INTERNATIONAL CENTRE FOR
THEORETICAL PHYSICS

FINITE TEMPERATURE SCALAR PERGEOMETRY

G. Denardo
and
E. Spallucci

1982 MIRAMARE-TRIESTE

INTERNATIONAL
ATOMIC ENERGY
AGENCY

UNITED NATIONS
EDUCATIONAL,
SCIENTIFIC
AND CULTURAL
ORGANIZATION

International Atomic Energy Agency

and

United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

FINITE TEMPERATURE SCALAR PERGEOMETRY *

G. Denardo

International Centre for Theoretical Physics, Trieste, Italy,
Istituto di Fisica Teorica dell'Universita' di Trieste, Italy,

and

Istituto Nazionale di Fisica Nucleare, Sezione di Trieste,

and

E. Spallucci

Istituto di Fisica Teorica dell'Universita' di Trieste, Italy.

MIRAMARE - TRIESTE

December 1982

* To be submitted for publication.

ABSTRACT

We study the temperature corrections to the induced Newton constant. We obtain the Einstein action from an effective action of matter at finite temperature by means of the heat-kernel expansion method. We find that the induced gravitational constant decreases as the temperature becomes higher. In the second part of the paper we investigate the consequences of this behaviour on the black hole thermodynamics. We deduce the existence of a minimum mass and entropy corresponding to the maximum allowed temperature for the black hole.

1. INTRODUCTION

Many attempts to quantize the gravitational field have been made but none of them has arrived to a fully satisfactory theory⁽¹⁾. This difficulty has suggested to some authors that actually gravity could be interpreted at very high energies in terms of quantum effects of matter fundamental fields. This approach pioneered by Sakharov⁽²⁾ and more recently adopted by many others⁽³⁾ is known as pregeometry or as induced quantum gravity⁽⁴⁾. The characteristic feature of this approach is that the classical Einstein action is obtained as a long wavelength approximation of an effective action for quantum matter fields, and the induced Newton constant G_{ind} is expressed through the characteristic parameters of the fundamental field. In general, a correct numerical estimate of G_{ind} has not yet been obtained in a self-consistent way⁽⁵⁾. One then fixes suitably the values of the masses, coupling constants and the cut-off of the pregeometric fields in order to reproduce at least the order of magnitude of G_N ⁽⁶⁾. Such an approach is supported by the fact that the masses one must introduce are of the order of the energy scales of grand unified theories.

In this paper we want to investigate the effects of temperature on G_{ind} .

One of the most appealing aspects of modern quantum field theory is the analogy with statistical mechanics which allows the introduction of the concept of temperature for the vacuum state⁽⁷⁾. Actually in finite temperature quantum field theory radiative corrections introduce an explicit temperature dependence in the physical masses and coupling constants, therefore the same will occur for G_{ind} which is essentially generated by quantum vacuum fluctuations. Such effects are expected to become important when temperature is high enough and this can be

realized either in early cosmologies or in the field of a black hole.

In Section 2 we shall recover the temperature dependence of the induced Newton and cosmological constant. In Section 3 we shall study the effect of the temperature dependent ϵ_{NW} in the process of particle emission of a black hole.

We shall use natural units $\hbar = c = 1$, the Planck mass defined as $M_P = (2\pi\epsilon_{\text{NW}})^{-1}$ where ϵ_{NW} is the zero temperature Newton's constant and K_B is the Boltzmann constant.

2. FINITE TEMPERATURE INDUCED ACTION

In the framework of Induced Quantum Gravity or Pregeometry the fundamental object one starts from is the action for matter quantum fields propagating in a classical (non-dynamical) background geometry. For a free scalar field the most general form of the euclidean action is

$$S_E \equiv \int_M d^4x \sqrt{g} L_E[\phi] \quad (2.1)$$

$$L_E[\phi] = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} (m^2 + \xi R) \phi^2, \quad (2.2)$$

where m is the mass of ϕ , and ξ is the n -dimensional coupling constant between ϕ and the Ricci scalar R of the metric $g_{\mu\nu}$. In (2.1) and (2.2) the Wick rotation $X^0 \rightarrow -i X_E^0$ has been performed in order to guarantee that the functional integral for the partition function

$$Z[g] = \int [d\phi] e^{-S_E[\phi;g]} \quad (2.3)$$

be well defined⁽⁸⁾. Therefore (2.1) and (2.2) describe a quantum field theory on a Riemannian (rather than pseudo-Riemannian) manifold M . At this stage $g_{\mu\nu}(x)$ has no dynamical meaning, (there is no kinetic term in the action for $g_{\mu\nu}$ itself) it simply plays the role of an auxiliary field which makes the theory invariant under general coordinate transformations. This approach is similar to the semiclassical theory of quantized electrons interacting with an external electromagnetic field⁽⁹⁾. In this framework the gauge field $A_\mu(x)$ is introduced in the

