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***In Situ* Calibration of Rotating Sensor Coils for Magnet Testing**

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Abstract

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I. INTRODUCTION

In particle accelerators, an accurate control of the beam is achieved by tight tolerances on the magnet's field. At this aim, high-quality magnetic measurements are crucial during all the phases of prototyping, construction, testing, and operation of the magnets.¹ In particular, main construction inaccuracies are detected by measuring magnetic field errors.

Rotating coil fluxmeters are the most accurate and precise solution for measuring the integral field and the harmonics of quasi-static magnetic fields.² In recent years, owing to digital integrators and angular encoders with improved performance, rotating coils showed to be the best solution for testing magnets with long and narrow apertures, such as for the large hadron collider (LHC) at the European Organization for Nuclear Research (CERN).^{3,4} A coil consists of several rectangular loops of conducting wire, usually stretched during the winding on a rigid core and then glued to assure a well defined and stable geometry. According to the Faraday's law, the coil generates a voltage by rotating in the magnetic field. The voltage is integrated over angular steps in order to measure the magnetic flux. The strength, the quality, and, for quadrupole magnets, the magnetic axis, are measured in terms of 2D harmonic components, expressed in a cylindrical coordinate system centered on the coil rotation axis.

The precision of the measurement relies on the mechanical quality of the coil, namely on its geometrical parameters: length, width, and rotation radius.³ For a dipole magnet, the main calibration parameter to measure the integral of the field over the length is the *magnetic equivalent area*. In addition, the *magnetic equivalent average rotation radius* of the coil must be calibrated to measure the gradient integral of a quadrupole magnet, and to assess the harmonic content, i.e., the field quality.

The area of a multilayer coil sensor can be assessed analytically as a function of the inner and outer width of the coil.⁵ Experimentally, the *magnetic equivalent area*, including all non-ideal effects, defined by the average position of the coil

windings, can be determined with a typical target uncertainty of $\pm 0.01\%$.⁶ However, at this aim, mechanical measurements are usually not adequate, even if carried out by advanced 3D sub-micrometer coordinate machines. Conversely, micro-photogrammetric methods, such as x ray or optical scans, provide high-quality measurements, but are expensive and time consuming.^{7,8} Far better results are achieved by calibration in a reference dipole field (stable and uniform), mapped by nuclear magnetic resonance (NMR),^{5,9,10} and yielding by definition of the equivalent area very accurately. This area can be measured along the longitudinal axis by means of localized field sources. As an example, the coil is held still in an ac reference field,⁹ or it is flipped/translated in a dc field.¹¹ Alternating current fields provide higher sensitivities,¹² while easy to operate dc magnets provide stable field conditions. In any case, the field source must provide good field uniformity in a sufficiently wide region transversal to the coil,¹³ in order to relax positioning constraints.^{14,15}

In rotating coils, another fundamental parameter is the rotation radius, i.e., the average distance between the longitudinal symmetry axis and the mechanical rotation axis.¹⁶ The uncertainty on the rotation radius affects all the measured harmonics exponentially with their order. The accuracy decreases according to the coil diameter, owing to the corresponding increase in the mechanical uncertainty impact.⁶ The *magnetic equivalent rotation radius* can be obtained in a reference quadrupolar field either by rotating the coil by a given angle or by measuring the focusing strength.

If the coil geometry is not regular, the equivalent area and radius vary appreciably along the coil, and the measurements depend on their average values weighted by the field. Therefore, averaged parameters, such as obtained by traditional calibration in reference magnets longer than the coil, lead to significant errors, sometimes in the percent range. This creates a measurement problem for testing magnets shorter than the measuring coil. Moreover, in traditional calibration procedures, two reference magnets are needed: a dipole to calibrate the area and a quadrupole for the radius.

