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TESTING QCD AT THE PROTON-ANTIPROTON COLLIDER

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ABSTRACT

The recent UA1 and UA2 data for large- p_T jet production are compared with the predictions of perturbative QCD.

Using the standard leading log QCD formula, and neglecting all corrections (higher orders, higher twists), we find a good agreement with the UA1 data. On the other hand, the UA2 data are below our predictions.

The theoretical uncertainties due to the value of Λ and to the detailed shape of the gluon distribution are within the experimental errors of the UA1 data.

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1. INTRODUCTION

Recently the first results on the jet production from the CERN $p\bar{p}$ collider have been presented [1,2]. In particular, the distributions of the total jet transverse energy E_T have been measured in the energy range $20 \leq E_T \leq 50$ GeV ($\sqrt{s} = 540$ GeV).

The large E_T events develop a very prominent two-jet structure, as expected by the QCD parton picture, and they provide us with the possibility of carrying out the quantitative tests of perturbative QCD in a new energy region.

The idea that there should be a hard component in the hadron-hadron collision at very high energies is as old as the parton model itself. Soon after the granular structure of the proton had been established in the deep inelastic lepton-hadron measurements [3], it was suggested [4] that the hard parton-parton collision should be visible in purely hadronic reactions.

At the quantitative level, this idea has been supported by a series of experiments [5], starting from the CERN ISR measurements [6], where indeed a significant hard component of the single-particle spectra in the large- p_T region has been observed.

However, the quantitative analysis of the QCD-parton ideas in the hard hadronic reactions remained on a less firm ground for a long time. The canonical behaviour for the p_T dependence of the particle or jet spectra,

$$E \frac{d\sigma}{d^3p} \sim \frac{1}{p_T^N} f(x_T) \quad (1)$$

$$x_T = 2p_T/\sqrt{s} ,$$

with $N = 4$, expected by dimensional arguments, was never visible. Instead, much larger ($N \approx 8$) and p_T, x_T -dependent values of N have been reported [5].

In the meantime, however, it became clear that the theory also does not predict such a simple form as in eq. (1). The QCD scaling violation effects [7,8] generate some additional p_T dependence [9],

$$E \frac{d\sigma}{d^3p} \sim \frac{1}{p_T^4} f\left(x_T, \frac{p_T^2}{\Lambda^2}\right), \quad (2)$$

which breaks the simple large- p_T scaling law in eq. (1) and makes the quantitative analysis of the large- p_T data much more cumbersome.

To analyse numerically the formula (2) one needs various ingredients which are either unknown or poorly determined by the perturbative QCD, such as:

- a) the shape of the parton densities (quarks, antiquarks, gluons) at some reference scale Q_0^2 ,
- b) the value of the effective scale $Q^2 = Q^2(p_T^2, x_T)$ at which the parton densities are probed in a given p_T, x_T range,
- c) the contamination of the higher twist contributions (CIM-type processes [10], intrinsic p_T effect, etc.).

A lot of effort has been devoted to studying the QCD predictions for the large- p_T scattering in the accessible energy range by playing with the unknown ingredients listed above [11]. The net conclusion was that the large- p_T data in the ISR energy region are more or less consistent with QCD. However, owing to the considerable freedom in adjusting the unknown elements of the QCD formula, the entire analysis could hardly be considered as a decisive test of perturbative QCD.

One obviously needs larger values of p_T to diminish the influence of higher orders and higher twists. The recent $p\bar{p}$ data, probing the p_T range $20 < p_T(\text{jet}) < 50$ GeV therefore provide the starting point for a new, hopefully less ambiguous, QCD analysis.

The present paper is the first step in this direction. A natural strategy for the initial analysis is just to neglect all non-leading effects and to calculate as precisely as possible the leading log predictions.

