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THE AXIAL FORM-FACTOR OF THE NUCLEON AND
THE PION-NUCLEON VERTEX FUNCTION

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A B S T R A C T

The axial form-factor of the nucleon in the Cloudy Bag Model is harder than the $NN\pi$ vertex function, and consistent with data for a bag radius about 1fm. We examine how these results are altered by excluding the pion field from a smaller, interior volume.

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There has recently been considerable discussion of the calculation of the axial charge of the nucleon in various chiral bag models¹⁻⁵). However little attention has been given to the axial-vector form-factor, $g_A(q^2)$, which is known from measurements of muon production cross-sections ($\nu_\mu + n \rightarrow \mu^- + p$) and pion electroproduction⁶). Phenomenologically the best fit is a dipole with mass $m_A = 0.95 \pm 0.14$ GeV. In fact Amaldi et al.⁶) quote an error of ± 0.07 GeV from the neutrino reaction, and ± 0.14 GeV from electroproduction. We take the more conservative error in order to improve the overlap with the pion production data from the ZGS, which yielded values of m_A between 1.0 and 1.4 GeV⁷). Of course these data are most sensitive to the low $-q^2$ behaviour, which can be parametrized as

$$g_A(q^2)/g_A(0) = 1 - q^2 r_A^2/6, \quad (1)$$

with $r_A = 0.72 \pm 0.12$ fm.

In the Cloudy Bag Model^{2,8}) the pion field is allowed to penetrate the interior of the MIT bag. The appropriate expression for the axial-vector current density is

$$\vec{A}_i(x) = \frac{1}{2} \bar{q}(x) \vec{\gamma} \gamma_5 \tau_i q(x) \theta(R-r) - f_\pi \vec{\nabla} \phi_i(x), \quad (2)$$

where q and ϕ are respectively the quark and pion fields, R the bag radius and f_π is the pion decay constant (93 MeV). For this model $g_A(q^2)$ is entirely determined⁸) as the expectation value of the Fourier transform of the first term in Eq. (2), in the physical nucleon. Ignoring the corrections arising from renormalization, which are expected to be small*), we find

$$g_A^{CBM}(q^2)/g_A^{CBM}(0) = 1.5 j_0^{-2}(\omega) \int_0^R dr r^2 \left\{ \left[j_0^2\left(\frac{\omega r}{R}\right) - j_1^2\left(\frac{\omega r}{R}\right) \right] j_0(qr) + 2 j_1^2\left(\frac{\omega r}{R}\right) j_1(qR)/qR \right\}. \quad (3)$$

As usual, $\omega = 2.04$ is the eigenfrequency for massless quarks.

*) In the static approximation they are identically zero for the ratio.

Expanding Eq. (3) to order q^2 for comparison with Eq. (1) we find $r_A^{\text{CBM}} = 0.62 R$, and hence $R = 1.16 \pm 0.20$ fm. Of course we must add a caution that we have also omitted corrections arising from possible spurious centre of mass motion and recoil. Surprisingly several groups⁹⁾ have found quite a large correction to $g_A(0)$ from c.m. corrections (of order 20%) -- using the Peierls-Yoccoz technique. Because the deviation of $g_A(0)$ from $5/3$ is a purely relativistic effect in the bag model we have limited confidence in the Peierls-Yoccoz procedure. In any case we expect the corrections in the ratio $g_A(q^2)/g_A(0)$ to be less than in $g_A(0)$ alone.

Recent lattice calculations of Kogut et al.¹⁰⁾ have suggested that the temperature at which confinement occurs (T), may be somewhat smaller than that at which chiral symmetry breaking occurs (T_{ch}). This has been translated by Chin and Miller⁴⁾ and Vento⁵⁾ into a simple modification of the CBM in which pions are coupled to the quarks at $r = R$, but excluded from the region $r < R_{\text{ch}}$ (where $R_{\text{ch}}/R \sim T/T_{\text{ch}} < 1$). As a caution we would note that the calculation of Kogut et al. omits fermion loops, so that it is hard to see how dynamical symmetry breaking could appear in the model. There is no doubt that their calculation gives an indication of the surface thickness of the confinement volume, but the naive association of the interior with the Wigner phase and the exterior as the Goldstone phase is not so straightforward¹¹⁾.