Dirac Lagrangian to implement $U(1)$ local phase invariance. The kinetic term $(F_{\mu\nu})^2$ for A_μ can be obtained by integrating out the electron field in the generating functional for the theory. This is essentially the Schwinger approach⁽⁹⁾ to QED. The same strategy is adopted in the induced quantum gravity programme.

In order to apply the usual formalism of finite temperature quantum field theory we shall assume M to be a static manifold, with topology $S^1 \times M_3$, where S^1 refers to the euclidean time coordinate and M_3 is the spatial, three-dimensional, homogeneous, section of M . A further simplification can be obtained by choosing M_3 without boundaries⁽¹⁰⁾, then no surface terms will appear in the induced action^(*). Therefore we can write (2.1) as^(**)

$$S_E \equiv \int_{S^1 \times M_3} d^4x \sqrt{g} L_E \equiv \int_0^\Lambda dx_4 \int_{M_3} d^3x \sqrt{g} L_E, \quad (2.4)$$

(*) All these hypotheses seem to be quite restrictive and could suggest that the final results would depend on them. Actually this does not seem to be the case. In fact, the induced cosmological and Newton constants arise as coefficients of the geometrical terms in the induced action, depending only on the ultraviolet cut-off Λ , the mass m , the coupling constant ξ , i.e. the parameters characterizing the quantum matter field vacuum. The only memory of M , the length β of the S^1 timelike dimension, enters the finite part of the induced action and is customarily interpreted as the inverse vacuum temperature. All other information about the structure of M is factored out of the induced gravitational constants. So we are confident that the temperature dependence we shall find will be actually independent of our choice of M and $g_{\mu\nu}$.

(**) For the sake of notational simplicity we shall use in the following the short-hand notation $\int_M d^4x \sqrt{g}(\dots)$ instead of the integration in (2.4).

where β is the length of the euclidean time axis. In the framework of finite temperature quantum field theory β is identified with the inverse temperature $\beta^{-1} = k_B T$, and the field configurations we sum over in the path integral (2.3) are the only ones which are periodic in x_4^0 with period β . Then Z is the partition function for a statistical system at the temperature T , and the effective action W

$$e^{-W[g]} \equiv Z[g] \quad (2.5)$$

becomes the free-energy for such a system. $W[g]$ formally reads

$$W[g] = \frac{1}{2} \ln \text{Det } \Delta \quad (2.6)$$

with $\Delta \equiv -\tilde{\square} + \xi R + m^2$. A useful representation for $W[g]$ is provided by the Schwinger De-Witt proper-time formalism⁽¹¹⁾

$$W[g] = -\frac{1}{2} \int_0^\infty \frac{ds}{s} \int_M d^4x \sqrt{g} K(x, x; s) \quad (2.7)$$

$K(x, x; s)$ is the heat-kernel of Δ in the coincidence limit. At finite temperature $K(x, x; s)$ can be expressed as a sum of zero temperature images⁽¹²⁾:

$$K(x, x; s) = \sum_{n=-\infty}^{+\infty} \frac{e^{-\beta^2 m^2/4s}}{(4\pi s)^{1/2}} K_3(\bar{x}, \bar{x}; s) \quad (2.8)$$

$K_3(\bar{x}, \bar{x}; s)$ is a solution of the three-dimensional diffusion equation

$$\left(\partial_{\bar{s}}^2 - \tilde{\square}_3 + \xi R \right) K_3(\bar{x}, \bar{x}; s) = 0 \quad (2.9)$$

where $\tilde{\square}_3$ is the covariant, D'Alembertian on M_3 and suitable

spatial boundary conditions on $K^{\lambda}(\vec{x}, \vec{x}', s)$ have to be imposed.

