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SCALE-SCHEME AMBIGUITIES IN THE BRODSKY-LEPAGE-MACKENZIE PROCEDURE

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A B S T R A C T

The recently proposed method of Brodsky, Lepage and Mackenzie for "eliminating scale ambiguities in perturbative QCD" yields different results in different renormalization schemes. We show that one cannot make a scheme-independent separation of fermion loop effects from other types of higher-order correction. The case of T decay is discussed particularly.

Recently, Brodsky, Lepage and Mackenzie¹⁾⁻³⁾ (BLM) have proposed a new procedure for dealing with the renormalization-scheme-dependence problem which plagues perturbative QCD. In this note we emphasize that the results obtained from the BLM procedure depend on the initial choice of renormalization scheme^{*)}. On applying their method to T decay, BLM found a next-to-leading order coefficient of -14.0, which led them to question the reliability of the perturbative prediction. However, we believe this large number to be merely a scheme artifact, and therefore not an indication of an intrinsic difficulty with perturbation theory. We intend, in this note, to keep strictly to a discussion of the BLM method, without prejudice to our own - separate - views on the wider issues^{4),5)}.

The motivation behind the BLM approach is the idea that, in an Abelian theory such as QED, the running of the coupling constant is basically due to vacuum polarization - i.e., to fermion-loop insertions in the vector propagator. In a non-Abelian theory, other effects will also contribute to the renormalization of the coupling constant. However, BLM argue that these effects may be accounted for by taking the fermion-loop contribution (proportional to n_f , the number of flavours) and replacing n_f by $(n_f - 33/2)$ - the ratio $n_f: -33/2$ being set by the leading β function coefficient, $\beta_0 \equiv 11-2/3 n_f$. Therefore, they argue, simply by studying the fermion-loop insertions for a given process, one can identify a natural scale " Q^* " for the coupling constant $\alpha(Q)$. The BLM algorithm can be described as follows²⁾.

One begins with the result of a next-to-leading order calculation of some physical quantity $\mathcal{R}$, in some scheme "i":

$$\mathcal{R} = [\alpha_i(Q)]^P \left\{ 1 + (A_i n_f + B_i) \frac{\alpha_i(Q)}{\pi} + \dots \right\}. \quad (1)$$

A shift of scale, $Q \rightarrow \lambda Q$, corresponds to the substitution

$$\alpha_i(Q) = \alpha_i(\lambda Q) \left\{ 1 + \frac{1}{2} \beta_0 \ln \lambda \frac{\alpha_i(\lambda Q)}{\pi} + \dots \right\} \quad (2)$$

*) Reference 1) is a revised version of an earlier paper submitted to the XX1st International Conference on HEP (July 1982). Other accounts of the method have been given previously^{2),3)}. These earlier references incorrectly claimed that the procedure led to renormalization-scheme-invariant results.

which follows from solving the β function equation. Recalling that $\beta_0 \equiv 11-2/3 n_f$, Eq. (1) becomes

$$\mathcal{R} = [\alpha_i(\lambda Q)]^P \left\{ 1 + \left[(A_i - \frac{1}{3}P \ln \lambda) n_f + (\frac{11}{2}P \ln \lambda + B_i) \right] \frac{\alpha_i(\lambda Q)}{\pi} + \dots \right\}. \quad (3)$$

The BLM procedure is to fix λ by requiring the coefficient of n_f in this equation to vanish; i.e., to set

$$\lambda = \lambda_i^* \equiv \exp(3A_i/P). \quad (4)$$

[There is a parallel with the method proposed by Grunberg⁶⁾, in which λ is chosen to make the whole next-to-leading order coefficient vanish. Here it is only the n_f part which is made to vanish]. The final second-order result in the BLM method is then

$$\mathcal{R}_{\text{BLM}}^{(2)} \equiv [\alpha^*]^P \left\{ 1 + r^* \frac{\alpha^*}{\pi} \right\}, \quad (5)$$

where

$$\alpha^* = \alpha_i(\lambda_i^* Q) = \alpha_i(e^{3A_i/P} Q), \quad (6)$$

$$r^* = \frac{33}{2} A_i + B_i. \quad (7)$$

Notice that the BLM result is immediately obtainable from a knowledge of the A_i and B_i coefficients in the initial "i" scheme.

