EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN/LEP-TH/83-2

PROGRAM BBI: Bunched-Beam Instabilities
in High-Energy Storage Rings

M. Gygi-Hanney, A. Hofmann, K. Hübner, B. Zotter

Geneva, January 1983
Abstract

A detailed description of the computer program BBI (Bunched Beam Instabilities) is presented in this report. This program has been developed to aid in the design of circular electron or proton storage rings. It calculates limits of longitudinal and/or transverse stability and several related effects with particular emphasis on coherent bunch oscillations.
CONTENTS

1. Introduction  
2. Program structure  
3. Program input  
4. Basic parameters  
5. Bunch lengthening by potential well  
6. Effective impedance of a resonator  
7. Turbulent bunch lengthening  
8. Longitudinal instability  
9. Transverse instability  
10. Resistive wall effect  
11. Laslett tune shifts  
12. Intrabeam scattering  
13. Conclusions  
14. References
1. Introduction

The computer program described in this report was originally written for the calculation of instabilities in electron beams, hence its original name EBI\[1\]. It has been extended to include the calculation of instabilities of bunched proton beams. In addition to coherent effects, a number of beam-dynamical calculations have been included such as "Laslett" tune-shifts and intra-beam scattering which are of interest to the machine designer.

The calculation of the coherent instabilities is mainly based on F. Sacherer's work on both longitudinal\[2\] and transverse\[3\] bunch oscillations. For coupled bunch oscillations, the bunches are assumed to be identical and equally spaced, but estimates of the required spread in bunch population to decouple the bunches are calculated. For electron beams a Gaussian distribution of particles is assumed, and Hermitian modes are used. For proton beams, which usually have a parabolic distribution, the approximate sinusoidal modes are taken for simplicity. Changes of the eigenmodes due to interaction with the wall impedance as well as higher order radial modes are neglected. Transverse mode coupling for single bunches and plots for tune and frequency shifts have been incorporated in Version 2 of the program. Details of these modifications are described in BBI Newsletters 2 and 3. Longitudinal mode-coupling is included indirectly through the turbulent bunch-lengthening calculations discussed hereafter.

Before stability is calculated, the bunchlength and synchrotron frequency are determined by the program using several methods: First the "natural" values for negligible space-charge are calculated, followed by a pure "turbulent" and a combined potential-well and turbulent bunch-lengthening calculation. The user can select the desired value by the choice of a parameter.

The wall impedance is given to the program in the form of up to three longitudinal and/or transverse resonators, characterized each by a resonant frequency, a quality factor and a shunt or transverse-impedance. Usually the first longitudinal resonator is taken to be a broad-band impedance with unit quality factor, a resonant frequency around the cut-off frequency of the vacuum chamber, and a shunt impedance adjusted to give the correct low-frequency inductance*. The other resonators can be chosen from the fundamental or higher-harmonics of the RF cavities or other resonant structures in the ring. In addition, the resistivity of the vacuum-chamber wall has to be given if the (transverse) resistive wall effect is to be calculated (optional).

* only the first (longitudinal) resonator is used for the potential-well bunch lengthening calculations.
Often the resonator frequencies are not exactly known, except for the fundamental one which is usually kept constant to high accuracy. It is therefore possible to vary the frequency of the third resonator over a range of several revolution frequencies in a number of steps by setting a logical variable (FVAR=).T.).

Since the number of coupled bunch modes is equal to the number of bunches for each bunch-shape mode, the output could become very large if there are many bunches in the machine and all the modes were printed. Printing is therefore restricted to 8 coupled bunch modes per case, but the program also searches and prints the mode with the fastest growth rate (for each m). By selecting a logical variable (MINOUT=).T.) the output can be limited to these summary lines.

Unavoidably, in a program of this size there will be errors. At present, the program has been tested on several machines, and most of the subroutines have been updated to make the use of the program easier and the output clearer. Nevertheless, the authors encourage further testing and welcome any suggestion for improvements.

2. Program structure

The main program starts by setting a number of default values, and then reads the input data (in Namelist format). Next it calls the subroutine BAS (for converting the number of particles into current or vice-versa) and subroutine PRINT which prints all input data (except for transverse resonator data which are printed on top of the output). Afterwards, the program calls the various subroutines sequentially (some of which are optional) and goes back to read further input. The cycle is repeated until the energy on the input is found to be zero.

The program has been written for use on a CDC computer 7600, where it usually takes only a few seconds to run for not too many bunches. The space requirements of the program are small and no LCM space is necessary even for up to 6000 bunches in the storage ring (however, the time requirement has to be established).

At present, seven library routines are used which are available on tape or are easily replaced:

- RZERO (zero finding routine in BLSC3)
- CHERF (complex error function in ZNS)
- DATIMH (time and date in PRINT)
- UFILL, UZERO, UCOPY (array handling in main)

and GAUSS (integration in INTRABI)
The program has been converted for use on other machines in other laboratories, but so far has not been implemented on the CERN-IBM machine. Also it has recently been converted to FORTRAN 5.

A flow-diagram of the program BBI shows its major steps (the numbers refer to the section where the subroutine is discussed).
The constants used in the program are

\[
C = 2.997925 \times 10^8 \text{ m/s light-velocity} \quad (c)
\]

\[
E_{CH} = 1.6021917 \times 10^{-19} \text{ Cb unit charge} \quad (e)
\]

\[
E_0 = 0.511004 \times 10^6 \text{eV electron rest energy} \quad (E_0)
\]

\[
M_0 = 938.2 \times 10^6 \text{eV proton} \quad (\mu_0)
\]

\[
\Phi_{ISR/PHIS} = 0.0174532925 \text{ degree-to-radian conversion factor}
\]

3. Program input

Generally in mks units, except for energy in eV. We quote first the name used in the program, then the units, the description and finally the commonly used symbol.

a) Machine parameters

- \(E(\text{eV})\) - energy of synchronous particle
- \(R(\text{m})\) - average machine radius (circumference/2\(\pi\))
- \(R_{\text{ho}}(\text{m})\) - bending radius (\(p\))
- \(\text{TUNE}(-)\) - betatron tune (usually vertical) (\(v\) or \(Q\))