The induced action is obtained by inserting into (2.7) the asymptotic expansion of $K(x, x', s)$ and by taking into account only the contributions from the suitably regularized divergent terms:

$$W[g] = -\frac{1}{2(4\pi)^2} \sum_{k=0}^{\infty} B_k(x) \int_0^{\infty} ds s^{\frac{k}{2}-3} e^{-ms} \sum_{n=-\infty}^{\infty} e^{-m^2 s/n^2} \quad (2.10)$$

with

$$B_k(x) = \int_M d^4x \sqrt{g} b_k(x) \quad (2.11)$$

b_k are k -degree polynomials in the curvature invariants of M . According to our hypothesis about M , $B_{2k+1} \equiv 0$. Then we find for the effective action

$$W[g] = W_D(\Lambda) + W(\beta) + \tilde{W} =$$

$$= -\frac{1}{2(4\pi)^2} \left\{ \sum_{k=0}^4 B_k \int_{1/\Lambda^2}^{\infty} ds s^{\frac{k}{2}-3} e^{-ms^2} + \sum_{k=0}^4 B_k \int_0^{\infty} ds s^{\frac{k}{2}-3} e^{-\beta^2 m^2 s} + \sum_{k=0}^4 B_k \int_0^{\infty} ds s^{\frac{k}{2}-3} e^{-\beta^2 m^2 s} \right\} \quad (2.12)$$

The first sum in (2.12) $W_D(\Lambda)$ is just the zero temperature induced action regularized by means of an ultraviolet cut-off Λ . The second sum in (2.12) $W(\beta)$ is finite and contains the temperature contribution to the induced action while the third series $\tilde{W}$ can be neglected at low energy according to the reasoning in ref. (6). $W_D(\Lambda)$ has been evaluated elsewhere⁽⁶⁾ it turns out to be:

$$W_D(\Lambda) = -\frac{1}{4(4\pi)^2} B_0 \left[\Lambda^4 - 2m^2 \Lambda^2 - m^4 \left(\gamma - \frac{3}{2} - \ln \Lambda^2/m^2 \right) \right] + \quad (2.13)$$

$$-\frac{1}{2(4\pi)^2} B_2 \left[\Lambda^2 + m^2 \left(\gamma - 1 \right) - m^2 \ln \Lambda^2/m^2 \right] - \frac{1}{2(4\pi)^2} B_4 \left(\ln \Lambda^2/m^2 - \gamma \right)$$

$$B_0 = \int_M d^4x \sqrt{g} \quad (2.14)$$

$$B_2 = \int_M d^4x \sqrt{g} \left(\frac{1}{6} - \frac{1}{3} \right) R \quad (2.15)$$

$$B_4 = \int_M d^4x \sqrt{g} \left[\frac{1}{180} (R^{\lambda\mu\nu\rho} R_{\lambda\mu\nu\rho} - R^{\mu\nu} R_{\mu\nu}) + \right. \quad (2.16)$$

$$\left. + \frac{1}{6} \left(\frac{1}{3} - \frac{1}{3} \right) \bar{\square} R + \frac{1}{2} \left(\frac{1}{6} - \frac{1}{3} \right) R^2 \right]$$

For later convenience let us define, without any loss of generality, the induced cosmological and Newton constants in terms of cut-off dependent quantities only, i.e. at zero temperature we have:

$$\frac{1}{16\pi G_{\Lambda_0}} = \frac{1}{2(4\pi)^2} \left(\frac{1}{6} - \frac{1}{3} \right) (\Lambda^2 - m^2 \ln \Lambda^2/m^2) \quad (2.17)$$

$$\frac{2\Lambda^0}{16\pi G_{\Lambda_0}} = V_0 - \frac{1}{4(4\pi)^2} (\Lambda^4 - 2m^2 \Lambda^2 + m^4 \ln \Lambda^2/m^2) \quad (2.18)$$

V_0 is a vacuum energy normalization constant^(*). We recall that in order to have the correct positive sign of G_{Λ_0} (attractive gravitation) we must require $\frac{1}{3} < \frac{1}{6}$. As we shall see in the following this requirement determines also the "asymptotically free" temperature behaviour of $G_{\Lambda_0}(\beta)$.

A compact representation for $W(\beta)$ can be obtained by means

(*) e.g. we can choose V_0 in such a way as to reproduce the observed almost vanishing value of Λ^0 in the present epoch of the universe. According to Sandage⁽¹³⁾ $\Lambda^0 \lesssim 3 \cdot 10^{-55} \text{ cm}^{-2}$.