As an illustration we consider T decay. It is convenient to define

$$\mathcal{R}_\gamma \equiv \frac{81\pi e_b^2}{10(\pi^2-9)} \alpha_{em}^2 \frac{\Gamma_g(\gamma \rightarrow \text{hadrons})}{\Gamma(\gamma \rightarrow \mu^+ \mu^-)}, \quad (8)$$

where Γ_g is the gluonic decay width of the T , α_{em} is the electromagnetic coupling constant, and $e_b = -1/3$ is the charge of the b quark. In the " $\overline{MS}$ "⁷⁾ scheme, with M , the T mass, as the renormalization scale, $\mathcal{R}_T$ has the perturbation expansion^{8),9)}

$$\mathcal{R}_\gamma = [\alpha_{\overline{MS}}(M)]^3 \left\{ 1 + (-1.847n_f + 16.5) \frac{\alpha_{\overline{MS}}(M)}{\pi} + \dots \right\}, \quad (9)$$

so that

$$P = 3, \quad A_{\overline{MS}} = -1.847, \quad B_{\overline{MS}} = 16.5. \quad (10)$$

This leads to

$$\alpha^* = \alpha_{\overline{MS}}(e^{-1.847} M) = \alpha_{\overline{MS}}(0.158 M), \quad (11)$$

$$r^* = \frac{33}{2}(-1.847) + 16.5 = -14.0,$$

as quoted by BLM¹⁾⁻³⁾.

However, we do not have to start with the $\overline{MS}$ scheme. Let us start instead from the result in a momentum subtraction ("mom") scheme¹⁰⁾. In particular, for the usual Landau-gauge, three-gluon-vertex version of "mom" the series has the form

$$\mathcal{R}_r = [\alpha_{\text{mom}}(M)]^3 \left\{ 1 + (0.716n_f - 3.39) \frac{\alpha_{\text{mom}}(M)}{\pi} + \dots \right\}, \quad (12)$$

so that

$$P=3, \quad A_{\text{mom}} = 0.716, \quad B_{\text{mom}} = -3.39. \quad (13)$$

This leads to

$$\alpha^* = \alpha_{\text{mom}}(e^{0.716} M) = \alpha_{\text{mom}}(2.05 M), \quad (14)$$

$$r^* = \frac{33}{2}(0.716) - 3.39 = +8.42.$$

The α^* , r^* of Eq.(14) are not the same as those of Eq. (11)! For α^* this is not immediately obvious, since one must first convert α_{mom} into $\alpha_{\overline{\text{MS}}}$. However, it is quite sufficient to note that the r^* 's are different. (Note the opposite sign.) Thus, $r^* \equiv 33/2 A_1 + B_1$ is not scheme independent. The next-to-leading coefficient - and, of course, the final prediction $\mathcal{R}_{\text{BLM}}^{(2)}$ [Eq. (5)] - remains dependent on the initial choice of renormalization scheme!

The underlying reason for this is as follows. There is nothing sacred about the decomposition of the coefficient r_1 into $A_1 n_f + B_1$: it may be altered in an arbitrary fashion by a change of scheme. Under $\alpha \rightarrow \alpha'$ with

$$\alpha' = \alpha \{ 1 + (v n_f + w) \alpha / \pi + \dots \}, \quad (15)$$

one has $r \rightarrow r'$

$$r' = (A - P_v) n_f + (B - P_w), \quad (16)$$

so that $r^* \rightarrow r^{*'}$

$$r^{*'} = \frac{33}{2} (A - P_v) + (B - P_w), \quad (17)$$

which is generally not equal to r^* . Now, it happens that $r^{*'}$ and r^* coincide if $w = (-33/2)v$. This is the case, for instance, for an n_f independent change of scale [cf. Eq. (2)]. The $\overline{MS}$ and $\overline{MS}$ schemes are related in this fashion, and so one does have $r_{\overline{MS}}^* = r_{\overline{MS}}^{*}$. However, in general, one may have arbitrary v and w 's in Eq. (15), and the $\overline{MS} \leftrightarrow \text{mom}$ relation illustrates this. In fact, the various forms of momentum subtraction discussed in Ref. 10) all have different $v:w$ ratios in relation to $\overline{MS}$.