In this paper, a method for calibrating equivalent area and rotation radius of rotating coils is proposed for testing accelerator quadrupole magnets. In particular, in Sec. II, the theory of the rotating coil magnetic measurement is illustrated. In Sec. III, the *in situ* calibration method is highlighted in the specific case of a quadrupole magnet. In Sec. IV, the metrological analysis is described. In Sec. V, experimental results of the comparison of the proposed calibration method with a reference magnet¹⁶ are illustrated.

II. THEORY OF ROTATING COILS

The 2D multipole solution of the Maxwell equations in use for accelerator magnets is a series expansion conveniently described in terms of a complex variable $z = x + iy$,¹⁷

$$B(z) = \sum_{n=1}^{+\infty} C_n \left(\frac{z}{R_{ref}} \right)^{n-1}, \quad (1)$$

where C_n are complex coefficients ($B_n + iA_n$) resulting from the series expansion and R_{ref} is the measurement radius, usually referred in literature as reference radius.⁴ For the sake of simplification, in this paper, R_{ref} is equal to the external radius of the rotating shaft. Under the assumption of static field, the radial flux Φ , intercepted by a coil rotating in the field as a function of the rotation angle θ is equal to,

$$\Phi(\theta) = N_t L_c \operatorname{Re} \left\{ \sum_{n=1}^{+\infty} \frac{C_n}{n R_{ref}^{n-1}} (z_1^n - z_2^n) e^{in\theta} \right\}, \quad (2)$$

where N_t is the number of coil windings of length L_c , with edges at z_1 and z_2 in the frame with origin in the coil rotation axis (Fig. 1). A Fourier analysis of the measured flux distribution allows the field parameters to be expressed in terms of harmonic coefficients. The field harmonic content is proportional to the discrete Fourier transform (DFT) coefficients Ψ_n of the flux samples Φ_k , resulting from the integration of the voltage between two angular positions during the coil

rotation,

$$\Psi_n = \frac{1}{N} \sum_{k=1}^N \Phi_k e^{kin \frac{2\pi}{N}}, \quad (3)$$

where N is the number of angular points per turn.

From Eqs. (1) and (3), the magnetic field harmonic coefficients C_n are derived,²

$$C_n = \frac{2 \cdot \Psi_n}{\kappa_n}, \quad n = 1, \dots, \frac{N}{2}, \quad (4)$$

where κ_n are complex coefficients of coil geometrical calibration ("sensitivity factors"). They are independent from the measured magnetic field,

$$\kappa_n = \frac{N_t L_c (z_1^n - z_2^n)}{n R_{ref}^{n-1}} \quad (5)$$

related to the position of the wire under the assumption of an infinitesimal winding cross section in $X'-Y'$. For the sake of simplification, z_1 and z_2 are defined as real parameters: the average radius $R_0 = \|z_0\|$, where z_0 is the position of the coil center (Fig. 1) and the equivalent width $W_{eff} = \|z_1 - z_2\|$. In practice, the total coil equivalent area is defined as $A_c = N_t L_c W_{eff}$. The angle α defines how the coil cut the flux lines, making the coil more sensitive to some harmonics rather than others.⁴ This is important when the sensitivity to higher order field components is critical.

The angle φ_0 is usually adjusted to zero for the main field component. R_0 , W_{eff} , and α completely define the coil geometry in $X'-Y'$. By expressing the distance $z_1 - z_2$ as function of R_0 , W_{eff} , and κ_n are defined as

$$\kappa_n = \frac{N_t L_c}{n R_{ref}^{n-1}} \left[\left(R_0 - i \frac{W_{eff}}{2} e^{i\alpha} \right)^n - \left(R_0 + i \frac{W_{eff}}{2} e^{i\alpha} \right)^n \right] e^{in\varphi_0}. \quad (6)$$

