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THE DEUTERON ASYMPTOTIC D STATE AS A PROBE
OF THE NUCLEON-NUCLEON FORCE

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A B S T R A C T

A critical discussion is given of the various contributions to the asymptotic D/S ratio from the πNN interaction. Particular emphasis is laid on the analysis of contributions from 2π exchange, the dependence on the πNN form factor, and on the πN coupling constant, so as to assess quantitatively the model-dependence of the result. From this analysis, significant restrictions are deduced on the shape of the πNN form factor and, in quark bag models, the value of the quark bag radius.

The quadrupole moment is also quantitatively analyzed in detail. Its value is proved to be an accurate and almost model-independent consequence of the experimental D/S ratio (apart from relativistic and meson-exchange corrections) in agreement with observations.

In the comparison with experiments, a critical survey of the relevant experimental data is given. In particular, a considerably improved value for the quadrupole moment is deduced.

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1. INTRODUCTION

An outstanding issue in present nuclear physics concerns how quark degrees of freedom can be made compatible with the classical nucleonic and mesonic description of nuclear forces and nuclear properties. The popular way to approach the problem is via phenomenological bag models in which the quarks in nucleons are assumed to move freely within a radius of approximately 1 fm. Since such models in the original MIT bag version¹⁾ do not incorporate the important special features associated with the low pion mass, improvements have led to chiral bag descriptions²⁻⁴⁾ with a somewhat smaller radius. When such nucleons are present inside the nuclei, two problems arise associated with the bag size. First, the interchanged pions will have a form factor associated with the bag size; this problem is no more difficult than the usual question of the range of the πNN vertex. Its effect is already implicitly incorporated in meson exchange models and associated with the 3π cut. The second and far more crucial problem is that at nuclear densities, i.e. at internuclear spacings of the order of 1 fm, the bags will either normally interpenetrate or at least do so frequently. The quarks can then fluctuate from one bag to another, and one would expect substantial modifications of the properties of "nucleons" in nuclei to occur, as compared to free nucleons. On the other hand, a large number of properties of nuclei are very nucleonic. Suffice it to mention β -decay, μ -capture, magnetic moments (Schmidt lines), multiple-scattering descriptions of various kinds both at lower and higher energies, etc., all providing evidence that nucleons inside nuclei are largely unmodified.

This evidence is convincing to a nuclear physicist, but is not in a form which makes it possible to quantify the restriction imposed on quark bag models. Furthermore, it is the normal development in science that a successful alternative in the approach permits a description of the previous knowledge as the result of approximations. It is thus crucial to establish quantitatively the specific conditions in which previous meson descriptions are valid for cases with indisputable interpretation. This requires a simple nuclear system amenable to high-precision

analysis. Only by such input can the nature of the bag-size problem be made quantitative enough to have a constructive impact on the nuclear physics discussion.

In this context the deuteron properties acquire exceptional importance, both because of the high-precision experimental data and of the clearness as to their exact information. In addition the deuteron is the simplest nuclear system from a theoretical view point. We have found that the asymptotic D/S ratio η is of particular significance in this context, as well as the closely related deuteron quadrupole moment Q .

In a recent brief communication⁵⁾, we demonstrated that inside the usual meson-exchange approach the D/S ratio can be described almost model-independently as being due to the pionic tensor force and its repulsive iterated effects in the D state. In fact, the precision of the description was mainly limited by the experimental knowledge of the πNN coupling constant. In order to achieve this controllable result, we introduced a Green function technique not previously used in the discussion of the NN interactions.

First we will give a detailed description of the method (section 2). The remarkable feature of our previous result was that only 30% of the total contribution to the D/S ratio comes from the region $1 < r < 2$ fm, with only a few per cent from the region inside 1 fm. At a precision of a few per cent and in agreement with experiments, it is thus extremely relevant that a very important contribution comes exactly from the region in which quark bags are supposed to overlap, while at the same time the uncontrollable short-range region is eliminated.

In order to further convince the reader of the stability of the result to assumptions, in sections 3 and 4 we will strengthen the previous argument for model independence. We analyse in detail the main sources of uncertainty. These are mainly the contribution of the 2π exchange to the nuclear potential, the uncertainty on the πNN coupling constant, and the dependence on the πNN form factor. The first two can be controlled to considerable precision, while the potential uncertainty from the last one is larger. The experiments impose rather rigorous constraints on the possible values of the form factor.

In this way we also establish a clear challenge to quark models which should be capable of giving an alternative and equivalent description. The restrictions on the πNN form factor are also amenable to independent experimental tests.

The only previous dedicated approach to the D/S ratio is the dispersive approach by Wong⁶). We show in Appendix II that his result is equivalent to a Born approximation.

As a related second test case, in section 5 we investigate the deuteron quadrupole moment. Inside any local potential description, only very limited additional information on the D/S structure function is needed so that Q follows almost model-independently from experimental data on η and on the asymptotic normalization of the S-wave. For this reason any discrepancy between theory and experiment for Q is evidence for relativistic or π -exchange effects provided the theoretical D/S ratio agrees with experiment.

None of the discussions will have any relevance if the experimental comparison material cannot be brought up to a similar standard of precision. We have therefore taken the opportunity to make a critical review of the relevant experimental data in Appendix I. In particular we give a recommended value of the D/S ratio. It also came as a pleasant surprise to discover that the quadrupole moment can be deduced from experiments to at least a factor of five higher precision than that quoted in the literature so that it is now known to one part per thousand (1 ppt).

2. FORMAL EXPRESSION FOR THE D/S RATIO

A very important property of the low-energy NN interaction is the possibility of expressing it to a very considerable precision in terms of static potentials. This formulation is particularly convenient when the interaction must be iterated. The modifications to this approach will occur as corrections which can be investigated separately. We will therefore adopt this approach here. The standard way of solving the potential equation is by straightforward integration of the differential equations. In the present problem it is vital to have clear control

of the uncertainties due to various aspects of the potential. As we demonstrated in our brief communication⁵⁾, a Green function approach is ideally suited to this purpose. Since NN interactions are not usually discussed using this method, we will for convenience describe it in some detail below.

With the potential assumption, the deuteron S- and D-wave functions are given by the usual coupled equations:

$$u''(r) = [\alpha^2 + U_{00}(r)]u(r) + U_{02}(r)w(r), \quad (1a)$$

$$w''(r) = [\alpha^2 + 6/r^2 + U_{22}(r)]w(r) + U_{20}(r)u(r). \quad (1b)$$

Here α is the appropriate wave number and $U = MV$. The potentials V_{LL} , for the S- and D-states are linear combinations of the central, tensor, spin orbit, and quadratic spin orbit potentials V_C , V_T , V_{LS} , V_{LL} (or V_{SO^2}):

$$V_{00} = V_C, \quad V_{02} = V_{20} = \sqrt{8} V_T, \quad V_{22} = V_C - 2V_T - 3V_{LS} - 3V_{LL} \quad (2)$$

(where alternatively $V_{LL} = -3 V_{SO^2}$).

The asymptotic wave functions for large r are

$$u(r) \xrightarrow{r \rightarrow \infty} \sqrt{2\alpha} N e^{-\alpha r}, \quad (3a)$$

$$w(r) \xrightarrow{r \rightarrow \infty} \eta \sqrt{2\alpha} N [1 + 3/(\alpha r) + 3/(\alpha r)^2] e^{-\alpha r}. \quad (3b)$$

By definition the asymptotic S-wave normalization can be expressed in terms of the effective range parameter $\rho(-\epsilon, -\epsilon)$ (see Appendix I, section 2):

$$(1 + \eta^2) N^2 = \frac{1}{[1 - \alpha \rho(-\epsilon, -\epsilon)]} \quad (4)$$

as the formal expression for the S-wave residue at the deuteron pole.

The D-state wave function is the formal solution of eq. (1b) in terms of the exact $L = 2$ Green function $G_2(r, r')$:

$$w(r) = \int_0^\infty G_2(r, r') U_{20}(r') u(r') dr'. \quad (5)$$

Here the Green function is the solution to the point source equation including $L = 2$ rescattering

$$G_2''(r, r') - [\alpha^2 + 6/r^2 + U_{22}(r)] G_2(r, r') = \delta(r - r') \quad (6)$$

with the boundary condition of regularity at the origin for $r \rightarrow 0$ and a vanishing value for $r \rightarrow \infty$ and fixed r' . It is convenient to express $G_2(r, r')$ in terms of the regular and irregular solutions to the exact homogeneous $L = 2$ equation.

Denote by $\mathcal{J}_2(r)$ the solution regular at the origin and by $\mathcal{H}_2(r)$ the solution vanishing for $r \rightarrow \infty$ normalized so that the Wronskian

$$W[\mathcal{J}_2(r), \mathcal{H}_2(r)] = \frac{1}{\alpha r^2} \quad (7)$$

A convenient choice is

$$r \mathcal{H}_2(r) \xrightarrow{r \rightarrow \infty} \alpha^{-1} e^{-\alpha r} \quad (8a)$$

$$r \mathcal{J}_2(r) \xrightarrow{r \rightarrow \infty} -\frac{1}{2} \alpha^{-1} e^{\alpha r} \quad (8b)$$

In the absence of D-wave rescattering (i.e. if we neglect the U_{22} potential) $\mathcal{H}_2(r)$ and $\mathcal{J}_2(r)$ coincide in this normalization exactly with the spherical Bessel functions $h_2(i\alpha r)$ and $j_2(i\alpha r)$.

