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FURTHER STUDIES OF CONVERGENCE IN THE CLOUDY BAG MODEL

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A B S T R A C T

We generalize an earlier bound on the virtual pion content of the physical nucleon to include non-relativistic recoil of the baryon core. In addition we discuss the possibility and physical meaning of obtaining such a bound for virtual ρ mesons.

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1. INTRODUCTION

At the present time there is a great deal of interest in the development of phenomenological models which link conventional nuclear physics with the quark substructure of the nucleon¹⁻³⁾. One of the more interesting of these models which has proven successful in a number of applications⁴⁻⁷⁾ is the Cloudy Bag Model (CBM). In an earlier study⁸⁾ (hereafter called I) we examined the convergence properties of the static CBM, and obtained rigorous bounds on the probability of finding n pions in the cloud about the nucleon (MIT-bag) core, as well as a bound on the average number of pions itself.

In this paper we first present a generalization of the results of I to the case where the baryon bag is allowed to recoil non-relativistically. (A preliminary account of this work was given at the Versailles conference -- ref. 9.) Note that we do not tackle the outstanding problem of spurious c.m. motion in the bag¹⁰⁻¹²⁾, and the associated corrections. We assume that such corrections would not change the basic shape of the static $NN\pi$ vertex function, although they may slightly alter the effective bag radius. Our aim here is simply to modify the CBM Hamiltonian in baryon space to incorporate non-relativistic recoil. The resulting bounds on the probability of finding n pions, and on the average number of pions are identical to those proven in I, and constitute an improvement with respect to the ones given in ref. 9.

In the second part of this paper we address the question of vector meson coupling to the bag. Under the assumption that it makes physical sense to talk about a virtual ρ -meson cloud surrounding the extended core (which has also been made more or less explicitly by other authors), we are able to obtain bounds analogous to those found for pions. Such bounds will turn out to be consistent, essentially, with previous phenomenological estimates of the ρ -meson contribution in various situations by other authors. At a more fundamental level, some arguments are also presented which question the real physical meaning of such ρ -meson couplings.

2. CBM WITH RECOIL

One of the attractive features of the CBM for the purposes of intermediate energy applications is that we arrive very naturally at a Hamiltonian involving only pion and baryon degrees of freedom. The considerations at the quark level serve to constrain the parameters of the theory -- bare masses, coupling constants and form-factors. Since the MIT bag model itself deals with a static, fixed bag the original CBM necessarily involved a static baryon source. The natural extension of such a model to include the non-relativistic recoil of the nucleon¹³⁾ and delta⁹⁾ (N and Δ) is

$$H_{R,CBM} = H_{R,0} + H_{R,I}, \quad (2.1)$$

$$H_{R,0} = \sum_{\alpha=N,\Delta} \left[m_{\alpha} + \frac{\vec{p}_{\alpha}^2}{2m_{\alpha}} \right] + \sum_k \omega_k a_k^{\dagger} a_k, \quad (2.2)$$

$$H_{R,I} = \sum_k \left\{ a_k \left[\sum_{\alpha,\beta=N,\Delta} v_k^{\alpha\beta} \exp(i \vec{k} \cdot \vec{x}_{\alpha\beta}) \right] + h.c. \right\}, \quad (2.3)$$

$$\left[\sum_{\gamma=N,\Delta} m_{\gamma} \right] \vec{x}_{\alpha\beta} = m_{\alpha} \vec{x}_{\alpha} + m_{\beta} \vec{x}_{\beta}, \quad \alpha,\beta=N,\Delta. \quad (2.4)$$

Here m_{α} , $\vec{x}_{\alpha}$, and $\vec{p}_{\alpha}$ are respectively the bare mass, position and momentum operators of the baryon bag state of type α ($= N, \Delta$), a_k ($a_k^{\dagger}$) destroys (creates) one pion of momentum $\vec{k}$, energy $\omega_k = (\mu^2 + \vec{k}^2)^{1/2}$ and isospin projection j ($k \equiv (\vec{k}, j)$), and $v_k^{\alpha\beta}$ is the bare interaction matrix element for the pion- α - β vertex. (Its detailed expression is given in refs. 2-5.)