In practice, two configurations are used: (i) the radial coil parallel to X' ($\alpha = \pi/2$), where κ_n from Eq. (5) are real numbers,

$$\kappa_n^{radial} = \frac{N_t L_c (R_1^n - R_2^n)}{n R_{ref}^{n-1}}, \quad (7)$$

where R_1 and R_2 (Fig. 1) are the coil internal and external radii, respectively, and (ii) the tangential coil, perpendicular to X' ($\alpha = 0$), where κ_n are imaginary numbers related to the coil opening angle $\Delta\theta$,

$$\kappa_n^{tangential} = \frac{2i N_t L_c R_0^n \sin\left(\frac{n\Delta\theta}{2}\right)}{n R_{ref}^{n-1}}. \quad (8)$$

In practice, after a preliminary estimation of the coil equivalent area and radius by means of geometrical measurements, the values of A_c and the radius R_0 are refined iteratively, by means of magnetic measurements in a reference dipole and quadrupole, respectively.⁶

III. IN SITU COIL CALIBRATION

The *in situ* calibration procedure is aimed at improving the accuracy of rotating coils, when the coil is longer than the

FIG. 1. Geometry of a coil rotating in a quadrupole field, coil frame $X'-Y'$, with origin in rotation axis, and magnet frame $X-Y$.

magnet, by exploiting directly in the magnet under test measurements of magnetic field and mechanical displacement. The values of A_c and R_0 are related to the full length of the measuring coil, while A_{cm} and R_{0m} will refer to the values integrated over the length of the measured magnet. The procedure is independent of the measuring shaft and quadrupole magnets can be extended easily to higher order magnets.

In a quadrupolar field (Fig. 1), a displacement Δz between the magnet axis and the coil rotation axis gives rise to a dipole component in the measured field series expansion. If the dipole and quadrupole terms and the displacement are measured, the two calibration parameters, the equivalent radius R_{0m} and area A_{cm} , can be determined by analytical considerations.

The field coefficients C_n of the multipole expansion [Eq. (4)] measured by a rotating coil system are referred to the frame $X'-Y'$ (Fig. 1). In general, a translation and a rotation in the complex plane are needed for referring the harmonic coefficients to the magnet frame $X-Y$.⁴ The distance $\Delta z = \Delta x + i\Delta y$ is calculated at first order as

$$\Delta z = -R_{ref} \frac{C_1}{C_2}. \quad (9)$$

In other words, a virtual dipole component C_1 is generated by a translation of the coil with respect to the magnet frame. In principle, if the magnet is moved transversally by a given distance Δx and the quadrupole component C_2 is known, the radius R_{0m} and the total coil area A_{cm} can be obtained directly, because the system of equations determined from Eqs. (4) and (6) for $n = 2$ is defined,

$$\begin{cases} \frac{\Psi_1}{\kappa_1} = -\frac{C_2 \Delta x}{R_{ref}} \\ C_2 = \frac{\Psi_2}{\kappa_2} \end{cases}. \quad (10)$$

By computing κ_1 and κ_2 from Eq. (6) and by substituting them in Eq. (10), R_{0m} and A_{cm} are derived,

$$\begin{cases} R_{0m} e^{i\varphi_0} = -\frac{\Psi_2 \Delta x}{\Delta \Psi_1} \\ A_{cm} e^{i\alpha} = i e^{i\varphi_0} \frac{\Delta \Psi_1 R_{ref}}{C_2 \Delta x} \end{cases}. \quad (11)$$

This method depends on the unknown initial offset Δz_0 between the frames. This can be avoided by considering the difference between two harmonic measurements with a given offset, i.e., ΔC_1 instead of C_1 ,

$$\Delta C_1 = \frac{\Delta \Psi_1}{\kappa_1} = -C_2 \frac{\Delta x}{R_{ref}}. \quad (12)$$

The value of C_2 , corresponding to the quadrupole gradient G_c at the longitudinal center of the magnet,⁴ must be determined by an independent measurement,

$$G_c = \frac{C_2}{R_{ref}}. \quad (13)$$

In practice, the best solution providing the lowest uncertainty on the focusing strength ($FStrength$) is the single stretched wire (SSW),¹⁸ where $FStrength$ is defined as the gradient integrated over the quadrupole axis,

$$FStrength = G_c \cdot L_{eff} = \int_{-\infty}^{\infty} G(l) dl, \quad (14)$$

where L_{eff} is the effective magnetic length.

