For convenience, the shape of $\eta(r)$ is indicated.
- Fig. 8 The model-dependence of Q_1 in going from pure OPEP to the Paris potential.

- Figure 1 -

- Figure 2 -

- Figure 3 -

- Figure 4 -

- Figure 5 -

- Figure 6 -

- Figure 7 -

- Figure 8 -

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