The Green function is thus in obvious notation

$$G_2(r_<, r_>) = \alpha r_< r_> \mathcal{J}_2(r_<) \mathcal{H}_2(r_>) \quad (9)$$

When the expression (9) for the Green function is substituted in eq. (5) and $r_>$ is taken larger than the range of the source we find

$$W(r) \xrightarrow{r \rightarrow \infty} \alpha r \mathcal{H}_2(r) \int_0^\infty r' \mathcal{J}_2(r') U_{20}(r') u(r') dr' \quad (10)$$

From the asymptotic limit (8) for $\mathcal{H}_2(r)$ and the asymptotic form (3b) for $w(r)$ we have therefore the formal solution for η :

$$\eta N \sqrt{2\alpha} = \int_0^\infty r \mathcal{J}_2(r) U_{20}(r) u(r) dr \quad (11)$$

So as to emphasize that the parameter η in eq. (11) is nearly independent of the asymptotic normalization N of the S-state, we define

$$\tilde{u}(r) \equiv (N\sqrt{2\alpha})^{-1} u(r) \xrightarrow{r \rightarrow \infty} e^{-\alpha r} \quad (12)$$

Equation (11) then takes the form

$$\eta = \int_0^{\infty} r J_2(r) U_{20}(r) \tilde{u}(r) dr \equiv \int_0^{\infty} \gamma(r) dr. \quad (11')$$

The physical reasons why η can be theoretically derived from eqs. (11) or (11') to a very high precision have been discussed by us in detail in ref. 5. The main point is the behaviour at the origin of the regular solution $\mathcal{J}_2(r)$. In the free case ($U_{22} = 0$) we find that the solution $j_2(i\alpha r)$ vanishes similarly to r^2 at the origin. For the static one-pion exchange potential (OPEP) the function $\mathcal{J}_2(r)$ is further suppressed at the origin, since U_{22} is strongly repulsive adding to the suppression by the centrifugal barrier. For a realistic situation U_{22} becomes attractive at about 1 fm due to the reduction of the tensor potential and to the presence of V_{LS} and V_{SO^2} . In spite of this behaviour the region inside 1 fm is still heavily suppressed so that the contribution to the integral (11) comes essentially (to more than 95%) from $r > 1$ fm.

3. PROPERTIES OF THE POINT-LIKE OPEP DESCRIPTION

Already in ref. 5 we emphasized that η is heavily dominated by the point OPEP interaction to over 95%. The behaviour of the integrand $\eta_{\pi}(r)$ for the OPEP is therefore of particular interest on the qualitative level, since it provides a convenient reference frame for discussions. The dominant feature of the OPEP interaction in the present context is the tensor potential, which in the static limit has the form

$$V_T^{\pi}(r) = - f^2 \left[1 + \frac{3}{(\mu r)} + \frac{3}{(\mu r)^2} \right] r^{-1} e^{-\mu r} \quad (13)$$

Relativistic corrections have consistently been neglected⁷⁾.

Since both the masses and couplings of neutral and charged pions are slightly different, this should be taken into account. The correct combination of charged

and neutral contributions to the pion potential is

$$V_{\pi} = \frac{1}{3} [2 V_n^{\text{charged}} + V_n^{\text{neutral}}] \quad (13)$$

While we will return to this question later, we observe that for the purpose of this section it is convenient to approximate this expression by using an average pion mass $\mu = \frac{1}{3}(2\mu_c + \mu_0)$ and an average coupling $f^2 = \frac{1}{3}(2f_c^2 + f_0^2)$. This is in agreement with common practice. The typical uncertainty due to this procedure is only of the order of a few parts per thousand (see Appendix IV).

In order to emphasize the central role played by the OPEP we show in fig. 1 the exact point OPEP integrand $\eta_{\pi}(r)$ of eq. (11') as compared to the same integrand $\eta_{\text{Paris}}(r)$ for the Paris potential⁸⁾ as an exponent of the present state of the art regarding NN interactions. So as to facilitate comparison the identical π NN coupling constant was used as well as the identical S-wave function, leading to a value

$$\begin{aligned} \eta_n &= 0.02762 \\ \eta_{\text{Paris}} &= 0.02643^* \end{aligned} \quad (14)$$

The remarkable feature of fig. 1 and eq. (14) is the very close agreement between the OPEP value and the far more elaborate Paris result. The correction is only -4.3% and is mainly due to intermediate range interaction as is seen in the figure. The model dependence of this correction term will be extensively discussed in Section 4. Its origin is mainly due to the weakening of the tensor potential at shorter distances, so that it reduces η .

The driving term $U_{20}(r)\tilde{u}(r)$ in eq. (11') contains the tensor interaction and the S-wave function $\tilde{u}(r)$ on an equal footing. The very similar values for $\eta(r)$ using the OPEP and the Paris potential in fig. 1, and eq. (14) demonstrates simply that the result depends only weakly on the non-OPEP behaviour of the potentials U_{22} and U_{20} . It gives no information on the sensitivity to the S-wave function

*) One notes that the value $\eta = 0.02608$ quoted by the Paris group⁸⁾ is correctly reproduced using their value $f^2 = 0.078$ for the coupling constant.

itself. It is crucial to the following that the wave-function dependence is very weak, and we will now demonstrate this.

To disentangle the wave-function dependence from the potential problem, we assume point OPEP and vary the S-wave function in the same term. Starting from the worst possible Ansatz for the wave function we take the asymptotic form $\tilde{u}(r) = \exp \{-\alpha r\}$ and display the result in fig. 2. Although this wave function is extremely rudimentary with vanishing effective range and violating unitarity by 60% it still gives $\eta_{\text{asymptotic}} = 0.033$, i.e. it gives η correctly to within 20%. Any acceptable wave function must necessarily satisfy unitarity and be consistent with the effective range expansion for low-energy scattering. In fact, we expect the S-wave normalization factor to be in the range of 0.93-0.96 to be consistent with the existence of a 4-7% D-state probability. When such wave functions are introduced in almost any form the value of η moves very close to the previously quoted $\eta_{\pi} = 0.02762$ provided there are no additional nodes. This can easily be seen, for example, using a Hulthén wave function $\tilde{u}(r) = (e^{-\alpha r} - e^{-\beta r})$, which gives η to within 0.5% of the Paris value with the same effective range and normalization.

The difference produced by wave functions with the identical asymptotic behaviour is mainly produced by differences in the S-state normalization. In fact, a 1% change in the normalization of $u(r)$ produces a change in η by 0.25 to 0.3%. Such differences can naturally arise in descriptions with different D-state probability. Assuming a liberal normalization uncertainty of $\pm 1.5\%$ from the D-state percentage indicates an uncertainty range of less than $\pm 0.5\%$ from S-wave function effects.

In order to visualize the importance of complete consistency with data we discuss the concrete case of the Reid Soft Case (RSC) wave function⁹⁾ as compared to the Paris one (P)¹⁰⁾ illustrated in fig. 2. These two wave functions differ as follows: the RSC effective range is only 1.720 fm (differing by 8 standard deviations from the experimental value) as compared to the Paris value 1.765 fm. The RSC asymptotic wave function is therefore automatically 1.4% higher than the Paris one. In addition, and less importantly, the RSC wave function has a

normalization 0.7% below that of the Paris one, since the RSC D-state percentage is 0.7% higher. The RSC S-wave function is therefore inconsistent with experimental data in its present form. If we scale the RSC S-wave function so as to give it a correct asymptotic form and the same D-state probability, its normalization integral is therefore 2.1% higher than the Paris one. The consequence of this is clear in fig. 2. This effect enhances $\eta_{\text{RSC}}(r)$ near its maximum by 2% compared to the Paris value. In spite of this the overall effect is only 0.6% in the total η .

A rapid "rule of thumb" expression to relate S states of different normalization P_S to η is

$$\frac{\Delta \eta}{\eta} \approx \frac{\Delta P_S}{3} \quad (15)$$

We note that this expression is approximately correct even for the extreme case of a purely asymptotic wave function (60% wrong normalization giving a 20% too high value for η).

In conclusion: the uncertainties produced by the S-wave function are quite small provided obvious physical constraints are reasonably well respected.

In addition to the requirements of binding energy, effective range and approximately correct normalization, there is the implicit one that a reasonable model wave function should be fairly consistent with evidence from form factors. With the present state of developments it is also reasonable to require that the model description of a wave function should at least approximately insert itself into a broader dynamical picture of NN interactions. Among the multitude of model deuteron wave functions given in the literature we urge special caution with respect to those with additional nodes, since in any potential description this phenomenon is the signal of a more deeply bound true ground state. The present discussion shows that even S-wave functions with manifest inconsistencies give good values for η .

4. MAIN CORRECTIONS TO THE D/S RATIO

The previous discussion was based on point-like iterated OPEP interaction with the D/S ratio generated from the S-state wave function. We established convincingly the crucial role of the pionic interaction to a considerable degree of precision. On the other hand, the only uncertainties and corrections discussed were those associated with the S-wave function $u(r)$. These were shown to be very small as long as the experimental effective range and unitarity are respected. The purpose of the present section is to introduce the main corrections and to control the uncertainties associated with these.

The principal ones are:

- i) 2π exchange. This correction corresponds to the dispersive terms generated by $N\bar{N} \rightarrow \pi\pi$ amplitudes. It is dominated by S- and P-wave $\pi\pi$ exchange. The correction is firmly based theoretically and amounts to -4%. It corresponds in essence to the theoretical part of the Paris potential^{8,11}).
- ii) Uncertainties in πNN coupling constants and other long range effects. The experimental uncertainty in the charged πNN coupling is $\pm 1.3\%$. Theoretically, the difference between pseudoscalar and pseudovector πNN coupling implies a possible difference in the effective coupling constant of 2%. Present independent information on the neutral coupling is not decisive. Other small effects of long-range behaviour are the magnetic-moment and charge-independence corrections to the OPEP potential. Both effects are very small. (See Appendix IV.)
- iii) Finite range of the πNN vertex. This form factor is not well known experimentally or theoretically. This is the largest uncertainty in the present study. In fact, it is so large that present data on the D/S ratio give valuable upper limits for the range of the form factor.