The total momentum operator is given by

$$\vec{P}_R = \sum_{\alpha=N,\Delta} \vec{p}_{\alpha} + \sum_k \vec{k} a_k^{\dagger} a_k. \quad (2.5)$$

Equations (2.2) and (2.3) differ from Eqs. (2.1b-c) in I (the static limit) by the inclusion of the non-relativistic, baryon kinetic energies $\sum_{\alpha} \vec{p}_{\alpha}^2 / 2m_{\alpha}$, and the important factor $\exp i \vec{k} \cdot \vec{x}_{\alpha\beta}$. In turn, the latter is responsible for a simple but key property, that is, $\vec{P}_R$ is conserved. To be specific, $[\vec{P}_R, H_{R,CBM}] = 0$, as a direct computation shows. We shall characterize the pionic contents of the physical, recoiling nucleon, by suitably generalizing techniques and results presented in I. For that purpose, it will be useful to introduce successively:

- the subspace $\mathcal{H}_{\vec{\pi}}$ of all eigenstates ψ of $\vec{P}_R$ with eigenvalue $\vec{\pi}$, $|\vec{\pi}|$ being suitably small (say $\vec{\pi}^2 / 2m_{\alpha} < \mu$);
- the following basis in $\mathcal{H}_{\vec{\pi}}$:

$$\psi(k_1, \dots, k_r; \alpha, \vec{q}_{\vec{\pi}}) = (r!)^{-1/2} a_{k_1}^{\dagger} \dots a_{k_r}^{\dagger} | \alpha, \vec{q}_{\vec{\pi}} \rangle, \quad (2.6)$$

$$\vec{q}_{\vec{\pi}} + \sum_{i=1}^r \vec{k}_i = \vec{\pi},$$

where $| \alpha, \vec{q}_{\vec{\pi}} \rangle$ denotes a bare bag state of type α ($= N, \Delta$) with given spin and isospin projections (not written explicitly) and momentum $\vec{q}_{\vec{\pi}}$;

- the restricted scalar product in $\mathcal{H}_{\vec{\pi}}$:

$$\langle \psi(k_1, \dots, k_r; \alpha, \vec{q}_{\vec{\pi}}) | \psi(k'_1, \dots, k'_r; \alpha', \vec{q}'_{\vec{\pi}}) \rangle_{\vec{\pi}} = \delta_{\alpha\alpha'} \delta_{rr'} (r!)^{-1} \left\{ \sum_{\text{permutations}} [\delta(\vec{k}_{\sigma(1)} - \vec{k}_1) \delta_{j_{\sigma(1)}j_1}] \dots [\delta(\vec{k}'_{\sigma(r)} - \vec{k}_r) \delta_{j_{\sigma(r)}j_r}] \right\}, \quad (2.7)$$

where $(\sigma(1) \dots \sigma(r))$ denotes a generic permutation of $(1 \dots r)$. Equation (2.7) defines immediately the restricted norm, $\|\psi\|_{\vec{\pi}}$, of a generic ket belonging to $\mathcal{H}_{\vec{\pi}}$. The usual scalar products and norms in the full Hilbert space are equal to the restricted ones introduced above times $\delta^{(3)}(\vec{0})$.

- d) the following restricted norm for an operator A which commutes with $\vec{P}_R$:
 $\|A\|_{\vec{\pi}} = \text{least upper bound of } \|A\psi\|_{\vec{\pi}} / \|\psi\|_{\vec{\pi}} \text{ as } \psi \text{ varies in } \mathcal{H}_{\vec{\pi}}.$

Let

$$|\tilde{n} s t; \vec{\pi}\rangle = Z_2^{1/2} |\alpha = n s t; \vec{\pi}\rangle + \sum_{r=1}^{\infty} \sum_{\beta} \sum_{k_1 \dots k_r} \quad (2.8)$$

$$\cdot \langle \psi(k_1, \dots, k_r; \beta, \vec{q}_{\vec{\pi}}) | \tilde{n} s t; \vec{\pi} \rangle_{\vec{\pi}} \psi(k_1, \dots, k_r; \beta, \vec{q}_{\vec{\pi}}),$$

be the physical one-nucleon state with spin and isospin projections s and t, total momentum $\vec{\pi}$, and energy $E(\vec{\pi})$:

$$\begin{aligned} [H_{R,CBM} - E(\vec{\pi})] |\tilde{n} s t; \vec{\pi}\rangle &= 0, \\ [\vec{P}_R - \vec{\pi}] |\tilde{n} s t; \vec{\pi}\rangle &= 0. \end{aligned} \quad (2.9)$$

