**DYNAMIC APERTURE COMPUTATION FOR THE AS-BUILT CERN
LARGE HADRON COLLIDER AND IMPACT OF MAIN DIPOLES
SORTING**

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DYNAMIC APERTURE COMPUTATION FOR THE AS-BUILT CERN LARGE HADRON COLLIDER AND IMPACT OF MAIN DIPOLES SORTING

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Abstract

During the design phase of the CERN Large Hadron Collider the dynamic aperture, i.e., the amplitude of the domain in phase space where the particle motion is stable, was used as one of the most important figures-of-merit to specify the field quality of the various types of superconducting magnets and to quantify the machine performance. The programme of magnetic measurements performed during the production and acceptance testing of the magnets generated a large amount of information, which was used to obtain a best estimate of the dynamic aperture of the actual machine. In this paper the results of massive numerical simulations based on the measured field quality of several optical configurations and beam energies, are presented and discussed. The effect of the sorting of the main dipoles on the final value of the dynamic aperture has also been studied and the results are reviewed in detail.

Keywords: Non-linear beam dynamics, dynamic aperture, LHC, magnetic field errors

1. INTRODUCTION

In the design phase, the dynamic aperture (DA), the amplitude of the region in phase space where stable motion occurs, has been a key quantity for the specification of the performance of the CERN LHC; as is also true for all modern colliders and storage rings, such as Tevatron [1–4], HERA [5–9], RHIC [10], or the Superconducting Super Collider (SSC) [11, 12]. The case of synchrotron light sources, damping rings or, in general, lepton machines is somewhat different and is not considered here; the presence of strong damping due to synchrotron radiation, implies that only the so-called “short-term” dynamic aperture, i.e. the particle stability over few thousands turns, is relevant. Furthermore, the main limitations on the DA are related to the strong chromatic sextupoles that dominate the non-linear dynamics in lepton machines. In the symplectic case

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of hadron machines, several hundred-thousand turns must be considered.

It is worth emphasising that the problem of determining the DA, whether analytically or numerically, has fostered knowledge-transfer from other scientific fields, such as non-linear dynamical systems (see, e.g., the introduction and application of normal forms [13–18] in the eighties). It has also demanded an increased effort in the field of massive numerical simulations to gain insight to the DA characteristics, and direct measurements in existing machines (see, e.g., Refs. [9, 19–21] for the case of hadron machines) and that comparisons be made between measurements and simulations (see, e.g., Ref. [22] for a review of this topic).

An accurate numerical computation is mandatory as well as a good estimate of the error associated with the protocol used; see Ref. [23] for a detailed account for the case of the LHC and, e.g., Ref. [24] for a general discussion on the definition of DA, its computation, and accuracy. The computation consists of numerical simulations, performed with tracking codes such as MAD-X [25] and SixTrack [26]. For the case of the LHC studies, where only single-particle effects are considered, the protocol foresees simulating the particle motion for 10^5 turns, using an ensemble of initial conditions uniformly distributed over a polar grid defined in the physical space (x, y) . Five angles, corresponding to different transverse emittance ratios ϵ_x/ϵ_y , are considered. Along each radial direction 30 initial conditions evenly spaced over an amplitude range of 2σ are used. Each initial condition is further split into two nearby conditions to allow the detection of chaos by means of the computation of the maximal Lyapunov exponent [27]. The momentum offset of the initial conditions is set to $3/4$ of the bucket half-height. The use of such an approach should guarantee the DA computation to be accurate to the level of about 0.5σ [28]. An example of the results of typical tracking simulations using the described protocol is shown in Fig. 1. This plot represents a set of initial conditions in the polar grid of normalised physical space for one configuration of the LHC machine at 3.5 TeV, Beam 1 and optics with β^* corresponding to the injection configuration (see the next Section for more details). The blue markers identify initial conditions that are not stable for up to 10^5 turns. The marker size is proportional to the number of turns for which the motion is bounded. The red markers identify initial conditions that are stable for up to 10^5 turns. The regions provide a direct indication of the size of the DA (red region) and of the chaotic area (blue region) where the particles feature an increasing amplitude, which eventually leads to the loss of the particles.

In the design stage, the target value for the DA was set to 12σ assuming a safety factor of two between the computed and the actual value of the DA. This rather large factor was based on a detailed breakdown of the various parameters determining the uncertainty in the DA estimate [23], such as the mesh size in phase space and the number of turns considered, as well as the effects neglected in the numerical simulations, such as the closed orbit and linear imperfections. It should also be noted that the best numerical estimates of the DA for the HERA machine were about a factor of two larger than the measured values [7].

The targeted DA has been used to define bounds on the field quality of the

various magnets, both super- and normal-conducting. The specification, analysis and validation of the so-called error tables, originally based on extrapolation from the field quality of existing machines such as Tevatron or HERA, required CPU-intensive tracking campaigns lasting several years, resulting in a number of key publications, such as Ref. [29] for the field quality specification of the main dipoles and Refs. [30–37] for the other types of magnet (summarised in Ref. [38]). It is worth emphasising the contribution from colleagues outside CERN to these studies aimed at linking the DA to the field quality of the various classes of magnets (see, e.g., Refs. [39–42]). In this context, a number of different magnetic error configurations of the LHC machine had to be taken into account in order to provide a reliable estimate of the DA of the actual machine. The number of different sets of magnetic error values, or “seeds”, used in the standard DA computations was fixed at 60.

