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LIGHT SINGLET, GAUGE HIERARCHY AND SUPERGRAVITY

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A B S T R A C T

The existence of light singlets to help the breaking of $SU(2) \times U(1)$ in supersymmetric grand unified theories destroys the gauge hierarchy. The phenomenon exists for both global and local SUSY-GUT models.

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Supersymmetric or "fermion-boson symmetric" theories have remarkable properties viewed as field theories. Because of their fermion-boson symmetry there are quite powerful theorems concerning their renormalizability. These so-called non-renormalization theorems seem to be the key to the solution of the notorious gauge hierarchy problem which is the most serious weakness of currently known grand unified theories (GUT) ¹⁾. Unfortunately construction of realistic supersymmetric GUTs is not as easy as that. Among the problems that model-builders face is the breaking of supersymmetry and the spontaneous breaking of the weak SU(2)×U(1). It seems now that soft global supersymmetry breaking emerging from supergravity may be adequate while the breaking of SU(2)×U(1) may be achieved at the tree level or by radiative corrections ²⁾⁻⁵⁾. In this framework tree level breaking of SU(2)×U(1) in all cases considered so far involves the existence of a light singlet. We are going to prove in this letter that light singlets are dangerous because they tend to destabilize the so "hardly won" gauge hierarchy ⁶⁾. For clarity we consider a simplified model which nevertheless bears all features of realistic models. This is actually the model considered by Polchinski and Susskind ⁷⁾ slightly modified which is globally supersymmetric and whose superpotential reads

$$W = \frac{f}{2} X(B^2 - \mu^2) + \frac{M}{2} B^2 + M' BC + \frac{h}{3!} B^3 + \frac{g}{2} B^2 L + \frac{k}{3!} L^3 + \frac{m}{2} L^2 \quad (1)$$

We have allowed for different couplings and for a mass term for the L field. Supersymmetry is spontaneously broken at the tree level and for a range of parameters one has

$$\langle F_x \rangle = \frac{f}{2} \mu^2, \quad \langle F_{B,C,L} \rangle = 0$$

so that ψ_x is actually the Goldstino field. We assume that $m^2, \mu^2 \ll M^2$ though μ^2 can be much larger than m^2 ^{*)}. At the classical level B, C are heavies with masses $\sim M, M'$ and L, X are lights with masses $\lesssim m$. Actually for $m = 0$ all L, X fields would be massless.

What one is interested in is the effective low energy theory at scales $p^2 \ll M^2$. We shall follow a systematic analysis using "effective field theory" methods ⁸⁾ which will be useful also for $N = 1$ local supersymmetries. Let us denote the heavy fields by Φ and the light ones by ϕ collectively. Their

^{*)} In most realistic models $\mu \sim 10^{11}$ GeV and $M \sim 10^{15}$ GeV while the low scale $m \sim 10^2$ GeV.

corresponding sources will be denoted by J and j . The generating functional is

$$W[j, J] = \int [d\varphi][d\Phi] \exp \{ i S_0[\varphi] + i S_0[\Phi] + i S_I[\varphi, \Phi] + \text{source terms} \} \quad (2)$$

The interaction parts are all in S_I . For an effective theory one puts $J = 0$ since the heavies never appear in external legs so that one has

$$W_{\text{eff}}[j] = \int [d\varphi] \exp \{ i S_0[\varphi] + i S_{\text{eff}}[\varphi] + i \int j\varphi + i \int j^\dagger \varphi^\dagger \} \quad (3)$$

where by definition

$$e^{i S_{\text{eff}}[\varphi]} \equiv \int [d\Phi] e^{i S_I[\varphi, \Phi]}$$

This consists of all loops with only heavies in internal lines and lights appear only in external lines. $S_{\text{eff}}[\varphi]$ is of non-renormalizable type but the non-renormalizable terms are suppressed by inverse powers of M relative to the renormalizable ones ^{*)}.