The probability for finding r pions in the physical nucleon state is:

$$P_{r,R} = \sum_{\beta} \sum_{k_1 \dots k_r} |\langle \psi(k_1, \dots, k_r; \beta; \vec{q}_{\vec{\pi}}) | \tilde{n} s t; \vec{\pi} \rangle_{\vec{\pi}}|^2, \quad (2.10)$$

if the physical nucleon state is normalized

$$\langle \tilde{n} s t; \vec{\pi} | \tilde{n} s t; \vec{\pi} \rangle = 1. \quad (2.11)$$

Notice that Eqs. (2.8) and (2.10) are the natural generalization of Eqs. (3.2) and (3.8) in I.

Reference to the arguments of Section 3 and the appendices of I makes it clear that the same arguments can be applied to the present model. The only caution necessary is that: i) one uses the restricted norm for kets and operators, and ii) one replaces $\tilde{m}_n$ by $E(\vec{\pi})$. Clearly the smallest eigenvalue of the new Hamiltonian $H_{R,CBM}$ in the restricted subspace $\mathcal{H}_{\vec{\pi}}$ corresponds to the energy of a physical nucleon with momentum $\vec{\pi}$. Therefore one easily finds the result

$$\|(E(\vec{\pi}) - \omega - H_{R,CBM})^{-1}\|_{\vec{\pi}} \leq (\omega + E(\vec{\pi}) - E(\vec{\pi}))^{-1} = \omega^{-1}. \quad (2.12)$$

One must also carry over the phase factors, $\exp(\pm i \vec{k} \cdot \vec{x}_{\alpha\beta})$ into the analogue of the C-factors -- see Eq. (3.13) in I. However, such factors cancel identically in the expression for bounds -- e.g. in Eq. (B.2) of I.

Thus, one finally obtains the following bounds for $P_{r,R}$, the mean number of pions ($\langle r \rangle_R$) and the uncertainty in the number of pions (Δr_R):

$$P_{r,R} \leq \Lambda^r / r! \quad , \quad \langle r \rangle_R \leq \Lambda \quad , \quad \Delta r_R \leq [\Lambda^2 + 1/4]^{1/2} \quad (2.13)$$

$$\text{where} \quad \Lambda = \frac{57}{25} \frac{27 f^2}{\mu^2 (2\pi)^2} \int_0^\infty dk k^4 \frac{\left[\frac{\sin(kR)}{(kR)^3} - \frac{\cos(kR)}{(kR)^2} \right]^2}{\omega_k^3} \quad (2.14)$$

The analytical expression for Λ turns out to coincide with that for the static CBM [see Eqs (3.24)-(3.25) in I] and constitutes a slight improvement with respect to the bound $\Lambda(\bar{q})$ obtained in ref. 9. As in I, R is the bag radius, and f is the unrenormalized pion-nucleon coupling constant -- which in the CBM is within 10% of the renormalized value⁵⁾.

If one uses values for R and f close to $R \approx 0.82 F$, $f^2/4\pi \approx 0.078$ (which were obtained in ref. 5 from a best fit of the static CBM predictions to the pion-nucleon scattering data), one finds $\Lambda < 1$. This suggests a rather rapid convergence of the perturbation expansion for $|n\pi; \bar{\pi}\rangle$, as in the static CBM case.

3. RHO MESON COUPLING TO THE BAG

In this section we extend the earlier bounds on the virtual (non-resonating) π -meson cloud to include virtual ρ -mesons (resonating pairs of pions) as well. We tackle this problem in two parts. First we assume that it makes sense to talk about the coupling of a point like ρ -meson field to the bag, with conventional strength¹⁴⁾. Even in this case we shall see that the average number of ρ -mesons about the dressed nucleon is bounded by a rather small number, consistent with other phenomenological studies. Having dealt with the problem in the conventional way, we then ask whether such an approach is consistent with the quark model itself. We shall in fact argue that within the context of a theory where the composite nature of the nucleon is taken seriously^{2,3)}, one should not perhaps introduce the ρ -meson explicitly. Such contributions are probably better treated as uncorrelated $q\bar{q}$ excitations in the bag -- or sea quarks^{15,16)}.