The start of magnet production began a new phase; although the target field quality of the magnets had been defined, the issue of allocating the magnets to an optimised slot, the sorting of the magnets, was now raised. This issue had been considered for other machines, such as the SSC, and some studies and results were available in the literature (see, e.g., Refs. [43–52] and references therein). For the LHC, the approach proposed and adopted is described in Ref. [53]. In general, a pre-requisite for an effective sorting procedure is the availability of a large number of magnets, not so easy to achieve due to the large amount of storage space required. Nonetheless, this space was available for the LHC installation, and the selection of the magnets for the available slots was co-ordinated by the Magnet Evaluation Board (MEB) [54].

After the installation of the magnets in their allotted slots, the results of a demanding programme of magnetic measurements, were available. This made possible a new approach to the numerical simulation of the LHC performance. A tool WISE [55, 56] was designed and implemented to perform data mining, in the numerous CERN databases, to extract information relevant to beam dynamics simulations. The capabilities include extraction of alignment data and magnetic field quality data together with the possibility of generating random magnetic errors based on different physical models. This latter capability is particularly relevant to the studies described in this paper. In fact, it is now possible to simulate not only configurations of statistically distributed field errors, but the machine exactly as assembled. The measured errors are assigned to the magnets at their actual location. In principle this should have removed completely the need to perform numerical simulations with different magnetic error configurations. However, because only a limited number of magnets were measured in cold conditions, whereas all the magnets were measured when warm, the use of warm-to-cold correlations was required. These quantities are affected by unavoidable statistical errors; the 60 seeds used in the simulations described in this paper therefore represent different warm-to-cold correlations within the range of experimental error.

Preliminary results based on this new approach have already been presented in Ref. [36]; here many more configurations are analysed. In addition, it is now possible to simulate not only the clockwise beam (Beam 1), but also the

counter-clockwise (Beam 2). This step was non-trivial and required the solution of a number of technical problems.

This paper has two principal topics. First a review of the status of the computed DA value of the LHC taking into account all the latest information, i.e. the measured field quality of all the magnets, normal and superconducting, their actual position in the ring, and the various optical configurations foreseen for the beam commissioning and early physics runs. The effect of the September 2008 incident is included. All computations have been performed for both beams. Second, the impact on the DA of the magnet sorting performed during the installation stage is carefully evaluated and discussed in detail.

The paper is split in three main parts. In section 2 the layout of the LHC ring is summarised together with the cell layout and the optical configurations for the proton and Lead ion runs. In section 3 the principal results of the numerical simulations of the DA for the LHC in many variants are presented and discussed. In section 4 the sorting of the main dipoles is addressed and an evaluation is made of its effect on some key beam dynamics quantities including the DA. Finally, our conclusions are reported in section 5.

2. LHC LAYOUT AND OPTICAL CONFIGURATION

The CERN LHC is a p-p collider with a layout featuring an eight-fold symmetry shown in Fig. 2. Eight long straight sections, also called interaction regions (IRs), are connected by eight arcs with dispersion suppressor sections used to match the dispersion and the global ring geometry. The parts of the ring in between two interaction points are the so-called sectors, which are named with the number of the IPs at the extremity, e.g., sector 1-2 is the section of the LHC machine in between IP1 and IP2.

Four experimental insertions house the four main experiments, ATLAS, ALICE, CMS, and LHCb at points 1, 2, 5, and 8, respectively. Injection of Beam 1 (clockwise) is performed at point 2, while Beam 2 (counter-clockwise) is injected at point 8. Another difference between points 2 and 8, and 1 and 5, is that the D1 separation dipole is one single super-conducting magnet in IR2 and IR8, while it consists of 6 normal-conducting magnets in IR1 and IR5. The remaining insertion points are used to perform momentum cleaning (point 3) and betatron cleaning (point 7), while in point 4 the RF system and most of the instrumentation are installed. Finally, the beam dump system is housed in point 6.

The layout of the FODO cell, the periodic structure of the arc, is shown in Fig. 3. Six dipoles and two quadrupoles form the heart of the cell together with a multitude of corrector magnets. The LHC main dipoles are made in two variants: *i*) MBA, featuring two spool pieces correctors, one at each end, to correct sector-by-sector the average sextupole component (downstream corrector) and the octupole and decapole components (upstream corrector) of the main dipoles; *ii*) MBB, featuring only the sextupolar corrector.

Next to the quadrupole are located the double plane beam position monitors and the closed orbit correctors, together with lattice correctors such as trim

quadrupoles (normal and skew) to correct the tune and the linear coupling, sextupoles (normal and skew) to correct chromaticity and chromatic coupling, and octupoles to combat instabilities by providing Landau damping. The details of the strategy used to set the strength of these correctors for the LHC can be found in Ref. [52]

The convention used to express the magnetic field of the LHC magnets in terms of multipolar expansion is

$$B_y + i B_x = B_{ref} \sum_{n=1}^{\infty} (b_n + i a_n) \left(\frac{x + i y}{R_r} \right)^{n-1} \quad (1)$$

where the reference radius $R_r = 17$ mm and the subscript $n = 1$ refers to a dipolar component. It is customary to express the b_n and a_n , i.e., the normal and skew components, respectively, of the multipolar expansion, in units of 10^{-4} of the reference field B_{ref} .