FIG. 2. Flow diagram of the *in situ* calibration.

In Fig. 2, the flow diagram of the calibration procedure is depicted. First, $FStrength$ is measured by means of SSW. By keeping the magnet in the same position, a first rotating coil measurement is carried out to evaluate the flux arising from the quadrupole and dipole components (Ψ_2 and Ψ_1 , respectively) at the starting position x_0 . Then, a displacement is applied to the magnet (or to the coil), the flux measurement is repeated, and $\Delta \Psi_1$ and Ψ_2 are used in Eq. (15) to compute A_{cm} and R_{0m} .

IV. METROLOGICAL ANALYSIS

A. Calibration issues

The most critical issue is the accurate measurement of $FStrength$. NMR is not suitable for measurement in inhomogeneous fields.⁸ Hall probes reach high accuracy but give only a local measurement¹⁹ and, moreover, needs for temperature compensation and frequent calibrations. Therefore, the SSW is the most accurate method,⁴ although limited by the high-order multipoles.^{18,20} The multipole influence increases according to the distance from the magnet axis. In particular, by expressing the estimation of Smirnov,¹⁸ at the radius R_{ref} , this influence can be estimated and compensated,

$$FStrength = \frac{FStrength^{measured}}{\left(1 + \sum_{n \text{ even}} \frac{2}{n} \left(\frac{\Delta}{R_{ref}}\right)^n 10^{-4} b_{n+2}\right)}, \quad (15)$$

where b_n is the real part of the C_n n -harmonic field coefficient expressed in unit of the main field⁴ and the displacement Δ applied to the wire.

B. Uncertainty

The relative measurement uncertainty on R_{0m} is obtained by propagation through Eq. (11),

$$u_{R_{0m}} = \frac{\sigma_{R_{0m}}}{R_{0m}} = \sqrt{\left(\frac{\Delta z}{\Delta \Psi_1}\right)^2 \frac{\sigma_{\Psi_2}^2}{\Psi_2^2} + \frac{\sigma_{\Delta z}^2}{\Delta z^2}}. \quad (16)$$

The uncertainty on the flux arises mainly from the signal-to-noise ratio of the acquisition system (few tens of ppm for the low-order harmonics²¹). The uncertainty on Δz in the millimeter range reaches a few micrometer by using commercially available mechanical systems based on linear encoders. Thus, $u_{R_{0m}}$ is dominated by the displacement uncertainty.

The uncertainty on A_{cm} is derived from Eq. (11),

$$u_{A_{cm}} = \frac{\sigma_{A_{cm}}}{A_{cm}} = \sqrt{\frac{\sigma_{\Delta z}^2}{\Delta z^2} + \frac{\sigma_{C_2}^2}{C_2^2} + \frac{\sigma_{\Psi_1}^2}{\Psi_1^2}}. \quad (17)$$

The main contribution to the uncertainty on C_2 arises from the SSW measurement. For a quadrupole magnet with an aperture of 20 mm and a length of 45 mm, a typical value is ± 100 ppm.

The uncertainty on κ_n is derived from Eq. (6),

$$u_{\kappa_n} = \frac{\sigma_{\kappa_n}}{\kappa_n} = n \left(\frac{\sigma_{R_{0m}}}{R_{0m}} + \frac{\sigma_{A_{cm}}}{2A_{cm}} \right). \quad (18)$$

The quality of the low order sensitivity factor determination is critical. At the same time, this gives an upper bound to the order of measurable multipoles.

The multipole precision depends mainly on the two calibration parameters, because they include all the uncertainty sources owing to the *in situ* calibration.