Actually we shall calculate only the renormalizable part of $S_{\text{eff}}[\varphi]$ which is of relevance at low energies $E \ll M$. We shall use supersymmetric techniques which we believe is the most economical and transparent way of seeing how things go. Actually we shall calculate the effective action Γ to which as is well known only the one-particle irreducible Green's functions (1PI) contribute. Since only D terms are perturbatively generated ⁹⁾ we seek for terms of the form

$$\int d^4\theta \varphi_i^\dagger \varphi_j, \int d^4\theta \varphi_i^\dagger \varphi_j^\dagger \varphi_k, \int d^4\theta \varphi_i^\dagger \varphi_j \varphi_k, \dots$$

where φ_i 's denote X or L fields.

Consider first the $X^\dagger L$ contribution to the effective action. The relevant one-loop graph is shown in Fig. 1. Its contribution to the effective action is

*) The terms involving superfields with dimension ≥ 4 are called non-renormalizable.

$$i\Gamma = \prod_{i=1}^2 \int \frac{d^4 p_i}{(2\pi)^4} (2\pi)^D \delta^{(4)}(p_1 + p_2) \left[\frac{g_f^2}{2} \frac{1}{(2\pi)^D} \int \frac{d^D k}{(k^2 - M^2)^2} \right] \int d^4 \theta L(p_1, \theta) X^\dagger(p_2, \theta) \quad (4)$$

Terms $O(p_i^2/M^2)$ have been neglected. Thus we have a contribution to the effective Lagrangian $\mathcal{L}^{\text{eff}}$ given by

$$\mathcal{L}^{\text{eff}} = I \frac{g_f^2}{2} \int (L X^\dagger) d^4 \theta \quad (5)$$

where

$$I = \frac{(-i)}{(2\pi)^D} \int \frac{d^D k}{(k^2 - M^2)^2}$$

The L^+X , L^+L , X^+X contributions are taken directly from the same graph by replacing X (or L) by L (or X) and properly taking care of the combinatorics. The final answer is

$$\mathcal{L}^{\text{eff}} = \frac{I}{2} \int (L^+, X^+) \begin{pmatrix} g^2 & g_f^2 \\ g_f^2 & f^2 \end{pmatrix} \begin{pmatrix} L \\ X \end{pmatrix} d^4 \theta \quad (6)$$

Consider now other D terms, as for instance L^+LX^+ . The relevant graph is shown in Fig. 2. Its contribution to the effective action is

$$i\Gamma = \prod_{i=1}^3 \int \frac{d^4 p_i}{(2\pi)^4} (2\pi)^D \delta^{(4)}(\sum p_i) \left(\frac{g_f^2}{32\pi^2} \frac{1}{M} \right) \int d^4 \theta X^\dagger(p_1, \theta) L(p_2, \theta) L^\dagger(p_3, \theta) \quad (7)$$

This gives rise to a term

$$\mathcal{L}^{\text{eff}} = \frac{g_f^2}{32\pi^2} \frac{1}{M} \int d^4 \theta (L^+ L X^+) \quad (8)$$

Actually this is all that is needed since all other contributions are taken from this by properly adjusting the combinatorics and replacing some L or X lines by X or L , respectively. The final result is

$$\mathcal{L}^{\text{eff}} = \frac{1}{32\pi^2} \frac{1}{M} \int d^4 \theta \left\{ \frac{g^2}{2!} L^2 L^+ + \frac{f^2}{2!} X^2 X^+ + \frac{g_f^2}{2!} L^2 X^+ + \frac{g_f^2}{2!} X^2 L^+ + g_f^2 L L^+ X + g_f^2 X X^+ L + (h.c.) \right\} \quad (9)$$

Expression (6) suggests a diagonalization of $L^\dagger L$, $L^\dagger X$, $X^\dagger X$ D terms. Actually defining new fields G , Y as

$$\begin{pmatrix} G \\ Y \end{pmatrix} \equiv (g^2 + f^2)^{-1/2} \begin{pmatrix} g & f \\ -f & g \end{pmatrix} \begin{pmatrix} L \\ X \end{pmatrix} \quad (10)$$

one gets for the effective Lagrangian

$$\mathcal{L}^{\text{eff}} = \frac{I}{2} (g^2 + f^2) \int d\theta G^\dagger G + \frac{1}{32\pi^2} \frac{1}{M} (g^2 + f^2)^{3/2} \int d\theta (G^2 G^\dagger + \text{h.c.}) \quad (11)$$