The beams cross at each experimental insertion and then exchange the beam tube in the arc, from the inside aperture to the outside one, thus equalising the path length. To avoid unwanted collisions, a crossing angle is generated: at points 1 and 2 the crossing occurs in the vertical plane, while in points 5 and 8, the crossing is in the horizontal plane. A parallel separation is generated in the plane orthogonal to the crossing plane and is used to prevent collisions during injection, ramp, and squeeze stages.

At the injection energy of 450 GeV/beam, the beta-functions $\beta_{x,y}^*$ at the experimental IRs are 11 m, 10 m, 11 m, 10 m, for points 1, 2, 5, and 8, respectively. The optics of the other insertions are constant from injection to top energy, whereas the experimental insertions are squeezed to provide the desired luminosity. In points 1 and 5 the β -functions are reduced to a value of 0.55 m, while in points 2 and 8 they remain at the injection value. These optical configurations refer to proton physics, but for Lead ion physics runs point 2 will also be squeezed down to 0.5 m. Furthermore, during the initial stages of the beam commissioning with protons, β^* in points 2 and 8 will also be squeezed, but at most to $\beta^* = 2$ m in order to provide sufficient luminosity. The actual values of β^* used during the 2010 run are quoted in Sec. 3.4.

3. DA COMPUTATION FOR THE LHC AS-BUILT

3.1. DA at injection energy

The value of the DA is particularly critical at injection. The beam fills most of the available aperture in the arcs and the magnetic errors peculiar to persistent current effects could have an adverse impact on the DA. The main source of reduction of the DA is the field quality of the main dipoles and the insertion quadrupoles; the dipoles because of their integrated strength and total number and the insertion quadrupoles, where, in some of the insertions, the beta-functions can be as large as 600 m (as compared to 180 m in the arcs) thus magnifying the harmful effects of the non-linear magnetic errors.

In Fig. 4 the DA averaged over 60 seeds is shown as a function of the angle in phase space. These results were obtained using 25 angles (instead of the standard 5) to increase the phase space coverage. The markers show the average over all seeds, while the error bars show the minimum and maximum, providing an estimate of the variation in the DA as a function of the statistical uncertainty in the warm-cold correlation. Both beams are plotted. The results are rather similar for each beam and no systematic difference is found. A small increase of the average DA is visible at larger angles, corresponding to pure vertical motion. The target value for the average is achieved for almost all angles and the error bars are all above 11σ apart from a single case for Beam 2.

3.2. Tune scan

It is customary to study the sensitivity of the DA to the fractional part of the tunes. In the design phase this highlighted a number of dips corresponding to resonances excited by the non-linear field errors [57]. The injection optics configuration was used with the initial fractional tunes set to 0.28, 0.31 for the horizontal and vertical planes, respectively. The fractional part of the tunes was changed in steps of 2×10^{-3} moving along the main diagonal. The results are shown in Fig. 5. The DA (using the minimum of five angles then averaged over 60 seeds) is reported as a function of the fractional part of the horizontal tune for both beams.

The error bars represent the minimum and maximum DA over the seeds and angles. No systematic difference is found between the two beams, even if Beam 2 features a slightly lower DA for a fractional tune around 0.265. Apart from this, the value of the DA is virtually constant facilitating the operation of the LHC. No dedicated beam experiment has been performed to verify this finding. However, a study aimed at testing the use of collision tunes [58] at injection in view of shorter squeeze sequence, less complexity in the operation of feed-backs, and absence of tune crossing the 7th and 10th order resonances, did not show any reduction in lifetime. Even if a direct link between DA and beam lifetime cannot be proved, a small DA is likely to generate losses, which was not observed in the beam experiment. Therefore, the reported observation, could be considered as an indirect confirmation that the working point at injection is not critical in terms of DA.

It is important to note that the average DA is essentially within the specified target value of 12σ for a wide range of tunes.

3.3. DA at the nominal collision energy

At the maximum energy, i.e. 7 TeV/beam, the situation is rather different. The DA is no longer dominated by the field quality of the main dipoles, but rather by the quadrupoles and in particular the triplets of the experimental insertions.

In addition, the beam-beam effect is a principal source of non-linear behaviour and strongly dominates the evolution of the DA (see, e.g., Ref. [38]). In this study at collision energy, only single-particle effects are taken into account,

and the beam-beam is completely neglected. Nevertheless, the computations reflect the quality of the magnets in the machine and, in general, indicate that the situation is within tolerances.

In Fig. 6 the minimum value of the DA for several configurations at top energy is plotted. In the case of injection optics, the DA is large but a direct comparison can be made. The optics are identical, but the beam size shrinks after acceleration. This implies that with a similar field quality of the main dipoles at injection and collision, the DA should be larger at collision energy by a factor of four. This is indeed demonstrated by the numerical simulations, demonstrating that the field quality of the LHC magnets has a similar impact on the DA at both energies.