We will now discuss the various corrections and their uncertainties in detail.

4.1 2 π exchange

The equivalent potentials corresponding to the exchange of 2 π can be constructed from dispersion relations as¹¹⁾

$$V_i^{2\pi}(r) = - \frac{1}{2\pi^2} \int_{\mu^2}^{\infty} \eta_i(t) \frac{e^{-\sqrt{t}r}}{r} \phi_i(\sqrt{t}r, t) dt \quad (16)$$

Here the index i refers to the central, tensor, etc., interaction with the weight functions $\phi_c(x,t) = 1$ and $\phi_T(x,t) = (1 + 3/x + 3/x^2)t$, while $\eta_i(t)$ is the corresponding weight function for the strength of exchange of a mass $\sqrt{t}$. The weight function $\eta_i(t)$ contains the dynamical information. It can be constructed from the $\bar{N}N \rightarrow 2\pi$ amplitudes. These are closely related to the physical $\pi N \rightarrow \pi N$ scattering. Both the Δ resonance and the nucleon poles play an important role in these amplitudes. In addition the known S- and P-wave 2 π interactions contribute to $\eta_i(t)$, which is also corrected for iterative processes.

In our present case we are primarily concerned with the $T = 0$ tensor potential. The characteristic form of the theoretical weight function is given in fig. 3. (The actual numbers are those of the theoretical values in the Paris potential; we are indebted to Drs. B. Loiseau and R. Vinh Mau for making these available to us in an unpublished, accurate form due to P. Gauron.)

So as to clarify how different regions of the tensor spectral function $\eta_T(t)$ contribute at various distances r we display for fixed r the ratio of the differential 2 π tensor potential to the π tensor potential in fig. 4. The striking result is that for $r \geq 1$ fm there are negligible contributions from $t \geq 35 \mu^2$. In this region the low mass exchanges of non-resonant P-wave pions and the ρ meson (at $t = 30 \mu^2$) are completely dominant. Even at distances as small as 0.5 fm the main part of the potential is still due to the low mass region. This is a very important feature: the high mass contributions are exponentially damped and contribute surprisingly little even for quite small values of r .

Dr. Loiseau informs us that while the shape of the weight functions are accurate, there may be a systematic normalization factor, which may scale the contribution. A cautiously chosen scale factor may be as much as $\pm 15\%$. Since

the whole correction is only 4.3%, this introduces an uncertainty in η of less than 0.75% (and this number should be possible to improve on).

The contributions of different mass regions to the -4% correction to η is shown in fig. 5. The integrated theoretical contribution for masses smaller than μ_0 are shown for increasing μ_0 (the iterated OPEP is, of course, excluded). One sees clearly that nearly half the correction comes from non-resonant P-wave $\pi\pi$ states below the ρ -meson region. Higher masses than the ρ -meson one are not very relevant. The Paris potential⁸⁾ corresponds well to the theoretical values for $\eta(r)$. A notable feature is that $1/3$ of the correction comes from regions with $r \leq 1$ fm. In spite of the increasingly phenomenological nature of the Paris potential from 0.8 fm inwards, the contribution from inside 1 fm is nearly model-independent. The reason is that continuity requires that the region $r \approx 1$ fm must be continuously connected to $\eta(0) = 0$; the detailed way in which this connection is made has little influence on the result.

4.2 Dependence on the πNN coupling constants

The effect of the difference in the masses and coupling constants of charged and neutral pions is easily accounted for by using the average values:

$$\mu = \frac{1}{3} (2\mu_c + \mu_0) \quad ; \quad f^2 = \frac{1}{3} [2f_c^2 + f_0^2].$$

The accuracy of this replacement is better than 0.2% for η (see Appendix IV, section 2).

Experimental values for the coupling constants and their average are given in Appendix I, section 5. The uncertainty in the average coupling is of the level of a few per cent and it is a major source of uncertainty in the theoretical prediction of η . The principal source of the uncertainty is the difficulty of determining the π^0 coupling accurately, and the lack of theoretical guidance to decide whether charge independence is valid more accurately in a pseudoscalar or a pseudovector description of the coupling. The most accurate statistical determination of f_0^2 gives $f_0^2 = 0.0790(16)$ consistent with pseudovector charge independence, but the systematic error is unknown. The average f^2 corresponding to this

is $f^2 = 0.0790(12)$, which agrees with the assumption of pseudovector charge independence. (Pseudoscalar charge independence would give $f_0^2 = 0.0772$. This is 2.3% smaller in f^2 , which would imply a 2.3% decrease in the predicted η .)

In view of the uncertainty in the coupling constant, we will later quote the final result for η in a form so that a change in f^2 can immediately be incorporated [see eq. (17)].

4.3 Effect of the πNN form factor

In analogy with the interaction between extended Coulomb charges, the point OPEP is modified by the extended pion source function. We recall that Coulomb charges interact similarly to point objects when the charges do not overlap. Similarly, the OPEP potential will have the point value when the pion source functions do not overlap.

In order to give a qualitative feeling for the sensitivity to form-factor effects, we shall first take a purely empirical approach. Assume for convenience a πNN source with a Yukawa distribution $(\Lambda^2/4\pi r) \exp[-\Lambda r]$, which corresponds to a monopole form factor $(\Lambda^2 - \mu^2)/(\Lambda^2 + t)$. The OPEP has then a dipole form factor $[(\Lambda^2 - \mu^2)/(\Lambda^2 + t)]^2$ which in r -space corresponds to an exponential source term $(\Lambda^3/8\pi) \exp[-\Lambda r]$. The OPEP tensor potential takes the form of eq. (A.III.4), which regularizes the origin. Figure 6 illustrates the sensitivity of the integrand $\eta(r)$ as well as η to various choices of Λ . For convenience, we also indicate the corresponding radius R of a uniform source function with the identical r.m.s. radius, so as to facilitate comparison with the predictions of bag models. The percentage change in η for various cut-offs is shown in fig. 7 together with corresponding modifications for the quadrupole moment (see section 5).

At this point we vigorously remind the reader that all other sources examined can modify η only by a few per cent, the most important uncertainty being the πNN coupling constant. The experimental values for η are also quite accurate and unlikely to have major modifications (see Appendix I, section 4). In addition, even tighter bounds are imposed by the experimental quadrupole moment. Present

experimental values are consistent with a point source description, as will be discussed in detail later. It is important to realize that a form factor reduction of η by 5% (and therefore of the quadrupole moment by 8%) is quite a large effect, which is very difficult to reconcile with experiments on η and Q unless other compensating modifications are concretely suggested.

With this limit as a guide the phenomenological analysis shows from fig. 7 that $\Lambda < 6.3 \mu$ or equivalently $R > 0.7$ fm deviate substantially from experimental observations. We have also evaluated the effect of the form factor for a dipole shape of the πNN form factor with nearly indistinguishable results for the equivalent R . For quark bags the form factor effect has been evaluated by de Kam¹²⁾ and improved on by Miller¹³⁾. We have re-evaluated the effect and find for an equivalent radius that the percentage changes are nearly the same as for monopole and dipole form factors.

There is very little concrete evidence on the behaviour of the πNN form factor. We briefly examine the present experimental and theoretical situation.

An attempt has been made to determine the form factor directly from data on the near forward np and $p\bar{p}$ charge exchange at 8 GeV/c¹⁴⁾. The cross-sections have undoubtedly an important pion component. The result is $\Lambda = 6.4 \mu$. In our opinion this conclusion is not possible for the following reasons: the qualitative behaviour of the np charge exchange and its relation to pions is very similar from 200 MeV upwards. In the low-energy region the data are extremely well described by the Paris potential¹⁵⁾ with point coupling in OPEP. We find it highly unlikely that such similar data should be described by quite different physical effects. While it is not known which is the crucial mechanism in the Paris description, M. Ericson has recently made the plausible suggestion¹⁶⁾ that the apparent form factor effect reflects the short-range shape of the NN correlation function. A. Gersten has performed a preliminary analysis in this sense for data below 700 MeV and informs us that the results seem to confirm this picture very well¹⁷⁾. The charge-exchange evidence for the πNN form factor should therefore be discarded for the moment, although it may serve still as a lower limit for Λ .

The $(6 \pm 2)\%$ discrepancy in the Goldberger-Treiman relation indicates a low value of $\Lambda \approx 4 \mu$ ¹⁸⁾. It has not been possible to explain the discrepancy as a form factor effect using theoretical dispersive methods in terms of 3π , $\rho\pi$, and $\sigma\pi$ contributions, and this in spite of a very considerable effort. At most 2% of the 6% seem obtainable in this way ($\Lambda > 7 \mu$). Since the discrepancy is a small correction only, this raises the issue of small additional contributions to the Goldberger-Treiman expression as could be produced by a heavy pion π' ¹⁸⁾. The evidence for the form factor from this source is therefore quite weak, even apart from the statistical uncertainty.