3.1 A new bound

Let us consider the following Hamiltonian which generalizes Eq. (2.1) to include a purely phenomenological interaction between a ρ -meson field and the confined quarks. If we define the formal sum over isospin and 3-momentum of the ρ

$$\sum_{\mathbf{q}} \equiv \sum_{n=1}^3 \int d^3 q, \quad (3.1)$$

then the new Hamiltonian becomes

$$H = H_0 + H_I, \quad (3.2)$$

with

$$H_0 = H_{R,0} + \sum_{\mathbf{q}, l} b_{\mathbf{q}}^{\dagger}(l) b_{\mathbf{q}}(l) \omega_{\rho, \mathbf{q}}, \quad (3.3)$$

where $H_{R,0}$ was given in Eq. (2.2), $\omega_{\rho, \mathbf{q}}$ is the kinetic energy of a ρ -meson of momentum $\mathbf{q}$, spin-projection l , and $b_{\mathbf{q}}$ the appropriate destruction operator. The interaction Hamiltonian has the form

$$H_I = H_{R,I} + \sum_{l=1}^3 \sum_{\mathbf{q}} \left\{ b_{\mathbf{q}}(l) \left[\sum_{\alpha\beta=N,\Delta} v_{\rho, \mathbf{q}}^{\alpha\beta}(l) \right. \right. \\ \left. \left. \times \exp(i \vec{q} \cdot \vec{x}_{\alpha\beta}) \right] + h.c. \right\}, \quad (3.4)$$

where again $H_{R,I}$ was given in Section 2 [Eq. (2.3)] and the interaction of the ρ with the bag is given by

$$v_{\rho, \mathbf{q}}(l) = \frac{i f}{(2\pi)^{3/2}} \frac{v_{\rho}(\mathbf{q}) f'}{m_N (2 \omega_{\rho, \mathbf{q}})^{1/2}} T_h \vec{S} \cdot \vec{a}(l), \quad (3.5)$$

where the matrix of coupling constants [in SU(6)] is

$$f = \frac{6}{5} \begin{pmatrix} 5 & 4\sqrt{2} \\ 4\sqrt{2} & 10 \end{pmatrix} = \begin{pmatrix} f^{nn} & f^{n\Delta} \\ f^{\Delta N} & f^{\Delta\Delta} \end{pmatrix}. \quad (3.6)$$

As in the work of Brown and Weise¹⁴⁾, we consider only the transverse coupling of the ρ -meson, which conventionally has the largest coupling constant,

$$f' = \frac{1}{2} g_{\rho} (1 + \kappa_v); \quad 0.4 < g_{\rho}^2 < 0.6; \quad \kappa_v = 3.7. \quad (3.7)$$

Of course the transverse coupling involves the vector $\vec{a}$ given by

$$\vec{a}(l) = \vec{q} \times \vec{\epsilon}(\vec{q}, l), \quad (3.8)$$

where $\vec{\epsilon}(\vec{q}, l)$ is the usual polarization vector for a non-relativistic, spin-1 particle satisfying

$$\sum_{l=1}^3 \left(\vec{\varepsilon}(\vec{q}, l) \right)_i \cdot \left(\vec{\varepsilon}^*(\vec{q}, l) \right)_j = \delta_{ij}, \quad (3.9)$$

(i, j = 1, 2, 3),

$$\vec{q} \cdot \vec{\varepsilon}(\vec{q}, l) = 0, \quad l = 1, 2. \quad (3.10)$$

The transition spins $\vec{T}$ and $\vec{S}$ are standard -- see, for example, Eqs. (2.3) in I. We also remark that the structure of the above model for the coupling of virtual ρ -mesons to N's and Δ 's is, essentially, the one considered by Niskanen¹⁷⁾.

Finally $v_\rho(q)$ is a new form-factor describing the coupling of the ρ -meson to the extended, composite nucleon. Unlike the pion coupling, which is completely determined by chiral symmetry, the coupling of the ρ -meson is not dictated by symmetry considerations. We can envisage two possible ways of describing this coupling, both of which lead to a relatively soft form-factor -- similar to that at the $NN\pi$ vertex. These arguments will be presented in Section 3.2. For the present we simply note that our expectations in this matter do agree with the findings of Niskanen in a phenomenological study of the ρ -meson on the width of the delta¹⁷⁾.