The other four configurations studied concern squeezed optics: for protons where IR1 and IR5 have $\beta^* = 0.55$ m, or for ions, with IR2 also squeezed to $\beta^* = 0.5$ m. The impact of the triplet field quality is clearly visible in the figure. The non-linear correctors next to Q3 are used to compensate the driving terms of the most dangerous resonances excited by the field errors in the triplets and the D1 separation dipole (see Ref. [34] for more details). This beneficial effect is only just apparent in the case of protons, while it is much more visible for ions as in this case the beta-functions at the super-conducting D1 are much larger, magnifying the effect of its magnetic errors.

3.4. DA after re-installation

As a consequence of the incident of September 2008, 39 main dipoles and 12 quadrupoles in sector 3-4 were re-installed. The LHC performance was calculated for this new configuration and the situation before and after is compared in Fig. 7, showing results for both injection and 7 TeV (top) as well as injection and 3.5 TeV (bottom).

The DA at both injection and top energy is essentially unchanged. This is because the re-installation affected only a small number of magnets.

As far as the 3.5 TeV/beam is concerned (see Fig. 7 bottom), it should be noted that this is not a planned configuration of the LHC machine, but rather one imposed by the decision to limit the current in the main dipoles. Error tables for the magnetic field quality had to be derived from the existing measurements in the field model, the so-called Field Description of LHC (FiDeL) (see Refs. [59–62] and references therein). The DA remains large for injection optics for the usual reasons. The ratio between DA at injection and at 3.5 TeV becomes roughly 2.8, corresponding exactly to the ratio of $\sqrt{\gamma}$ between injection and 3.5 TeV. Therefore, it can be deduced that the effect of the field quality of the LHC magnets is comparable to that at injection and at 7 TeV.

The results shown for squeezed optics are for $\beta^* = 2$ m in the 4 experimental insertions. The value of β^* is limited by the available beam aperture at this lower-than-nominal energy. A similar value had been used during the early stage of the beam commissioning in 2010, but was then abandoned for a value of 3.5 m, safer in terms of aperture margin, as soon as a non-zero crossing angle is used, which occurs for multi-bunch operation. Under these conditions, with $\beta^* = 2$ m there is no need to perform any correction of the non-linear errors in

the triplets. It is important to stress that the non-negligible difference between the DA for Beam 1 and Beam 2 with squeeze optics is in reality dominated by very few bad seeds.

4. SORTING OF THE MAIN DIPOLES

4.1. The plan

All the possible positions to which a magnet of a given type can be assigned in the LHC ring (the so-called *slot*) are not equivalent with respect to the beam requirements for mechanical aperture and field quality. An obvious example would be the three possible slots for the main dipoles in the regular cells of the LHC arcs (see Fig. 3): the two end-cell slots adjacent to an arc quadrupole are suitable for magnets with a rather well-controlled geometry, as the β -functions are large in one of the two planes. On the other hand, the third, in the middle of the half FODO cell (the so-called *mid-cell slot*), substantially relaxes the aperture requirements. Starting with this observation, the installation of the LHC magnets was planned using the two considerations: *i*) Out-of-tolerance magnets should be installed at ring positions which minimise their effect; *ii*) Magnets should be sorted so as to control and optimise the beam dynamics.

For the arc quadrupoles (MQ), the strategy applied was a pairing at $\pi/2 [\text{mod}\pi]$, corresponding to an odd number of cells for the LHC arc optics. The pairing was applied to magnets with similar transfer function (expressing the strength of the magnet as a function of current), but substantially different from average. This pairing then minimised their impact on the β -beating [63] as the almost equal and opposite beta-waves would generate a non-zero beta-beating only between the members of each pair.

For the main dipoles (MB), there was a list of potential issues which had been rapidly identified in the early stages of production. This list mainly concerned the control of the MB geometry and their random sextupole components. Fig. 8 shows the evolution of the b_3 component of the MBs, after two successive changes of cross-section in the early stage of the production. An installation strategy was therefore defined for the dipoles aimed at preserving the mechanical aperture of the LHC arcs and minimising the third-order resonance driving terms excited by b_3 [53, 65]. Later, the skew quadrupole component (a_2) of the main dipoles was also taken into account in the sorting algorithm in order to further optimise the installation sequence.

In practice, the strategy used consisted of an up and down local pairing of the main dipoles. An average value of the sextupole component Δb_3 based on collared coil measurements had been calculated. The magnets were paired based on the deviation of their sextupole component from this average. The integrated b_3 component of the MBs is compensated on average in the eight LHC sectors by using dedicated sextupole spool-piece correctors attached to each dipole magnet, with only one corrector circuit per beam and per sector. However, some magnets featured a sextupole component located in the tail of the global distribution, in particular for the first ~ 100 pre-series magnets with a

higher-than-specified geometric b_3 (see Fig. 8) and there were no dipole magnets of opposite Δb_3 in stock. A π -pairing strategy was applied in order to cancel, to the first order in b_3 , the excitation of the third-order resonance driving terms.