- \(\xi(-)\) - chromaticity \(\xi = \frac{\partial}{\partial \nu} \frac{\Omega}{\nu^2}\)
- \(\text{ALF}(-)\) - momentum compaction factor \(\frac{\alpha}{1/\gamma^2}\)
- \(\text{TAXU}(-)\) - horizontal damping time \(\tau_X\)
- \(\text{FRF}(\text{Hz})\) - RF frequency \(f_{RF}\)
- \(\text{WRF}(-)\) - RF voltage \(V_{RF}\)
- \(\Phi_{\text{ISRF}}(\text{deg})\) - synchronous phase-angle \(\Psi_S\)

b) Beam parameters

- \(N(-)\) - total number of particles \((N)\)
- \(I_0(\text{A})\) - average beam current \((I_0)\)
- \(K_B(-)\) - number of bunches \((k)\)
- \(E_X(\text{m})\) - horizontal emittance \((E_X = \sigma_X^2/B_X)\)
- \(E_Z(\text{m})\) - vertical \((E_Z = \sigma_Z^2/B_Z)\)
- \(\text{SIGS}(\text{m})\) - RMS bunch-length for electrons \((\sigma_X)\)
  half-width at half height for protons
- \(\text{DELE}(-)\) - relative RMS energy spread \((\sigma_E/E)\) for electrons
  relative momentus spread half-width and
  half-height \((\Delta p/p)\) for protons
- \(\text{SL}(-)\) - relative spread of synchrotron frequencies
- \(\text{ST}(-)\) - relative spread of betatron frequencies

c) Chamber parameters

- \(W(\text{m})\) - (full) chamber width \((w)\)
- \(H(\text{m})\) - (full) chamber height \((h)\)
- \(W_{\text{D}}(\text{m})\) - wall thickness \((d)\)
- \(Z_{\text{N}}(\Omega)\) - low-frequency wall-impedance \((Z/N)_0\)
- \(\text{RESIV}(\Omega \text{ m})\) - wall resistivity \((p_w)\)
d) Resonant impedances (up to 3 longitudinal and 3 transverse)

- FRS (H2) (long) resonant frequencies (f_r)
- QRS (-) (long) quality factors (Q_r > 1/2)
- ZRS (Ω) (long) shunt impedances (R_s)
- FRT (H2) (transv) resonant frequencies (f_t)
- QT (-) (transv) quality factors (Q_t > 1/2)
- ZT (Ω/m) (transv) resonant impedance (R_t)

e) Options

- ELEC = .T. electrons
  .F. protons (default)
- LONG = .T. longitudinal instabilities (default)
  .F. transverse
- LASL = .T. calculate "Laslett" tune shift
  .F. not calculated (default)
- INTRA = .T. calculate inter-beam scattering
  .F. not calculated (default)
- MINOUT = .T. minimum output (default)
  .F. full output (up to 8 coupled bunch modes for each m)
- NMIN (integer) > 0, <K_b-1 lowest coupled bunch mode (default 0)
- NPR (integer) > K_b/8 print every NPRth mode (default 1+[K_b-1]/8)
- FVAR = .F. no frequency variation (default)
  .T. varies FRS (3) in NS steps on either side of input value over a total range of RS x fo
- NS = integer number of steps on either side (default 4)
- RS = value of the total range (default KB)
- CH = centre harmonic
- RESIST = .T. calculates the transverse resistive wall effect
  .F. not calculated (default)
- NCASE = 0 σ = σ_o, f_s = f_s0 (natural values)
  1 σ = σ_1, f_s = f_s1 (turbulent bunch-lengthening)
  2 σ = σ_2, f_s = f_s1 (turbulent + potential well bunch-lengthening)
  3 σ = σ_data, f_s = f_s1 (input data)
- SUM = .T. complex frequency shifts of several sets of up to 3 resonant impedances are added up over consecutive runs
  .F. (default) independent calculation for each set of resonators.
4. Basic parameters

a) Before any calculation of bunch-lengthening, stability etc. the program computes a number of basic parameters which are derived mainly from the input data. In order to permit the input to be expressed either in (average) current or the total number of particles, the subroutine BAS, which gives both of these quantities, is called before the PRINT subroutine. In addition, BAS calculates (computer variables in brackets)

\[ \gamma = \frac{E}{E_0} \]  
beam energy factor \hspace{1cm} (GAM)

\[ \beta = \left(1-\gamma^{-2}\right)^{\frac{1}{2}} \]  
beam velocity factor \hspace{1cm} (BETA)

\[ f_0 = \frac{\beta c}{2\pi R} \]  
revolution frequency \hspace{1cm} (FO)

\[ h = \frac{f_{RF}}{f_0} \]  
harmonic number \hspace{1cm} (SH)

\[ \eta = \alpha^{-2} \]  
frequency slip factor \hspace{1cm} (AMETA)

\[ |\eta| = |\alpha^{-2}| \]  
absolute frequency slip factor \hspace{1cm} (ETA)

\[ I_0 = \frac{N_0}{\beta c e} \]  
(average) beam current \hspace{1cm} (IO)

\[ N = \frac{2\pi R I_0}{\beta c e} \]  
total number of particles \hspace{1cm} (N)

\[ N_b = N/k \]  
particles per bunch \hspace{1cm} (NB)

b) The subroutine SYN calculates the parameters for vanishing space-charge

\[ f_{SO} = f_{RF} \left( \frac{n R F \cos \phi_S}{2 \sqrt{2 \pi \hbar E}} \right)^{\frac{1}{2}} \]  
synchrotron frequency \hspace{1cm} (FSO)

\[ v_{SO} = \frac{f_{SO}}{f_0} \]  
synchrotron tune \hspace{1cm} (QSO)

\[ \sigma_{SO} = \frac{m c \sigma_E}{\sqrt{2 \pi f_{SO} \beta}} \]  
(natural) bunch length \hspace{1cm} (SIGSO)

\[ I_{po} = \frac{\sqrt{2 \pi R I_0}}{k \sigma_{SO}} \]  
peak current for electrons \hspace{1cm} (IPT)

\[ I_{po} = \frac{\pi^2 R I_0}{3 k \sigma_{SO}} \]  
peak current for protons \hspace{1cm} (IPT)

\[ \sigma_{SO} = \frac{m c \sigma_E}{\sqrt{2 \pi f_{SO} \beta}} \]  
(gaussian)

\[ \sigma_{SO} = \frac{m c \sigma_E}{\sqrt{2 \pi f_{SO} \beta}} \]  
(sinusoidal)

\[ \tau = \frac{2 \pi}{\sigma} \]  
bunch-length in secs \hspace{1cm} (TAU)

\[ b = (W+H)/\pi \]  
equivalent chamber radius \hspace{1cm} (B)

\[ f_w = \frac{w}{\xi w_0} \]  
frequency where skindepth = wall thickness \hspace{1cm} (FW)

\[ f_n = \frac{\xi w_0}{\pi} \]  
chromatic frequency \hspace{1cm} (FXI)

\[ X = \xi w_0 / \eta \]  
phase-shift across bunch \hspace{1cm} (CHI)

\[ h_r = \frac{f_r}{f_0} \]  
harmonic-number of resonant frequency \hspace{1cm} (HFR)

\[ (Z/n) = \frac{Z f_0}{h_r Q} \]  
longitudinal low-frequency impedance \hspace{1cm} (ZNO)

A number of other parameters are calculated in the other subroutines, and will be discussed there.
5. Bunch-lengthening by potential-well

The calculations are based on a generalization of the expressions given in ref.4, for the incoherent synchrotron frequency in the presence of a purely inductive wall with $Z/N = \text{constant}$ (we use capital $N$ in order to avoid confusion with the coupled-bunch mode-number $n$).

$$\omega_s^2 = \omega_{s0}^2 \left[ 1 + A \left( \frac{J_0}{x} \right)^3 \Im \left( \frac{Z}{\sqrt{N}} \right) \right]$$

(4.1)

where $A = \frac{3J_0}{\pi^2 h k v R \cos \theta}$

(4.2)

For $\gamma > \gamma_T$, $\cos \theta \leq 0$ and $A \leq 0$, hence $\omega_s < \omega_{s0}$ for $\Im(Z/N) > 0$.

