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THE TWISTED EGUCHI-KAWAI MODEL FAILS TO REPRODUCE
THE WEAK COUPLING OF THE WILSON MODEL

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A B S T R A C T

We perform a saddle point analysis of the twisted Eguchi-Kawai model in 2, 3 and 4 dimensions. Our conclusion is that this model does not reproduce the usual Wilson model results in the weak coupling region at the large N limit. However, we stress that the deviations between the two models are small and decrease with increasing dimension of space.

1. INTRODUCTION

A recent major progress in the analysis of the large N behaviour of lattice gauge theories has been accomplished through the reduction technique proposed by Eguchi and Kawai ¹⁾. Unfortunately, this method, while appropriate in order to describe the strong coupling regime of the Wilson theory, fails to reproduce the weak coupling phase because of the spontaneous breakdown of the $[U(1)]^d$ symmetry of the reduced Lagrangian ^{2),3)}.

An effective, but computationally unpleasant way of eliminating this trouble is the quenching procedure ²⁾. It would be appealing to consider modifications of the model preserving the original simplicity of the Eguchi-Kawai (EK) reduction. The twisted model proposed by Gonzales Arroyo and Okawa ⁴⁾ is therefore especially attractive.

It is, however, necessary to check that the weak coupling behaviour of this model does actually reproduce the weak coupling of the Wilson theory and no phenomena such as the collapse of eigenvalues identified by Bhanot et al. ²⁾ in the EK model do occur in the twisted theory.

We investigated the weak coupling expansion of the twisted Eguchi-Kawai (TEK) model and came to the rather unpleasant conclusion that, albeit in much less dramatic form, the collapse of (pseudo) eigenvalues occurs also in the twisted case, and the leading weak coupling behaviour of the Wilson theory is not exactly reproduced. The deviation is, however, smaller than in the original EK model, and in particular we notice that the leading term in the TEK weak coupling tends to mimic the behaviour of small N $SU(N)$ theories. The values of N for which the approximation is sensible grow with the dimensionality of space-time (roughly like $2^{d/2}$ for the $z_{\mu\nu} = -1$ twist), which means that the TEK scheme could be a sensible approximation in a large d , large N expansion.

This paper is organized as follows : we first show how the TEK model works in exactly soluble two-dimensional cases $[SU(2)$ and large $N]$ then perform the computation of the corrections due to the quadratic integration in the three- and four-dimensional case, showing explicitly the phenomenon we have discussed above.

2. NOTATIONS AND GENERAL RESULTS

The partition function for the TEK model ⁴⁾ in d dimensions of space time is

$$Z_{\text{TEK}} = \int \prod_{k=1}^d dU_k \exp S_{\text{TEK}} \quad (2.1)$$

$$S_{\text{TEK}} = \beta N \sum_{\substack{i,j=1 \\ i \neq j}}^d t_{ij} z_{ij} U_i U_j U_i^+ U_j^+ \quad (2.2)$$

where U_i denotes the matrices in the gauge group $(U(N))$, dU denote the Haar measure over this group and z_{ij} are twist phases

$$z_{ij} = \exp \frac{2i\pi}{N} n_{ij}, \quad n_{ij} = -n_{ji}, \quad n_{ij} \in \mathbb{Z}_N$$

In order to study the TEK model in the weak coupling region (i.e., for large β) by a saddle point method, a first step is to construct the configurations of matrices U_i^0 which minimize the action (2.2); it can be shown that such matrices necessarily obey

$$U_i^0 U_j^0 = z_{ij}^* U_j^0 U_i^0 \quad (2.3)$$

Then, considering fluctuations around these solutions

$$U_j = U_j^0 A_j \quad (2.4)$$

$$A_j = 1 + i a_j - \frac{1}{2} a_j^2 + \dots, \quad a_j = a_j^+ \quad (2.5)$$

the second step is to compute the quadratic order in a_j and emphasize the possibility of performing Gaussian integrations in the variables a_j .