of the generalized Riemann ζ function⁽¹⁴⁾:

$$F(\alpha, \mu^2) = \sum_{n=0}^{\infty} \frac{1}{n!} (m^2 + \mu^2)^{-\alpha} \quad (2.19)$$

$$W(\beta) = -\frac{1}{2} \sum_{k=0}^{\infty} B_k \left[\frac{1}{(4\pi)^{3/2} \beta} \left(\frac{\beta}{2\pi} \right)^{k-3} \Gamma(k/2 - 3/2) F(k/2 - 3/2, \mu^2) + \right. \\ \left. + \frac{(m^2)^{3-k}}{2(4\pi)^{3/2} \beta} \Gamma(k/2 - 3/2) - \frac{(m^2)^{2-k}}{2(4\pi)^2} \Gamma(k/2 - 2) \right] \quad (2.20)$$

with $\mu \equiv (\beta m / 2\pi)$. For $k = 0, 2, 4$ the pole part of $F(k/2 - 3/2, \mu^2)$ exactly cancels against the pole of the third term in (2.20), as it must be because all the ultraviolet divergences are contained in the zero temperature part. This cancellation can be easily verified using the Laurent expansion of (2.19):

$$F(\alpha, \mu^2) = \frac{F_1(m; \mu^2)}{\alpha + m^{-1/2}} + F_0(m; \mu^2) + O(\alpha + m^{-1/2}) \quad (2.21)$$

In such a way (2.20) becomes

$$W(\beta) = -B_0 \left[\frac{4\pi^2}{3\beta^4} F_0(2; \mu^2) + \frac{m^3}{12\pi\beta} - \frac{m^4}{4(4\pi)^2} \left(\frac{3}{2} - \gamma \right) \right] + \\ + B_2 \left[\frac{1}{2\beta^2} F_0(2; \mu^2) + \frac{m}{8\pi\beta} + \frac{m^2}{2(4\pi)^2} (\gamma - 1) \right] - \frac{B_4}{16\pi^2} \left(\frac{3}{2} \gamma + \frac{\pi}{m\beta} \right). \quad (2.22)$$

By inserting (2.22) and (2.13) in (2.12) and comparing with the Einstein action one finds

$$\frac{2\lambda(\beta)}{16\pi G_{\text{ind}}(\beta)} = \frac{2\lambda^{(1)}}{16\pi G_{\text{ind}}^{(0)}} - \frac{m^3}{16\pi\beta} - \frac{4\pi^2}{3\beta^2} F_0(2; \mu^2) \quad (2.23)$$

$$\frac{1}{16\pi G_{\text{ind}}(\beta)} = \frac{1}{16\pi G_{\text{ind}}^{(0)}} - \left(\frac{1}{6} - \xi \right) \left[\frac{m}{8\pi\beta} + \frac{1}{2\beta^2} F_0(4; \mu^2) \right] \quad (2.24)$$

The asymptotic form⁽¹⁴⁾ of $F_0(m; \mu^2)$ for small μ^2 is

$$F_0(4; \mu^2) \sim -\frac{1}{12} + \frac{1}{2}(\gamma - 1)\mu^2$$

$$F_0(2; \mu^2) \sim \frac{1}{120} - \frac{1}{8}\mu^2 + \left(\frac{3}{8}\gamma - \frac{1}{2} \right) \mu^4 + \dots$$

and provides us with the high temperature behaviour of $\lambda(\beta)$ and $G_{\text{ind}}(\beta)$

$$\frac{2\lambda(\beta)}{16\pi G_{\text{ind}}(\beta)} \sim \frac{2\lambda^{(1)}}{16\pi G_{\text{ind}}^{(0)}} - \frac{\pi^2}{40\beta^4} + \frac{m^2}{24\beta^2} + \dots \quad (2.25)$$

$$\frac{1}{16\pi G_{\text{ind}}(\beta)} \sim \frac{1}{16\pi G_{\text{ind}}^{(0)}} + \left(\frac{1}{6} - \xi \right) \left[\frac{1}{24\beta^2} - \frac{m}{8\pi\beta} - \frac{m^2}{16\pi^2} (\gamma - 1) \right] \quad (2.26)$$

(2.26) represents the main result of this paper: $G_{\text{ind}}(\beta)$ decreases as the temperature becomes larger and larger. The physical consequences of this behaviour during the late stage of black hole evaporation will be discussed in the next section. We remark that the choice $\xi < 1/6$, required for reproducing the correct sign of $G_{\text{ind}}^{(0)}$, also fixes the overall positive sign in front of the temperature dependent part in (2.26). This plus sign is crucial because it guarantees that $G_{\text{ind}}(\beta)$ will never become infinite or negative (= antigravity?) for any finite value of the temperature⁽¹⁵⁾. Therefore we see that induced quantum gravity seems to exhibit a physically more reasonable temperature behaviour than ordinary field theoretical models containing the Einstein action, besides matter and gauge fields, from the very beginning.