Even within the context of dimensional renormalization schemes, one can have essentially arbitrary n_f dependence. For instance, one is perfectly entitled to subtract an n_f dependent constant together with the $1/\epsilon$ poles. The same effect also arises in a very natural way, as follows. The renormalization scheme known as " $\overline{MS}$ " ⁷⁾ uses dimensional continuation rules in which the trace of the unit Dirac matrix in N dimensions is set equal to four. That is merely a convention, and one may equally well take $\text{Tr}(\mathbf{1}) = 2^{N/2}$, or indeed any analytic function $d(N)$ with $d(4) = 4$. [See Ref. 11), for example.] One would thereby define a new renormalization scheme, different from the usual $\overline{MS}$. Such a modification would affect only the renormalization of fermion loops. Thus, the relation of such a scheme to the usual $\overline{MS}$ would involve $v \neq 0, w = 0$ in Eq. (15)^{*)}. It would be very hard to argue that such schemes are any more, or any less, artificial than the conventional $\overline{MS}$.

Consequently, we cannot see a way to choose, a priori, a scheme which is " n_f independent" or "flavour neutral". There is no obviously "canonical" way of treating the n_f terms. Surprising as it may seem at first, one cannot meaningfully separate the contributions of quark loops from other types of next-to-leading corrections: any such split-up is necessarily scheme-dependent. To say the least, this fact must cast some doubt on the motivating philosophy behind the BLM approach.

In the light of the prescription dependence that we have described above, BLM adopt the following strategy¹⁾. They find a renormalization convention for which their method leads to a small value of r^* for some physical process - say, $R_{e^+e^-}$. (Conveniently for them, $\overline{MS}$ turns out to have such a property.) They then

*) These remarks echo the last paragraph of Section V of Ref. 10).

observe that "most" other processes have small r^* 's if one works in that convention. The glaring exception, BLM claim, is T decay, since any convention that yields a small r^* for e^+e^- will give a large, negative r^* for $\Gamma_g(T \rightarrow \text{hadrons})/\Gamma(T \rightarrow \mu^+\mu^-)$. BLM conclude that therefore low-order perturbation theory is unreliable for T decays. We do not agree.

For one thing, there is a lot of "slop" left in the BLM strategy. Just how small is "small"? For instance, we can exhibit a renormalization convention in which $r_{e^+e^-}^* = 3.38$ and $r_T^* = -4.1$. [Such a scheme is related to $\overline{MS}$ by Eq. (15) with $(33/2)v + w = -3.3$.] Is one to regard either or both of these coefficients as being "large" or "small"?

However, suppose that we were to accept the dubious hypothesis that one must insist on a convention for which $r_{e^+e^-}^*$ is essentially zero ($\ll 3$). Then r_T^* would indeed be large and negative. One could interpret this in two ways:

- (a) (BLM's view) fermion loops infallibly indicate the proper momentum scale for all processes (provided the convention is such that the corrections to e^+e^- are very small); in this case, large coefficients must be interpreted as signalling a failure of perturbation theory;
- (b) for some processes fermion loops are a worse guide than for others.

We consider option (b) more likely. After all, if the contribution of fermion loops is quite different from one scheme to another (as we showed above), then it is most unlikely that they contribute with exactly the same weight to all physical quantities. One has only to glance at the Feynman diagrams for T decay [see Fig. 1) of Ref. 9)] to see that fermion loop diagrams are much less representative than in the e^+e^- case: - T decay involves many more kinds of gluon-self-coupling diagrams. Thus, it would not be at all surprising if, in the T case, the fermions "mis-probe" the momentum scale somewhat. Even a factor 2 change in momentum scale would alter the next-to-leading coefficient by ± 9 . Consequently, we interpret the large, negative r^* obtained by BLM as an indication of a failure of their fermion-loop-absorption method, not as a breakdown of perturbation theory.

Our conclusion on T decay echoes that of Mackenzie and Lepage in Ref. 9), page 190: "It is our opinion that when renormalization scheme and scale have been treated in a sensible way, there is little evidence that the QCD perturbation series is not reasonably well-behaved".

In summary: the basis of the BLM procedure is the idea that the radiative corrections to be absorbed into the running coupling constant can be identified

by isolating the contributions of fermion loops. Unfortunately, the physics of these fermion contributions is inextricably entangled with renormalization and regularization ambiguities. Faced with this convention dependence, BLM appeal to the old idea that "good schemes are the ones in which corrections are 'usually' small". This still leaves a considerable amount of ambiguity. Thus, even if one believes in the ideas behind the BLM method, it is difficult to see that it provides any advantage over - or phenomenologically significant contradictions with - existing approaches. BLM claim that T decay is an exception, since only their procedure "correctly" indicates the unreliability of perturbation theory. However, we believe it is much more reasonable to interpret the T case as a failure of their method, not a failure of perturbation theory.

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