C. Systematic errors due to higher order multipoles

Equation (10) is only a first-order approximation of a more general equation,⁶

$$C'_1 = \sum_{n=1}^{+\infty} C_n \left(\frac{\Delta z}{R_{ref}} \right)^{n-1}, \quad (19)$$

where C'_1 is the dipole in the frame $X'-Y'$ (Fig. 1), and C_n is the n th order field component in $X-Y$. Therefore, at the n th order, Eq. (11) begins,

$$A_{cm} = \left\| \frac{\Delta \Psi_1 R_{ref}}{C_2 \Delta z} \frac{1}{1 + \sum_{n=3}^{+\infty} \frac{C_n}{C_2} \left(\frac{\Delta z}{R_{ref}} \right)^{n-2}} \right\|, \quad (20)$$

where C_n/C_2 is the n th field error component. The multipoles are less than 1% for the useful aperture of most accelerator magnets. In the worst case of a quadrupole magnet with 1% of C_3/C_2 and a displacement of 1 mm over 8 mm radius, the systematic effect due to the multipole on the area calibration is less than 1%.

The calibration of R_{0m} does not depend on the higher multipoles, but only on the gradient. It is therefore not affected by systematic multipole errors.

V. EXPERIMENTAL RESULTS

The *in situ* procedure was validated experimentally at the European Organization for Nuclear Research (CERN) on the magnets of the new linear particle accelerator Linac4,²² namely quadrupoles with aperture 22 mm and length 450 mm, designed to intensify the beam brightness in the accelerating chain of the large hadron collider. Two coil shafts were manu-

FIG. 3. (Color online) Coil design of the shaft with two radial coils (a) and the shaft with three tangential coils (b).

factured to experiment the validity of different measuring coil concepts for testing small aperture magnets.

At this aim, the coils were calibrated by the traditional¹⁶ and the *in situ* procedures, in order to obtain two sets of sensitivity coefficients κ_n^T and κ_n^m , respectively. The quadrupole magnet was further tested by means of the rotating coil and the single-stretched wire methods. The flux measured by the coil is processed by exploiting the κ_n^T and κ_n^m , resulting in the gradient measurements $FStrength^T$ and $FStrength^m$, respectively. These values were finally compared with the reference gradient $FStrength^{SSW}$ obtained by the single-stretched wire method.

A. The coil shafts

In the two rotating coil shafts, the small dimensions of the support (200 mm long, less than 10 mm wide, and 0.5 mm for the winding groove) make arduous the winding. Thus, a multi-strand wire, allowing a precise layering over multiple turns,⁶ was used. However, problems of low rigidity of the coil arise, because the wire has to be wound manually. As an example, for an 8-mm wide coil, the typical precision of the wire positioning is ± 0.2 mm, resulting in $\pm 2\%$ of uncertainty on A_c . Furthermore, this non-uniformity affects the

FIG. 4. (Color online) Calibration setup: (a) 2D translation stage to move the magnet in the coil cross section plane, (b) rotating shaft under test, and (c) permanent quadrupole.

TABLE I. Results from the validation applied to several magnets of the Linac4 drift tube.

Magnet	Magnet length (mm)	$FStrength^{SSW}$ (Tm/m)	$\frac{FStrength^m - FStrength^{SSW}}{FStrength^{SSW}}$ (%)		$\frac{FStrength^T - FStrength^{SSW}}{FStrength^{SSW}}$ (%)	
			(three-coil)	(two-coil)	(three-coil)	(two-coil)
107	45	2.334	-0.1	-0.3	0.4	5.8
108	45	2.331	-0.2	-0.3	0.6	6.2
109	45	2.317	0.1	0.2	0.4	6.2
905	45	2.328	0.2	-0.3	0.6	5.4
R_1	45	2.431	0.0	0.0	0.3	6.4
R_2	45	2.449	0.1	0.2	0.5	4.6
1 637 691	80	6.861	-0.1	-0.3	0.4	4.7
1 637 690	80	6.833	0.1	0.1	0.3	5.1

measurement quality when the magnetic field L_{eff} is shorter than the measuring coil L_c .

