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DYNAMICAL GENERATION OF GAUGE BOSONS

IN CP^{n-1} MODELS

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The two-dimensional $\mathbb{CP}^{n-1}$ models^{1),2)} have proved useful to study many theoretical ideas like chiral symmetry breaking and the U(1) problem^{2),3)} and non-perturbative methods. They have recently been applied to study dynamical gauge symmetry breaking by fermion condensates⁴⁾ and vacuum alignment⁵⁾. In all these cases the model displays a remarkable property: classically the model has a local gauge invariance, but no explicit gauge field; quantum mechanically the gauge field is generated dynamically.

The $\mathbb{CP}^{n-1}$ model consists of n complex scalar fields z_i , $i = 1, \dots, n$, subject to the constraint $\bar{z}^i z_i = n/2f$, where f is a dimensionless constant. The Lagrangian is

$$\mathcal{L} = \overline{D_\mu z^i} D_\mu z_i + \frac{i\lambda}{\sqrt{n}} \left(\bar{z}^i z_i - \frac{n}{2f} \right), \quad (1)$$

where

$$D_\mu = \partial_\mu + \frac{i}{\sqrt{n}} A_\mu, \quad (2)$$

and λ is a Lagrange multiplier imposing the constraint in the path integral. The Lagrangian (1) is invariant under global SU(n) and local U(1) symmetry. However, the gauge field A_μ has no kinetic terms and can be eliminated by the equation of motion

$$A_\mu = \frac{if}{\sqrt{n}} \bar{z}^i \overleftrightarrow{\partial}_\mu z_i. \quad (3)$$

By integrating over z in the path integral (1) can be solved in the $1/n$ expansion. λ acquires a non-zero vacuum expectation value (VEV). Expanding around the VEV in powers of $1/n$ we find that kinetic terms are generated for A_μ . A_μ has become a dynamical gauge field via quantum corrections. Since gauge fields are confining in two-dimensions, the effective theory consists of interactions between z and A_μ to give $\bar{z}z$ bound states. This is a remarkable result.

The model can be generalized to include fermions²⁾ and to include a non-Abelian (auxiliary) gauge invariance⁶⁾. Further, we can construct a model where the left- and right-handed fermions are in different representations of the gauge group⁴⁾. The model has a U(n) global and U(ℓ) local invariance with $n > \ell$, and can be used to study gauge symmetry breaking. If we put ψ_R in ℓ and ψ_L in $\bar{\ell}$ of the U(ℓ) gauge group, then the generalized $\mathbb{CP}^{n-1}$ Lagrangian is

$$\begin{aligned} \mathcal{L} = & (\overline{D_\mu z^i})^\alpha (D_\mu z_i)_\alpha + \bar{\Psi}_L^i D_{-\beta}^\alpha \Psi_{Li}^\beta + \bar{\Psi}_R^{i\alpha} D_{+\alpha}^\beta \Psi_{Ri\beta} \\ & - \frac{1}{\sqrt{n}} (\bar{\Psi}_L^\alpha \Psi_{R\beta} \bar{\phi}^{\alpha\beta} + \bar{\Psi}_R^\alpha \Psi_L^\beta \phi_{\alpha\beta}) + \frac{1}{g_A} \bar{\phi}_{(A)}^{\alpha\beta} \phi_{\beta\alpha}^{(A)} + \frac{1}{g_S} \bar{\phi}_{(S)}^{\alpha\beta} \phi_{\beta\alpha}^{(S)} + \end{aligned} \quad (4)$$

$$+ \frac{i\lambda_\alpha^\beta}{\sqrt{n}} (\bar{z}^{i\alpha} z_{i\beta} - \frac{n}{2f} \delta_\beta^\alpha),$$

(4)
cont.

where $i = 1, \dots, n$ and is the index for global $U(n)$ and $\alpha = 1, \dots, \ell$ is the index for local $U(\ell)$. In two-dimensions $\psi_R(\psi_L)$ couples only to $D_+(D_-)$ where

$$D_\pm = D_0 \pm iD_1; \quad D_{\mu\alpha}^\beta = \partial_\mu \delta_\alpha^\beta + \frac{i}{\sqrt{n}} A_\mu^a (\tau^a)_\alpha^\beta,$$