Using the formulae (2.2)-(2.5), we have obtained the following result where the phases of the twists have been removed by use of (2.3)

$$S_{\text{TEK}} = \beta N \left\{ d(d-1)N + t \sum_{i,j=1}^N [(U_i^0 a_j U_i^{0+} a_j - a_i^2) + (i,j) + (a_i - U_i^0 a_i U_i^{0+})(a_j - U_j^0 a_j U_j^{0+})] \right\} \quad (2.6)$$

The choice of the relation

$$\sum_{i=0}^d (a_i - U_i^0 a_i U_i^{0*}) = 0 \quad (2.7)$$

as a gauge fixing term greatly simplifies (2.6); this left us with the relevant action below :

$$S_{TEK} = \beta N \left\{ d(d-1)N + 2 \ln \sum_{i,j=1}^d (U_i^0 a_j U_i^{0*} a_j - a_j^2) \right\} \quad (2.8)$$

where the sum has to be taken over all couples of indices i, j . This gauge fixing function (2.7) leads to a Faddeev-Popov determinant which plays a major rôle in the final result.

In the following, we will treat in detail the cases $d=2, 3$ and 4 with the twist

$$Z_{ij} = -1 \quad i, j = 1, \dots, d \quad (2.9)$$

As pointed out in Ref. 4) this choice of twists has a direct physical meaning since it can be seen as a change of the sign of the coupling constant β .

3. $d=2$

An amusing feature of the two-dimensional case is the possibility of computing explicitly the partition functions in all the models of interest to our discussion ⁵⁾.

In particular, if we consider the case $N=2$ we have four possibilities: Wilson $U(2)$ model (WU), Wilson $SU(2)$ model (WS), Eguchi-Kawai model (EK) and twisted model (TEK).

Due to the factorization property of two-dimensional lattice gauge theories, we only need to compute the one-plaquette partition functions. We collect here the results, recalling that $I_k(x)$ are the Bessel functions of imaginary argument and $U = -(\partial \ln z) / \partial \beta$

$$Z_{WU} = I_0^2(4\beta) - I_1^2(4\beta) \quad U_{WU} = \frac{2 I_2^2(4\beta)}{\beta(I_0^2(4\beta) - I_1^2(4\beta))} \quad (3.1)$$

$$Z_{WS} = \frac{I_1(8\beta)}{4\beta} \quad U_{WS} = \frac{8I_0(8\beta)}{I_1(8\beta)} - \frac{2}{\beta} \quad (3.2)$$

$$Z_{EK} = \frac{e^{8\beta} - I_0(8\beta)}{8\beta} \quad U_{EK} = \frac{8(e^{8\beta} - I_1(8\beta))}{e^{8\beta} - I_0(8\beta)} - \frac{1}{\beta} \quad (3.3)$$

$$Z_{TEK} = \frac{I_0(8\beta) - e^{-8\beta}}{8\beta} \quad U_{TEK} = \frac{8(I_1(8\beta) + e^{-8\beta})}{I_0(8\beta) - e^{-8\beta}} - \frac{1}{\beta} \quad (3.4)$$

These results are best analyzed through their graphical plot (Fig. 1).

It is apparent that, while the EK model is quite different from both Wilson models, the TEK model converges rapidly to the SU(2) Wilson expectation value in the weak coupling region ($\beta \gg 1$).

A direct $1/\beta$ expansion of the results shows that a difference between TEK and SU(2) emerges only at $O(\beta^{-2})$. This is only a first hint of a more general trend.

In the EK model $d=2$ turned out to be a critical value for the dimensionality. A straightforward analysis shows that the same happens in the twisted model. Actually, by considering the weak coupling expansion around the classical anticommuting saddle points, one verifies that there is neither attraction nor repulsion among the phases characterizing the classical solutions, and the large N limit may therefore be recovered in a way similar to the one employed originally by Eguchi and Kawai :

$$\begin{aligned} Z_{TEK} &= \int dV \sum_n \frac{\chi_n(V)}{d_n} \exp - N\beta \, t(V+V^*) \\ &= \int dV \sum_n (-1)^{|r|} \frac{\chi_n(V)}{d_n} \exp N\beta \, t(V+V^*) \end{aligned} \quad (3.5)$$

where χ_r are the characters of the irreducible representations of $U(N)$ and d_r the corresponding dimensionality; $|r|$ is the number of boxes in the Young tableau. The same expression with the factor $(-1)^{|r|}$ removed represents Z_{EK} .

