Ref.TH.3478-CERN
ADDENDUM

CHIRAL PROPERTIES OF SUPERSYMMETRIC VACUA

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1. - It is possible to show that analyticity in m is a general property of the expectation value of a chiral superfield, by noticing that $\partial/\partial m^* \langle Q \rangle = T \langle Q F^* \rangle$ (F^* is the θ^2 component of an antichiral superfield) which vanishes, if Q is chiral, as a consequence of supersymmetric Ward identities.
2. - If the above result is coupled to the diagrammatic heuristic arguments given at the end of the paper, the only consistent solution is that all v.e.v.s vanish identically (i.e., also at $m \neq 0$).
3. - Disregarding the diagrammatic arguments, we can take two attitudes:
 - i) exclude as unphysical v.e.v.s that go to infinity as $m \rightarrow 0$. In this case, $\langle \phi \phi \rangle \equiv 0$, $\langle \lambda \lambda \rangle \sim m^{1/N}$ (or $\equiv 0$), $\langle \bar{\psi} \psi \rangle \leq m^*$ and the conclusions of the paper hold;
 - ii) allow singular v.e.v.s such as (12). In such a case we have no reason to accept $\langle \phi^* \phi \rangle \rightarrow \text{constant}$ and we should allow the possibility that $\langle \bar{\psi} \psi \rangle \xrightarrow{m \rightarrow 0} \text{constant}$. Such a scenario, not considered above, was recently studied by M. Peskin (private communication and SLAC preprint to appear) in the context of a non-linear effective Lagrangian. If the supercurrent anomaly could be implemented in such scheme and the diseases found in Ref. 7) could be avoided, this would represent an appealing alternative to the one discussed here. It is amusing to notice that, in this scheme, G_s couples to $\phi^* \phi$ but not to $\bar{\psi} \psi$, the opposite being true of course for G_p .

Ref.TH.3478-CERN

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ABSTRACT

Arguments, suggesting that chirality is (at least partly) unbroken in a class of gauge theories with exact or softly broken supersymmetry, hint at the possible relevance of such theories for constructing composite models of quarks and leptons.

Ref. TH.3478-CERN

6 December 1982

Composite models of quarks and leptons¹⁾ based on ordinary, colour-like gauge interactions suffer from several difficulties. On the one hand, one has to argue that chiral symmetries, supposedly keeping quarks and leptons light, are not dynamically broken in contrast with the QCD example, as well as with $1/N$ and fermion loop expansion arguments²⁾. Even if chiral symmetry of the vacuum is assumed, the necessity of fulfilling 't Hooft anomaly matching conditions³⁾ by the at-least-three-preon composites, seems to prevent a phenomenologically appealing picture from emerging.

It looks very plausible that supersymmetric gauge theories, by having fermionic and bosonic preons, can open new possibilities by allowing quarks and leptons to be bound states of a fermion and a scalar. Indeed some prototype models of this type have already been discussed in the literature⁴⁾.

At a more fundamental level, we can ask whether supersymmetry (SUSY) could play a fundamental rôle by preventing at least part of the dynamical chiral symmetry breaking expected in ordinary gauge theories. If this could be proved, the appeal of a supersymmetric preon dynamics would be greatly enhanced. In this respect, an encouraging result has been obtained⁵⁾ in a SUSY version of the Nambu-Jona Lasinio model where no chirality breaking solution of the gap equation was found to exist. For SUSY gauge theories, the only results we are aware of come from the effective Lagrangian study of SUSY-Yang-Mills⁶⁾ and SUSY-QCD⁷⁾ where several puzzling results originated from trying to extend supersymmetrically the chirality breaking effective Lagrangians of QCD.