τ^a being $U(\ell)$ matrices and

$$\phi_{\alpha\beta}^{(A)} = \left(\frac{g_A}{g_S} \right) \frac{1}{2\sqrt{n}} (\bar{\psi}_{L\alpha}^i \psi_{Ri\beta} + \bar{\psi}_{L\beta}^i \psi_{Ri\alpha}); \quad \phi_{\alpha\beta} = \phi_{\alpha\beta}^{(A)} + \phi_{\alpha\beta}^{(S)}.$$

$g_S(g_A)$ is a coupling constant for the symmetric (antisymmetric) channel. The fermion composite, ϕ is a scalar of $U(n)$, allowing the $1/n$ expansion to be performed, but is not a scalar of $U(\ell)$. Again, A_μ is an auxiliary field and can be eliminated by the equation of motion, which is more complicated than in the case of (1). To solve (4) we integrate over z and ψ in the path integral and solve for the remaining fields in a saddle-point approximation. We find that λ and ϕ acquire non-zero VEV. Since $\phi_{\alpha\beta}$ is not a singlet of the $U(\ell)$ gauge group, the group is broken by the VEV, $M_{\alpha\beta}$. In particular, detailed analysis in the $1/n$ expansion shows that if $g_S > g_A$ then $U(\ell)$ is broken to an $O(\ell)$ gauge group and if $g_A > g_S$ then $U(\ell)$ is broken to an $Sp(\ell)$ gauge group. Specializing to the case $g_S > g_A$, the results can be summarized in terms of the effective action

$$\begin{aligned} S_{\text{eff}} = \int d^2x & \left[F_{\mu\nu}^a{}^2 + G_{\mu\nu}^i{}^2 + 4M^2 W_\mu^i{}^2 \right. \\ & + (\partial_\mu D^a)^2 + (\partial_\mu E^i)^2 + 4M^2 (D^a{}^2 + E^i{}^2) + (\partial_\mu A^a)^2 \\ & \left. + 4Mi(\epsilon_{\mu\nu} \partial_\mu A_\nu^a) A^a - \epsilon (A^a{}^2 + D^a{}^2) \right], \end{aligned} \quad (5)$$

where a [i] runs over components in $O(\ell)$ [not in $O(\ell)$] and we have projected all fields into the unbroken $O(\ell)$ and broken components, represented by the matrices T^a and t^i respectively,

$$A_{\mu\alpha}^\beta \rightarrow A_\mu^a T^a + W_\mu^i t^i,$$

$$(\phi_{\alpha\beta} M^{\beta\gamma} + M_{\alpha\beta} \phi^{\beta\gamma}) \rightarrow D^a T^a + E^i t^i,$$

$$(\phi_{\alpha\beta} M^{\beta\gamma} - M_{\alpha\beta} \phi^{\beta\gamma}) \rightarrow A^a T^a + B^i t^i,$$

and ϵ is $1/g_S - 1/g_A$. B^i does not appear in the physical spectrum, having been eaten in a dynamical Higgs mechanism to give mass to W_μ . $F_{\mu\nu}$ and $G_{\mu\nu}$ are the kinetic terms for A_μ and W_μ . All fields except the $O(\ell)$ gauge field A_μ acquire mass, the mass obtained by dimensional transmutation. There is also a $U(1)$ anomaly coupling A_μ^a to the pseudoscalar A^a . Similar results hold for $g_A > g_S$. For $g_A = g_S$ there is a vacuum alignment problem, discussed in detail elsewhere^{4),5)}.

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In this model we have generalized the $\mathbb{CP}^{n-1}$ mechanism. That is, we have dynamically generated a $U(1)$ gauge boson via the $1/n$ expansion and, by a fermion composite acquiring non-zero VEV, dynamically broken the gauge group to $O(1)$. For both cases we have discussed, quantum corrections have induced dynamics for an auxiliary gauge field at the classical level.

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