With few exceptions, the b_3 component of the main dipoles at each of the two apertures was found to be similar, a feature essential to the simultaneous optimisation of both beams. The situation was much more difficult for the a_2 component, where there was no correlation between apertures nor with the b_3 . So, after defining optimised dipole pairs with respect to b_3 (up and down pairs or π -pairs), the basic principle consisted of

- selecting and ordering a certain number of dipole pairs (typically 5 or 6) with respect to their skew quadrupole component in order to minimise in each aperture the deviations of the running averages of a_2 calculated separately for the first and second dipoles of each pair.
- defining an installation sequence, compatible with the aperture constraints, and then separating by 2π in betatron phase (i.e. 8 half-cells for the arc optics of the LHC) the dipole pairs belonging to the ordered sequence obtained above.

For it to work, this algorithm required the availability of a substantial stock of magnets which had already been cold tested and waiting to be lowered to the tunnel, as was the case in practice.

This strategy was applied during the entire machine installation phase, not only to globally optimise the installation sequence, but also to minimise locally the effect of the pre-series magnets with, e.g., high sextupole component, or, less frequently, to accept magnets outside the specified tolerances. The latter case refers to situations with non-conforming geometry or with multipoles, such as a_2 , beyond the 3σ threshold. Even more importantly, a clear definition and control of the installation sequence was crucial to preventing any build up of worst cases. For the dipoles, this could have occurred, for instance, if magnets with similar and large, even if within tolerance, b_3 components, had been paired in sequence at 2π , instead of π . Similarly, for the quadrupoles, a build up could have been generated by pairing magnets with a similar transfer function at π , instead of $\pi/2$.

4.2. Numerical results

The beneficial effect of this sorting in terms of mechanical aperture is clear, knowing that a certain number of dipoles exceed their mechanical tolerances by up to 1 mm or 2 mm. Such a non-conformity would have induced a loss of physical aperture of the order of 1-2 σ , were they installed randomly.

To analyse the field quality and more specifically the impact of a_2 and b_3 on the beam dynamics, the following numerical studies were carried out:

- Considering only the field imperfections of the main dipoles and the actual installation sequence, 60 different possible values of the field quality of each individual magnet were generated, using, whenever available, the

magnetic measurement data obtained at cold, or by extrapolating to cold the collared coils data including an uncertainty different for each seed. For the a_2 and b_3 components, the uncertainty on the warm-cold offset at 450 GeV is estimated to be 0.4 units r.m.s. at a reference radius of 17 mm, which is considered below the level of significance for beam dynamics [66]. It is appropriate to compare this uncertainty with the natural a_2 and b_3 spread calculated at the end of production, corresponding to 0.9 units and 1.5 units, respectively, at injection energy. As a result, since the slot assignment process was based on raw data, without considering any source of error in the available field quality data, the beneficial impact of the sorting is expected to be less important for a_2 (e.g. in terms of coupling or vertical dispersion) than it will actually be for b_3 (e.g. in terms of third-order resonance driving terms).

- For seven reference seeds, specifically the seeds 1, 10, 20, 30, 40, 50 and 60 generated above for the actual machine, 60 different permutations were randomly chosen, the same for all seeds, in order to define 60 different installation sequences for the 1232 main dipoles of the LHC.

4.2.1. Coupling and third-order resonance driving terms

One specific reference seed (seed 10) has been analysed with and without reordering the installation sequence of the main dipoles. Figures 9 and 10 illustrate the beneficial effect of sorting on a_2 and b_3 respectively. More specifically, the following beam dynamics quantities have been analysed individually for both beams:

- the local difference coupling coefficient after correction:

$$c_{-}(s) \equiv \frac{1}{2\pi} \int_s^{s+C} ds K_{\text{skew}}(s') \sqrt{\beta_x(s') \beta_y(s')} e^{j[(\mu_x(s') - \mu_x(s)) - (\mu_y(s') - \mu_y(s))]} , \quad (2)$$

$K_{\text{skew}}(s)$ being a function describing the distribution of skew quadrupole errors in the main dipoles and including the contribution of the corresponding corrector magnets (one circuit per beam and per sector, therefore acting only on the average a_2 component of the MBs in each sector).

- the local $x - y$ coupling coefficient of the beam matrix related to the sum coupling resonance driving term:

$$\frac{\langle xy \rangle}{\sqrt{\langle x^2 \rangle \langle y^2 \rangle}} = \frac{1}{2 \sin[\pi(Q_x + Q_y)]} \int_0^C ds K_{\text{skew}}(s') \sqrt{\beta_x(s') \beta_y(s')} \times \cos[\pi(Q_x + Q_y) - (|\mu_x(s) - \mu_x(s')| + |\mu_y(s) - \mu_y(s')|)] . \quad (3)$$

- the vertical normalised dispersion:

$$\frac{D_y(s)}{\sqrt{\beta_y(s)}} = \frac{1}{2 \sin(\pi Q_y)} \int_0^C ds K_{\text{skew}}(s') D_x(s') \sqrt{\beta_y(s')} \cos[\pi Q_y - |\mu_y(s) - \mu_y(s')|] , \quad (4)$$

then expressed as a percentage of a similar quantity $(D_x/\sqrt{\beta_x})_{\text{QF}}$ referring to the unperturbed horizontal dispersion in the focusing quadrupoles of the LHC arcs ($D_x \approx 2$ m and $\beta_x \approx 180$ m at the QFs for the nominal LHC optics).