In both types of shaft, the radii and the equivalent areas of the coils are optimized in order to have the same sensitivity to dipolar and quadrupolar components. The first shaft has two nested radial coils on the same plane (Fig. 3(a)), with equal equivalent area, for the sake of compensation.⁶

The second shaft (Fig. 3(b)) has two tangential coils in series, compensating for the quadrupole, and a third one (coil 3), compensating for the dipole.

Equation (5) is valid for coils with point-like windings. The finite dimension of the section produces an error increasing according to the multipole to measure. Squared-shaped windings give the best approximation of the infinitesimal size assumption⁴ and a multi-strand wire leads to very uniform geometries.

B. The calibration setup

The calibration bench for the *in situ* procedure is depicted in Fig. 4. A 2D translation stage (a), mounted between the shaft ends (b), permits a controlled motion to the permanent quadrupole of Linac4 (c) in the cross section plane of the coil, with less than $\pm 2 \mu\text{m}$ of uncertainty. This system exploits the same mechanics and electronics²¹ to calibrate and test the magnet, thus all the errors are taken into account by the coil sensitivity factors. The magnet displacement is

limited to $\pm 1 \text{ mm}$, i.e., clearance between the coil and the magnet aperture. The origins of magnet and coil frames are made coincident by minimizing iteratively the dipolar flux through the 2D stage. This allows the calibration results to be cross-checked, because the position of the rotation axis after a given displacement is related to the ratio between C_1 and C_2 .

C. Validation results

The two shafts were calibrated according to the *in situ* and the traditional calibrations on a Linac4 quadrupole. Then, the calibrated coils were used to test several other Linac4 quadrupoles. The values obtained from both methods are directly compared to the magnet strength measured by the SSW. In Table I, the focus strength results obtained through the *in situ* ($FStrength^m$) and the traditional ($FStrength^T$) procedures are compared with the stretched-wire ($FStrength^{SSW}$). The *in situ* procedure reduces the average error to 0.1% (two coils) and 0.03 % (three coils), with respect to the 5.5% (two coils) and 0.5% (three coils) of the traditional calibration. For the three-coil shafts, the traditional calibration performs better owing to the improved machining and design. The *in situ* results met the requirements of $\pm 0.5\%$ from the Linac4 beam even if the type-B uncertainty, as defined in Ref. 23, of the rotating coil ($\pm 0.3\%$) is higher than the stretched wire ($\pm 0.1\%$).

FIG. 5. (Color online) Difference in m^2 between the locally measured A_{cm} and nominal equivalent area A_c vs the longitudinal position along the coil for (a) the two-coil and (b) the three-coil shafts.

FIG. 6. (Color online) Difference in mm between the locally measured R_{0m} and nominal equivalent radius R_0 vs the longitudinal position along the coil for (a) the two-coil and (b) the three-coil shafts.

The deviation of the traditional procedure was confirmed by applying the *in situ* calibration in different longitudinal positions (Figs. 5 and 6) to show the difference between the nominal parameters A_c , R_0 and the local A_{cm} , R_{0m} . For the two-coil shafts (a), the coil 1 (Fig. 3) in the central part shows a high variation in the surface profile. Being nested, the coil 2 is more uniform. Nevertheless, the radius variation increases along the rotation axis: 5% for coil 1 and 3% for coil 2, on 200 mm. Therefore, the longitudinal measurement position is chosen far from the coil end, namely at 100 mm from the left coil edge, where the width is almost uniform. For the three-coil shafts (Fig. 5(b)), the enhancement in coil quality is highlighted by a smaller difference among the values of A_c and R_0 averaged over the length, given by the traditional calibration, and the local results A_{cm} and R_{0m} . In Figs. 6(a) and 6(b), the differences between the measured and nominal values of the coil equivalent radius are shown at different longitudinal positions. The coils of the two-coil shafts are affected by higher irregularities owing to the winding, the support machining, and the sag effects.