For electrons, the factor 3 is replaced by $4/\pi$. Radiation damping yields the relation

$$\frac{x}{x_0} = \frac{\omega_{s0}}{\omega_s} \quad \text{or} \quad xy = 1$$

(4.3)

while for protons phase-space-area conservation requires

$$\left( \frac{x}{x_0} \right)^2 = \frac{\omega_{s0}}{\omega_s} \quad \text{or} \quad x^2 y = 1$$

(4.4)

where we put

$$\frac{x}{x_0} = \frac{\sigma}{\sigma_0} = x \{ \text{bunch-lengthening factor} \} \quad \frac{\omega_s}{\omega_{s0}} = \frac{f_s}{f_{s0}} = y \{ \text{Synch. frequency factor} \}$$

(4.5)

Elimination of $y$ yields the equations

for electrons

$$x^3 - x + A \Im \left( \frac{Z}{N} \right) = 0$$

(4.6)

for protons

$$x^4 - 1 + A x \Im \left( \frac{Z}{N} \right) = 0$$

For an inductive impedance, $Z/N$ is constant and the equations are resp. cubic or quartic. For the general case, we take the effective impedance of the single bunch $(k=1, n=0)$ for mode-number $m=1$, but evaluated at the revolution frequency harmonics $p\omega_0$ and with $\sigma_{\text{eff}} = \sigma/\sqrt{2}$

$$\left( \frac{Z}{N} \right)_{m=1} = \sum_{p=1}^{N_{\text{eff}}} \frac{Z(p\omega_0)}{h_p(p\omega_0)}$$

(4.7)

Usually a large number of narrow-band resonances are replaced by a "broad-band" resonator impedance, to which is added the space charge term

$$\Im \left( \frac{Z}{N \, \text{Sc}} \right) = - \frac{Z_0}{2B y_T^2} \left( 1 + 2 \ln \frac{N}{2a} \right)$$

*Eqs. (16a) and (17a) of ref. (5) are incorrect and should be replaced by Eqs. (4.4) and (4.6) of this report.*
where $H$ is the full chamber height and $BR$ the full (average) beam height given by

$$2a = 2/2\sigma_z = 2\sqrt{\frac{R}{v}} E_z$$

Since the effective impedance is a function of the bunch length, the algebraic equations for the bunch-lengthening factor $x$ become transcendental. In order to find their solutions, we call the zero-finding library routine RZERO between 0.1 and 1 for negative Im$(Z/N)$, and between 1 and 10 if it is positive. If no zero is found, we set $a_1 = a_0$ and $f_{s1} = f_{s0}$, otherwise we have $a_1 \times a_0$, $f_{s1} f_{s0}/x$ (resp. $f_{s0}/x^2$ for protons). The peak current is $I_p = I_{p0}a_0/a_1$ in either case. These values as well as the value of the effective impedance are printed in the output.

**Subroutine BLSC3**

$$Z_0 = \eta_0 C = 4\pi 10^{-7} c $$ free-space impedance  
(ZO)

$$g = 1 + 2\pi n(H/2a)$$ g-factor  
(G0)

$$(X/N)_{SC} = gZ_0(2\pi)^2$$ space-charge capacitance  
(XNK)

$$\lambda_0 = 2\sqrt{2a_{s0}}$$ natural bunch-length at bottom  
(LO)

$$A = \left(\frac{2\pi R}{\lambda_0}\right)^3 \left(\frac{3I_0}{\pi^2 kN_{RF}\cos \phi_s}\right)$$ space-charge factor for protons (multiplied by $4/3\pi$ for electrons)  
(CCC)

$$I_{p1} = I_{p0}/a_1$$ peak current  
(IP1)

**Function CUBIC (x)**

We can rewrite the "cubic" for electrons and the "quartic" for protons in the common form

$$CUBIC(x) = x^3 - x^2 + A \cdot \text{Im}(Z) = 0$$

with $m=1$ for electrons and $m=4$ for protons. From its solution we obtain

$$\sigma = x \cdot a_0$$ bunch-length  
(SIGZN)

$$f_s = \left[\frac{f_{s0}}{x^2}\right]$$ synchrotron frequency for $\left[\frac{e}{p}\right]$  
(FSZN)

$$(X/N)_{eff} = \text{Im} ZNS (1,0)$$ (imaginary part of) effective impedance for mode $m=1$, $n=0$  
(See next section for ZNS)  
(ZNBX)

$$(X/N)_{tot} = (X/N)_{SC} - (X/N)_{eff}, \text{ (neg.) total reactance}$$  
(XN)
6. Effective impedance of a resonator

a) The effective impedance for longitudinal oscillations is defined by

\[
(Z_m)^{\text{eff}} = \sum_{p=\infty}^{\infty} \frac{Z(\omega_p)}{p} \frac{h_m(\omega_p)}{\sum_{p=\infty}^{\infty} h_m(\omega_p)}
\]

where \( \omega_p = \omega_0 (kp + n + m v_s) \) are the longitudinal oscillation frequencies. For the transverse case, we have

\[
(Z_\perp)^{\text{eff}} = \sum_{p=\infty}^{\infty} Z_\perp(\omega_p) h_m(\omega_p - \omega_\xi) / \sum_{p=\infty}^{\infty} h_m(\omega_p - \omega_\xi)
\]

where \( \omega_p = \omega_0 (kp + n + v + m v_s) \) are the transverse oscillation frequencies and \( \omega_\xi = \xi \omega_0 / n \) is the "chromatic frequency".

Here \(- \infty < p < \infty\) is the revolution-frequency harmonic mode-number \( m > 0 \) is the bunch-shape mode-number (\( m=0 \) stationary, \( m=1 \) dipole, \( m=2 \) quadrupole etc.) \( 1 \leq m \leq \text{max longitudinal}, 0 < m \leq \text{max transverse} \) (the program permits up to 4 values of \( m \), with \( \text{max}=2 \) as default). \( 0 < n < k \) is the coupled-bunch mode-number

\[ \Delta \phi = \frac{2 \pi}{k} n \]

is the phase-shift between bunches.