- the local third-order resonance driving terms

$$c_{m,n}(s) \equiv \frac{1}{2\pi} \int_s^{s+C} ds K_2(s') \beta_x^{\frac{m}{2}}(s') \beta_y^{\frac{n}{2}}(s') e^{j[m(\mu_x(s') - \mu_x(s)) + n(\mu_y(s') - \mu_y(s))]}, \quad (5)$$

for $(m,n) = (3,0)$, $(1,2)$ or $(1,-2)$, $K_2(s)$ being a function describing the distribution of sextupole errors in the main dipoles, including the contribution of the corresponding corrector magnets (one circuit per beam and per sector, therefore acting only on the average b_3 component of the MBs in each sector).

Finally, average figures-of-merit corresponding to the r.m.s. deviations of these quantities over the machine are computed and the corresponding statistical distributions are presented in Fig. 11 and Fig. 12. For each average quantity the results are presented in the form of a histogram containing 7×60 entries for the 7 reference seeds and the 60 random re-orderings of the MB installation sequence. The vertical dotted lines superimposed on each histogram indicate the position of the 7 reference seeds.

Even if 60 is a rather small number compared to the $2 \times 616!$ permutations of the 1232 main dipoles, taking into account the two different hardware types of the LHC MBs, the beneficial impact of the sorting is already evident from the results. The relative gain is a factor of 3 to 4 in terms of third-order resonance driving terms with respect to the worst seed and installation sequence considered, and a factor of slightly more than 2 for the optics distortions induced by the skew quadrupole imperfections of the main dipoles. These relative gains correspond almost exactly to the ratios between the natural spreads of a_2 and b_3 calculated over the MB population (0.9 and 1.5 units, respectively) and the uncertainty of 0.4 units r.m.s. on the value of these two multipole components for each individual magnet. Therefore, the sorting algorithm has been efficient enough to cancel out the random a_2 and b_3 components of the main dipoles, leaving a residual below the significance level given by the error bars of the data. It is worth emphasising that there is no correlation between a_2 and b_3 , and also no correlation for a_2 between the two apertures of each main dipole, contrary to the case of b_3 . Hence it is unlikely to have configurations corresponding to random re-arrangements of the installation sequence that are on the lowest edge of both coupling-related quantities (for both apertures) and sextupoles-related quantities.

Furthermore, it should not be forgotten that even if the simulations might indicate the existence of some magic configurations, which is in any case very unlikely, the actual installation sequence also took into account constraints that are neglected by the simple approach of random rearrangements. In fact, details such as mechanical aperture constraints and availability of magnets have been

tackled by the magnet assignment strategy, while they are not included in the re-ordered sequences shown in Figs. 11, 12. Hence, once more, the realised installation sequence is, almost by definition, the optimal one, being based on sensible optimisation of the key beam dynamics quantities.

4.2.2. *Dynamic aperture*

The expected gain in terms of dynamic aperture was analysed in Ref. [53] and evaluated to be between 1 and 2σ depending on the initial polar coordinates of the tracked particles. This estimate was, however, obtained considering only the field imperfections of the main dipoles, as specified in Ref. [29], but with the random b_3 component increased by 1 unit with respect to the r.m.s. target of 1.4 units. In the end, the situation turned out to be much better than anticipated before the two changes to the dipole cross-sections. Indeed, most of the MB multipole imperfections are very close or within tolerances, and even the random b_3 component of 1.5 units r.m.s. is very close to the initial target. On the other hand, the distribution of b_3 , and the strongly correlated b_5 , feature long non-Gaussian tails containing around 10 % of the magnet population. These tails correspond to the first ~ 150 magnets produced with the so-called cross-sections of type 1 and 2.

To assess the dynamic aperture of the actual machine and the possible gain due to the MB sorting, a strategy similar to that of the previous section was applied, but this time assigning the known field imperfections to all LHC magnet types, namely

- 60 different seeds for the actual machine have been generated. These correspond to those presented in section 3.
- 7 reference seeds have been selected, each of them being further modified by defining 60 random re-arrangements of the MB installation sequence.
- Tracking studies were performed to compute the DA at injection energy for both beams and for each of the $7 + 7 \times 60 = 427$ LHC machines.

The results obtained have been analysed in two ways. First of all, considering globally the statistics obtained for the 427 machines described above, i.e. mixing the statistics related to the reference seeds and the permutations, the tracking results are reported in Fig. 13. The histograms show the DA distribution obtained for each of the two beams and for a few selected angles in the $(x-y)$ phase space. As in the case of the analytical figure-of-merit, the 7 vertical dotted lines indicate the DA of the 7 reference seeds, which are well distributed around the vertical red line representing the average calculated dynamic aperture of the actual machine. At low angles, where the DA is smaller, the gain is still of the order of $1 - 1.5\sigma$ with respect to the worst case.