Models for NN interactions generally ignore the effect of the πNN form factor, and therefore tacitly assume point coupling. The implicit reason for this procedure is that a πNN form factor with a low value for Λ causes serious problems. A specific investigation by Holinde using OBEP¹⁹⁾ concludes that "a small value for Λ would have disastrous effects for lower partial waves". A particularly dramatic case is the 3D_1 phase shift, which is experimentally well determined, but also 3P_1 gives very serious difficulties. The 3D_1 phase shift has its energy dependence profoundly modified to the extent that it even changes sign at 400 MeV with $\Lambda = 7 \mu$. The data militate for a cut-off $\Lambda \geq 10 \mu$.

On the theoretical level the form factor is associated with 3π contributions, and, as usual, the threshold region near 3μ is kinematically suppressed. Models for the form factor give characteristically $\Lambda \approx 7 \mu$ ²⁰⁾ at least or larger in the range $(8-10) \mu$ ²¹⁾.

The strongest argument for a relatively low value for Λ comes from the axial form factor $g_A(t)$. This quantity is well determined from electroproduction of pions and has a dipole shape with $\Lambda_{\text{dipole}}^{\text{axial}} \approx 7 \mu$ (the equivalent monopole $\Lambda_{\text{axial}} \approx 5 \mu$)²²⁾. This form factor is dispersively associated with the same quantum numbers as the πNN form factor. Although the two are not identical and may be unrelated, the argument that nature tends to produce similar scales under similar circumstances must be taken quite seriously. From fig. 7 we conclude that the axial form factor reduces η by 10% (and the quadrupole moment by 16%). These

reductions are extremely large and disagree sharply with experiments. We conclude that the axial form factor is an unlikely approximation to the π NN form factor.

One may now try to take the attitude that the π NN form factor effect is in fact non-negligible, but that its effects are incorrectly attributed to something else. The main effect of the form factor is to weaken the tensor force. This can also be produced by the P-wave 2π exchange (" ρ -exchange"). Although we previously argued convincingly that this interaction is very well based theoretically in the present context, let us for the moment phenomenologically assume that its effects are due to the form factor. This assumption is by no means physically equivalent, since the form factor is a 3π phenomenon and not a 2π phenomenon. If the 2π P-wave contributions are artificially quenched, serious problems will occur in the LS potential, the SS potential and the SO_2 potential, which all have their long range behaviour dominated by this exchange. The 2π contributions to the tensor force are not interchangeable with a form-factor effect.

In spite of the inconsistency of such a replacement let us anyway compare the form factor modified OPEP tensor potential with the Paris potential. Figure 8 demonstrates that the approximate shape of the Paris tensor potential corresponds to an equivalent monopole π NN form factor with $\Lambda \simeq 5.5 \mu$. Therefore, even if the entire sophistication of the Paris description is described by OPEP alone, the π NN form factor is no more spread out than the axial form factor.

4.4 Magnetic moment interaction and velocity-dependent interactions

A well defined small contribution to η comes from the tensor interaction of the neutron and proton magnetic moments⁵⁾. The contribution is $\delta\eta_{\text{mm}} = -0.76 \times 10^{-4}$ (-3 ppt). This term is derived in Appendix IV, section 1 and has been added separately to η in the final result.

Velocity-dependent corrections are of relativistic origin, and we will not discuss their details. They are expected to be small however. The Paris potential⁸⁾ includes such contributions for the central potential. Explicit evaluation confirms the smallness in this case [$\delta\eta = 0.17 \times 10^{-4}$ (+0.6 ppt)].

The conclusion of this section is therefore that contributions to η beyond the point OPEP interaction in general are very well understood and controllable. The glaring exception is the πNN form factor, on which experimental and theoretical indications are very unsatisfactory. All present data and indications are as far as we know consistent with a coupling that is more point-like than the axial coupling to the nucleon.

Collecting the results of this section we can therefore write the theoretically deduced D/S parameter (including a -1 ppt correction from velocity dependence, magnetic moment and pseudovector charge symmetry breaking) as

$$\eta_{th} = 0.02641 \left[1 + \frac{(f^2 - 0.0790)}{0.0790} - \epsilon_\eta(\Lambda) \right] \quad (17)$$

with a theoretical error of 0.8% from other sources. The form-factor correction function $\epsilon_\eta(\Lambda)$ is the one given in fig. 7. The theoretical value should be compared with the experimental value [eq. (A1.11)]

$$\eta_{exp} = 0.0271(4). \quad (18)$$

The theoretical value is in agreement with the experimental one within 2 standard deviations for a nearly point-like coupling, but is somewhat on the low side (and it would be lower still if a form-factor effect were included).

5. THE POTENTIAL QUADRUPOLE MOMENT

Within a potential model description of the nucleonic motion the quadrupole moment becomes nearly model-independent to a precision of better than 1%. We demonstrate below that the decisive quantities are on the one hand the asymptotic S- and D-wave normalizations (i.e. the experimentally determined effective range and η) and on the other hand the approximate shape of $\eta(r)$ as given by iterated OPEP and the r.m.s. radius of the πNN form factor. Other features have only a marginal influence. For example, this level of accuracy is easily reached without even respecting unitarity for the S-wave function (although this is a minimal theoretical constraint). The quadrupole moment is defined as

$$Q = Q_1 + Q_2 = \frac{1}{\sqrt{50}} \int_0^{\infty} r^2 u w dr - \frac{1}{20} \int_0^{\infty} r^2 w^2 dr. \quad (19)$$

The main term Q_1 contributes about 106% and it must be treated very carefully. There are only minor contributions inside 1.5 fm, which is a strong hint that mainly asymptotic properties matter. The second term Q_2 is only a -6% correction. It is also determined mainly by the external region. It introduces very little uncertainty compared to effects in Q_1 .

The value of Q_1 is readily expressed in terms of the tensor interaction and the integrand $\eta(r)$ of eq. (11') using the Green function $G_2(r, r')$. We have:

$$\begin{aligned} Q_1 &= 2\alpha \frac{N^2}{\sqrt{50}} \int_0^{\infty} \int_0^{\infty} r^2 \tilde{u}(r) G_2(r, r') U_{20}(r') \tilde{u}(r') dr dr' \\ &\equiv \frac{2\alpha N^2}{\sqrt{50}} \int_0^{\infty} r' U_{20}(r') J_2(r') \tilde{u}(r') dr' \left[\frac{1}{r' J_2(r')} \int_0^{\infty} G_2(r', r) r^2 \tilde{u}(r) dr \right]. \end{aligned} \quad (20)$$

Note that according to eq. (11')

$$\eta(r) = r J_2(r) U_{20}(r) \tilde{u}(r). \quad (21)$$

It is convenient to define the dimensionless function $F(r)$ by

$$F(r) = \frac{\alpha^3}{r J_2(r)} \int_0^{\infty} G_2(r, r') r'^2 \tilde{u}(r') dr' \quad (22)$$

We may therefore rewrite the expression (20) identically as

$$Q_1 = \frac{N^2 \sqrt{2}}{5 \alpha^2} \int_0^{\infty} \eta(r) F(r) dr. \quad (23)$$

The model dependence of Q_1 in eq. (23) comes mainly from the variation of $\eta(r)$. The weight function $F(r)$ is nearly independent of detailed assumptions about the wave function $\tilde{u}(r)$ and the potential. We now investigate these effects.

5.1 Effect of model dependence of the weight function $F(r)$

It is clearly important that the weight function is accurately known in regions where $\eta(r)$ is large, while regions with small $\eta(r)$ can be considered more leisurely. Since the approximate shape of $\eta(r)$ is well known, we will assume it fixed in the present discussion with the shape given in fig. 9. The weight function $F(r)$ depends on the one hand on the wave function $\tilde{u}(r)$ and on the other on the potential interaction in the Green function. We will investigate these effects separately.

Figure 9 shows first the behaviour of $F(r)$ for free propagation in $G_2(r, r')$ using the "worst possible" wave function $\tilde{u}(r) = \exp(-\alpha r)$ [i.e. violating unitarity by 60%(!)] as compared to a realistic wave function. The asymptotic wave function gives a 5% too large value for Q_1 . The wave-function dependence is far weaker than in $\eta(r)$ (where it is very weak).

We conclude that the wave-function uncertainty in the quadrupole moment is completely negligible and $\lesssim 0.2\%$.

To estimate the dependence of the weight function on the potential we take a realistic but fixed S-wave function $\tilde{u}(r)$ and vary the interaction. Comparing the result for no interaction and realistic potentials, we find very closely similar results in fig. 9. The difference between these extreme cases is only 1.7% in Q_1 . Since iterated OPEP is a vast improvement on no interaction and a realistic interaction is better still, the model dependence of Q from this source is very small ($\lesssim 0.2\%$). Although the dependence of $F(r)$ in the integrand (23) is nearly negligible it is still interesting to note that the main corrections are concentrated near the maximum of the integrand $F(r)\eta(r)$, i.e. in the region $1 \lesssim r \lesssim 2$ fm.

The reason for the model independence of $F(r)$ is clear from eq. (22). The weight factor r^2 in the integrand emphasizes contributions to $F(r)$ from large r , for which the propagation is nearly free and the wave function nearly asymptotic.

5.2 Dependence on $\eta(r)$

As a consequence of the model independence of $F(r)$, the value of Q_1 (and indirectly Q) is entirely determined by the properties of $\eta(r)$ discussed previously in sections 3 and 4. We will therefore only touch on those properties which are of special importance for the quadrupole moment. The outstanding feature of $\eta(r)$ is that both its value and its shape are given to a quite good approximation by iterated OPEP. Since Q_1 dominates the quadrupole moment an immediate consequence is that Q is rather accurately proportional to f^2 : a 1% change in the effective coupling constant provokes both a 1% change in η and in Q .