Nevertheless it is interesting to pursue the interpretation of Chin and Miller⁴⁾ and Vento⁵⁾. As shown by those authors the axial charge of the nucleon was modified to

$$\tilde{g}_A(0) = g_A^{\text{CBM}}(0) \left[1 + \frac{\xi^3}{2} \right], \quad (4)$$

with $\xi = R_{\text{ch}}/R$. For the case $\xi = 2/3$ suggested by Kogut et al.¹⁰⁾ this means a 15% increase over the CBM value of 1.09 -- which is identical to that of the original MIT bag model¹²⁾. The effect on charge radii and magnetic moments is expected to be much smaller because these quantities get their biggest contribution from pions outside the bag^{2,8)}.

Clearly it is interesting to check whether this new version of the CBM significantly alters the predictions for $g_A(q^2)/g_A(0)$. Following Chin and Miller we write the expectation value of the pion field as

$$f_\pi \langle \tilde{\phi}(\vec{r}) \rangle = g_A(0) \Delta_1(r, R) \left(3/4\pi R \right) \langle N | \frac{\vec{\sigma} \cdot \hat{r}}{2} \tau | N \rangle, \quad (5)$$

where $\Delta_1(r, R)$ is the static Green function for p-wave pions excluded from the region $r < R_{ch}$. This Green function has the form⁴⁾

$$\Delta_1(r, r') = \frac{m_\pi}{3} \left[j_1(i m_\pi r_<) + \gamma h_1^{(1)}(i m_\pi r_<) \right] h_1^{(1)}(i m_\pi r_>), \quad (6a)$$

where

$$\gamma = - \left[\frac{\partial j_1(i m_\pi r)}{\partial r} / \frac{\partial h_1^{(1)}(i m_\pi r)}{\partial r} \right]_{r=R_{ch}}. \quad (6b)$$

It is now straightforward to show that in this model the q^2 -dependence is given by

$$\tilde{g}_A(q^2) = g_A^{CBM}(q^2) + g_A^{CBM}(0) \frac{\xi^3}{2} u(q R_{ch}) (1 - y^2/2), \quad (7)$$

where $u(x) = 3j_1(x)/x$ is the familiar CBM form factor, and $y \equiv m_\pi R$. The $q^2 = 0$ limit of Eq. (7) differs from Eq. (4) because the pion mass (m_π) has not been taken to zero. In Eq. (7) small terms of order y^3 or higher are ignored.

If we expand Eq. (7) to order q^2 the new mean square radius of the axial-vector charge distribution ($\tilde{r}_A^2$) is

$$\tilde{r}_A^2 = \left(1 + \xi^3/2\right)^{-1} \left[0.38 + 0.30 \xi^5 (1 - y^2/2)\right] R^2. \quad (8)$$

As shown in Fig. 1a $\tilde{r}_A$ is almost independent of ξ over its entire range. [The two curves refer to two values of y ($y = 0$ and $y^2 = 1/2$) used in Eq. (8).] For small $m_\pi R$ and for $\xi = 1$, $\tilde{r}_A/(r_A)_{CBM} = 1.05$. For all other parameters $\tilde{r}_A/(r_A)_{CBM}$ is closer to unity. Thus $g_A(q^2)$ offers no discrimination between models with different values of ξ .

Another property of the nucleon which has been intensely discussed in the context of chiral bag models is the form-factor for coupling to a pion, $g_{\pi NN}(q^2)$. As shown in Eq. (3), in such models $g_A(q^2)$ arises from an integral over the bag volume. On the other hand, $g_{\pi NN}(q^2)$ is associated with the surface of the bag, and one might intuitively expect it to fall more rapidly with q^2 . In the CBM this is certainly the case, because the form-factor there is

$$\begin{aligned} g_{\pi NN}^{CBM}(q^2) &= 3 j_1(q R) / (q R) \\ &= 1 - q^2 (0.6 R^2) / 6, \end{aligned} \quad (9)$$

and hence $r_\pi [g_{\pi NN}(q^2) \equiv 1 - q^2 r_\pi^2/6 + O(q^4)]$ is $0.77R$. This should be compared with the value of $r_A (= 0.62R)$ calculated from Eq. (3). That is in the CBM (r_A/r_π) equals 0.80.