The total momentum operator in this theory is

$$\vec{P} = \vec{P}_R + \sum_{l=1}^3 \sum_{\vec{q}} \vec{q} b_{\vec{q}}^{\dagger}(l) b_{\vec{q}}(l), \quad (3.11)$$

and again one finds that $\vec{P}$ is a constant of the motion

$$[\vec{P}, H] = 0. \quad (3.12)$$

In order to determine the ρ -meson content of the physical nucleon, we generalize directly steps (a) through (d) of Section 2, by replacing $\vec{P}_R$ by $\vec{P}$, and using the following basis vectors with any numbers of π 's and ρ 's:

$$\psi(k_1, \dots, k_r; q_1 l_1, \dots, q_s l_s; \alpha; \vec{q}_{\pi}) = (s!)^{-1/2} b_{q_1}^{\dagger}(l_1) \dots b_{q_s}^{\dagger}(l_s) \psi(k_1, \dots, k_r; \alpha; \vec{q}_{\pi}) \quad (3.13)$$

The wave function $\psi(k_1, \dots, k_r; \alpha; \vec{q}_{\pi})$ is given in Eq. (2.6), but now $\vec{q}_{\pi} = \vec{\pi} - \sum_{i=1}^r \vec{k}_i - \sum_{i=1}^s \vec{q}_i$. The new restricted scalar product for these basis vectors is similar to the right-hand side of Eq. (2.7), but a new factor for ρ -mesons $[\delta_{ss}/s! \sum_{\text{permut.}} \delta^{(3)}(\vec{q}'_{\sigma(1)} - \vec{q}_1) \delta_{h\sigma(1)}; h_1 \delta_{\ell'_{\sigma(1)}, \ell_1}, \dots]$ has to be included. The normalized physical one-nucleon state $|\tilde{n}st; \vec{\pi}\rangle$ satisfies Eqs. (2.9), with $H_{R,CBM}$, $\vec{P}_R$ replaced by H , $\vec{P}$, and is given by an expansion similar to Eq. (2.8), with $\psi(k_1, \dots, k_r; \alpha; \vec{q}_{\pi})$ replaced by $\psi(k_1, \dots, k_r; q_1 l_1 \dots q_s l_s; \alpha; \vec{q}_{\pi})$. Now, we

are interested in the probability for finding r pions and s ρ 's in $|\tilde{n}st; \vec{\pi}\rangle$, which reads [compare with Eq. (2.10)]:

$$P_{r,s} = \sum_{\beta} \sum_{k_1 \dots k_r} \sum_{q_1 \dots q_s} \sum_{l_1 \dots l_s} \left| \langle \psi(k_1 \dots k_r; q_1 l_1 \dots q_s l_s; \alpha, \vec{q}_\pi | \tilde{n}st; \vec{\pi}) \rangle_{\vec{\pi}} \right|^2 \quad (3.14)$$

By generalizing the techniques in I, one finds:

$$P_{r,s} \leq \frac{\Lambda^r}{r!} \frac{\Lambda^s}{s!} e, \quad (3.15)$$

where Λ is given in Eq. (2.14), and

$$\Lambda_e = \sum_{\vec{q}} \sum_{\ell} \left\| \frac{\sum_{\alpha, \beta = N, \Delta} v_{e, \vec{q}}^{\alpha\beta} \exp(-i \vec{q} \cdot \vec{x}_{\alpha\beta})}{\omega_{e, \vec{q}}^2} \right\|^2, \quad (3.16)$$

which should be compared to Eq. (3.20) in I. Λ_e can be evaluated by extending the arguments given in Appendix B of I. By choosing linearly polarized $\vec{E}$'s and integrating over the angles of $\vec{q}$, one finds [compare with Eq. (B2) in I]

$$\begin{aligned} \sum_{\ell, \ell'=1}^3 \int d\Omega_{\vec{q}} \langle s_\alpha s | \vec{S} \cdot \vec{a}(\ell) | s_\gamma s' \rangle \langle s_\gamma s' | \vec{S} \cdot \vec{a}(\ell') | s_\beta s'' \rangle \\ = \frac{8\pi}{3} |\vec{q}|^2 \sum_{\ell=1}^3 \langle s_\alpha s | s_\ell | s_\gamma s' \rangle \langle s_\gamma s' | s_\ell | s_\beta s'' \rangle. \end{aligned} \quad (3.17)$$

After this, the calculation proceeds as in Appendix B in I, and one finds

$$\Lambda_e = \frac{57}{25} \frac{6 f'^2}{(2\pi)^2 m_\rho^2} \int_0^\infty \frac{dq q^4 v_e^2(q)}{\omega_{e, \vec{q}}^3}. \quad (3.18)$$

Similarly, the mean number of ρ 's and the uncertainty in the number of ρ 's are bounded as $\langle s \rangle \leq \Lambda_\rho$, $\Delta s \leq (\Lambda_\rho^2 + 1/4)^{1/2}$.