The spectral power density of the \( m \)-th mode for sinusoidal modes is

\[
h_m(\omega) = (m+1)^2 \frac{1 + (-)^m \cos y}{y^2 + (m+1)^2} ; \ y = \frac{\omega}{\pi}
\]

while for Hermitian modes it becomes

\[
h_m(\omega) = \frac{1}{\Gamma(m+\frac{1}{2})} y^{2m} \exp(-y^2) ; \ y = \frac{\omega}{\beta c}
\]

b) The complex function \( Z_{NS}(m,n) \) uses analytic summation formulae which have been derived for sinusoidal\(^5\) and Hermitian modes\(^6\). For a resonator with impedance \( R_s \), quality factor \( Q_r \), and resonant frequency \( \omega_r = 2\pi f_r \), we write the argument of the impedance as

\[
\omega = \frac{\omega_r}{\omega_p} \quad \text{with} \quad b = \frac{\omega_r}{\omega_p}, a = \frac{1}{k} \left[ \frac{n+m v_s}{\nu} \right] \quad \text{(longitudinal)}
\]

\[
\text{while the argument of the power-spectrum is written as} \quad y = \frac{\omega_r}{\pi} = \frac{p+\varepsilon}{d} \quad \text{with} \quad c = \left[ a - \frac{\omega_\xi}{\omega_p} \right] \quad \text{(transverse)}
\]
For protons, d depends on the "total" bunch-length \( t = \frac{L}{v_c} \). For a parabolic distribution, we take \( L = 2\sqrt{2}a \) and \( d = \frac{\pi R}{2\sqrt{2}k\sigma} \), when we designate with \( \sigma \) the half-length at half-height.

For electrons with a Gaussian distribution, \( \sigma \) designates the standard deviation, and \( d \) is simply \( R/k\sigma \). We also have to define an (equivalent) total bunch-length. The most logical procedure consists of requiring the same bunching factor \( B = I_{av}/I_{peak} \) for both distributions. For a parabolic distribution, \( B = kL/3\pi R \), while for a Gaussian \( B = k\sigma^2/k_2 \). By taking \( L_G = \frac{3\sigma}{\sqrt{\pi / 2}} \) the two expressions agree. Since \( 3\sqrt{\pi / 2} = 3.71 \), this agrees approximately with the standard practice of using a bunch-length of \( \pm 2\sigma \) which contains \( 95\% \) of the beam.

c. The summation of the series uses the auxiliary quantities such as

\[
\begin{align*}
  u &= b \left(1 - \frac{1}{4Q_r^2}\right) \frac{1}{2} \\
  v &= b/2Q_r \\
  (U) & (V)
\end{align*}
\]

(the first expression limits the input to \( Q_r > 1/2 \), but later it is even limited to \( Q_r > 1/\sqrt{2} \). From these we form for \( i = 1,2 \)

\[
\begin{align*}
  p_i &= -a \pm u + jv \\
  q_i &= -c \pm (m+i)d \\
  r_i &= (p_i + c)/d \\
  B_i &= \cot \theta \cdot p_i \\
  (P) & (Q) & (R) & (BB)
\end{align*}
\]

Since \( p \) is a complex quantity, the cotangent must be calculated as the ratio of complex sine to cosine. For a too large imaginary part of the argument \( \text{Im}(xp_i) = \nu = \nu b/2Q_r \), the evaluation of exponentials in these routines leads to overflow. For \( z = x + iy \)

\[
B_i = \cot Z = \frac{\cos x \cdot \text{Coshy} - j\sin x \cdot \text{Sinh}y}{\sin x \cdot \text{Coshy} + j\cos x \cdot \text{Sinh}y}
\]

For \( y > 1 \), \( \text{Coshy} = \text{Sinh}y = e^{y/2} \), we find \( \cot Z = -j \)

We take \( B_i = -j \) for \( \text{Im}(p_i) > \text{EXP\_MAX} \)

The maximum number for the exponent (\( \text{EXP\_MAX} \)) in the CDC7600 is about 740, but we usually take only half that value (370) which gives more than sufficient accuracy (\( e^{370} = 10^{160} \)).

A common factor required for the effective impedance is \( F_1 = \frac{bZ_r}{2kQ_r} \) \( \text{(FAC)} \).
d) Sinusoidal modes

The effective impedance of a resonator can be summed exactly for these modes\(^5\) by the expression

\[
\left( \frac{Z}{N} \right)_{\text{eff}} = j F_1 (F_2 S_2 - S_1)
\]

where

\[
F_2 = \frac{2}{\pi} \frac{d^3}{U} (m+1)^2
\]

\[
S_1 = \frac{1}{p_{11} - p_{21}} + \frac{1}{p_{12} - p_{22}} (p_{ij} - p_{ij})
\]

\[
S_2 = \frac{T_1}{Z_{12}} - \frac{T_2}{Z_{22}} (Z_{12} = p_{11}^2 - p_{12}^2)
\]

and

\[
T_i = \left[ 1 + (-)^m \cos n_i \right] B_i + (-)^m \sin n_i
\]

Again we run into overflow problems if \(\text{Im}(\pi n_i) = \frac{dV}{d}\) is too large. Since \(d\) is always larger than 1, this implies that \(\text{Im}(\pi n_i) = \pi V\) is even larger, and thus \(B_i = -j\). Then

\[
T = -j \left[ 1 + (-)^m e^{jn_i} \right] = -j
\]

has to be used for \(\text{Im}(\pi n_i) > \text{EXPMAX}\).

e) Hermitian modes

The effective impedance for a resonator can be summed\(^6\) to a very good approximation\(^*\) by the expression

\[
\left( \frac{Z}{N} \right)_{\text{eff}} = \frac{j F_1}{\text{udr}(m+1)} (S_2 - S_1)
\]

where for \(m=0\): \(S_1 = \pi \left( j \left[ W(r_i) - E_i \right] - E_i B_i \right)\)

Here \(W(Z)\) is the complex error function (library routine CHERF), and

\[
E_i = \begin{cases} 
\exp(-r_i^2) & \text{for } |\text{Re}(r_i)| < \text{EXPMAX} (=370) \\
0 & \text{for } \text{Re}(r_i) > \text{EXPMAX} \\
\text{error message for } \text{Re}(r_i) < -\text{EXPMAX} 
\end{cases}
\]

\(^*\) The approximation consists of replacing the series for the Theta-function by its first term, which corresponds to neglecting terms of the order of \(\exp(-\pi^2 d^2)\). For \(d > 1\), the approximation is thus very good (e.g. for \(d=10\), the second term is of the order \(10^{-400}\)).
For \( m > 1 \), we have to multiply \( S_1 \) by \( r_1^{2m} \) and add the sum (SFN)

\[
F_m = \sum_{l=0}^{m-1} \frac{1}{2} \Gamma \left( \frac{l+\frac{1}{2}}{2} \right) \Gamma \left( \frac{2m-2l-1}{2} \right) r_i^2
\]

where

\[
\Gamma \left( \frac{l+\frac{1}{2}}{2} \right) = \left( \frac{l-\frac{1}{2}}{2} \right) \Gamma \left( \frac{l-\frac{1}{2}}{2} \right)
\]

and

\[
\Gamma \left( \frac{1}{2} \right) = \sqrt{\pi}
\]

is used to evaluate the gamma-function.

