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COMMENT ON THE PAPER : "NECESSARY CONDITIONS ON POTENTIAL FUNCTIONS
FOR NON-RELATIVISTIC BOUND STATES" BY GERALD ROSEN

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We were astonished to discover that the recent paper of G. Rosen ¹⁾ with the above-mentioned title contains no new result.

The sufficient condition for the absence of bound state in a three-dimensional potential

$$\int |V|^{3/2} d^3x < \frac{3\sqrt{3}}{4\pi^2} \quad (\text{units } \hbar = 2m = 1) \quad (1)$$

was established in 1976 by H. Grosse, W. Thirring and ourselves ²⁾ and tested on various examples. Reference 2) presents a self-contained proof of the optimality of the relevant Sobolev inequality as well as one based on the old result of Bliss ³⁾. The previous work of Rosen ⁴⁾ does not contain a complete proof since spherical symmetry is postulated without the justification which we supplied in Ref. 2) and since it is only the existence of a local minimum which is established.

The second result of Ref. 1) is not original either. It is a bound on the binding energy of the form

$$|E| < C_\gamma \left(\int |V|^{3\gamma/2} d^3x \right)^{\frac{2}{3(\gamma-1)}} \quad (2)$$

This inequality is a particular case of inequality (25) in the paper that we wrote in collaboration with H. Grosse ⁵⁾.

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This being said, let us indicate what kind of results in this area of physics would, in our opinion, justify a publication in Phys.Rev. Letters, i.e., "be important and deserve urgent publication". It would be :

- i) a proof that the total number of bound states N , in three dimensions, satisfies

$$N < \frac{4\pi^2}{3\sqrt{3}} \int |V|^{3/2} d^3x, \quad (3)$$

i.e., an inequality with the same constant as in the one-bound state case, while the best numerical constant obtained by Lieb ⁶⁾ is somewhat larger ;

- ii) a generalization of the above result to n dimensions and a proof (or a destruction) of the conjecture made in Ref. 5), which replaces the conjecture of Lieb and Thirring ⁷⁾ which turned out to be wrong ;
- iii) a proof that the sum of the binding energies, in absolute value, is given by the semi-classical estimate, a result which would bring simplification and improvement in the proof of stability of matter by Lieb and Thirring ⁸⁾.

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