Since $F(r)$ is not a constant there is also a weak shape dependence on $\eta(r)$ when it is modified by additional potential contributions, the π NN form factor, etc. The effect of such modifications is clearly seen in fig. 10, which compares contributions to Q_1 from OPEP and the Paris potential. The effects are qualitatively very similar to those of fig. 1 for $\eta(r)$, with the difference between OPEP and the Paris potential mainly in the region around 1 fm. While the decrease in η is -4.3%, the decrease in Q_1 is -6.1%, the reason being that $F(r)$ gives somewhat more emphasis to the inner region. This effect of the 2π potential is typical of a number of other modifications. The π NN form factor also modifies contributions in the region 1 to 2 fm, as seen in fig. 6 and will therefore be more emphasized in Q_1 , but with essentially the same scale factor as for 2π contributions. The constancy of such corrections is well demonstrated in fig. 7, which compares the effect of form-factor modifications in η and in Q . Over a large range of form-factor values which lead to modifications in the intermediate range region, there is an approximate relation

$$\frac{\Delta Q}{Q} \approx 1.6 \frac{\Delta \eta}{\eta} . \quad (24)$$

As long as $\eta(r)$ is modified in the intermediate region by any mechanism, the approximate relation (24) is applicable.

5.3 Model-dependence of Q_2

The contribution from Q_2 of eq. (19) to the quadrupole moment is only a -6% correction. Similarly to Q_1 it contains a term in r^2 in the wave-function integrand. As a consequence it is mainly dependent on asymptotic properties and is only weakly model-dependent. A straightforward value for Q_2 , using the Paris potential as a reference value is

$$Q_2 = -0.0182 \text{ fm}^2 \quad (25)$$

In order to demonstrate its insensitivity to assumptions we decided to make drastic changes in the D-state wave function in the term. Assume arbitrarily that the D-state probability P_D can be varied in a substantial range but with the asymptotic D-state normalization fixed by experiment. When the P_D probability varies from 5 to 7% the Q_2 value changes from -0.017 to -0.019, or equivalently an error of 20% in the P_D reflects in a 5% change of Q_2 and in an overall uncertainty on Q of less than 0.3%. We emphasize that this procedure is internally inconsistent, since a change in P_D implies a change in the tensor interaction and hence a change in Q_1 which in fact will be far more important. The model dependence of Q_2 is therefore indeed negligible.

5.4 Minor corrections to Q

The quadrupole moment is proportional to N^2 , which is experimentally known to 2 ppt according to eq. (AI.8). No significant uncertainty is associated with this term.

The magnetic moment interaction contributes $-4.4 \times 10^{-4} \text{ fm}^2$ (see Appendix IV, section 1).

Velocity- and energy-dependent terms in the interaction fall strictly speaking outside the potential approach. We note, however, that the velocity dependence of the Paris potential contributes only -0.03% and a negligible amount to the weight function $F(r)$, so that its overall effect on Q is negligible.

The conclusion is therefore that the local potential description gives a very simple relation between Q and η . Since both η and Q are OPEP dominated, both scale in the same way with f^2 . As far as corrections to this picture occur, whether by additional intermediate-range interactions or form factor, the shape correction can be simply summarized by the formula (24) (or taken from the more accurate fig. 7). Assuming no anomalous contributions we can express the potential quadrupole moment in a form analogous to eq. (17) for η_{th} :

$$Q_{pot} = 0.2821 \left[1 + \frac{(f^2 - 0.0790)}{0.0790} - E_Q(\Lambda) \right] f_m^2, \quad (26)$$

where the form-factor correction function is given in fig. 7. From the normalization uncertainty in the 2π contributions to η of less than 0.6%, we expect the corresponding uncertainty in Q in eq. (26) to be less than 1%. The expression (26) can be improved very substantially inside the local potential description provided η is empirically known to sufficient precision. Any deviation of η from the theoretical value (17) due to any reasonable modification of the intermediate range will manifest itself in a modification of Q according to the relation (24). Once such corrections are introduced the potential relation between η and Q is model-independent to better than 0.5%.

The value (26) for the quadrupole moment should be compared with its experimental value [Appendix I, section 3, eq. (AI.9)]

$$Q_{exp} = 0.2859(3) f_m^2$$

Clearly, the result (26) with the experimental value for f^2 and Λ large is very close to the experimental quadrupole moment. We will return to this point in a simultaneous discussion of η , Q , f^2 , and Λ later.

Owing to relativistic and meson-exchange current effects the experimental quadrupole moment is not expected to correspond exactly to the potential prediction. This is not the place to discuss such effects in detail. We observe, however, that our choice of representation for the OPEP (like the Paris choice) corresponds to a maximally static representation [a Dyson transformation with

$\mu = -1^{23})]$. The isoscalar meson-exchange corrections are then expected to be dominated by the pion-pair term. Typical meson-exchange current (MEC) corrections to the quadrupole moments are given in the literature as $+0.013 \text{ fm}^2^{24})$ and $0.009 \text{ fm}^2^{25})$. These values include minor corrections (recoil-renormalization terms), which require complete consistency in the definition of wave functions and their corrections. Apart from the weak velocity dependence in the central Paris potential definitions are consistent and we have no reason to doubt the results obtained to an accuracy of about 25% in the correction. On the other hand, we know of no unambiguous experimental test of the isoscalar exchange currents. A critical study of relativistic and meson-exchange current (MEC) contributions both to η and Q is desirable. Isobar contributions have been calculated in addition in both refs. 24 and 25, and shown to contribute mainly in an indirect way, by the modification of the nucleon wave function normalization due to isobar probability. The asymptotic normalization of the nucleon wave functions is of course unchanged, so that the effect is equivalent to a change in the S-wave normalization. According to eqs. (15) and (24) a 1% isobar probability decreases η by 0.3% and Q by 0.5%. The effect is therefore analogous to any other modification of intermediate-range contributions to η .

6. CONCLUSIONS

The present work had four objectives, which have been mainly achieved.

Firstly, we wanted to demonstrate explicitly the dominant role played by the pion in the description of the deuteron D/S ratio and quadrupole moment. This result, partly reported previously in ref. 5, was achieved by avoiding the usual pitfall of iterating the interaction both in S- and D-states, which introduces apparent model dependence in order to be consistent with binding energy and asymptotic normalization. Instead we used the trick of generating the D-state from the S-state, which allowed us to demonstrate both a remarkable independence to the S-wave function itself and the quantitative role of OPEP to a precision of better than 5%. The reason for the high precision was the suppression of the inner regions due to the strong repulsion.

Secondly, we wanted to control quantitatively the corrections to the OPEP picture, so that uncertainties and assumptions in the approach would be clearly separated and understood. The purpose of this procedure is to restrict future discussions to relevant points, and, more constructively, to establish the crucial quantities on which more accurate independent experimental information is necessary, linking the D/S ratio and quadrupole moment to other physical phenomena.

We found three major effects of this kind.

- a) 2π exchange beyond iterated OPEP. In section 4 we demonstrated convincingly that these contributions are very well understood. All the contributions of any importance come from the region of mass $\leq 6 \mu$, i.e. up to and including the ρ mass. The uncertainty in the correction is mainly due to an overall normalization factor. It should be relatively easy to decrease this theoretical uncertainty when necessary.
- b) Experimental precision of the πNN coupling constants. A somewhat larger uncertainty is that of the basic coupling constants. An unnecessary source of uncertainty is that of the $\pi^0 NN$ coupling, which could be improved by a careful re-analysis of pp forward dispersion relations.
- c) The πNN form factor. In section 4 we discussed the present experimental information on this quantity extensively and came to the conclusion that no satisfactory evidence exists. Consequently this form factor is the main uncertainty in our analysis. The crucial parameter is the equivalent r.m.s. radius of the source, and much less its detailed shape. The present lack of information is so great that one can turn the argument around and use η and Q to establish constraints on the form factor (see below). Data favour a source function more short-ranged than the axial form factor, similar to indications from NN scattering in the intermediate range region.

A major conclusion of the present work is therefore that a deviation between theory and experiments for the D/S ratio is almost certainly due either to the πNN form factor or to the πNN coupling constant.

Thirdly, we wished to establish an explicit, accurate and model-independent relation between the quadrupole moment and the D/S ratio. This goal was explicitly achieved in section 5 provided the interaction can be described in terms of a local, energy-independent potential consistent with experimental effective range and OPEP. In this case, the knowledge of η and f^2 leads to a knowledge of Q_{pot} to a precision of $\frac{1}{2}\%$. It is therefore not reasonable to consider Q and η as independent quantities unless the purpose is to explore the consequences of relativistic corrections and meson-exchange currents, or, alternatively, to explore the effects of using non-local descriptions.

Fourthly, our ultimate objective was to exploit the new level of accurate understanding and the accurately known experimental properties of the deuteron for information on the πNN form factor and coupling constants, as well as on the isoscalar meson-exchange current effect. We considered the form factor to be of particular significance, since it is predicted by quark bag models and gives rise to large effects. The pionic properties of the deuteron should therefore be well suited to establishing quantitative constraints on quark bag radii.

At this point, we found the present theoretical accuracy so high that we were compelled to make a critical examination of the experimental quantities, in particular Q , η , $\rho(-\epsilon, -\epsilon)$ and f^2 . This discussion is found in Appendix I. A consequence of this discussion is a new, highly accurate value for Q and a confirmation of the high accuracy of $\rho(-\epsilon, -\epsilon)$. Systematic uncertainties in both η and f^2 may amount to several per cent and should be brought under tighter control.