7. Turbulent bunch-lengthening

Since no fully satisfactory theory exists, several complementary methods are used to find the bunchlength, width and synchrotron frequency. When NCASE = 1, the results of subroutines BOUS and BLTU are compared and only the one with the smaller bunchlength and width is printed (the selection is done in subroutine SIGFS).

a) Subroutine BOUS uses a local coasting beam criterion where the average is replaced by the peak current (Boussard criterion). This is usually good for bunches which are long compared to the wavelength of oscillation. The threshold impedance is given by

\[
\left( \frac{Z}{N} \right)_{th} = 4F \frac{E_0}{\varepsilon} \frac{|\eta| \beta^2 \gamma}{I_p} \left( \frac{\Delta p}{p} \right)^2 \frac{N}{HH}
\]

where \( F \) is a formfactor (one for parabolic, 1.063 for Gaussian distribution). For a Gaussian, we use RMS energy deviation rather than half-height of momentum, so we multiply \( F \) with the factor \( \sqrt{2 \ln 2} / \beta^2 \) to obtain \( 4F=5.896/\beta^4 \).

If the threshold impedance is larger than the low-frequency impedance \( Z_N \), there is no turbulent bunchlengthening and the threshold current \( I_{th}=I_0(Z/N)th / Z_N \) is printed. In the opposite case, \( (Z/N)th < Z_N \), the bunchlengthening factor \( x = \sigma_3 / \sigma_0 \) is given by

\[
x = \left( \frac{I_0}{I_{th}} \right)^{\frac{1}{3}} > 1
\]

The momentum or energy spread increases by the same amount

\[
\delta E_3 = x \delta E_0
\]
b) Subroutine BLTU is based on the impedance-model of SPEAR with a power-law \( Z(\omega) = Z \omega^{-\alpha}, (\omega \gg \omega_1) \) assumed to be valid for shorter bunches which sample higher frequencies. A "critical frequency" \( \omega_c = c/\alpha \) is chosen to obtain the equilibrium bunchlength\(^3\)

\[
x_2 = \frac{\sigma_2}{\sigma_0} = \left( \frac{c}{\omega_f} \right) \left[ \frac{\eta I_0 R^3}{k v_{s0}^2 E/e} \left( \frac{\omega_c R}{Z/N} \right) \right]^{\frac{1}{2-\alpha}}
\]

If \( x_2 < 1 \), there is no turbulent bunch-lengthening and the natural values are taken for \( \sigma, E \) and \( I_p \). However, if \( x_2 > 1 \), it is compared with \( X_3 \) and the smaller bunchlength and energy-spread (and larger peak-current) are printed.

c) entry BLSC33: this uses subroutine BLSC3 (with flag TURB = 1) to combine the effects of potential-well and turbulent bunch-lengthening. The values obtained are kept for further calculation if one chooses NCASE=2.

The routine solves essentially the transcendental equation

\[
x^3 + A \left( \text{Im} \left( \frac{Z}{N} \right)_{\text{eff}} + \left| \frac{Z}{N} \right|_{\text{crit}} \right) = 0
\]

For protons

\[
A = \frac{24 \pi R^3 I_0}{\kappa_n k_b V_{RF} \cos \phi_s}
\]

and it is multiplied by \( 4/3\pi \) for electrons. As for pure potential-well calculations, the effective impedance is evaluated for a broad-band resonator. The impedance and the power-spectrum of the dipole mode (with \( \sigma_{0e} = \sigma/\nu_2 \)) are evaluated at revolution frequency harmonics \( p \nu_o \). For the "critical impedance" we take the broad-band resonator impedance evaluated at the "critical frequency" \( \omega_c \)

\[
\omega_c = \sqrt{\frac{c}{\sigma}}, \text{ i.e. with } p = \omega_c \nu_0
\]

\[
\frac{Z}{N}_{\text{crit}} = \frac{\omega_0 R S}{\omega_c Q_s} \left[ \frac{1}{p^4} \left( \frac{2-1/q_s^2}{p^2+1} \right)^2 \right] V_2
\]

For long (electron) bunches, these expressions have been found to be in agreement with measurements.

* In order to avoid numerical difficulties in solving the transcendental equation for \( x \), the "critical" frequency is multiplied by \( \sqrt{2} \).
8. Longitudinal stability

a) If the logical variable LONG = .T. (default), the program will calculate the stability of single-bunch and coupled-bunch oscillations in the presence of up to three resonators, for which the resonant frequencies \( f_r \), quality factors \( Q_r \), and shunt-impedances \( R_s \) are specified in the input.

The first resonator should always be the "broad-band" resonance since it is also used for potential-well bunch-lengthening. For this one usually takes a quality factor of unity, a resonant frequency near the pipe cut-off, and a shunt impedance chosen to give the correct low-frequency inductance, which is also given in the input as parameter ZN which is assumed to be known from measurements. We thus have to take

\[
R_s = \frac{f_r}{f_0} Q_r ZN \quad \text{[ZRS(1)]}
\]

The other two resonators can be chosen from the fundamental or higher modes of the RF or incidental cavities in the ring. However, one should be careful when using the fundamental RF cavity mode, since usually there exists one or more feed-back loops which reduce the impedance seen by the beam. By setting the logical variable FYVAR = .T. (default = .F.), the resonant frequency can be varied. The range of variation is \( R_S = f_0 \) (default \( R_S = k \)) in NS (default 4) steps on either side (hence 2\( N_S+1 \) total). The centre frequency of the variation can be chosen by prescribing a value CH, otherwise it is set at the first sideband of the nearest bunch-frequency harmonic \( (n \times f_S) \) which is probably the frequency with the fastest growthrate.