One can also compute the $NN\pi$ form factor in the model of Chin and Miller. This is more complicated than in the CBM because the pion field cannot be expanded in plane waves. The $NN\pi$ form factor can be identified from the coefficient of the pion pole term which appears in the expectation value of the pion field operator for the physical nucleon. By using the field equations, or by considering directly

$$(-\nabla^2 + m_\pi^2) \partial_\mu \tilde{A}^\mu = (-\nabla^2 + m_\pi^2) \theta(r - R_{ch}) m_\pi^2 f_\pi \tilde{\phi}, \quad (10)$$

one finds

$$\begin{aligned} & (\vec{q}^2 + m_\pi^2) \int d^3r e^{i\vec{q}\cdot\vec{r}} \langle \tilde{\phi}(\vec{r}) \rangle \\ &= \int d^3r e^{i\vec{q}\cdot\vec{r}} \langle \frac{i}{2f_\pi} \bar{q}(r) \gamma_5 \tau q(r) \delta(r-R) - i \vec{q}\cdot\hat{r} \delta(r-R_{ch}) \tilde{\phi}(\vec{r}) \rangle. \end{aligned} \quad (11)$$

Direct evaluation of the integrals in Eq. (11) gives the result

$$\begin{aligned} g_{\pi NN}(q^2)/g_{\pi NN}(0) &= \left[u(q, R) + \frac{\xi^3}{2} \left(u(q, R_{ch}) \right. \right. \\ &\quad \left. \left. - 3j_2(q, R_{ch}) \right) (1 - y^2/2) \right] / \left[1 + \frac{\xi^3}{2} (1 - y^2/2) \right]. \end{aligned} \quad (12)$$

Using the small q^2 expansions of the spherical Bessel functions it is then easy to show that

$$\tilde{r}_\pi^2 = \frac{1 + \frac{3}{2} \xi^5 (1 - y^2/2)}{1 + \frac{1}{2} \xi^3 (1 - y^2/2)} r_\pi^2. \quad (13)$$

For $\xi \lesssim 0.75$ (Fig. 1a) the variation with ξ is rather mild just as for $\tilde{r}_A$, and indeed $\tilde{r}_\pi$ is essentially equal to r_π . For values of ξ closer to one, on the other hand $\tilde{r}_\pi/r_\pi$ tends towards 1.28. Thus for a given bag radius, in models with pions totally excluded from the bag interior ($R_{ch}/R = \xi = 1$) the πNN vertex function is significantly softer than for the cloudy bag model.

At this stage we recall the infamous discrepancy in the Goldberger-Treiman relation¹³⁾

$$5.97 \pm 0.03 = g_A^{(0)} m_N \neq g_{\pi NN} f_\pi = 6.36 \pm 0.10, \quad (14)$$

where we have used the measured value of the $NN\pi$ coupling constant¹⁴⁾ ($g^2/4\pi = 14.3 \pm 0.4$) at the nucleon pole ($q^2 = -m_\pi^2$). On the other hand one should use $g_{\pi NN}$ at $q^2 = 0$. It is amusing to ask what radius (r_π^{GT}) would give enough variation between $q^2 = -m_\pi^2$ and $q^2 = 0$ to resolve all of this discrepancy. Clearly that would be $(r_\pi^{G-T})^2 m_\pi^2/6$ equal to $6.1 \pm 1.7\%$, or

$$r_\pi^{G-T} = 0.86 \pm 0.12 \text{ fm.} \quad (15)$$

Using the experimental value for r_A given in Eq. (1), and the CBM ratio $r_A/r_\pi = 0.80$, we find $r_\pi = 0.90 \pm 0.15$ fm. (We hope that the use of the ratio of these quantities will reduce the unknown error associated with centre of mass motion.)

Thus, within the rather large experimental errors, the variation of the $NN\pi$ form-factor with q^2 in the CBM is enough to remove all of the Goldberger-Treiman discrepancy. From the discussion of the modified version of the CBM, with pions excluded from a region $r < R_{ch}$, and in particular using Eqs. (8) and (13) (see also Fig. 1b), it is easily seen that this agreement is maintained for all values of ξ . For example, with $\xi = 2/3$ as suggested by the first calculations of Kogut et al.¹⁰⁾ $\tilde{r}_A/\tilde{r}_\pi = 0.75$ and hence $\tilde{r}_\pi = 0.96 \pm 0.16$ fm. Even in the extreme case where $\xi = 1$ ¹⁵⁾, $\tilde{r}_A/\tilde{r}_\pi = 0.65$ and $\tilde{r}_\pi = 1.11 \pm 0.18$ fm, there is no discrepancy within the errors.