A display of η as a function of f^2 (fig. 11) visualizes both the internal consistency of data as well as the global implications of these. The η - f^2 plane is first divided by diagonal, nearly straight and parallel lines of constant cut-off Λ . The line $\Lambda = \infty$ represents the theoretical line for point-like nucleons with the theoretical 2π correction included. The position of this line has a nearly negligible uncertainty compared to the experimental numbers. Points to the left of the line are incompatible with any form factor. For convenience the line corresponding to the axial form factor is explicitly indicated.

A second set of nearly parallel lines is given by the potential model quadrupole moment. These are shown for several assumptions about meson-exchange effects. The quadrupole constraint is extremely accurate provided the MEC correction is known.

The experimental η - f^2 point is shown with contour plots corresponding to 39%, 87%, and 99% probability.

From fig. 11 one concludes at once that the present η - f^2 measurements are inconsistent with any form factor, but not to an unreasonable degree. The closest acceptable (η, f^2) points give $f^2 \approx 0.080$ and $Q = Q_{\text{pot}}$ with $\Lambda \approx 10 \mu$. In the region of the 90% probability line a quadrupole moment with a theoretically plausible meson-exchange correction and $\Lambda \approx 7 \mu$ becomes possible with $f^2 \approx 0.080$. In all cases the quadrupole constraint makes $\Lambda = \Lambda_{\text{axial}}$ extremely unlikely: it requires $f^2 \gtrsim 0.086$ ($g^2 \gtrsim 15.5$) with normal meson-exchange currents or alternatively, nearly four times the expected MEC correction, $f^2 = 0.079$, but $\eta \approx 0.024$. Intermediate values are no more satisfactory.

While it would clearly be very important to have the experimental value for η , but also f^2 , more constrained it is even now quite clear that the (effective) πNN coupling must be much more point-like than the axial form factor. A plausible value is $\Lambda \approx 7-10 \mu$. Although the present experimental η - f^2 value is strictly speaking inconsistent (most likely owing to an overoptimistic experimental error estimate for η) it seems unlikely that it could take such small values that the MEC correction could be more than 50% beyond the largest present estimate.

Figure 11 also makes it very clear that it will not be easy to use $f^2 \approx 0.077$ which is characteristic of pseudoscalar coupling. This would correspond both to point coupling and increased meson-exchange effects.

The present experimental situation is not ideal. Our conclusions would be much more clear cut with better data on η and f^2 . In spite of this, the present constraints, particularly on the πNN form factor are quite strong*). In particular, bag models with a πNN source radius similar to the axial radius are not compatible with data.

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*) To prevent any misunderstanding we insist that our conclusion concerns the effective πNN vertex in NN interactions. It is possible to save a πNN form factor of, say, the axial form-factor range if a compensating intermediate range effect is introduced (for example a "heavy pion" π' with a judiciously chosen coupling strength). This would automatically give correct compensation in both η and Q , but at the price of an arbitrary *ad hoc* assumption. In the absence of other evidence we find such *ad hoc* approaches unattractive.

APPENDIX I

A CRITICAL SURVEY OF BASIC EXPERIMENTAL PARAMETERS OF THE DEUTERON

The high accuracy of the present discussion makes it imperative that the relevant experimental quantities are accurately defined and the origin of uncertainties understood. This concerns, in particular, the quantities η , Q , and $\rho(-\epsilon, -\epsilon)$. We therefore give a brief critical discussion of these quantities. Other basic quantities for the present discussion are also given here for convenience. Precision is quoted in parts per million (ppm) or parts per thousand (ppt).

1. DEUTERON BINDING ENERGY ϵ ²⁶⁾

$$\epsilon = 2.224575(9) \text{ MeV} \quad (4 \text{ ppm}). \quad (\text{AI.1})$$

From this quantity and the nucleon masses the relativistic value for the deuteron radius R is derived

$$R \equiv \alpha^{-1} = 4.31888(2) \text{ fm} \quad (5 \text{ ppm}). \quad (\text{AI.2})$$

The non-relativistic definition $\alpha = (\epsilon M_{\text{red}})^{\frac{1}{2}}$ gives

$$R_{NR} = 4.31768(2) \text{ fm}$$

The difference of 0.28 ppt is not relevant in the present context.

2. EFFECTIVE RANGE AND ASYMPTOTIC NORMALIZATION

From the np coherent scattering length $-3.738(1) \text{ fm}$ ²⁷⁾ and the incoherent total cross-section at threshold $20491(19) \text{ b}$ ²⁸⁾, the 3S_1 scattering is found to be $a_t = 5.424(4) \text{ fm}$ (0.8 ppt). From this value and α the mixed effective range is obtained to high accuracy from $\rho(-\epsilon, 0) = 2\alpha^{-1}[1 - (\alpha a_t)^{-1}]$.

$$\rho(-\epsilon, 0) = 1.760(5) \text{ fm} \quad (3 \text{ ppt}). \quad (\text{AI.3})$$

The diagonal effective range parameters $\rho(-\epsilon, -\epsilon)$ and $\rho(0, 0)$ cannot be directly determined to similar experimental precision. Their determination depends therefore on a threshold extrapolation. In all modern descriptions consistent with

correct pion-exchange contributions and low-energy scattering, they are very close to the value of $\rho(-\epsilon, 0)$. A number of such models give

$$[\rho(-\epsilon, -\epsilon) - \rho(-\epsilon, 0)] = 0.0052 (2\sigma). \quad (\text{AI.4})$$

The sign and magnitude are well established. We have rather liberally assigned a 40% uncertainty to the extrapolation difference. The small variation above reflects mainly that π exchange is well incorporated, since the leading correction to the effective range expansion vanishes in the limit of dominating π singularity in the model-independent Cini-Fubini-Stanghellini dispersion approach²⁹⁾. We recommend the value

$$\rho(-\epsilon, -\epsilon) = 1.765 (5) \text{ fm} \quad (3 \text{ ppt}). \quad (\text{AI.5})$$

The corresponding value for A_s is given by

$$A_s^2 (1 + \eta^2) = \frac{2\alpha}{[1 - \alpha \rho(-\epsilon, -\epsilon)]} \quad (\text{AI.6})$$

The recommended value for A_s is

$$A_s = 0.8846 (8) \text{ fm}^{-1/2}. \quad (\text{AI.7})$$

We note that the accurately known charge radius is proportional to A_s^2 , to leading order, but does not lead to the same accuracy for A_s owing to uncertainties about meson-exchange corrections. The value for A_s recently recommended by Kermode et al.³⁰⁾ is both based on old experimental data and is partially inconsistent with OPEP, so it must be discarded. An often used quantity related to A_s is N^2 defined as follows:

$$(\text{AI.8})$$

3. QUADRUPOLE MOMENT Q

The generally quoted value is that of Reid and Vaida³¹⁾ $Q = 0.2860(15) \text{ fm}^2$. This is a theoretically deduced number based on hyperfine splittings in HD and D₂ molecules as measured by atomic-beam technique³²⁾. The experimental precision is mainly given by the state D₂ ($J = 1$) with a splitting known to 0.1 ppt. The

experimental uncertainty is at present negligible compared to the theoretical one. The quadrupole interaction is deduced from the experimental values by theoretical calculations of the quadrupole hyperfine splitting using variational wave functions.

Bishop and Cheung³³⁾ have very carefully re-examined the Reid and Vaida analysis. They find the results to be very stable, and some very minor errors are corrected. In addition, they correct the result for non-adiabatic motion (+1.0 ppt) and they find vibrational coupling negligible. Reference 33 gives no value for the quadrupole moment and emphasizes only that the result is in excellent agreement with ref. 31. On examining ref. 33 we were struck by the high quality of the approach with excellent variational wave functions. In addition, the contributions to the matrix elements comes from regions which make important contributions to the energy, so that the variational approach should be exceptionally accurate. Dr. Bishop agrees with this and has authorized us to quote as their result for Q:

$$Q = 0.28590(30) \text{ fm}^2. \quad (\text{AI.9})$$

In spite of the improvement by a factor of 5 on the value of ref. 24, we believe this error estimate to be quite conservative and that the precision on the theoretically deduced number may be substantially more accurate.

In addition we estimate the following effects to be negligible:

Relativistic effects: $0[(e^2/4\pi)^2] < 0.1 \text{ ppt}$

Deuteron electric tensor polarizability $[(e^2/4\pi) m_e \alpha_D^{\text{tensor}}/Q] < 0.1 \text{ ppt}$

Deuteron finite size: $0[(m_e R_D)^2] \ll 0.1 \text{ ppt}.$

A conservative recommended value for Q is therefore (AI.9).

4. THE ASYMPTOTIC D/S RATIO η

Very recently a breakthrough has occurred in the experimental techniques for determining quantities closely related to η . Very accurate measurements are now available using three different and independent methods. As a consequence of the

sudden improvement in precision, smaller correction terms in the theoretical analysis are still under investigation, so that the value recommended below must be considered an interim one.

The methods are the following:

- a) $(\vec{p}, d)$ and $(d, \vec{p})$ elastic scattering. By extrapolation in $\cos \theta$ to the deuteron pole, which is close to the physical region in the backward direction, one extracts $A_S^d \eta$. It is therefore important to use the correct effective range. The problems in analysis are mainly Coulomb corrections and the stability of the extrapolation procedure.
- b) ${}^2\text{H}(\vec{d}, p){}^3\text{He}$. By analytic extrapolation in $\cos \theta$ the near-lying deuteron pole η is directly determined. The main problems are the same as above, although the Coulomb problem is simpler in the absence of the forward singularity.
- c) Sub-Coulomb stripping of polarized d. This determines η directly. The main uncertainty is probably nuclear distortion effects, which are however claimed to be small. A series of small physical effects, such as deuteron electric polarizability, quadrupole scattering in the Coulomb field, etc., have recently been included in an improved analysis³⁴⁾.