This precision is required because of the kinematic region of the present jet collider data. The large- p_T range $20 < p_T < 50$ GeV is actually the small- x_T range $0.07 < x_T < 0.019$ owing to the high c.m.s. energy, $\sqrt{s} = 540$ GeV. As a consequence, the parton densities are effectively probed in the small- x region, where the QCD scaling violation effects are very prominent. Since the approximate analytic parametrizations of the (Q^2 -dependent) parton densities are not very efficient at very small x , we decided to use the Monte Carlo method [12] to simulate the QCD evolution. Hence we treat the scaling violation effects exactly (with the exception of very small Monte Carlo errors) and the only ambiguities of our predictions are due to the value of Λ and to the shapes of the input parton densities.

The quark distributions are rather well determined from the deep inelastic data (we use the CDHS parametrization [13]). The gluon distribution is not so well determined but we find a surprising result: the dominant contribution to the jet cross-section comes from the gluon-gluon interaction but the result is only weakly sensitive to the input gluon distribution, at least within the generally used parametrization

$$xG(x, Q_0^2) = 0.55 (\beta + 1)(1 - x)^\beta. \quad (3)$$

The reason is that $G(x, Q_0^2)$ is probed by the UA1 data in the crossover point, where the curves for various values of β are rather similar.

Another surprising result is that our leading log formula depends very weakly on the value of Λ and/or on the choice of the effective scale $Q^2(p_T^2, x_T)$. This is due to the cancellation effects between the increasing gluon bremsstrahlung and the diminishing coupling constant α_s , which fixes the normalization of the hard scattering.

As a consequence, we obtain an almost unambiguous prediction for the QCD jet cross-sections, which agrees rather well with the recently published collider data. The experimental results, being the first ones, are still subject to systematic errors, which may be around 30 to 50%. In spite of this fact we consider the agreement between the QCD predictions and the data as a first considerable success

of QCD in this new energy region because all possible discrepancies between data and QCD prediction are contained within a factor of 2, but from the ISR to the collider energy regions the cross-section rises by many orders of magnitude.

This paper is organized as follows. In section 2 we predict some details of our numerical technique. The discussion and analysis of the numerical results is given in section 3. The comparison with data together with the conclusions is the content of section 4.

2. MONTE CARLO ALGORITHM FOR THE QCD FORMULA

The inclusive cross-section for the two-jet process is given in QCD by the following expression:

$$d\sigma_{AB}^{J_1 J_2 + X} = \sum_{a,b} \int dx_1 dx_2 F_A^a(x_1, t) F_B^b(x_2, t) \int d\eta \frac{d\hat{\sigma}_{ab}^{J_1 J_2}}{d\eta} . \quad (4)$$

Here A and B are the incoming hadrons, a and b are partons (q, $\bar{q}$, g), and J₁ and J₂ are the final-state jets. The precise definition of a jet, adopted in the UA1 measurement, will be discussed in section 3. For the present purposes, we assume that, as far as the kinematics of eq. (4) is concerned, jets can be just identified with the final-state partons, i.e. $d\hat{\sigma}_{ab}^{J_1 J_2}$ is given by the elastic parton-parton scattering:

$$d\hat{\sigma}_{ab}^{J_1 J_2} = \sum_{a', b'} d\hat{\sigma}_{ab}^{a' b'} .$$

The structure functions F_A^a and F_B^b describe the densities of partons a and b inside the hadrons A and B, respectively. Their Q^2 evolution is governed by the evolution equations [8]:

$$\frac{d}{dt} F_A^a(x, t) = \int dx_1 dx_2 \delta(x - x_1 x_2) \times \sum_{q'} P_{q'}^q(x_1) F_A^{q'}(x_2) , \quad (5)$$

where $P_{q'}^q(x)$ are the Parisi-Altarelli probabilities for the transitions $q' \rightarrow q + x$ and the evolution variable t is related to Q^2 as follows:

$$t = \frac{2}{\beta_0} \ln \frac{\ln Q^2/\Lambda^2}{\ln Q_0^2/\Lambda^2} \quad (6)$$

$$\beta_0 = \frac{11}{3} N_c - \frac{2}{3} N_f ,$$

where N_c = number of colours, and N_f = number of flavours.