In order to go deeper into the analysis, each reference seed was investigated separately and the DA calculated as a function of the phase space angle before and after MB permutation. The results obtained are illustrated in Figs. 14 and 15 for Beam 1 and Beam 2, respectively. In each figure, the first plot illustrates

the situation of the actual machine, with the 7 selected reference seeds represented by red solid lines, with the black dotted lines with error bars showing the minimum, average and maximum DA calculated over the 60 seeds. It should be noted that the lines joining the markers are used only as a guide for the eye. The results obtained for each reference seed taken separately are shown in the 7 remaining plots, using a similar convention for the symbols, but with the black dotted lines (with error bars) now representing the DA statistics obtained over the 60 MB permutations. With only one exception, each of the 7 reference seeds features a DA that is very close or higher than the average DA calculated over the 60 random permutations of the MB installation sequence. This is another clear sign of the beneficial impact of the sorting of the LHC main dipoles. It is also clear that the improvement in DA due to the sorting is certainly less important than what was found for the figures-of-merit considered earlier, such as coupling and third-order resonance driving terms. The only exception is the DA of Beam 1 at 30° and for a few reference seeds. This pathological case existed already, for the same seeds, before permutation (see first plot of Fig. 14). In this case, the origin is certainly a non-MB type class of magnets. Hence, randomising the MB installation sequence could be a cure, for instance leading to a self-compensation of the tune spread induced by two different types of magnet. In the case of b_3 , it should be noted that the induced tune spread has a well-determined sign, regardless of the details of its distribution over the ring, as soon as the machine operates close to the third-order resonance, as is the case of the LHC. Therefore, an LHC ring unsorted with respect to the b_3 component of the main dipoles, would feature a more or less important tune spread, but always driving the betatron tunes of particles with non-zero betatron amplitudes in the same direction.

5. CONCLUSIONS

The meticulous work of specifying the field quality of the LHC magnets, in particular the main dipoles, the overall high-quality design of the magnets, together with the professional follow-up of the magnet production, and in particular the speedy modifications to the cross-section of the main dipoles at the beginning of the production, made it possible to almost completely reach the desired field quality.

The patient and accurate work done during the allocation phase of the LHC magnets made it possible to implement the sorting strategy devised earlier.

Dipole and quadrupole field imperfections, induced linear optics distortion, such as closed orbit, beta and dispersion beating, and linear coupling, together with the random sextupole component of the dipoles were taken into account in the installation strategy. A clear signature of the beneficial impact of this optimisation is the actual residual beta and dispersion beating all along the machine, which is well within specification after identification and correction of localised errors found in the insertions [67].

The non-linear multipoles were analysed and specified in terms of detuning

(e.g., impact of the b_4 or b_6 harmonics of the main dipoles and quadrupoles, respectively), non-linear chromaticity (e.g., b_5), and various non-linear quantities such as chromatic coupling for a_3 . Nevertheless, the final acceptance was based on the evaluation of the corresponding dynamic aperture, whether to assess the overall situation, or to analyse some field quality trend in the production and/or to justify some hardware modification, such as the magnet cross-section.

This paper focused mainly on this last aspect with other additional technical details given in order to describe and show the beneficial impact of the sorting strategy which was applied to the LHC main dipoles.

It must be emphasised that the strategy followed for the optimisation of the LHC performance in terms of DA is completely general and can be applied to other machines where relevant, i.e., to optimise the single-particle dynamics. Needless to say, when the potential performance limitations stem from other sources, e.g., space charge, the approach discussed here needs to be adapted. It is also clear that the sorting algorithm could be applied not only to hadron machines, but also to synchrotron light sources.

The LHC machine was designed with a target dynamic aperture (single-particle, without beam-beam effects) ranging between 12σ at injection and 10σ in collision, which is larger than or equal to the mechanical acceptance of the ring. In practise the absence of lifetime problems or sporadic losses in the LHC ring during mechanical aperture measurement sessions [68], when the beams are pushed towards very large transverse amplitudes, is a clear proof that this goal has been fully achieved in the actual machine. To be able to measure the dynamic aperture in the LHC, very specific perturbations need to be applied to the nominal machine settings, such as switching on the very strong Landau octupoles at injection energy. Comparisons of benchmarking simulation results with true measurements of dynamic aperture is not covered in this paper.

Here, emphasis has been given to the simulation principles themselves, the analysis of the sorting strategy that was applied to the arc magnets, and to the results obtained in various operation modes of the machine (injection, collision, etc.) using the field quality data extracted from the actual machine, emphasising that the dynamic aperture of the LHC is actually similar or larger than the 10σ mechanical aperture of the ring.

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Figure 1: Plot of the stable initial conditions defined on a polar grid in the normalised physical space for one realisation of the LHC machine at 3.5 TeV, Beam 1 and optics still with β^* corresponding to the injection configuration. The blue markers identify initial conditions that are not stable up to 10^5 turns. The marker size is proportional to the number of turns for which the motion is still bounded. The red markers identify initial conditions that are stable up to 10^5 turns.

Figure 2: Layout of the LHC ring (from Ref. [38]). The eight-fold symmetry of the ring is clearly visible, together with the arcs and the long straight sections. The insertion housing the experimental detectors are highlighted, as well as the injection point of Beam 1 (through the transfer line TI2) and Beam 2 (through the transfer line TI8). The key systems located in the non-experimental insertions are also mentioned.