3. BLACK-HOLE EVAPORATION

From now on we shall assume (2.26) as the general law describing the temperature dependence of $G_{\mu\nu}(\beta)$ in the framework of scalar pregeometry. A natural arena for testing this kind of effects could be the hot-black hole during the late stage of its evaporation. The relation between the mass M_H and the temperature T_H of such an object reads

$$T_H = \frac{1}{8\pi k_B} \frac{1}{G_{\mu\nu} M_H} = \frac{M_P^2}{4k_B M_H} \quad (3.1)$$

(3.1) is the famous Hawking formula⁽¹⁶⁾⁽¹⁷⁾ obtained by quantizing a scalar, massless, minimally coupled, i.e. $\xi = 0$, matter field in the Schwarzschild background geometry. The temperature corrections to (3.1) can be obtained by substituting $G_{\mu\nu}^{-1}$ with $G_{\mu\nu}(\beta)^{-1}$ given by (2.26) taken with $M = 0$ and $\xi = 0$. In this way a quadratic equation for T_H is obtained. Its solution is

$$k_B T_H = 36 M_H \left[1 - \left(1 - M_P^2 / 42 M_H^2 \right)^{1/2} \right] \quad (3.2)$$

The solution with the plus sign in front of the square root must be discarded because it does not reproduce (3.1) (it increases linearly with M_H) in the limit $M_H \gg M_P$, that is when the temperature is very small and $G_{\mu\nu}(\beta) \approx G_{\mu\nu}$. On the contrary, this boundary condition is naturally satisfied by (3.2).

The most remarkable feature of (3.2) is that the black hole mass cannot decrease below a minimum value in order to keep the temperature a real number. Therefore we find a lower bound for the mass of a quantum black-hole

$$M_H \geq \frac{M_P}{\sqrt{2}} \simeq 0.117 M_P \equiv M_H^* \quad (3.3)$$

It has already been suggested that the relic of a black hole explosion should be massive object around the Planck mass M_P ⁽¹⁸⁾. The usual motivation is that near the Planck energy the semiclassical approximation, used to obtain (3.1), necessarily breaks down because the quantum fluctuations of the metric itself become stronger than the matter field fluctuations, so a fully quantum treatment is compelling. As far as we know this is the first time that a lower bound on the black hole mass is provided self-consistently by the theory itself and not simply guessed on the basis of dimensional arguments. From the numerical point of view M_H^* results to be around 1/10 of the Planck mass and this is consistent with the decrease of $G_{\mu\nu}(\beta)$ with the temperature. A smaller $G_{\mu\nu}(\beta)$ means a smaller gravitational radius, i.e. mass, below which the quantum gravitational effects dominate.

To the minimum M_H^* mass the maximum temperature corresponds (it is easy to check that (3.2) is a monotonically decreasing function of M_H):

$$T_H^* = \frac{6 M_P}{k_B \sqrt{2}} \simeq 3 \cdot 10^{32} K \quad (3.4)$$

it is very similar to the maximum temperature found in ref. (19).

The dynamics of the evaporation process is described, in the geometrical optics approximation, by equating the mass loss of the hole with the energy flux emitted to infinity⁽¹¹⁾

$$\frac{dM_H}{dt} = - \frac{\pi k_B^2 T_H^2}{60} \quad (3.5)$$

Actually (3.5) is correct in the adiabatic approximation: the flux of energy is evaluated ignoring the time dependence of M_H . A self-consistent calculation is precluded until the back-reaction problem will not be solved. Some attempts in this direction have been recently carried out by modelling a radiating black-hole by means of the Vaidya metric (18). Anyway the problem is still far from being fully clarified. Therefore what follows has to be considered with some caution.

By inverting the functional relation between M_H and T_H we find

$$M_H(T_H) = \frac{1}{8\pi k_B} \frac{1}{T_H} \left(2\pi M_P^2 + \frac{\pi}{9} k_B^2 T_H^2 \right). \quad (3.6)$$

Then, expressing time derivative in terms of temperature derivative we get

$$\frac{dT_H}{dt} = \frac{\pi k_B^2}{60} T_H^2 \left(\frac{M_P^2}{4k_B T_H^2} - \frac{k_B}{T_H} \right)^{-1}. \quad (3.7)$$