The distribution $d\sigma_{AB}^{J_1 J_2 + X}$ can be derived using the following Monte Carlo algorithm:

- a) generate x_1 and x_2 according to the distributions $F_A^a(x_1, t)$ and $F_B^b(x_2, t)$, respectively;
- b) generate $\eta = \frac{1}{2}(1 + \cos \theta_{cm})$ (θ_{cm} = polar angle in the parton-parton c.m. frame) according to the distribution $d\hat{\sigma}_{ab}^{J_1 J_2}/d\eta$;
- c) for given values of x_1 , x_2 , and η , derive the final-state jet momenta; e.g.

$$p_T^2 = x_1 x_2 \eta (1 - \eta) s$$

[here $s = (p_A + p_B)^2$ is the total c.m. energy squared].

However, the simple algorithm presented above turns out to be very inefficient in the large- p_T region. Most of the generated events populate the relatively low p_T region owing to the strong fall-off of the distribution $d\sigma_{AB}^{J_1 J_2}$ with the jet transverse momentum. To increase the efficiency of the program in the large- p_T region, we modify eq. (4) as follows:

$$d\sigma_{AB}^{J_1 J_2} = \left(\frac{s}{p_T^2} \right)^\omega \int dx_1 dx_2 \times F_A^a(x_1, t; \omega) F_B^b(x_2, t, \omega) \frac{d\hat{\sigma}_{a,b}^{J_1 J_2}}{d\eta}(\eta, \omega) , \quad (7)$$

where

$$\begin{aligned} F(x, t; \omega) &= F(x, t) x^\omega \\ d\sigma/d\eta(\eta, \omega) &= d\sigma/d\eta(\eta) [\eta(1-\eta)]^\omega . \end{aligned} \quad (8)$$

As seen, the modification consists just in multiplying eq. (4) by the factor

$$1 = (s/p_T^2)^\omega [x_1 x_2 \eta (1 - \eta)]^\omega , \quad (9)$$

where ω is an arbitrary parameter ($\omega > 0$). The weighting factor $[x_1 x_2 \eta(1-\eta)]^\omega$ provides the damping in the regions $x_1 \approx 0$, $x_2 \approx 0$, $\eta \approx 0$, $\eta \approx 1$ which give rise to the small p_T events. It is because a part of the strong fall-off of the distribution $d\sigma_{AB}^{J_1 J_2}$ has already been factored out explicitly in the form $(s/p_T^2)^\omega$.

The effective p_T^2 distribution $[p_T^2 = x_1 x_2 \eta(1-\eta)s]$, where x_1 , x_2 , and η are generated according to the modified distributions in eq. (8)] has now a form of a broad bump and the position of the maximum $\langle p_T(\omega) \rangle$ increases with ω . We can therefore tune ω in such a way that our sampling becomes efficient in any required p_T -range.

The only non-trivial technical point is the Monte Carlo sampling of the parton densities $F(x, t; \omega)$. We illustrate our method using the example of the non-singlet parton density:

$$F(x, t; \omega) = \sum_i F^{qi}(x, t; \omega) - F^{\bar{q}i}(x, t; \omega) . \quad (10)$$

The evolution equation is

$$\frac{d}{dt} F(x, t; \omega) = \int P_{FF}(x_1, \omega) F(x_2, t, \omega) \times \delta(x - x_1 x_2) , \quad (11)$$

where

$$P_{FF}(x, \omega) = C_F \left(\frac{1+x^2}{1-x} \right)_+ x^\omega , \quad (12)$$

$$C_F = \frac{1}{2N_c} (N_c^2 - 1) .$$

Solving eq. (10) in terms of moments, expanding the result in a power series in t , and inverting back to the x space, we find the following representation:

$$F(x, t; \omega) = \int_{n=0}^{\infty} \frac{[t P_{FF}(\omega)]^n}{n!} e^{-t P_{FF}(\omega)} \times \int dx dy \delta(x - yz) \Gamma_n(y; \omega) F(z, 0; \omega) W(t) , \quad (13)$$

where

$$\begin{aligned}
 P_{FF}(\omega) &= \int_0^{1-\varepsilon} dx \, x^\omega \frac{1+x^2}{1-x} . \\
 \Gamma_n(y;\omega) &= \int dx_1 \dots dx_n P_{FF}(x_1;\omega) P_{FF}(x_2;\omega) \dots P_{FF}(x_n;\omega) \delta(y - \prod_{i=1}^n x_i) \\
 P_{FF}(x;\omega) &= \frac{1}{P_{FF}(\omega)} x^\omega \frac{1+x^2}{1-x} \theta(1-\varepsilon-x) \\
 W(t) &= e^{t[P_{FF}(\omega) - P_{FF}(n)]} .
 \end{aligned} \tag{14}$$