Therefore the summation is more convergent in the twisted model. This fact is reflected in the weak coupling behaviour :

$$\ln Z_{TEK} = N^2 \left(2\beta - \frac{1}{2} \left(1 - \frac{1}{2N} \right) \ln \beta \right) + O(\beta^{-1}) \quad (3.6)$$

to be compared with

$$\ln Z_{EK} \cong N^2 \left(2\beta - \frac{1}{2} \left(1 - \frac{1}{N} \right) \ln \beta \right) \quad (3.7)$$

$$\ln Z_{WU} \cong N^2 \left(2\beta - \frac{1}{2} \ln \beta \right) \quad (3.8)$$

$$\ln Z_{WS} \cong N^2 \left(2\beta - \frac{1}{2} \left(1 - \frac{1}{N^2} \right) \ln \beta \right) \quad (3.9)$$

4. d = 3

In three dimensions, Eqs. (2.3) and (2.9) can be satisfied only if the matrices U_i^0 are even-dimensional, so the gauge group is restricted to $U(2K)$ or $SU(2K)$. The matrices U_j^0 can be written in terms of the Pauli 2x2 matrices in a suitable gauge :

$$U_j^0 = D_j \otimes \sigma_j \quad , \quad j = 1, 2, 3 \quad (4.1)$$

$$D_j^{ab} = \delta_{ab} \exp i \theta_j^a \quad , \quad a, b = 1, \dots, K \quad (4.2)$$

in Eq. (4.1)-(4.2) D_j are arbitrary diagonal matrices in $U(K)$, so the saddle point solutions depend on $3K$ [respectively $3(K-1)$] arbitrary phases in $U(2K)$ [respectively $SU(2K)$].

The decomposition (2.4) factors out of the matrices U_j their part in the subgroup $U(1)^K$ defined by elements

$$1_i \otimes \text{diag} (\exp i \varphi_1, \dots, \exp i \varphi_K) \quad (4.3)$$

the part A_j being in the right coset space $U(K)/U(1)^K$.

In agreement with this decomposition, the Haar measure dU_i will factorize into the flat measure on $U(1)^K$ times the measure on the coset.

It is convenient to parametrize the generators a_j [see (2.5)] in 2×2 dimensional block matrices in the following way :

$$a_j = \sum_{\rho=0}^3 a_{j\rho} \tau_{\rho} \quad , \quad \tau_{\rho} = (1, \sigma_1, \sigma_2, \sigma_3) \quad (4.4)$$

$$a_j^{ab, \alpha\beta} = \sum_{\rho=0}^3 a_{j\rho}^{ab} \tau_{\rho}^{\alpha\beta} \quad , \quad a, b = 1, \dots, \kappa \\ \alpha, \beta = 1, 2. \quad (4.5)$$

The Hermiticity condition is satisfied if $a_{ip}^{ab} = (a_{ip}^{ba})^*$; however, in order for the A_j to remain in the coset space, we have to put

$$a_{io}^{bb} = 0 \quad , \quad b = 1, \dots, \kappa \quad (4.6)$$

These degrees of freedom are taken into account by the integration over the arbitrary phases θ_j^a .

Straightforward calculations then lead to the expression above for the determinant of the quadratic form in $a_i a_j$ (Δ_M) and for the Faddeev-Popov determinant (Δ_{FP}) due to the gauge fixing (2.7)

$$\Delta_M = \beta^A \Delta^d \quad , \quad A = d \left(\frac{4\kappa^2 - \kappa}{2} \right) \quad (4.7)$$

$$\Delta_{FP} = \beta^B \Delta^2 \quad , \quad B = \frac{4\kappa^2 - \kappa}{2} \quad (4.8)$$

$$\Delta = \prod_{a < b} \left\{ \left(\sum_{i=1}^d \sin^2 \theta_i^{ab} \right) \prod_{k=1}^d \left(\sum_{\substack{i=1 \\ i \neq k}}^d \cos^2 \theta_i^{ab} + \sin^2 \theta_k^{ab} \right) \right\} \quad (4.9)$$

with

$$\theta_i^{ab} = \frac{\theta_i^a - \theta_i^b}{2}$$

Before going further, let us make two remarks :