Let us summarize briefly. We have calculated the q^2 -dependence of the axial-vector form-factor of the nucleon, and the $NN\pi$ vertex function, using both the original CBM and a recent modification due to Chin and Miller⁴⁾ and Vento⁵⁾. These calculations used a static bag, without c.m. or recoil corrections. All of these approximations should be improved in future work. Nevertheless our central result is very simple, and probably correct independent of these approximations. That is, in both versions of the theory the $NN\pi$ form-factor is considerably softer than the axial form-factor. (The corresponding r.m.s. radius is at least 25% larger.) If one then uses the experimental axial form-factor, the q^2 dependence of the resulting $NN\pi$ form-factor is enough to entirely remove the Goldberger-Treiman discrepancy -- within the rather large experimental uncertainty. At this stage we might remark that the $NN\pi$ form-factor obtained in this fashion is rather soft, and certainly raises interesting questions in the context of the nucleon-nucleon interaction¹⁶⁾. Clearly it would be very valuable to have more precise data on $g_A(q^2)$, which is of fundamental importance in chiral bag models. Finally we must point out that there may well be additional contributions to the Goldberger-Treiman discrepancy -- see, for example, Refs. (17-19). Our point is merely that given the rather large experimental errors at present, these additional corrections are not essential.

REFERENCES

- 1) R.L. Jaffe, Erice Summer School, "Ettore Majorana" Lecture (1979) MIT CTP #814.
- 2) A.W. Thomas, S. Thberge and G.A. Miller, Phys. Rev. D 24 (1981) 216.
- 3) C. Detar, Phys. Rev. D 24 (1981) 752, 762.
- 4) S.A. Chin and G.A. Miller, Physics Letters (to be published).
S.A. Chin, Nucl. Phys. A382 (1982) 355.
- 5) V. Vento, preprint (1982).
- 6) E. Amaldi, S. Fubini, G. Furlan, Berlin, Springer 1979 (Springer Tracts in modern physics, 83) "Pion-electroproduction at low energy and hadron form factors".
- 7) J. Bell et al., Phys. Rev. Lett. 41 (1978) 1008, *ibid.* 1012.
- 8) S. Thberge, A.W. Thomas and G.A. Miller, Phys. Rev. D 22 (1980) 2838.
A.W. Thomas, TH3368-CERN, TRI-PP-82-29, to be published in Adv. in Nucl. Phys., eds. J. Negele and E. Vogt, 13 (1983).
- 9) J. Donoghue and K. Johnson, Phys. Rev. D 21 (1980) 1975.
C.W. Wong, Phys. Rev. D 24 (1981) 1416.
M. Betz, private communication.
C.E. Carlson and M. Chachkhunashvili, Nordita preprint, August (1981).
- 10) J. Kogut et al., Phys. Rev. Lett. 48 (1982) 1140.
- 11) C.G. Callan, R.F. Dashen and D.J. Gross, Phys. Rev. D 19 (1979) 1826.
E.V. Shuryak, Phys. Lett. 107B (1981) 103.
- 12) T. De Grand et al., Phys. Rev. D 12 (1975) 2060.
- 13) M.L. Goldberger and S.B. Treiman, Phys. Rev. 111 (1958) 354.
- 14) M.M. Nagels et al., Nucl. Phys. B147 (1979) 189.
G. Hhler et al., Phys. Data 12-1 (1979).
- 15) J. Kogut et al., U. of Illinois preprint ILL-(TH)-82-39 (1982).
- 16) T.E.O. Ericson and M. Rosa-Clot, CERN TH.3462 (1982), to be published.
- 17) H. Pagels, Phys. Rep. 16 (1975) 219.
- 18) C.A. Dominguez, Phys. Rev. D 25 (1982) 1937.
- 19) S.A. Coon and M.D. Scadron, Phys. Rev. C23 (1981) 1150.

Fig.1a

Fig.1b

- a) $\tilde{r}_\pi$ and $\tilde{r}_A$ in units of R .
- b) $\tilde{r}_A/\tilde{r}_\pi$. This quantity is essentially independent of y .