Equation (13) has a simple probabilistic interpretation. Gluons are radiated with the Poisson distribution

$$P_n(\langle n \rangle) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle} ,$$

where $\langle n \rangle = tP_{FF}(\omega)$. Each emission amounts to an energy loss of the radiation quark described by the (normalized) probability $P_{FF}(x,\omega)$.

The Monte Carlo sampling of the distribution $F(x,t;\omega)$ goes as follows:

- a) generate n with the Poisson distribution $P_n[tP_{FF}(\omega)]$;
- b) generate $x_1, x_2, \dots, x_n$ with the distribution $P_{FF}(x,\omega)$;
- c) generate z with the input density $F(x,0;\omega)$;
- d) calculate $x = yz$, $y = \prod_{i=1}^n x_i$;
- e) ascribe the weight $W(t)$ to any event.

The infrared singularities of $P_{FF}(x;\omega)$ at $x = 1$ are regularized by a cut-off $[x < (1-\varepsilon)]$. The limit $\varepsilon \rightarrow 0$ exists for any smeared quantity, such as moments of $F(x,t;\omega)$ or the bin averages:

$$\frac{1}{\Delta x} \int_{x_0}^{x_0+\Delta x} dx \, F(x,t;\omega)$$

if only $\varepsilon \ll \Delta x$.

An additional complication is that, for any typical choice of Q^2 , such as $Q^2 \sim \hat{s} = x_1 x_2 s$ or $Q^2 \sim p_{Tj}^2 = x_1 x_2 \eta(1-\eta)s$, the variable t in eq. (3) is some function of x_1 , x_2 , and η . The simple algorithm for generating $do_{AB}^{J_1 J_2}$, presented above, cannot be used then, since the variables x_1 , x_2 , and η are now coupled in a complicated way. However, using the representation (13), one can solve this problem in a simple way. For this reason we rewrite eq. (13) as follows:

$$F(x, t; \omega) = \int_{n=0}^{\infty} \frac{[\langle t \rangle P_{FF}(\omega)]^n}{n!} e^{-\langle t \rangle P_{FF}(\omega)} \times \int dy dz \Gamma_n(y; \omega) F(z, 0; \omega) W_n(t, t_0),$$

where

$$W_n(t, \langle t \rangle) = \left(\frac{t}{\langle t \rangle} \right)^n \left[e^{\langle t \rangle P_{FF}(\omega) - t P_{FF}(\omega)} \right].$$

The algorithm for generating $F(x, t; \omega)$ is now modified as follows: n is generated according to the Poisson distribution $P_n[\langle t \rangle P_{FF}(\omega)]$, whereas the weight ascribed to a given event is equal to $W_n(t, t_0)$. In this way the whole dependence on $t = t(x_1, x_2, y)$ is contained in the weight which is calculated after x_1 , x_2 , and η have been generated in the way described above.

In a similar manner one can construct the Monte Carlo algorithms for generating gluon and sea-quark distributions. The procedure is now more involved because of mixing, so we only sketch the relevant points. Denoting by E the 2×2 evolution matrix for the singlet case, we have the evolution equation

$$\frac{d}{dt} E = P E, \quad (15)$$

where P is the 2×2 probability matrix ($P_{11} = P_{FF}$, $P_{12} = P_{FG}$, etc.). [The product PE in eq. (15) and in what follows denotes the matrix multiplication, and the convolution in the x space.] The formal solution of eq. (15) is

$$E = e^{tP},$$

but this form is not very suitable for the Monte Carlo calculation. Instead, we write eq. (15) as follows

$$\frac{d}{dt} E = (D+M)E , \quad (16)$$

where D is the diagonal part of P (P_{FF}, P_{GG}) and M is the off-diagonal part (P_{FG}, P_{GF}). We now solve eq. (16) as in the old-fashioned perturbation theory, treating M as a perturbation. The solution is

$$E = e^{tD} + \int_0^t dt e^{(t-\tau)D} M e^{\tau D} + O(M^2) . \quad (17)$$

The formula (17) gives E as a formal series in M , i.e. in the number m of mixing steps.