The Y field has completely decoupled from $\mathcal{L}^{\text{eff}}$. The field G which is subject to renormalization due to the heavies is just the linear combination of L and X which couples to B^2 properly normalized. One could have defined G , Y from the very beginning and got the result (11). Since $\langle F_G \rangle \sim \mu^2$ we have terms from Eq. (11) $\sim \mu^2 F_G$ and $\mu^4/M S_G^*$ which destroy the two-stage hierarchy giving G masses much larger than m . Though Y is not renormalized in the presence of heavies, nevertheless it does become heavy $\sim \mu$ too. The reason is that the superpotential (1) when written in terms of G , Y contains a linear term $fg \mu^2 Y$. Therefore we have proved that in order to have a stable hierarchy we need $g = 0$; that is to say, no coupling of the form $L B^2$ should appear. This proof can be generalized to include an arbitrary number of light and heavy fields. One should notice that this effect is not due to radiative corrections. The correct content of the theory is easily guessed already at the classical level by isolating the combination which couples to B^2 ; then the orthogonal combinations to it are all multiplied by the scale μ^2 ($\gg m$), as the term $\mu^2 Y$ above, for instance. Hence all are actually heavy except those which do not couple to B^2 in the superpotential.

We now pass on to examine what happens in local $N = 1$ supersymmetries ⁶⁾. Recently much effort has been expended in this direction ²⁾⁻⁵⁾. As is well known breaking of $N = 1$ local supersymmetry leads to a softly broken supersymmetric theory where the soft breaking terms contain the gravitino mass $m_{3/2}$. In the limit of $m_{3/2} \rightarrow 0$ one has a supersymmetric theory. As in most realistic models we shall assume that $m_{3/2} \sim M_W \ll M$ and therefore mass terms $m_{3/2}^2 |s|^2$ or interaction terms containing only the scale $m_{3/2}$ are not dangerous in the sense that they do not upset the hierarchy. In fact the results stated for the $N = 1$ rigid supersymmetry hold true up to corrections $O(m_{3/2}/M)$, where the supersymmetry breaking scale is $\frac{2}{2}$.

*) S_ϕ will denote the scalar of the multiplet ϕ throughout this paper.

actually $\mu^2 \sim m_{\frac{3}{2}} M$. However, there may be a class of terms which if present would be disastrous and turn up the whole scheme. For instance, consider a term ^{*)}

$$M m_{\frac{3}{2}} (S_B^2 + h.c.) \quad (12)$$

which mixes both scales $m_{\frac{3}{2}}$ and M and which does not vanish in the limit M becoming large. One can write this term as

$$\mathcal{L}^{soft} = \frac{\alpha}{2!} \int d^3\theta \, S B^2 + (h.c.) \quad (13)$$

where " α " is a typical coupling and S a spurion field given by

$$S = M m_{\frac{3}{2}} \theta\theta \quad (14)$$

Note that (13) is just an F term which can be considered as part of the superpotential containing a superfield S which, however, does not propagate. S can also be viewed as a source of a B^2 term. In the low energy theory there are graphs analogous to those in Figs. 1 and 2 with X replaced by the spurion field. The evaluation of these is exactly the same ^{**)} and one gets for the $LL^+S + (h.c.)$, for instance

$$\mathcal{L}^{eff} = \frac{\alpha g^2}{32\pi^2} \frac{1}{M} \int d^4\theta \, L^+ L S + (h.c.) \quad (15)$$

This term is, however, mild, not spoiling the hierarchy since it actually gives $\sim m_{\frac{3}{2}} (L^+ L)_F$.