Although our arguments are easy to extend to other $N = 1$ SUSY gauge theories with real matter, we shall only discuss explicitly here, as in Ref. 7), the SUSY-QCD theory, with an $SU(N)$ gauge group and M flavours. The relevant left-handed (chiral) superfields of this theory are :

$$W^a(x, \theta) : (\lambda^a, F_{\mu\nu}^a) , \quad a = 1 \dots N^2 - 1 , \text{ metacolour gauge superfield}$$

$$\mathcal{M}_\alpha^i(x, \theta) : (\phi_\alpha^i, \psi_\alpha^i) , \quad \alpha = 1 \dots N, \quad i = 1 \dots M, \text{ matter superfield}$$

$$\bar{\mathcal{M}}_i^\alpha(x, \theta) : (\bar{\phi}_i^\alpha, \bar{\psi}_i^\alpha) , \text{ antimatter superfield} \quad (1)$$

The Lagrangian has, in addition to the usual kinetic term for W and to the gauge interaction terms for $\mathcal{M}$ and $\bar{\mathcal{M}}$, a mass term:

$$\mathcal{L}_{mass} = - \int d^2\theta \sum_{i=1}^M m_i \mathcal{H}_\alpha^i \bar{\mathcal{H}}_i^\alpha + h.c. \quad (2)$$

where, for reasons that will be clear below, we consider complex m_i for the time being.

The maximal classical global symmetry of SUSY-QCD is reached for $m_i = 0$ and corresponds to the group

$$G_g = SU(M)_R \otimes SU(M)_L \otimes U(1)_V \otimes U_A(1) \otimes U_A^{PQ}(1) \quad (3)$$

where the first four factors are basically those of ordinary QCD [see also Ref. 7)] while the last $U(1)$ factor is an R transformation rotating gluinos and scalars, but not the gluons or the matter fermions. This symmetry is of the Peccei-Quinn (PQ)-type⁸⁾ in the sense that it is unbroken classically (even if $m_i \neq 0$) but suffers from a quantum level anomaly^{*)}:

$$U_A^{PQ}(1): \lambda_L^a \rightarrow e^{i\alpha} \lambda_L^a, \phi(\bar{\phi}) \rightarrow e^{i\alpha} \phi(\bar{\phi}), \mathcal{L} \rightarrow \mathcal{L} + 2\alpha N Q(x) \quad (4)$$

where $Q(x) \equiv g^2/32\pi^2 F^a \bar{F}^a$ is the topological charge density. The other $U_A(1)$ factor in (3) can be combined with $U_A^{PQ}(1)$ to give an anomaly-free axial symmetry $\hat{U}_A(1)$ defined by:

$$\hat{U}_A(1): \begin{cases} \mathcal{H}_i(x, \theta) \rightarrow e^{i(1-M/N)\alpha} \mathcal{H}_i(x, \theta e^{iM/N\alpha}) \\ W(x, \theta) \rightarrow e^{-iM/N\alpha} W(x, \theta e^{iM/N\alpha}) \end{cases}, \text{ same for } \bar{\mathcal{H}} \quad (5)$$

which is only softly broken by the mass term. We are interested in what happens to axial symmetries of this type in the $m_i \rightarrow 0$ limit, starting from finite m_i .

Our first assumption is that, for all $m_i \neq 0$, SUSY is not broken, in accordance with arguments based on the analysis of Witten's index⁹⁾ Δ . According to these arguments $\Delta = N$ in this case. An immediate consequence is that the expectation value of the various mass terms in Eq. (2) must vanish. Eliminating the auxiliary fields of $\mathcal{M}$, and $\bar{\mathcal{M}}$ one finds easily⁷⁾:

*) Throughout the paper we assume SUSY not to be explicitly broken by the anomaly and the Adler-Bardeen construction of a conserved non-gauge invariant axial current to be valid.

$$\langle \bar{\Psi}_i \Psi_i \rangle = m_i^* \langle \phi_i^* \phi_i + \bar{\phi}_i^* \bar{\phi}_i \rangle \quad (6)$$