In general, there is some distribution P_m of the number m of non-diagonal transitions P_{GF}, P_{GG} and some distribution $\Phi(\tau)$ of τ , i.e. of the fraction of the "evolution time", spent by a given diagonal transition. Both probabilistic distributions can be extracted from eq. (17) after normalizing all sums and integrals to unity.

The final Monte Carlo algorithm is as follows:

- a) generate m from the P_m distribution,
- b) split the evolution time t into pieces $\tau_1, \tau_2, \dots$ using $\Phi(\tau)$ distribution,
- c) for each piece, corresponding to a given diagonal evolution with $t = \tau_i$, use the non-singlet algorithm.

3. QCD COMPUTATION

We performed the QCD computation using the parton density functions as recently determined by the CDHS Collaboration [13]. As a characteristic scale of Q^2 we took $Q^2 = \frac{1}{2} p_T^2$ and for Λ we assumed $\Lambda = 0.2$. The predictions are made for the angular and p_T distributions of the final partons. Partons are, of course, not directly observed in the experiments; they appear as jets. A jet is a burst of particles with small relative transverse momenta with respect to each other. In the $p\bar{p}$ experiments they are mainly detected in the calorimeters where they appear as energy clusters which can be well distinguished from the background

above ≈ 20 GeV. In this energy range the cluster energy is equal to the sum over all particle energies in a jet. The position in space of the cluster together with the position of the event vertex gives the direction of the jet. For comparison with QCD we identify the jet energy and direction with the corresponding quantities for scattered partons. This identification is not completely correct owing to the fact that partons in QCD calculation are massless, whereas the observed jets have non-zero invariant mass. This leads to a systematic lowering of the jet energy as compared to the initial parton energy because some part of the jet energy has to convert itself into a jet mass. Since we are dealing here with high-energy jets, we expect that the corrections due to this effect are small so we will neglect them in the forthcoming discussion.

The QCD prediction for the parton p_T distribution is shown together with the measured jet cross-sections from the UA1 and UA2 detectors in fig. 1. In this figure we have also plotted the QCD predictions obtained by Horgan and Jacob [14]. We observe a very good agreement between the two computations in spite of the fact that the evolution was treated differently.

Our exact treatment of the evolution equations allows us to investigate easily the uncertainties in the QCD computation due to the poorly known parameter Λ and the Q^2 scale. We show in fig. 2 the QCD predictions for the p_T inclusive jet distributions for two values of Λ which differ by an order of magnitude. In fig. 3 we show the ratio of these cross-sections. We note that the change of Λ by an order of magnitude leads to changes in the cross-sections which are smaller than a factor of 1.5. The relatively small difference in the computed cross-sections is due to the cancellation of two opposite effects. On the one hand, the increase of Λ leads to the increase of the gluon bremsstrahlung effects, which tends to deplete the higher x parton density distribution tails. On the other hand, it increases the elastic parton-parton cross-sections which partly compensates the losses due to the increased evolution. This is illustrated in fig. 3, where we show the cross-section ratio computed artificially with different Λ 's in the evolution equation but not in the elastic scattering. The weak dependence of

the predicted cross-sections Λ means that we can neglect the effects due to the uncertain scale of the effective Q^2 . The naïve choice of Q^2 as $Q^2 = \hat{s}$ differs from the value suggested by higher order QCD computations [15] $Q^2 = \frac{1}{2} p_T^2$, by a factor of ~ 8 . This corresponds to the effective change of Λ by only a factor of 3.