b) The (complex) coherent frequency shift is then calculated for several bunch-shape mode numbers 1 to MAX (default 2, maximum 4) and for each coupled bunch mode-number \( G \times n \times k \). It is only printed for up to 8 values of \( n \), which are either evenly spaced between 0 and \( k \), or can be chosen by prescribing the lowest desired value of \( n \) (NMIN) and the step-size (NPR). The frequency shift is calculated by

\[
\Delta \omega_{m,n} = \sum_{m=1}^{m_{max}} A \left( \frac{\omega_{so}}{\omega_s} \right)^2 \left( \frac{Z}{N} \right)^{m,n}
\]

where

\[
A = \frac{I_0}{3 \hbar V_{RF} \cos \phi_s} \left( \frac{2 \pi R}{L} \right)^2
\]

and the effective impedance is calculated with the subroutine function \( ZNS(m,n) \) described above.
The program prints $\Delta \nu_{mn} = \text{Re} \omega_{mn}/\omega_0$ and $1/\tau_{mn}$ for each of the three resonators as well as their sums. If $\text{Im} \omega_{mn}$ is positive, it prints "S" (stable). If it is negative, it is compared with the radiation damping rate (input), and prints "D" (damped) if it is smaller. If it is larger, we compare the absolute value of the frequency shift with the synchrotron frequency spread. The relative spread $SL$ is either given as input (e.g. for Landau cavities), or calculated from the nonlinearity of the RF potential.

$$\frac{S}{\omega_S} = \frac{\pi^2}{16} \frac{1+\frac{2}{1-\Gamma^2}}{\left(\frac{\hbar L}{2\pi R}\right)^2} \quad [\text{SL}]$$

where $\Gamma = \sin \gamma_S$.

If $|\Delta \omega_{mn}| < mS/(m+1)$, we print "L" (Landau damped), otherwise "U" (unstable). The program searches for the unstable mode with the fastest growth rate for each value of $m$, and prints it in a summary line. If none of the modes is unstable it prints a corresponding message.

9. Transverse Stability

a) If the logical variable LONG = .F., the program calculates the stability of transverse oscillations in the presence of up to 3 resonators. Again the resonant frequencies, quality factors, and transverse resonant impedances are given in the input. Usually the first resonator is again broad-band with $Q_1 = 1$, $f_1$ at the chamber cut-off, and the resonant impedance can be found from the longitudinal one with the relation

$$Z_1 = \frac{2R_1}{(H/2)^2} \left( \frac{Z_0}{N} \right)$$

which is strictly true only for smooth chamber walls, and tends to overestimate $Z_1$ for cavities.

The other two resonators are usually chosen from (higher) modes of the RF or incidental cavities. Again the resonant frequency of the third resonator can be varied as described in the last section.

b) The (complex) frequency shift for the coupled bunch modes is found from

$$\Delta \nu_{mn} = j \frac{A_1}{m+1} \left( \frac{f_1}{fo} \right) (Z_1)^{\text{eff}}$$

..//..
where

\[ a_{l} = \frac{I_{0}R}{2Qe\epsilon k r} \]

and \((Z_{s})_{\text{eff}}\) was defined in section 5.

Here we had to include the factor \(f_{l}/f_0\) because we evaluated the effective impedance with the function \(Z_{NS}\) which was originally written for the longitudinal case (in the program, we multiply the transverse impedance with \(f_{l}/f_0\), and include the revolution frequency in the factor \(a_{l}\)).

Again, the program prints only up to 8 coupled-bunch modes for each \(m\) (which start at 0 for the transverse case), as well as the one with the fastest growth-rate. The test for stability is done in the same way as in the longitudinal case, except that the relevant spread for Landau damping is now the betatron frequency spread \(\Delta T\) which can be read in (as relative spread) but is not calculated inside the program.

10. Resistive wall effect

a) By setting the logical variable \(\text{RESIST} = .T.\) (default \(\text{F.}\)), the transverse resistive wall effect is calculated. The longitudinal resistive wall effect is usually negligible since \(v_s < 1\) and the longitudinal chromaticity is very small.

When the skin-depth is smaller than the wall-thickness, the transverse resistive impedance is given by

\[ Z_{l}(\omega) = (\text{sgn} \omega + j) \frac{R}{(H/2)^3} \frac{2p}{\epsilon_0 |\omega|} \]

When the skin-depth is larger, the impedance will depend on the medium outside the vacuum-chamber, which is usually not specified. Since this happens if at all only for the spectral lines with the lowest frequency, we use the above expression in any case but print a warning message when the skin-depth is larger than the wall thickness.

b) Since the impedance as function of frequency now contains a square-root we cannot sum the infinite series analytically. In order to avoid time-consuming summations, the sum is approximated by an integral ("single-turn effect") plus the contribution of the spectral line with the smallest (absolute) frequency ("multi-turn effect"). For this we rewrite the expression for the complex frequency shift in the form
\[ \Delta \omega_{mn} = \frac{j A_\perp}{m+1} \sum_{m+1} Z_{\perp}(\omega_p) h_m(\omega_p - \omega_c) \]

where

\[ A_\perp = B A_\perp = \frac{\beta I_0 c}{4 \pi v E/c} \]

and

\[ B = k_f \tau \]

is the "bunching factor".

We replace the sum by an integral and the term with the lowest frequency \( \omega_{po} \), which is found from

\[ \omega_{po} = \omega_0 \left( \frac{n+v+m v_s}{k} + \frac{1}{2} \right) \]

where the square bracket stands for the integer part.

\[ \omega_{mn} = \frac{j A_\perp}{m+1} \left[ \frac{1}{8 \omega_0} \int_{\infty}^{\infty} \frac{Z_{\perp}(\omega) h_m(\omega - \omega_c) d\omega}{h_m(\omega_p - \omega_c)} + Z_{\perp}(\omega_{po}) \frac{h_m(\omega_{po} - \omega_c)}{8 \omega h_m(\omega_p - \omega_c)} \right] \]

The first term is the "single-turn" contribution and is independent of the coupled-bunch mode-number, while the second term is the "multi-turn" contribution which depends on \( n \) through \( \omega_{po} \).

c) The spectral density function \( h_m(\omega) \) may be normalized such that

\[ \sum_{m} h_m(\omega_p - \omega_c) = \sum_{m} h_m(\omega_p) = \frac{\pi^2}{4 B} \]

and then the multi-turn contribution can be written

\[ Z_{\perp, eff} = Z_{\perp}(\omega_{po}) F_m(\chi - \omega_{po}) \]

where \( \chi = \omega_c \) is the chromatic phase shift across the bunch, and

\[ F_m = \frac{4}{\pi^2} h_m(\omega_c) \]

is a form-factor of order unity which is evaluated in the subroutine function FMD.