D. Metrological analysis

According to Eqs. (16) and (17), the total uncertainty is equal to $\pm 0.3\%$ for the equivalent radius and $\pm 0.6\%$ for the area. About half of this uncertainty arises from the repeatability of the calibration, assessed by fixing the displacement Δz and by using the same permanent quadrupole (i.e., fixed $FStrength$). The effect of higher multipoles [Eq. (20)] is one order of magnitude lower than the repeatability.

According to Eq. (18), the uncertainty on A_{cm} and R_{0m} affects also the higher order harmonic coefficients proportionally to the multipole order. In Fig. 7, the percentage uncertainty on κ_n^m in a given longitudinal position is shown, obtained by varying the displacement between -1.1 and 1.1 mm and the $FStrength$ between 2.0 and 2.4 Tm/m, for the three-coil shafts. For coil 3, the uncertainty is linear, such as expected. Conversely, for the other two coils, the uncertainty on κ_6^m and κ_8^m increases by a factor 3. These two coils are tangential, therefore insensitive to multipole order having an angular period, or an integer number of periods, correspond-

ing to the coil opening angle $\Delta\theta$ (Fig. 1),

$$\Delta\theta = 2 \arctan\left(\frac{W_{eff}}{R_0}\right). \quad (21)$$

These two coils have an aperture close to the 12-pole and to the 16-pole, corresponding to 1.047 and 0.78 rad, respectively. The sensitivity to harmonics higher than the quadrupole is relevant only for coil 1, because the two other coils are only used to compensate the dipole and quadrupole terms. Coil 1 is optimized to measure the multipoles up to $n = 8$ (16-pole), i.e., the field errors in a quadrupole magnet. In practice, calibrating the amplitude of high order multipoles ($n > 2$) to better than 10% is useless.

The influence of the SSW error in Eq. (15) was proved by (i) *in situ* calibrating the three-coil shafts by a “bad” quadrupole with strong multipole errors of 46, 32, and 4 units for b_4 , b_6 , and b_8 , respectively, (ii) by testing successively 21 magnets, (iii) *in situ* calibrating the same three-coil shafts by a “good” quadrupole, with low errors of less than 14 units of b_4 and less for the higher orders. For the “bad” quadrupole, the deterministic error computed from Eq. (15) was equal to 0.67%, and the measured value was 0.58%, while, for the “good” quadrupole, the computed error was 0.14%, and the measured error 0.03%.

FIG. 7. (Color online) Uncertainty of the coil sensitivity factor for each n -pole (three-coil shafts).

VI. CONCLUSIONS

A method is proposed to calibrate directly in the magnet under test (*in situ*) the coil sensitivity parameters, namely the coil equivalent area and rotation radius. It was applied to the coils developed for testing the permanent quadrupoles of the accelerator Linac4 at CERN. The precision of the calibration is 0.3%, in spite of the small aperture and the little clearance between coil and magnet bore. In addition, the sensitivity to higher multipole terms is correctly estimated. The tolerance typically required for the focusing strength of the Linac4 magnets is about 1%, thus the new procedure was verified to be useful in the series measurements of magnets for the latest generation of linear accelerators.

The following drawbacks of the traditional calibration are overcome: (i) the winding tolerances of small coils are relaxed, by calibrating them in the magnet to measure against a separated reference measurement; (ii) the systematic measurement errors arising from the calibration of the averages of the coil radius and width are removed, by calibrating a specific longitudinal portion of the coil; and (iii) by calibrating the coils directly on the field, the twofold steps of the traditional calibration for evaluating the area (by rotating the coil in a reference dipole magnet) and the rotation radius (by rotating in a reference quadrupole) become unnecessary.

The *in situ* calibration only requires the knowledge of the quadrupole strength, based on the single stretched wire, and the displacement between the magnet and the coil rotation axes. The method in addition provides a calibration of the sensitivity to the first higher order multipoles. Multipoles above the dodecapole, first allowed harmonic, are irrelevant for linear accelerator magnets.

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