- i) to compute Δ_{FP} , we have used the transformation of A_μ [see (2.4)] under a gauge transformation Ω ; infinitesimally, this transformation law is written, for instance,

$$a'_\mu = a_\mu - i\omega + i U_\mu^{\sigma\dagger} \omega U_\mu^\sigma \quad (4.10)$$

- ii) both Δ_M and Δ_{FP} have zeros which appear in different classes; given a pair a, b ($a \neq b$), the zeros of Δ occur if

$$\theta_i^a = \theta_i^b \quad i = 1, 2, 3 \quad (4.11a)$$

$$\begin{aligned} \theta_i^a &= \theta_i^b + \pi & i = 1, 2, 3; \quad i \neq k \\ \theta_i^a &= \theta_i^b & i = k \end{aligned} \quad (4.11b)$$

The zeros are unavoidable; they occur especially at values of the phases θ_i^a so that all the associated U_i^0 commute with one generator of the gauge group which does not belong to $U(1)^K$.

So far, the effective measure on the minimum action manifold can be written

$$\begin{aligned} d\mu(\theta) &= \prod_{j=1}^3 \prod_{a=1}^K d\theta_j^a \frac{\Delta_{FP}}{\Delta_M} \\ &= \beta^{(2-d)(\frac{4K^2}{2}-K)} \prod_{j=1}^3 \prod_{a=1}^K d\theta_j^a \Delta^{2-d}, \quad (d=3) \end{aligned} \quad (4.12)$$

We see that this effective measure suffers the same singularity problem as its analogue on the non-twisted EK model ^{2),3)}. However, in the TEK model, the situation is less dramatic because, whatever the values of the phases, a non-zero number of Gaussian integrations can be performed despite the singularities.

In the case of equal phases [see (4.11a)], this number is equal to $3(K)^2$, it corresponds to integration over the a_{ip}^{ab} , $p \neq 0$; the number of quartic integrations being $K(K-1)$ as justified in the following. Therefore we argue that the leading contribution to the free energy $F(\beta)$ ($= \ln Z(\beta)$) is, for $SU(N)$

$$\begin{aligned}
 F_{TEK}(\beta) &= 6N^2\beta - 2 \left[3 \left(\frac{N}{2} \right)^2 + \frac{1}{2} \cdot \frac{N}{2} \cdot \left(\frac{N}{2} - 1 \right) \right] \frac{\ln \beta}{2} + O\left(\frac{1}{\beta}\right) \\
 &= 6N^2\beta - \frac{7N^2 - 2N}{8} \ln \beta + O\left(\frac{1}{\beta}\right)
 \end{aligned} \tag{4.13}$$

a comparison with the Wilson theory analogue of this function

$$\begin{aligned}
 F_W(\beta) &= d(d-1) N^2\beta - \frac{c}{2} (d-1) \ln \beta + O\left(\frac{1}{\beta}\right) \\
 c &= N^2 \text{ in } U(N) \quad \text{and} \quad N^2 - 1 \text{ in } SU(N)
 \end{aligned} \tag{4.14}$$

shows that agreement is rather good and is full in $SU(2)$.

5. d = 4

As pointed out in Ref. 4), Eqs. (2.3) and (2.9) can be satisfied in $d=4$ iff the dimension of the gauge group $U(N)$ is a multiple of 4. ($N=4K$, $K=1,2,\dots$). Then, the minimizing configurations U_j^0 are, up to a gauge transformation given by

$$U_j^0 = D_j \otimes \gamma_j, \quad j=1,\dots,4 \tag{5.1}$$

In (5.1) γ_j are 4×4 Dirac matrices in some representation which we have chosen to be Hermitian and D_j are as in (4.2). We have found it convenient to parametrize a_i in (2.5) in terms of the 16 matrices Γ_ρ of the algebra :

$$\Gamma_\rho = \mathbb{I}, \gamma_j, \frac{i}{2} [\gamma_i, \gamma_j], i \gamma_5 \gamma_j, \gamma_5 \tag{5.2}$$

$$\gamma_5 = \gamma_1 \gamma_2 \gamma_3 \gamma_4, \quad \Gamma_\rho^\dagger = \Gamma_\rho, \quad \epsilon \gamma_\rho \gamma_\sigma = 4 \delta_{\rho\sigma} \tag{5.3}$$