, There is no symmetry preventing $\phi_i^* \phi_i$ from acquiring a v.e.v.; furthermore, in perturbation theory, the left hand side of (6) is indeed $O(m_i^*)$. We shall then assume:

$$\langle \bar{\Psi}_i \Psi_i \rangle = O(m_i^* \Lambda^2) \quad (7)$$

Our next observation is that the $U_A^{PQ}(1)$ symmetry allows one to get rid of the apparent CP violating phases of m_i :

$$m_i \equiv |m_i| e^{i\beta_i} \quad (8)$$

through a redefinition of the various elementary fields. This implies that the original composite fields have v.e.v.s which depend of β_i in a well defined way. A simple calculation gives:

$$\langle \bar{\Psi}_i \Psi_i \rangle \sim e^{-i\beta_i} \quad (9)$$

$$\langle \bar{\phi}_i \phi_i \rangle \sim e^{-i\beta_i + i/N \sum_j \beta_j} \quad (10)$$

$$\langle \lambda_L \lambda_L \rangle \sim e^{i/N \sum_j \beta_j} \quad (11)$$

An obvious non-anomalous transformation of type (5) then gives real masses and v.e.v.s for the transformed composite operators. Clearly Eq. (9) is consistent with the assumed behaviour (7). The question is whether (10) and (11) can give us information about the $|m|$ behaviour of the corresponding order parameters. A constant limit implies, of course, that $\hat{U}(1)$ is spontaneously broken.

We are now in a position to understand the surprising results of Ref. 7) where we represented the effective theory of SUSY-QCD by a Wess-Zumino-type Lagrangian depending on the chiral superfields $\bar{\mathcal{M}}\mathcal{M}$ and W^2 . In this case, the v.e.v.s of the lowest components of these superfields, which are those of Eqs. (10) and (11) respectively, must be analytic functions of the complex parameters m_i , so that Eqs. (10) and (11) will imply the following m dependences:

$$\langle \bar{\phi}_i \phi_i \rangle \sim m_i^{-1} \prod_{j=1}^M (m_j)^{1/N} \quad (12)$$

$$\langle \lambda_L \lambda_L \rangle \sim \prod_{j=1}^M (m_j)^{1/N} \quad (13)$$

as indeed was found in Ref. 7). Equation (12) in particular, leads to very peculiar (not to say unphysical) results⁷⁾.

The argument of analyticity is not sacred, however. As soon as fields like $D^2(W^2)$ or $D^2(\bar{M}M)$ enter in $L_{\text{eff.}}$, analyticity in m no longer needs to hold. Such "higher derivatives" are expected to occur and do occur in certain soluble models¹⁰⁾.

In order to see in more general terms what would be required in order that either $\langle \bar{\phi}\phi \rangle$ or $\langle \lambda_L \lambda_L \rangle$ approach a constant for $m \rightarrow 0$ consider (in the one-flavour case for simplicity) the Ward-type identity:

$$|m| \frac{\partial}{\partial |m|} \langle Q \rangle = T \langle Q \int d^4x (-m \bar{\psi} \psi + \text{h.c.} - 2|m|^2 (\phi^* \phi + \bar{\phi}^* \bar{\phi})) \rangle \quad (14)$$

If $\langle Q \rangle \xrightarrow{m \rightarrow 0} \text{const.}$, it follows from (14) that, necessarily:

$$\langle Q \rangle^{-1} T \langle Q \int d^4x (-m \bar{\psi} \psi + \dots) \rangle \xrightarrow{m \rightarrow 0} 0 \quad (15)$$

On the other hand if $\langle Q \rangle \neq 0$ breaks $\hat{U}_A(1)$ (i.e., if the chirality $\chi_Q \neq 0$), the corresponding Nambu-Goldstone pseudoscalar has a degenerate scalar partner. Indeed, the Goldstone field $G = G_S + iG_P$ is the complex lowest component of a chiral superfield and, as such, can be expanded in the lowest components of the chiral superfields of the theory, giving:

$$\langle 0 | Q | G_S \rangle = i \langle 0 | Q | G_P \rangle = \chi_Q F_G^{-1} \langle Q \rangle \quad (16)$$

where F_G is the analogue of the usual pion decay constant F_π . It follows from (16) that G_S contributes to Eq. (15) with a term which is, potentially, finite as $|m| \rightarrow 0$. Hence we must require:

$$\chi_0 = \frac{\langle G_S | \int d^4x (m \bar{\Psi} \Psi + h.c. + \dots) | 0 \rangle}{\langle G_S | \int d^4x i (m \bar{\Psi} \Psi - h.c.) | 0 \rangle} \xrightarrow{m \rightarrow 0} 0 \quad (17)$$

where $F_G m_G^2$ has been expressed in terms of the divergence of the $\hat{U}_A(1)$ current.