However, one has contributions as follows ^{***)}

$$\mathcal{L}^{eff} = \frac{I}{2} \alpha g \int d^4\theta (L^+ S + h.c.) + \frac{\alpha^2 g^2}{32\pi^2} \frac{1}{M} \int d^4\theta (L^+ S^+ S + h.c.) \quad (16)$$

which are potentially dangerous. In fact (13) gives terms

$$M m_{\frac{3}{2}} (F_L + F_L^+) , \quad M m_{\frac{3}{2}}^2 (S_L + S_L^+)$$

^{*)} See Barbieri, Ferrara and Savoy in Ref. 3), and also Ref. 5).

^{**)} One actually calculates Green's functions in the presence of the source S .

^{***)} More powers on S will bring down more powers of $(1/M)$ and hence are suppressed.

which upset the hierarchy. Hence for a hierarchy to be kept stable in perturbation one has either to demand that no (light)(heavy)² couplings appear or that only soft terms which vanish in the limit $M \rightarrow \text{large}$ are present. This may be proved to be a severe constraint for model builders. Very recently, Ferrara, Nanopoulos and Savoy¹⁰⁾ have shown a general way to solve the hierarchy problem at the one-loop level in theories with light singlets. This requires a technically natural fine-tuning on the parameters of the theory which renders the hierarchy one-loop stable. Their fine-tuning avoids large mass splittings $O(m_{\frac{3}{2}}M)$ for the heavy fields which couple to the light singlets. The two-loop hierarchy can be easily maintained with a suitable choice of the superpotential couplings and higher loop corrections are completely harmless.

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REFERENCES

- 1) For reviews, see :
P. Fayet and S. Ferrara - Physics Reports C32 (1977) 249 ;
J. Wess and J. Bagger - Princeton Lectures (1981) ;
E. Witten - Erice and Trieste lectures (1981).
- 2) E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello and
P. van Nieuwenhuizen - Nuclear Phys. B147 (1979) 105 ;
E. Cremmer, S. Ferrara, L. Girardello and A. van Proeyen - Physics Letters
116B (1982) 219 and CERN Preprint TH. 3348 (1982).
- 3) H.P. Nilles - Phys.Letters 115B (1982) 193 and CERN Preprint TH. 3398 (1982) ;
R. Barbieri, S. Ferrara and C. Savoy - CERN Preprint TH. 3365 (1982) ;
L. Ibáñez - CERN Preprint TH. 3374 (1982) ;
J. Ellis, D.V. Nanopoulos and K. Tamvakis - CERN Preprint TH. 3418 (1982)
- 4) R. Arnowitt, P. Nath and A.H. Chamseddine - Phys.Letters 49 (1982) 970 and
Northwestern University Preprint 2565 (1982).
- 5) H.P. Nilles, M. Srednicki and D. Wyler - CERN Preprint TH. 3432 (1982).
- 6) H.P. Nilles, M. Srednicki and D. Wyler . CERN Preprint TH. 3461 (1982).
- 7) J. Polchinski and L. Susskind - SLAC Preprint PUB 2924 (1982).
- 8) S. Weinberg - Phys.Letters 91B (1980) 51 ;
L. Hall - Nuclear Phys. B178 (1981) 75 ;
B. Ovrut and H. Schnitzer - Phys.Rev. D21 (1980) 3369 ; ibid. D22 (1980) 2518 ;
Nuclear Phys. B179 (1981) 381 ;
P. Binétruy and T. Schücker - Nuclear Phys. B178 (1981) 293, 307.
- 9) M.T. Grisaru, M. Roček and W. Siegel - Nuclear Phys. B159 (1979) 429.
- 10) S. Ferrara, D.V. Nanopoulos and C.A. Savoy - CERN Preprint TH. 3442 (1982).

FIGURE CAPTIONS

Figure 1 Graph contributing to a $\int d^4\theta \, LX^+$ term.

Figure 2 Graph contributing to a $\int d^4\theta \, L^+LX^+$ term.

Fig. 1

Fig. 2