In complete analogy with the $d=3$ case, we put

$$a_i = \sum_{\rho=0}^{15} a_{i\rho} \Gamma_\rho \tag{5.4}$$

$$a_i^{a\ell, \alpha\beta} = \sum_{\rho=0}^{15} a_{i\rho}^{a\ell} \Gamma_\rho^{\alpha\beta} \quad \begin{array}{l} \alpha, \beta = 1, \dots, 4 \\ a, \ell = 1, \dots, K \end{array} \tag{5.5}$$

$$a_{ip}^{ab} = (a_{ip}^{ba})^* \quad (5.6)$$

$$a_{i0}^{bb} = 0 \quad , \quad b = 1, \dots, K \quad (5.7)$$

this last formula being necessary in order for A_μ to stay in the coset $U(4K)/U(1)^K$. Using these notations, the quadratic part of the action (2.8) can be written as follows

$$\begin{aligned} & t \sum_{i,j=1}^d (a_i U_j a_i U_j^\dagger - a_i^2) \\ &= \sum_{p,\sigma=0}^{15} \sum_{i,j=1}^d a_{i\sigma}^{ab} a_{j\sigma}^{ba} t (\Gamma_p D_j \delta_j \Gamma_\sigma D_j^\dagger \Gamma - \Gamma_p \Gamma_\sigma) \\ &= \sum_{p,\sigma=0}^{15} \sum_{i,j=1}^d a_{i\sigma}^{ab} a_{j\sigma}^{ba} (\exp i(\theta_j^b - \theta_j^a) \cdot t (\Gamma_p \delta_j \Gamma_\sigma \delta_j) - t (\Gamma_p \Gamma_\sigma)) \end{aligned} \quad (5.8)$$

Using the normalization (5.3) of the Γ matrices and the identities among traces of Γ matrices of the Appendix, we have obtained the Δ_M and Δ_{FP} determinants :

$$\Delta_M = \beta^A \Delta^d \quad A = \frac{d(16K^2 - K)}{2} \quad (5.9)$$

$$\Delta_{FP} = \beta^B \Delta^2 \quad B = \frac{16K^2 - K}{2} \quad (5.10)$$

$$\begin{aligned} \Delta = \prod_{a < b} & \left\{ \left(\sum_{i=1}^4 \sin^2 \theta_i^{ab} \right) \left(\sum_{i=1}^4 \cos^2 \theta_i^{ab} \right) \right. \\ & \prod_{k=1}^4 \left(\sum_{\substack{i=1 \\ i \neq k}}^4 \sin^2 \theta_i^{ab} + \cos^2 \theta_k^{ab} \right) \left(\sum_{\substack{i=1 \\ i \neq k}}^4 \cos^2 \theta_i^{ab} + \sin^2 \theta_k^{ab} \right) \\ & \left. \prod_{k < l} \left(\sum_{\substack{i=1 \\ i \neq k \\ i \neq l}}^4 \sin^2 \theta_i^{ab} + \cos^2 \theta_k^{ab} + \cos^2 \theta_l^{ab} \right) \right\} \end{aligned} \quad (5.11)$$

In this case, again zeros of Δ in (5.11) are present and lead to a singular effective measure on the minimum action manifold. The conclusions we draw for the three-dimensional case are still valid; in particular, we can now perform $15 K^2$ quadratic integrations and $K(K-1)$ quartic ones. These numbers improve the leading contribution to the free energy; it is now for $SU(N)$

$$\begin{aligned} F_{\text{TEK}}(\beta) &= 12 N^2 \beta - 3 \left[15 \left(\frac{N}{4} \right)^2 + \frac{1}{2} \left(\frac{N}{4} \right) \left(\frac{N}{4} - 1 \right) \right] \frac{\ln \beta}{2} + O\left(\frac{1}{\beta}\right) \\ &= 12 N^2 \beta - \frac{3}{2} \left[\frac{37}{32} N^2 - \frac{N}{8} \right] \ln \beta + O\left(\frac{1}{\beta}\right) \end{aligned} \quad (5.12)$$

and it can be seen from (4.14) that it coincides with the Wilson theory in $SU(4)$.