The various results are given in Table AI. Based on the starred entries we find a recommended value consistent with all the three methods

$$\eta = 0.0271(4) \quad (13 \text{ ppt}). \quad (\text{AI.10})$$

This interim value of η is dominated by the accurate data of ref. 35.

5. THE πNN COUPLING CONSTANTS

The coupling constants are cited in the pseudoscalar (PS) version as g or in the pseudovector (PV) version as f with

$$f_{c,0} = \frac{\mu_{c,0}}{2M} g_{c,0}. \quad (\text{AI.11})$$

The charged coupling constant is determined from πN scattering with the generally accepted value³⁶⁾

$$g_c^2 = 14.28(18) ; f_c^2 = 0.0790(10) \quad (13 \text{ ppt}). \quad (\text{AI.12})$$

The difference between PS and PV coupling makes a 7% difference in g^2 or f^2 assuming one or the other of the couplings to obey charge independence. It is therefore important to have a good independent determination of the neutral coupling.

Viollier et al.³⁷⁾ have used forward dispersion relations for pp scattering at lower energies and find the pole residue

$$g_o^2 = 15.3(3) ; f_o^2 = 0.0790(16) \quad (20 \text{ ppt}). \quad (\text{AI.13})$$

The method used is basically very accurate, but no critical discussion exists of possible smaller corrections and systematic errors. From pp scattering in the 200-500 MeV range Bugg et al.³⁸⁾ deduce from differential cross-sections

$$g_o^2 = 14.10(65) ; f_o^2 = 0.0728(33) \quad (46 \text{ ppt}). \quad (\text{AI.14})$$

This determination has intrinsically larger errors at present than the dispersion-relation method. The Viollier value for g_o^2 favours charge-independence in PV coupling, while the Bugg value is consistent with both PS and PV charge independence.

In view of the importance of having a good value for the π^0 coupling not only in the present work, but also for its relevance to charge-independence, we recommend a critical re-evaluation of the forward dispersion relation for pp scattering including the best modern data.

We note in passing that the breaking of charge independence in the singlet NN scattering lengths for np, nn and pp does not permit a distinction between PS or PV charge-independence. Although the data somewhat favour PS coupling a major part of the discrepancy remains unexplained^{39,40)}.

Based on the accurately known charged coupling constant (AI.12) the average coupling constant $f^2 = \frac{1}{3}(2f_c^2 + f_0^2)$ takes the following values:

$$\begin{aligned} f^2 &= 0.0773(10) \quad [PS \text{ charge independence}] \\ f^2 &= 0.0790(10) \quad [PV \text{ charge independence}] \end{aligned} \quad (AI.15)$$

If f^2 , on the other hand, is based on the experimental charged and neutral values (AI.12) and (AI.13), one finds

$$f_{exp}^2 = 0.0790(12). \quad (AI.16)$$

6. PHYSICAL PARAMETERS NEEDED IN THE CALCULATIONS

We give here for the convenience of the reader a set of physical parameters which have been used in the calculations

$$\begin{aligned} \hbar c &= 197.32858(51) \text{ MeV fm} \quad (2.5 \text{ ppm}) \\ M_n &= 938.2796(27) \text{ MeV} \quad (2.8 \text{ ppm}) \\ M_p &= 939.5731(27) \text{ MeV} \quad (2.8 \text{ ppm}) \\ M_{\text{reduced}} &= 938.926 \text{ MeV} \quad (2.8 \text{ ppm}) \\ \mu_c &= 139.5669(12) \text{ MeV} \quad (8.5 \text{ ppm}) \\ \mu_0 &= 134.9626(39) \text{ MeV} \quad (28 \text{ ppm}) \\ \mu &= \frac{1}{3}(2\mu_c + \mu_0) = 138.032 \text{ MeV} = 0.69950 \text{ fm}^{-1} \quad (10 \text{ ppm}) \end{aligned}$$

Table AI

Experimental values for η

(Starred entries are current best values for each method)

Method	Quantity	η	Remark	Reference
$(\vec{p}, d)$ elastic	$A_S^4 \eta$	0.0263(13)	No Coulomb; $\rho(-\epsilon, -\epsilon)$ not stated	a)
Re-analysis	-	0.0270(7)	Coulomb correction	b)
$(d, \vec{p})$ elastic	$A_S^4 \eta$	0.0259(7)	No Coulomb; $\rho(-\epsilon, -\epsilon) = 1.75$ fm	c)
Re-analysis of a) and b)	-	0.0263(9)*	Coulomb correction $\rho(-\epsilon, -\epsilon) = 1.764$ fm	d)
Sub Coulomb stripping	η	0.0263(4)		e)
Re-analysis	-	0.0271(8)*	Replaces above; improved analysis with various minor effects included	f)
$^2\text{H}(\vec{d}, p)^3\text{He}$	η	0.0268(7)	Superseded	g)
$^2\text{H}(\vec{d}, p)^3\text{He}$	η	0.0272(4)*	Coulomb correction	h)

- a) H.E. Conzett, F. Hinterberger, P. v. Rossen, F. Seiler and E.J. Stephenson, Phys. Rev. Lett. 43 (1979), 572.
b) F.D. Santos and P.C. Colby, Phys. Lett. 101B (1981), 291 and Nucl. Phys. A367 (1981), 197.
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PREVIOUS APPROACHES TO THE D/S RATIO

The only specific study of the asymptotic D/S parameter we have found is a partial wave dispersion relation approach by D. Wong⁶⁾. Although his work derives the parameter η using only on-shell scattering parameters, it contains a number of approximations. It is in essence equivalent to a Born-approximation to the OPEP using a deuteron wave function consistent with the physical effective range parameter. His value $\eta = 0.029$ has an intrinsic error of about 15%.

In view of the interest of this approach using only on-shell ingredients we briefly examine the input. Wong⁶⁾ starts from the D-S mixing parameter ϵ_1 and mixing amplitude α , given in terms of the $L = 0$ and 2 eigenphases as

$$\alpha' = i \sin(2\epsilon_1) [e^{i\delta_{0,1}} \sin \delta_{0,1} - i e^{i\delta_{2,1}} \sin \delta_{2,1}]. \quad (\text{AII.1})$$

He specifically assumes that the mixing parameter is small and that the D-state phase shift is negligible. Wong then writes a partial wave dispersion relation in energy with a known branch point in the physical region, and a branch point for the unphysical negative energy side; the residue of the deuteron pole contains η . He approximates the unphysical branch by one-pion exchange, since the singularity is close (-5 MeV) and opts for an unsubtracted dispersion relation. He implicitly neglects all contributions from iteration of pion exchange as well as from other exchanges. Consequently he obtains a linear relation between η and the πNN coupling constant, since there are no rescattering effects in his model.

From the beginning Wong makes the convenient approximation that the S-wave phase shift is described by the effective range expansion. From the preceding it should be quite clear that within the same set of approximations any potential interaction with a correct effective range expansion will give the same result in this approach, and in particular any of the standard potential models for the NN interactions. As one may guess from the absence of rescattering, Wong's description is closely related to the Born approximation. In the zero-range approximation

corresponding to $\tilde{u}(r) = e^{-\alpha r}$, Wong's result and the Born result are indistinguishable. For a finite range there is a minor difference, presumably due to his neglect of all singularities in the unphysical region except for the tensor OPE.

Since Wong's approach has only the precision of a Born approximation, and in addition does not seem to lend itself readily to systematic improvements, it is at present of limited predictive value. It does, however, make it quite clear that the contributions to η are governed by the physical scattering processes in the range below ≈ 10 MeV with little other contributions, and it does implicitly insist on the necessity that the deuteron wave function used must correctly describe the effective range expansion in a natural way. This very minimal requirement is of course true of any reasonable modern deuteron wave function.

A large number of calculations of η exist in the literature. A partial list of results is given in Table AII. Essentially all of these results are a byproduct of descriptions or models of the NN interaction. The main purpose of listing the value of η (and the quadrupole moment) is to give a general impression of the "goodness" of the description as compared to other models. We find no attempts to evaluate the model dependence of the results*). It is, however, extremely striking that all the descriptions with an adequate OPEP behaviour of the long-range part of the interaction give closely similar results. Descriptions which give strongly different values for η are quite unrealistic. All currently used models with π exchange give reasonable values for η , once more confirming the general model independence albeit in an imprecise way.

*) The first approach to η in a model with OPEP seems to be that of Hamada⁴¹⁾. His discussion demonstrates the general stability of η with respect to assumptions giving values from 0.024 to 0.028.

Table AII

Value of η and Q in various potential models

A_S (fm) ^{-1/2}	η	Q (fm) ²	$Q/(A_S^2 \eta)$ (fm) ³	Reference
0.875	0.048	0.276	7.5	a)
0.893	0.0092	0.092	12.54	b)
0.878	0.0324	0.283	11.33	c)
0.8776	0.0262	0.280	13.88	d)
0.878	0.0259	0.277	13.87	e)
0.8943	0.0255	0.2774	13.60	f)
0.8976	0.0262	0.2860	13.55	g)
0.8894	0.0261	0.279	13.27	h)
0.9009	0.0258	0.279	13.32	i)
0.9026	0.0254	0.275	13.26	j)

- a) T.R. Mongan, Phys. Rev. 178 (1969), 1597.
b) S. Kahana et al., Phys. Rev. 185 (1969), 1378.
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g) M.M. Nagels et al., Lecture Notes in Physics (Springer, Berlin, Heidelberg, New York, 1978), Vol. 82, p. 17.
h) M. Lacombe et al., Phys. Rev. C21 (1979), 861.
i) H.F.U. Zingl, L. Mathelitsch and M.I. Haftel, Acta Phys. Austriaca 53 (1981), 29.
j) H.F.U. Zingl, L. Mathelitsch and M.I. Haftel, Acta Phys. Austriaca 53 (1981), 29.