From Eq. (17) we deduce that, in order to have operators like $\bar{\phi}\phi$ or $\lambda_L \lambda_L$ have finite v.e.v.s, G_S must have an unexpectedly small coupling to the scalar operator appearing in Eq. (14). Although this "decoupling" of G_S looks unlikely, we have not yet found an argument to reject it: this possibility can be shown to correspond, in the massless theory, to the decoupling of the energy momentum tensor from G_S :

$$\langle G_S | T_{\mu\nu} | 0 \rangle = (g_\mu g_\nu - g^2 g_{\mu\nu}) F_S, \quad F_S / F_T \rightarrow 0 \quad (18)$$

The above arguments suggest (in our opinion quite strongly) that dynamical chiral symmetry breaking is not trivially achieved in a SUSY vacuum. We shall now give some diagrammatic arguments which, besides confirming the above remark, offer a way to estimate the small mass behaviour of the various order parameters.

SUSY-QCD possesses effective interactions caused by the anomaly (via instantons, for instance). A typical vertex is shown in Fig. 1a and gives

$$\Gamma_{\text{eff}}^{\text{an.}} \simeq \Lambda^{4-3(N+M)} (\lambda_L \lambda_L)^N \det(\bar{\Psi}_R^i \Psi_L^j) + h.c. \quad (19)$$

There is also a perturbative vertex, also discussed in Ref. 7), which gives (Fig. 1b):

$$\Gamma_{\text{eff}}^{\text{pert.}} \simeq g^2 \Lambda^{-4} (\lambda_L \lambda_L) \bar{\Psi}_R^i \Psi_L^j (\bar{\phi}^i \phi^j)^* + h.c. \quad (20)$$

Using the two above vertices others can be constructed, e.g., for $M < N$:

$$\Gamma_{\text{eff}}^{\text{an.}} \simeq \Lambda^{4-(3N-M)} \det(\bar{\phi}^i \bar{\phi}^j) (\lambda_L \lambda_L)^{N-M} + h.c. \quad (21)$$

Let us start with the case $M = 1$. Because of Eq. (19) we get a bound^{*)}

^{*)} Throughout these diagrammatic arguments we shall assume that :

- i) the v.e.v.s of a product of operators is at least as big as the product of the v.e.v.s of its factors.
- ii) if there is a coupling Γ_{12} between two operators $\mathcal{O}_1, \mathcal{O}_2$, then $\langle \mathcal{O}_1 \rangle \geq \Gamma_{12} \langle \mathcal{O}_2 \rangle$.

$$\langle \lambda_L \lambda_L \rangle \lesssim m^{1/N} \quad (22)$$

Indeed, if $\langle \lambda_L \lambda_L \rangle = O(1)$, the vertex (19) would give $\langle \psi \psi \rangle \gtrsim O(1)$ in violation of SUSY. On the other hand, from the vertex (20) we get:

$$\langle \bar{\psi} \psi \rangle \simeq m^* > \langle \bar{\phi} \phi \rangle \cdot \langle \lambda_L \lambda_L \rangle \quad (23)$$

In order to have $\langle \bar{\phi} \phi \rangle = O(1)$, we would need $\langle \lambda \lambda \rangle = O(m)$. However, in the absence of matter (or equivalently, if we take the $N \rightarrow \infty$ limit before letting $m \rightarrow 0$), we need $\langle \lambda \lambda \rangle$ to be non-zero in order to reproduce and interpret the known value N for the index⁹⁾ [see also Refs. 6) and 7)]. Thus $\langle \lambda \lambda \rangle$ should change from a non-vanishing value to zero as a result of the switching on of matter loops, in spite of the fact that these are unimportant^{*)} at large N .

On the other hand, by assuming the bound (22) to be saturated, we are immediately consistent with Eq. (11), with large N behaviour and we get $\Delta = N$, the N vacua being given by the N th order roots of m .