6. CONCLUSIONS

The appearance of undesirable zeros (not removed by the Faddeev-Popov determinant) in the coefficients of the quadratic form appears to be an unavoidable feature of the TEK model in more than two dimensions and for the twists $z_{ij} = -1$. An analysis of higher orders of weak coupling perturbation theory (which we performed explicitly for $d=3$) shows us that, focusing on the saddle $\theta_\mu^i = 0$, all cubic interaction terms containing a_{i0} vanish in the background gauge and the first term containing contributions from a_{i0} is the quartic form

$$\hbar (a_{i0}^2 a_{j0}^2 - a_{i0} a_{j0} a_{i0} a_{j0}) \quad (6.1)$$

Therefore the rescaling of a_{i0} contributes a factor $-\frac{1}{4} \ln \beta$ in the free energy for each degree of freedom, which justifies our previous formulae.

As for the reliability of these results, we mention that we have not taken into account the non-trivial zeros, but this might offset only the non-leading $O(N)$ terms, and we believe the $O(N^2)$ coefficient is fixed by our analysis. We believe that we can extrapolate to larger d with some confidence, thus obtaining

$$\begin{aligned} F &= d(d-1) N^2 \beta - \left(\frac{d-1}{2} \right) \left[(2^d - 1) \left(\frac{N}{2^{\frac{d}{2}}} \right)^2 + \frac{1}{2} \left(\frac{N}{2^{\frac{d}{2}}} \right) \left(\frac{N}{2^{\frac{d}{2}}} - 1 \right) \right] \ln \beta \\ &= d(d-1) N^2 \beta - \frac{d-1}{2} \left(N^2 \left[1 - \frac{1}{2^{\frac{d}{2}+1}} \right] - \frac{N}{2^{\frac{d}{2}+1}} \right) \ln \beta \end{aligned} \quad (6.2)$$

We stress that the deviation from the exact Wilson result is exponentially damped for large d . For the $d=4$ case, the deviation is $\frac{1}{32}$, a figure that does not disagree with the numerical Monte-Carlo plots presented in the literature ⁴⁾. As a by-product of Eq. (6.2), we observe that by focusing on the group $N=2^{d/2}$ we find $O(1/\beta)$ agreement in weak coupling between $SU(2^{d/2})$ and the corresponding TEK model. This is a completely reliable result, due to the fact that there is only one anticommuting solution in d dimensions when $N=2^{d/2}$ and no collapse of phases. This observation also constitutes an independent check of the statement that the TEK scheme becomes rapidly exact in larger space-time dimensions.

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A P P E N D I X

Here are the identities we have used to compute (5.9)

$$\frac{1}{4} \text{tr} (\Gamma_\rho \gamma_j \Gamma_\sigma \gamma_j) = \quad \text{if}$$

$\begin{aligned} &1 \\ &\delta_{\rho\sigma} (-1 + 2 \delta_{\rho j}) \\ &(\delta_{\mu\lambda} \delta_{\nu\theta} - \delta_{\mu\theta} \delta_{\nu\lambda}) (1 - 2 \delta_{j\mu} - 2 \delta_{j\nu}) \end{aligned}$	$\begin{aligned} &\Gamma_\rho = \Gamma_\sigma = 1 \\ &\Gamma_\rho = \gamma_\rho, \Gamma_\sigma = \gamma_\sigma \\ &\left\{ \begin{aligned} \Gamma_\rho &= \frac{i}{2} [\Gamma_\mu, \Gamma_\nu] \\ \Gamma_\sigma &= \frac{i}{2} [\Gamma_\lambda, \Gamma_\theta] \end{aligned} \right. \end{aligned}$
$\begin{aligned} &\delta_{\rho\sigma} (1 - 2 \delta_{\rho j}) \\ &- 1 \\ &0 \end{aligned}$	$\begin{aligned} &\Gamma_\rho = i \gamma_5 \gamma_\rho, \Gamma_\sigma = i \gamma_5 \gamma_\sigma \\ &\Gamma_\rho = \Gamma_\sigma = \gamma_5 \\ &\text{otherwise.} \end{aligned}$

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FIGURE CAPTION

Plot of $U(\beta)$ versus β . We have, respectively
[see (3.1) and (3.4)]. A : U_{WU} , B : U_{TEK} ,
C : U_{WS} , D : U_{EK} .