In the energy region of the collider the jets are produced by the hard scattered gluons. This is shown in fig. 4, where the predicted inclusive cross-section for the quark jet is plotted separately. The gluon density function is only poorly determined in deep inelastic scattering. Therefore it is important to show how the QCD predictions are affected by the uncertainties in its determination. We show in fig. 5 the QCD curves computed with different parton densities. We assume the parton densities to be of the form of eq. (3) at $Q_0^2 = 5 \text{ GeV}^2$ and we change the exponent β from 3 to 8. We see that the corresponding changes in the cross-section are below a factor of 1.5 in the whole measured p_T range. This is due to the fact that the p_T range of the collider jets corresponds to the x -range between ~ 0.1 and 0.2 . In this x -range, as shown in fig. 5, all the density functions have a crossover point so that their effective values are similar. We expect the existence of such a crossover point because we changed only the shape of the gluon density but not its integral.

4. COMPARISON WITH DATA AND CONCLUSIONS

Our QCD predictions are compared with data in figs. 1 and 6. Figure 1 shows the comparison for the inclusive jet cross-sections and fig. 6 that for the invariant mass of the two-jet system. We first discuss the comparison with the UA1 data with which the predictions are in good agreement. We observe that the predictions for the inclusive cross-section of fig. 1 lie slightly but systematically below the UA1 data. This discrepancy may be due to the lack of higher order corrections in our calculation but may also be due to the systematic effects in the experimental cross-section determination. The lack of a systematic shift in the two-jet invariant mass cross-section of fig. 6 indicates that the observed discrepancy in fig. 1 is due, at least mainly, to the experimental systematic errors.

The higher order corrections would raise the prediction for both cross-sections: the single-jet inclusive and the two-jet invariant mass one.

The UA2 inclusive cross-sections are systematically lower than the UA1 data. As can be seen from figs. 1, 2, and 4, our predictions are in poorer agreement with the UA2 data even if we allow the variation of the input parameters in a considerable range. We consider this lack of full agreement as significant, especially in view of the fact that we have not considered in our calculation intrinsic p_T , higher orders and higher twist effects, which all would further increase the predicted cross-sections and therefore also the discrepancy. However this disagreement may be due to the assumption about the width of the jet, made by the UA2 collaboration, which might have been rather restrictive.

In conclusion, we can say that the QCD makes predictions which are remarkably stable under the change of free parameters. These predictions agree well with the UA1 data and not so well with the UA2 data. In view of the fact that both groups have taken a considerable amount of new data the differences between the results should soon be resolved. We found the agreement of the QCD predictions with one set of data encouraging and indicating that $p\bar{p}$ collider physics will provide us with valuable quantitative QCD tests.

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Figure captions

- Fig. 1 : Comparison of the UA1 and UA2 data with the QCD predictions. The H+J curve describes the Horgan and Jacob prediction [14] computed with $\Lambda = 0.5$. The F+K curve is our prediction computed with the standard structure function [13] and $\Lambda = 0.2$. y denotes the pseudo-rapidity, E_T the jet transverse energy (with the assumption $E_T = p_T$).
- Fig. 2 : This figure shows the sensitivity of the QCD predictions to the changes of Λ .
- Fig. 3 : This figure shows the ratio of the predicted cross-sections from fig. 2 (full line). The dotted line shows the same ratio computed with different Λ changed artificially only in the evolution part of the computation but not in the elastic scattering part.
- Fig. 4 : This figure shows the sensitivity of the QCD predictions to the changes of the shape of the gluon density function. The gluon density is assumed to have a form $\approx (1-x)^B$.
- Fig. 5 : The shape of different gluon density functions with the same integral. Note the crossover point in the region of data on collider jets.
- Fig. 6 : Comparison of the UA1 data for the invariant jet-jet mass with the QCD predictions. The cross-sections are evaluated in the pseudo-rapidity interval $-1.4 < y < 1.4$ and for jets (partons) with $E_T > 15$ GeV/c.
- Fig. 7 : QCD predictions for inclusive cross-sections for all partons (i.e. quarks and gluons), and quarks alone. The upper curve is a QCD prediction for all partons computed without the Altarelli-Parisi evolution equation.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7