Eq. (3.7) can be easily integrated from the initial time t_0 , the instant from which we start to study the evaporation process, to the generic time

$$\frac{\pi k_B^2}{15} t = \frac{M_P^2}{3k_B} \left(\frac{1}{T_0^3} - \frac{1}{T_H^3} \right) + \frac{k_B}{18} \left(\frac{1}{T_H} - \frac{1}{T_0} \right), \quad (3.8)$$

where T_0 is the hole temperature at the initial time t_0 . For $T_H = T_H^*$ (3.8) gives us the time needed to reach the maximum temperature

$$\Delta t = \frac{5M_P^2}{\pi k_B^3 T_0^3} + \frac{5\sqrt{2}}{54\pi M_P} - \frac{5}{6\pi k_B} \frac{1}{T_0}. \quad (3.9)$$

If the initial temperature is very low, i.e. $T_0 \ll T_H^*$, the first term in (3.9) provides the main contribution to Δt , which means that the black-hole spends most of its "life" in a cold phase. The final stage of the evaporation is even shorter than in the case studied by Davies (19), in fact, by expanding (3.9) in Taylor series for $T_0 \gg T_H^*$ one finds a quadratic instead of a linear law

$$\Delta t \sim \frac{5}{T_0 \rightarrow T_H^*} \frac{1}{6\pi k_B} T_H^{*3} (T_0 - T_H^*)^2. \quad (3.10)$$

This is because the first derivative of Δt is identically vanishing at $T_0 = T_H^*$. Like in ref. (19) the final explosive burst of energy is suppressed, the emitted flux approaching a constant as $T \rightarrow T_H^*$

$$- \frac{dM_H}{dt} \xrightarrow{T \rightarrow T_H^*} \frac{\pi k_B^2 T_H^{*2}}{60}. \quad (3.11)$$

The original Hawking theory corresponds to the limit $T_H^* \rightarrow \infty$. Another physical quantity which approaches a finite constant value, instead of vanishing, is the entropy S_H of the black hole:

$$S_H = \frac{1}{16\pi k_B} \frac{1}{T_H^2} \frac{1}{G_{\text{WD}}(\beta)}. \quad (3.12)$$

Using again for $G_{\text{WD}}(\beta)$ (2.26) with $m=0$ and $\xi=0$, we obtain

$$S_H = \frac{1}{16\pi k_B} \frac{1}{G_{\text{WD}}} \frac{1}{T_H^2} + \frac{k_B}{144}. \quad (3.13)$$

Hence $dS_H/dt < 0$ and therefore using (3.5) also $dS_H/dt < 0$ as in the usual thermodynamics of black hole. We have nevertheless the peculiarity that S_H never vanishes, because T_H cannot exceed T_H^* , which implies a non-zero minimum value $S_{\text{MIN}} = k_B/72$. This looks consistent

with the other limitations on the physical parameters characterizing the black hole.

The result obtained in this paper relies on a simple model using a scalar field to represent the matter. Anyway our conclusions seem to be meaningful and this suggests us to investigate a more realistic model in which the fundamental pregeometry fields are only fermions and gauge fields. This will be the subject of a forthcoming paper.

REFERENCES

1. C.J. Isham: "Quantum Gravity: An Overview" in Quantum Gravity 2. A Second Oxford Symposium, Eds. C.J. Isham, R. Penrose and D.W. Sciama, Clarendon Press-Oxford 1981.
2. A.D. Sakharov: Sov. Phys. Dokl. 12, 1040 (1968).
3. T. Matauki: Progr. Theor. Phys. 59, 235 (1978);
K. Akama, Y. Chikashige, T. Matsuoka, ibidem, 60, 868 (1978);
K. Akama: ibidem 60, 1900 (1978);
P. Minkowski: Phys. Lett. 71B, 419 (1977);
L. Smolin: Nucl. Phys. B160, 253 (1979);
A. Zee: Phys. Rev. Lett. 42, 417 (1979);
A. Zee: Phys. Rev. D23, 858 (1981);
S.L. Adler: Phys. Rev. Lett. 44, 1567 (1980);
S.L. Adler: Phys. Lett. 95B, 241 (1980);
D. Amati, G. Veneziano: Phys. Lett. 105B, 358 (1981);
D. Amati, G. Veneziano: Nucl. Phys. B204, 451 (1982);
A. Aurilia, M. Martellini: "A Model for Quantum Gravity: an Effective Approach to Einstein Theory", Alberta - Thy -33 -81, preprint.
4. A pedagogical review of the subject is: S. Adler: Rev. Mod. Phys. 54, 729 (1982).
5. An estimate for $\langle \sigma_{\text{up}} \rangle$ by using a Yang-Mills instantons gas has been given by B. Hasslacher, E. Mottola: Phys. Lett. 95B, 237 (1980).
6. G. Denardo, E. Spallucci: N. Cim. 59A, 151 (1982).
7. L. Lolan, R. Jackiw: Phys. Rev. D9, 3320 (1974);
S. Weinberg: Phys. Rev. D9, 3357 (1974);
C. Bernard: Phys. Rev. D9, 3308 (1974).
8. A new approach to the functional formulation of Euclidean quantum field theory has been recently proposed by V. de Alfaro, S. Fubini,