Figure 4: 10^5 turns dynamic aperture vs. angle for the nominal injection optics. The standard computation protocol of the DA has been used, but considering 25 instead of 5 angles for the polar grid in the normalised physical space. Both Beam 1 and Beam 2 values are shown. The markers indicate the average DA value over the sixty realisations, while the error bars represent the total interval of variation over such realisations.

Figure 5: 10^5 turns dynamic aperture vs. fractional part of the horizontal tune at injection (assuming a fractional tune split of 0.03, namely $q_y = q_x + 0.03$ where $q_{x,y}$ stands for the fractional part of the tune.). The standard computation protocol of the DA has been used. Each marker represents the minimum stable amplitude over the angle of the polar grid, averaged over the sixty realisations. The error bars represent the total interval of variation of such a minimum stable amplitude over such realisations.

Figure 6: Global situation in terms of DA for various LHC machine configurations at top energy. The standard computation protocol of the DA has been used, but considering 11 instead of 5 angles for the polar grid. The minimum DA over realisations and angles is shown for both Beam 1 and Beam 2.

Figure 7: Global situation in terms of DA for injection and 7 TeV (top). The impact of the magnets replacement due to the consequences of the September 2008 incident is visible. Global situation in terms of DA for injection and 3.5 TeV (bottom). In both cases the standard computation protocol of the DA has been used, but considering 11 instead of 5 angles for the polar grid. The minimum DA over realisations and angles is shown for both Beam 1 and Beam 2. In the bottom plot the DA for nominal collision configuration, e.g., squeezed optic at 7 TeV is shown for reference.

Figure 8: Evolution of the b_3 component in the collared coils of LHC main dipoles in the early stage of the production (see also Ref. [64] for more details). The change of the average value of b_3 following the modifications of the LHC main dipole cross-section is clearly visible. The two horizontal lines represent the beam dynamics limits for b_3 .

Figure 9: Difference coupling resonance driving term (top), $\langle xy \rangle$ coupling coefficient (middle), and normalised vertical spurious dispersion (bottom) (see Eqs. 2, 3 and 4) along the machine at the location of the beam position monitors. These quantities are induced by the skew quadrupole imperfections of the main dipoles, for a typical seed before (red curve) and after (black curves) sixty random permutations of the installation sequence (the contribution of the lattice skew quadrupole correctors is included). The left plots refer to Beam 1 while the right ones to Beam 2. The red curve corresponds to the situation of the as-built machine.

Figure 10: Third-order resonances driving terms $|c_{3,0}|$ (top), $|c_{1,2}|$ (middle), and $|c_{1,-2}|$ (bottom) (see Eq. 5) along the machine at the location of the beam position monitors. These quantities are induced by the sextupole imperfections of the main dipoles along the machine for a typical seed, before (red curve) and after (black curves) sixty random permutations of the installation sequence (the contribution of the sextupole spool-pieces is included). The left plots refer to Beam 1 while the right ones to Beam 2. The red curve corresponds to the situation of the as-built machine.

Figure 11: Statistical distribution of the optics distortions due to a_2 (difference coupling resonance driving term - top -, $\langle xy \rangle$ coupling coefficient - middle -, and normalised vertical spurious dispersion - bottom showed in Fig. 9 and averaged over the whole machine circumference of seven reference realisations and sixty random permutations of each of them. The vertical dotted lines indicate the position of the seven reference seeds in the distribution. A rather long tail of the distribution towards large values of the beam dynamical quantities is clearly visible.

Figure 12: Statistical distribution of the third-order resonances driving terms ($|c_{3,0}|$ - top -, $|c_{1,2}|$ - middle -, and $|c_{1,-2}|$ - bottom) showed in Fig. 10 and averaged over the whole machine circumference of seven reference realisations and sixty random permutations of each of them. The vertical dotted lines indicate the position of the seven reference seeds. Also in this case a rather long tail of the distribution towards large values of the beam dynamical quantities is clearly visible.

Figure 13: 10^5 turns DA for Beam 1 (left) and Beam 2 (right) and three different angles of the $(x - y)$ phase space, namely 15° (top), 45° (middle), and 75° (bottom). Each histogram contains the statistics obtained for the seven reference seeds before and after a random re-ordering of the MB installation sequence (sixty permutations considered for each reference seed). For each picture, the vertical dotted lines illustrate the position of the seven reference seeds before permutation (as-built machine), while the vertical red line shows the average dynamic aperture of the as-built machine calculated over sixty reference seeds. This provides an information about the relationship between the reference seeds and the average over the realisations. In this case the DA distribution is skewed towards small values, thus indicating a positive impact of the MB sorting.

Figure 14: 10^5 turns DA for Beam 1 as a function of the $(x - y)$ phase space angle: as-built machine (first plot) and analysis of the impact of re-ordering the MB installation sequence for a few selected reference seeds. The markers correspond to calculated values, while the straight lines between markers are only used to guide the eye.

Figure 15: 10^5 turns DA for Beam 2 as a function of the $(x - y)$ phase space angle: as-built machine (first plot) and analysis of the impact of re-ordering the MB installation sequence for a few selected reference seeds. The markers correspond to calculated values, while the straight lines between markers are only used to guide the eye.