d) Since \( Z_{\perp}(\omega) = (\text{sgn} \omega + j) \sqrt{\frac{\omega}{|\omega|}} \text{Re} Z_{\perp}(\omega) \), we can express the single-turn contribution as

\[ Z_{\perp, eff} = \frac{4 \text{Re} Z_{\perp}(\omega)}{m^2 k v \omega_0} \int_{\infty}^{\infty} \frac{\text{sgn} \omega + j}{\sqrt{|\omega|}} h_m(\omega - \omega_c) d\omega \]
Changing the integration variable to \( y = \frac{\omega x}{\pi} \) we get with \( H_m(y) = h_m(\omega) \)

\[
Z_{\perp}^{S.T.} = \sqrt{\frac{2\pi}{K_B}} \Re Z_{\perp}(\omega_o) F_m(\pi)
\]

where

\[
F_m(\pi) = \frac{2\pi}{\pi^{5/2}} \int_{-\infty}^{\infty} \frac{\text{sgn} y + i}{\sqrt{|y|}} H_m(y - \frac{\pi}{\pi}) dy
\]

is a complex form-factor. By splitting the integral it can be written

\[
\text{Re} F_m(\pi) = 2(G_1 - G_2)/(\pi^{5/2})
\]

with

\[
G_1,2 = \int_{0}^{\infty} \frac{dy}{y} H_m(y \pm \frac{\pi}{\pi})
\]

This form-factor is evaluated in the subroutine function FM.

e) For sinusoidal modes, the spectral density

\[
H_m(y) = (m+1)^2 \frac{1+(-)^m \cos y}{[y^2 - (m+1)^2]^2}
\]

fulfills the normalization condition directly\(^5\). For Hermitian modes, we have

\[
h_m(\omega) = C_m(\omega)^4 e^{-(\omega t)^2}
\]

and we can determine \( C_m \) such that \( \sum h_m(\omega_p) = \frac{\pi^2}{48} \) where \( B = k_0 \alpha \).

However, we first have to define what we mean by the "full bunch length" for Gaussian bunches, and we use the same reasoning as in section 5: For \( L = 3\sqrt{m/2} \), the ratio of average to peak-current is given by the same expression for the Gaussian beam and the parabolic distribution. We then get

\[
C_m = \frac{\pi^3}{2^7 (m+1)^2}
\]

and

\[
H_m(y) = C_m y^2 e^{-y^2}
\]

The expressions for \( H_m(y+\pi/\pi)/\pi \) are needed for integration by the subroutine FM, and are programmed as function FMP (Sinusoidal modes) for protons and FME (Hermitian modes) for electrons.

f) The sum of the single-turn and multi-turn contributions yields the complex frequency shift for the coupled bunch modes. Again, we print only up to 8 selected modes for each \( m \), but print the mode with the fastest growth-rate. By adding up over all coupled-bunch mode-numbers and dividing by the bunch-number \( k \) we also get the single-bunch effect, which is printed above the multi-bunch effect.
Finally the frequency shifts of the 8 selected coupled bunch-modes for the transverse resonant impedances are added to those for the transverse resistive wall effect and printed in the last column of the multi-bunch output.

11. Laslett tune-shifts

a) If the logical variable LASL=.T. (default .F.), the subroutine DQLAS is called which calculates coherent and incoherent tune-shifts\textsuperscript{12} and decoupling parameters\textsuperscript{13}. It distinguishes between "local" contributions which are independent of the bunching factor B, and "DC" contributions which are proportional to it. It further calculates the coherent tune-shift both for penetrating ($\Delta Q_{C2}$) and non-penetrating ($\Delta Q_{C1}$) AC magnetic fields. For guidance of the user, it calculates the skin-depth for the transverse mode with the lowest frequency, and compares it to the wall-thickness.

b) The incoherent tune-shift is calculated from

$$\Delta Q_i = -\frac{\text{NRr}_O F_i}{\pi (a+b)b\beta^3 y^3 B v}$$

where

$$F_i = F_{loc} + F_{DC} \quad (a=\sqrt[4]{2}\sigma_x, \quad b=\sqrt[4]{2}\sigma_y)$$

with

$$F_{loc} = 1 + \frac{b(a+b)}{(H/2)^2} \epsilon_1$$

$$F_{DC} = B \frac{b(a+b)}{(H/2)^2} \beta^2 y^2 \left[ \epsilon_1 + \frac{\epsilon_2}{(V/H)^2} \right]$$

The distance V between pole-pieces is assumed to be 20% larger than the chamber height H.

$$\Delta Q_{i\text{loc}} = \Delta Q_i \frac{F_{loc}}{F_i}$$

c) The coherent tune shift is calculated from the expression

$$Q_C = -\frac{\text{NRr}_0 F_C}{\pi \beta^2 y^3 B (H/2)^2}$$

where

$$F_C = F_{loc} + F_{DC}$$

$$F_{loc} = \xi_1$$

$$F_{DC} = B \beta^2 y^2 \left\{ \xi_1 + \frac{\epsilon_2}{(V/H)^2} \right\}$$

non-penetrating

$$\left\{ \xi_1 + \frac{\epsilon_2}{(V/H)^2} \right\}$$

penetrating fields
and \( \Delta Q_{C10c} = \Delta Q_c \frac{F_{loc}}{F_c} \)

d) The "image coefficients" \( \epsilon_{i2} \) and \( \xi_{i2} \) are calculated by using approximations to the expressions for elliptic chambers\(^{16}\), which are valid for \( H/W < 0.7 \)

\[
\begin{align*}
\epsilon_1 &= -0.156 \left( \frac{H}{W} \right)^2 + 0.21 \\
\epsilon_2 &= 0.41 \left( \frac{W}{R} \right) \\
\xi_1 &= -0.10 \left( \frac{H}{W} \right)^2 + 0.617 \\
\xi_2 &= 0.617 \left( \frac{W}{R} \right)
\end{align*}
\]

The skindepth for the transverse mode with the lowest frequency is found from

\[
\delta \sim 500 \left( \frac{\rho_w}{f_0|n-q|} \right)^{1/2}
\]

where \( q \) is the non-integer part of the tune, and \( n=0 \) for \( q < 1/2 \), \( n=1 \) for \( q > 1/2 \). \( \rho_w[\Omega m] \) is the chamber resistivity given in the input.

c) There are two alternative expressions for the decoupling parameter:

\[
\begin{align*}
D_1 &= 2\pi f_0 \Delta Q_{C10c} \\
D_2 &= 2\pi f_0 \left( Q_{C10c} - 1/2\Delta Q_1 \right)
\end{align*}
\]

where the first expression appears to be in better agreement with experiment\(^{11}\). When multiplied with the growth time of the fastest growing transverse multi-bunch instability, one obtains an estimate of the required spread (e.g. in bunch population) in order to suppress the instability.