- G. Furlan: "On the functional formulation of quantum field theory" Ref. Th. 3426 - GERN preprint (1982).
9. J. Schwinger: Phys. Rev. 82, 664(1951).
10. Surface terms have been introduced into the Einstein Action by G.W. Gibbons, S.W. Hawking: Phys. Rev. D15, 2752 (1977); Surface action renormalization has been discussed by G. Kennedy, R. Critchley, J.S. Dowker: Ann. Phys. 125, 346 (1980).
11. B. De Witt: Phys. Rep. 19C, 295 (1975).
12. J.S. Dowker, G. Kennedy: J. Phys. A11, 895 (1978); J.S. Dowker, R. Critchley: Phys. Rev. D15, 1484 (1977).
13. A. Sandage: Observatory, 88, 91 (1968).
14. This hypothesis has been recently suggested by A.D. Linde: Phys. Lett. 93B, 394 (1980).
16. S.W. Hawking: Comm. Math. Phys. 43, 199 (1975).
17. An extensive discussion about particle creation by black-holes is given in the book by N.D. Birrell, P.C.W. Davies "Quantum Fields in Curved Spacetime", Cambridge University Press, 1982.
18. P. Hajicek, W. Israel: Phys. Lett. 80A, 9 (1980); W.A. Hiscock: Phys. Rev. D23, 2813, 2823 (1981); R. Balbinot, R. Bergamini: N. Cim. 68B, 104 (1982).
19. P.C.W. Davies: "Gauge theories, black-hole evaporation and Cosmic Censorship" Phys. Lett. 101B, 399 (1981).
- IC/82/63 N.S. CRAIGIE, V.K. DOBREV and I.T. TODOROV - Conformal techniques for OPE in asymptotically free quantum field theory.
- IC/82/64 A. FRYDRYSZAK and J. LUKIERSKI - $N = 2$ massive matter multiplet from quantization of extended classical mechanics.
- IC/82/65 TAHIR ABBAS - Study of the stomic ordering in the alloys Ni-IR using diffuse X-ray scattering and pseudopotentials.
- IC/82/66 E.C. NJAU - An analytic examination of distortions in power spectra due to sampling errors. INT.REP.*
- IC/82/67 E.C. NJAU - Power estimation on sinusoids mounted upon D.C. background: Conditional problems. INT.REP.*
- IC/82/68 E.C. NJAU - Distortions in power spectra of signals with short components. INT.REP.*
- IC/82/69 E.C. NJAU - Distortions in two- and three-dimensional power spectra. INT.REP.*
- IC/82/70 L. SCHWARTZ and A. PAJA - A note on the electrical conductivity of disordered alloys in the muffin-tin model.
- IC/82/71 D.G. FAKIROV - Mass and form factor effects in spectrum and width of the semi-leptonic decays of charmed mesons. INT.REP.*
- IC/82/72 T. MISHONOV and T. SARIISKY - Acoustic plasma waves in inversion layers and sandwich structures. INT.REP.*
- IC/82/73 T. MISHINOV - An exactly averaged conductivity in a disordered electronic model. INT.REP.*
- IC/82/74 S.M. MUJIBUR RAHMAN - Structural energetics of noble metals.
- IC/82/75 E. SEGGIN' and P. van NIEUWENHUIZEN - Ultraviolet finiteness of $N = 8$ supergravity, spontaneously broken by dimensional reduction.
- IC/82/76 JERZY RAYSKI and JACEK RAYSKI, Jr. - On a fusion of supersymmetries with gauge theories.
- IC/82/77 A. BOKHARI and A. QADIR - A prescription for n-dimensional vierbeins. INT.REP.*
- IC/82/78 A. QADIR and J. QUAMAR - Relativistic generalization of the Newtonian force. INT.REP.*
- IC/82/79 B.E. BAAQUIE - Evolution kernel for the Dirac field.
- IC/82/80 S. RAJPOOT AND J.G. TAYLOR - Broken supersymmetries in high-energy physics.
- IC/82/81 JAE HYUNG YEE - Photon propagators at finite temperature.
- IC/82/82 S.M. MUJIBUR RAHMAN - Roles of electrons-per-atom ratio on the structural stability of certain binary alloys. INT.REP.*
- IC/82/83 D.K. SRIVASTAVA - Geometrical relations for potentials obtained by folding density dependent interactions. INT.REP.*
- IC/82/84 C.A. MAJID - Glass forming tendencies of chalcogenides of the system $(As_2 Se_3)_{1-x} (Ti_2 Se)_x$. INT.REP.*
- IC/82/85 C.A. MAJID - Surface photoconductivity in amorphous $As_2 Se_3$. INT.REP.*