To obtain a numerical estimate of Λ_ρ we use one of the form-factors of Niskanen¹⁷⁾, namely:

$$v_e(\vec{q}) = \frac{\Lambda_0^2 - m_\rho^2}{\Lambda_0^2 + \vec{q}^2}, \quad (3.19)$$

with $\Lambda_0 = 1$ GeV. This leads to the result $\Lambda_\rho = 0.037$, which is extremely small. Using the somewhat larger value of $K_V = 6.7$, this would be $\Lambda_\rho = 0.10$ -- again a very small number. Of course this would increase rapidly if the cut-off Λ_0 were increased, but both Niskanen's phenomenological results¹⁷⁾, and the arguments given in the next section, strongly suggest that our choice is most reasonable.

3.2 Discussion of rho-meson coupling to an extended nucleon

In the previous section we suggested that one might *a priori* expect a relatively soft form factor at the ρNN , $\rho N\Delta$ vertices -- in agreement with Niskanen's phenomenological findings¹⁷⁾. Suppose, for example, we couple the ρ -meson directly to the quarks. As the quarks are structureless there is no $\sigma^{\mu\nu}$ term, but we can write a pure vector coupling, $\gamma^\mu \rho_\mu$. After taking bag model matrix elements this will reduce to the general form given in Eqs. (3.5)-(3.8). The key point with regard to the form factor is that one does not expect the $\bar{q}q$ constituents of the ρ -meson to propagate as a coherent pair inside the baryon. Indeed the one-gluon exchange interaction is quite strongly repulsive for this case^{18,19)} -- unlike the case of the pion^{2,20)}. Thus it seems much more appropriate to put the $\bar{q}q$ pair into bag eigenstates once the ρ -meson and baryon overlap.

In that case the phenomenological ρ -meson field should vanish inside the baryon bag radius and one naturally finds a Cloudy Bag type of form factor for the ρ , viz.

$$v_\rho(\vec{q}) = 3 j_1(qR) / qR, \quad (3.19)$$

(where R is the radius of the baryon bag) which for practical purposes can be approximated as²¹⁾

$$v_\rho(\vec{q}) \approx \exp(-0.106 \vec{q}^2 R^2). \quad (3.20)$$

This can be compared with the best fit form factor of Niskanen¹⁷⁾

$$v_\rho^{Niskanen}(\vec{q}) = \exp(-0.077 [m_\rho^2 + \vec{q}^2]), \quad (3.21)$$

with m_ρ and q in fm^{-1} corresponding to $R \sim 0.82 \text{ fm}$.

An alternate possibility would be to allow the ρ -meson to couple to the bag only through a two-pion intermediate state. Using the Gaussian approximation (3.20), it is relatively easy to show that if we do not exclude the ρ -meson from the hadronic bag $R \rightarrow R_{\text{eff}} = R/\sqrt{2}$ -- that is the ρ form factor becomes a little harder. However, the most probable effect of allowing the pions to form a ρ only outside the nucleon bag would be to soften it again.

So far our discussion has only served to explain the choice of ρNN and $\rho N\Delta$ form factors in the numerical work of Section 3.1. However, there is an even more fundamental issue, namely whether it makes sense to talk about a cloud of virtual ρ -mesons at all. Since the typical range of such a cloud is

$(m_\rho)^{-1} \sim 0.2$ fm, the answer to this question is probably no. The static surface of the MIT bag model is a phenomenological simplification of the true dynamical situation. One naturally expects a hadron to have some surface thickness, and a few tenths of a fermi is not an unreasonable scale for such a transition region -- see, for example, the soliton bag models^{22,23}). On this sort of scale [unlike the 1.4 fm ($\equiv \lambda_\pi$) associated with a virtual pion] there is no clean separation between bag model phenomenology and the meson cloud. Consequently it may be far more meaningful to simply deal with such effects as isovector $\bar{q}q$ fluctuations in baryon-bag eigenstates. Clearly it will not be possible to resolve this question completely until the problem can be formulated quantitatively. Nevertheless, it is reassuring that in either extreme such effects are relatively small, as the comparison of the estimates in Ref. 17 with ours suggests.

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