12. Intuboam scattering

a) If the logical variable INTRA = .T. (default F), the subroutine INTRAB is called and calculates the growth-rates in three dimensions due to single-beam scattering\(^{16}\). Compared to the reference, it differs only in dividing the growth-rates by factor two\(^{17}\) and in taking the beam-diameter as impact parameter\(^{18}\) rather than the average distance between particles. For the evaluation of the function \( f(a,b,c) \), which was originally given by a triple integral, it uses an analytic reduction to a single integration\(^{19}\) which is performed numerically with the library routine GAUSS calling the function subroutine FUNC.
This is much faster and can be done more accurately than the former double integral of a series expansion. In order to check the accuracy of the result, the relation

\[ F_1 + F_2/a^2 + F_3/b^2 = 0 \]

is used (see below for \( F_1 \)) and if the sum on the LHS is larger than 10^{-2}, a warning statement is printed.

b) The growth rates in the 3 dimensions for bunched beams are calculated from

\[
\begin{align*}
\frac{1}{\tau \varpi} &= \frac{A(1-d^2)}{E_3} \\
\frac{1}{\tau_X} &= \frac{A(F_2 + d^2F_1)}{F_3} \\
\frac{1}{\tau_Z} &= \frac{A F_3}{F_3}
\end{align*}
\]

where

\[
A = \frac{\kappa^2}{64\pi^2 \beta \gamma^4 \sigma_x \beta \sigma_z \sigma_\varpi^2 \sigma_x \sigma_z}
\]

\[
d = \frac{\sigma_E}{\sigma_x} D^2 \quad (D = \text{average dispersion})
\]

\[
F_1 = f \left( \frac{\sigma_y}{\sigma_x}, \frac{\sigma_y}{\sigma_z}, q \sigma_y \right) F_2 = f \left( \frac{\sigma_x, \sigma_y^2}{\sigma_z}, q \sigma_y \right) 
\]

\[
F_3 = f \left( \frac{\sigma_z, \sigma_z^2, q \sigma_z}{\sigma_x, \sigma_x}, q \sigma_x \right)
\]

Here \( \sigma_0 \) is the standard deviation of the bunch-length

\[
\sigma_x = \left( \frac{\beta_x E_x}{2} \right)^{1/2}
\]

\[
\sigma_z = \left( \frac{\beta_z E_z}{2} \right)^{1/2}
\]

are the standard deviations of the radial and vertical beam-dimensions due to betatron motion. (\( \beta_x = \beta_z = R/\gamma \))

\[
\sigma_x^* = \left( \frac{E_x}{\beta_x} \right)^{1/2}
\]

\[
\sigma_z^* = \left( \frac{E_z}{\beta_z} \right)^{1/2}
\]

are the standard dev. of the total radial width

\[
\sigma_x^* = \left( \frac{E_x}{\beta_x} \right)^{1/2}
\]

\[
\sigma_z^* = \left( \frac{E_z}{\beta_z} \right)^{1/2}
\]

are the standard dev. of the betatron angles in radial and vertical directions.

\( \sigma_\varpi \) is the relative energy spread.

\[
\sigma_y = \frac{\sigma_{xp} \sigma_x}{\gamma \sigma_x}
\]

\[
q = \frac{\beta \gamma^2 b}{\gamma_0}
\]

where \( b \) is the "impact parameter" for which we take the beam-radius \( \sigma_x \sqrt{2} \).
c) The function \( f_{m}(a,b,c) \) was originally defined by triple integral

\[
f(a,b,c) = 2 \int_0^{2\pi} d\theta \int_0^{\pi} d\phi \sin \phi \int_0^{1-3\cos^2 \mu} d\nu \rho D(\mu, \nu)
\]

with \( D(\mu, \nu) = \cos^2 \mu + \sin^2 \mu \left( a^2 \cos^2 \nu + b^2 \sin^2 \nu \right) \).

Performing an integration first over the \( \rho \) and then over \( \nu \) one obtains the simple integral

\[
f(a,b,c) = 8\pi \int_0^1 dx \frac{1-3x^2}{p(x)q(x)} \left[ 2\ln \left( \frac{C}{2} \sqrt{\frac{1}{p} + \frac{1}{q}} \right) - \gamma \right]
\]

where \( p(x) = x^2 + a^2(1-x^2) \)
\( q(x) = x^2 + b^2(1-x^2) \)
and \( \gamma = 0.577... \) is Euler's constant.

Conclusions

The computer program BBI has been written to facilitate the evaluation of bunched-beam stability using existing beam-dynamical theories. In addition to the stability calculations proper, bunch-lengthening by potential-well or turbulence, Laslett tune-shifts and inter-beam scattering are calculated. The stability of coupled-bunch modes is evaluated both in the longitudinal and the transverse plane using Sacherer's formalism (without mode coupling) in the presence of up to three resonant impedances and/or wall-resistivity. Only the transverse resistive wall effect is calculated since the longitudinal one is usually negligible. If there are more than three resonant impedances, the running sum of the complex frequency shifts can be obtained from several input data-sets by setting the logical variable SUM = .T. The stability of the oscillations is evaluated in the presence of radiation and Landau-damping, and the program searches automatically for the unstable mode with the fastest growth-rate. If one of the resonator frequencies is variable or not exactly known, it can be changed in several steps around a center frequency in order to find the best (or worst) value.

Due to the complexity of both the equations evaluated and the theories which produced them, the user should nevertheless check the validity of each formalism for a given set of input-data, i.e. an understanding of the beam-dynamics is necessary for proper interpretation of the results.
References

(1) A. Hofmann, K. Hübner, B. Zotter IEEE-NS26, p.3514 (1979)

(2) F. Sacherer IEEE-NS20 (1973) p.825, also CERN/PS/BR73-3
   " " " 24 (1977) p.1393 also CERN/PS/BR77-5,6

(3) F. Sacherer Proc.9th Internat Conf. High Energy Accelerator
    CERN 77-13, p.198, also CERN/PS/BR76-4

(4) S. Hansen et al IEEE-NS 22 (1975) p.1381

(5) B. Zotter CERN/ISR-TH/78-16 (1978)


(7) D. Boussard CERN/Lab.II-RF/75-2 (1975)

(8) A. Chao, J. Gareyte PEP 224 (1976)

(9) A. Hofmann LEP 70/74 (1978), LEP Note 168 (1979)

(10) C. Pellegrini, M. Sands PEP-258 (1977)

(11) A. Hofmann, private communication


(13) E. Brouzet, A. Cappi, J. Gareyte CERN/PS/OP-DL/78-14

(14) B. Zotter NIM 129 (1975) 377.

(15) D. Möhl LBL-570 (1971)

(16) A. Piwinsk Proc 9th International Conf. High Energy Accel
    (1974) 405

(17) " " private communication

(18) H.G. Hereward, private communication


***********************************************************************