APPENDIX III

THE EFFECT OF THE π NN FORM FACTOR

A π NN vertex with a monopole form factor

$$K(t) = \frac{\Lambda^2 - \mu^2}{\Lambda^2 + t} \quad (\text{AIII.1})$$

modifies the π exchange in NN scattering into

$$\frac{K^2(t)}{t + \mu^2} = \frac{1}{t + \mu^2} \left(\frac{\Lambda^2 - \mu^2}{\Lambda^2 + t} \right)^2 \quad (\text{AIII.2})$$

This introduces a dipole form factor in the OPEP. The point source OPEP in $\underline{r}$ -space is modified with

$$V_c^\pi \propto \frac{e^{-\mu r}}{r} \longrightarrow \left[\frac{e^{-\mu r} - e^{-\Lambda r}}{r} - \frac{(\Lambda^2 - \mu^2)}{2\Lambda} e^{-\Lambda r} \right], \quad (\text{AIII.3})$$

$$V_T^\pi \propto \frac{1}{r^3} \left[1 + \mu r + \frac{(\mu r)^2}{3} \right] e^{-\mu r} \longrightarrow \frac{1}{r^3} \left\{ \left[1 + \mu r + \frac{(\mu r)^2}{3} \right] e^{-\mu r} - \left[1 + \Lambda r + \frac{(\Lambda r)^2}{3} \right] e^{-\Lambda r} - \frac{1}{6} (\Lambda^2 - \mu^2) r^2 (1 + \Lambda r) e^{-\Lambda r} \right\}. \quad (\text{AIII.4})$$

The monopole form factor (AIII.1) has in $\underline{r}$ -space an r.m.s. radius equivalent to the r.m.s. radius of a uniform sphere of radius R with

$$R = \frac{\sqrt{10}}{\Lambda}. \quad (\text{AIII.5})$$

The π NN form factor for a source function which is a uniform sphere of radius R is

$$K(t) \propto \frac{3 j_1(\sqrt{t} R)}{\sqrt{t} R}. \quad (\text{AIII.6})$$

This expression is identical to the π NN form factor in the cloudy bag model⁴⁾ with R and Λ related by eq. (AIII.5).

MINOR CORRECTIONS TO η AND Q

Several minor corrections have been calculated for η and Q and included in the final results without further comments.

1. Magnetic moment interaction

The proton and neutron interact also by the classical magnetic-moment interaction,

$$V_{mm} = -\mu_p \mu_n S_{12} / r^3, \quad (\text{AIV.1})$$

where $\mu_p = 2.793$ and $\mu_n = -1.913$ Bohr magnetons and we have neglected central contact interaction. The V_{mm} potential does not give any sizeable contribution to the binding energy of the deuteron; however, thanks to its long range behaviour and tensor character it modifies the value of η and Q .

The results can be given simply in terms of the integral

$$\delta \eta_{mm} = \mu_p \mu_n \int_0^\infty r J_2(r) r^{-3} \tilde{u}(r) dr \quad (\text{AIV.2})$$

and the numerical contributions to η and Q are, respectively,

$$\delta \eta_{mm} = -7.6 \times 10^{-5} \quad (-2.8 \text{ ppt}) \quad (\text{AIV.3})$$

and

$$\delta Q_{mm} = -4.4 \times 10^{-4} \text{ fm}^2 (-1.4 \text{ ppt}) \quad (\text{AIV.4})$$

2. Charge independence breaking corrections to OPEP

The OPEP in the deuteron is the weighted average of charged and neutral pion exchange, each with its coupling constant f_c^2 and f_0^2 and each with its mass μ_c and μ_0 . The potential is

$$V_\pi = \frac{1}{3} (2 V_{\pi^c} + V_{\pi^0}). \quad (\text{AIV.5})$$

Each of these terms can be written

$$V_n(r) = f^2 \frac{1}{r} g(z) \quad (\text{AIV.6})$$

with $z = \mu r$. We follow the usual procedure and introduce an average mass $\mu = \frac{1}{3}(2\mu_c + \mu_0)$ and an average coupling constant $f^2 = \frac{1}{3}(2f_c^2 + f_0^2)$, as well as the deviation from these quantities $\delta = (\mu_c - \mu_0)/\mu \approx \frac{1}{30}$ and $\epsilon = (f_c^2 - f_0^2)/f^2$. With $z = \mu r$ we then have

$$V_n^{C,T} = \frac{f^2}{r} \left[g_{C,T}(z) + \frac{4}{3} \epsilon \delta z g'_{C,T}(z) + \frac{1}{9} \delta^2 z^2 g''_{C,T}(z) + \dots \right] \quad (\text{AIV.7})$$

For the tensor potential

$$g_T(z) = \left(1 + \frac{3}{2}z + \frac{3}{2}z^2 \right) e^{-z} \quad (\text{AIV.8})$$

The corrections in ϵ and δ occur mainly in the long-range region. To estimate the dependence on these terms we chose

i) Pseudoscalar coupling: $g_c^2 = g_0^2$

For this case $\epsilon = 2\delta$. The changes in η and Q are

$$\begin{aligned} \delta \eta_{ciB} &= 0.71 \times 10^{-5} \quad (0.3 \text{ ppt}) \\ \delta Q_{ciB} &= 0.45 \times 10^{-4} \text{ fm}^2 \quad (0.2 \text{ ppt}). \end{aligned} \quad (\text{AIV.9})$$

ii) Pseudovector coupling: $f_c^2 = f_0^2$

For this case $\epsilon = 0$. The changes in η and Q are

$$\begin{aligned} \delta \eta_{ciB} &= 3.7 \times 10^{-5} \quad (1.4 \text{ ppt}) \\ \delta Q_{ciB} &= 3.4 \times 10^{-4} \text{ fm}^2 \quad (1.2 \text{ ppt}). \end{aligned} \quad (\text{AIV.10})$$

In both cases the corrections are negligible at present. This demonstrates that it is a good approximation to use the average coupling constant f^2 and the average mass μ .

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Figure captions

- Fig. 1 : Comparison of $\eta(r)$ for OPEP and Paris potentials as well as the difference between them. The total area of the difference is -4.3%.
- Fig. 2 : Dependence of $\eta(r)$ on the choice of S-wave functions $\tilde{u}(r)$ all normalized so that $\tilde{u}(r) = \exp(-\alpha r)$ for large r .
- Fig. 3 : Behaviour of the $I = 0$ tensor weight function $\eta_T(t)$ as a function of the square of the exchanged mass. The isoscalar and isovector contributions $3\eta_T^{(+)}$ and $-6\eta_T^{(-)}$ are shown separately. We are indebted to Dr. Loiseau for the numerical input.
- Fig. 4 : The 2π tensor potential contributions from different mass regions for fixed r . The potential is taken in units of the corresponding OPE potential..
- Fig. 5 : The theoretical correction to $\eta_\pi(r)$ from dispersive 2π exchange as a function of r . The mass exchange has been summed over all mass exchanges smaller than μ_0 . The curves are based on the non-phenomenological 2π exchange of the Paris potential.
- Fig. 6 : Dependence of $\eta_\pi(r)$ on monopole form factors for the πNN vertex $g_{\pi NN}(t, \Lambda) = (\Lambda^2 - \mu^2)/(t + \Lambda^2)$. The integrand decreases by 3.1%, 8.5%, and 19%, for the three cases. The corresponding uniform source radii are $R = 0.63$ fm, 0.84 fm, and 1.26 fm.
- Fig. 7 : The modification functions $\epsilon_\eta(\Lambda)$ and $\epsilon_Q(\Lambda)$ for η [eq. (17)] and Q [eq. (26)].
- Fig. 8 : The OPE tensor potential for various πNN monopole form factors compared with the Paris tensor potential. The corresponding uniform source radii are $R = 0.63$ fm, 0.84 fm, and 1.26 fm.

Fig. 9 : Model dependence of the quadrupole weight function $F(r)$ using a) asymptotic wave function $\tilde{u}(r)$ and no interaction, b) realistic wave function $\tilde{u}(r)$ with no interaction as compared to c) realistic $\tilde{u}(r)$ and OPEP d) full Paris potential. For convenience the shape of $\eta(r)$ is indicated. The curves are from above: a) $\tilde{u} = \exp \{-\alpha r\}$; no interaction b) $\tilde{u} = \tilde{u}_{\text{Paris}}$ and OPEP. c) $\tilde{u} = \tilde{u}_{\text{Paris}}$ and V_{Paris} ; d) $\tilde{u} = \tilde{u}_{\text{Paris}}$ and no interaction.

Fig. 10 : Dependence of the integrand (23) for Q_1 on the interaction. The difference between the OPEP and Paris potential value is -6.3% of the OPEP result.

Fig. 11 : The η - f^2 plane. The diagonal lines represent constant Λ contours with the area left of $\Lambda = \infty$ unphysical. The near horizontal lines are constant Q_{pot} lines, based on the following assumptions on relativistic and meson-exchange contributions: a) none, b) 0.009 fm^2 , c) 0.013 fm^2 . The contours represent 39%, 87%, and 99% probability.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

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Fig. 10

Fig. 11