The conclusion thus seems to be that $\langle \lambda \lambda \rangle$ and $\langle \bar{\phi} \phi \rangle$ must both vanish as $m \rightarrow 0$ [presumably as $m^{1/N}$ and $(m^*)^{1-1/N}$ respectively], with the result that chirality remains unbroken by such order parameters as a consequence of supersymmetry^{**) .}

The case of general M , which looks rather rich and non-trivial is at present under study. For $M = 2$, for instance, the most likely possibility seems to be:

$$\begin{aligned} \langle \bar{\psi}_i \psi_i \rangle &\sim m_i^* & \langle \lambda_L \lambda_L \rangle &\sim (m_1 m_2)^{1/N} \\ \langle \bar{\phi}_i \phi_i \rangle &\sim (m_i^*)^{1-1/N} (m_2)^{1/N}, & \text{same with } 1 \leftrightarrow 2 \end{aligned} \quad (24)$$

For the special case $N = M = 3$ the vertex of Fig. 1c implies

$$\langle \det (\bar{\phi}_i \phi_j) \rangle = O(1) \quad (25)$$

*) No Witten type¹¹⁾ enhancement of matter loops is expected here since the η -like particle has a mass of $O(\Lambda)$ and the true Goldstone particle is essentially decoupled from $\lambda \lambda$.

**) For $M > 1$ part of the chiral symmetry could still be broken by objects like $\langle \phi^* \phi \rangle$ and $\langle \bar{\phi}^* \bar{\phi} \rangle$.

which, since in this special case $\phi, \bar{\phi}$ carry no $\hat{U}_A(1)$ charge, could possibly imply:

$$\langle \varepsilon_{\alpha\beta\gamma} \varepsilon_{ijk} \phi_\alpha^i \phi_\beta^j \phi_\gamma^k \rangle = O(1) \quad (26)$$

This last condensate was recently considered by Taylor¹²⁾ in one possible low energy realization of SUSY-QCD with anomaly matching light composite fermions. Notice that the vector $U(1)$ symmetry is broken by (26). In general, it seems¹³⁾ that insisting on the $\hat{U}_A(1)$ symmetry does put strong constraints on the composites.

The picture emerging from all these arguments is one in which chirality is protected by a supersymmetric vacuum. As we said in the introduction, this result suggests that we should look seriously at supersymmetric preon dynamics for composite quarks and leptons, presumably along some modification of the schemes of Ref. 4).

A possible shortcoming of our scheme appears to be that we have forced ourselves to work in a supersymmetric vacuum and thus to have around degenerate scalar partners to our composite quarks and leptons. It is not difficult to see, however, that the addition of an extra scalar mass term, while softly breaking SUSY, preserves all the chiral symmetries discussed here and, at least perturbatively in the breaking, leaves the vacuum chiral invariant. To see this, note that $\langle Q = \lambda\lambda \text{ or } \bar{\phi}\phi \rangle$

$$\left. \frac{\partial}{\partial m_s^2} \langle Q \rangle \right|_{m_s = m_F = m} = \langle Q \int d^4x (\phi^* \phi + \bar{\phi}^* \bar{\phi}) \rangle \Big|_{\text{SUSY limit}} \xrightarrow{m \rightarrow 0} 0 \quad (27)$$

where we have observed that also operators like $Q\phi^*\phi$ would break chirality in the supersymmetric theory and are required to have vanishing v.e.v.s from the diagrammatic arguments given above.

This last observation could offer an elegant mechanism for removing unwanted scalars while keeping quarks and leptons light by virtue of unbroken chiral symmetries. The crucial remaining problem is that of constructing a phenomenologically appealing realization of the above general ideas.

ACKNOWLEDGEMENTS

Discussions with T.R. Taylor and S. Yankielowicz have been very important to the development of the ideas presented here.

I am also grateful to D. Amati, R. Barbieri, I. Bars, A. D'Adda, S. Ferrara, J.M. Gérard, H. Nilles, E. Rabinovici, G. Shore and D. Wyler for interesting criticism and/or encouragement.

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FIGURE CAPTIONS

- Fig. 1a Non-perturbative effective interaction due to the axial anomaly.
- Fig. 1b Perturbative process allowed in SUSY-QCD.
- Fig. 1c Another effective vertex obtained by combining (a) and (b).