- IC/82/86 FAHEEM HUSSAIN and A. QADIR - Quantization in rotating co-ordinates revisited.
- IC/82/87 G. MUKHOPADHYAY and S. LUNDQVIST - The dipolar plasmon modes of a small metallic sphere.
- IC/82/88 A.P. BAKULEV, N.N. BOGOLIOBOV, Jr. and A.M. KURBATOV - The generalized Mayer theorem in the approximating Hamiltonian method.
- IC/82/89 R.M. MOHAPATRA and G. SENJANOVIC - Spontaneous breaking of global B-L symmetry and matter-anti-matter oscillations in grand unified theories.
- IC/82/90 PRABODH-SHUKLA - A microscopic model of the glass transition and the glassy state.
- IC/82/91 WANG KE-LIN - A new vacuum structure, background strength and confinement.
- IC/82/92 G.A. CHRISTOS - Anomaly extraction from the path integral.
- IC/82/93 V. ALDAYA and J.A. DE AZCARRAGA - Supergroup extensions: from central charges to quantization through relativistic wave equations.
- IC/82/94 ABUS SALAM and E. SEZGIN - Maximal extended supergravity theory in seven dimensions.
- IC/82/95 G. SENJANOVIC and A. SOKORAC - Observable neutron-antineutron oscillations in SO(10) theory.
- IC/82/96 Li TA-tsein and SHI Jia-hong - Global solvability in the whole space for a class of first order quasilinear hyperbolic systems.
- IC/82/97 Y. FUJIMOTO and ZHAO Zhi Yong - Avoiding domain wall problem in SU(N) grand unified theories.
- IC/82/98 K.G. AKDENIZ, M. ARIK, M. HORTACSU and M.K. PAK - Gauge bosons as composites of fermions.
- IC/82/100 M.H. SAFFOURI - Treatment of Cerenkov radiation from electric and magnetic charges in dispersive and dissipative media.
- IC/82/101 M. OZER - Precocious unification in simple GUTs.
- IC/82/102 A.N. ERMILOV, A.N. KIREEV and A.M. KURBATOV - Random spin systems with arbitrary distributions of coupling constants and external fields. Variational approach.
- IC/82/103 K.H. KHANNA - Landau's parameters and thermodynamic properties of liquid He II.
- IC/82/104 H. PUSZKARSKI - Effect of surface parameter on interband surface mode frequencies of finite diatomic chain.
- IC/82/105 S. CECOTTI and L. GIRARDELLO - Local Nicolai mappings in extended supersymmetry.
- IC/82/106 K.G. AKDENIZ, M. ARIK, M. DURGUT, M. HORTACSU, S. KAPTANOGLU and N.K. PAK - Quantization of a conformal invariant pure spinor model.
- IC/82/107 A.M. KURBATOV and D.P. SANKOVIC - On one generalization of the Fokker-Planck equation.
- IC/82/108 G. SENJANOVIC - Necessity of intermediate mass scales in grand unified theories with spontaneously broken CP invariance.
- IC/82/109 NOOR MOHAMMAD - Algebra of pseudo-differential operators over C^* -algebra.
- IC/82/111 M. DURGUT and N.K. PAK - SU(N)-QCD₂ meson equation in next-to-leading order.
- IC/82/112 O.P. KATYAL and K.M. KHANNA - Transverse magneto-resistance and Hall resistivity in Cd and its dilute alloys.
- IC/82/113 P. RACZKA, JR. - On the class of simple solutions of SU(2) Yang-Mills equations.
- IC/82/114 G. LAZARIDES and Q. SHAFI - Supersymmetric GUTs and cosmology.
- IC/82/115 B.K. SHARMA and M. TOMAK - Compton profiles of some 4d transition metals.
- IC/82/116 M.D. MAIA - Mass splitting induced by gravitation.
- IC/82/117 PARTHA CHOSE - An approach to gauge hierarchy in the minimal SU(5) model of grand unification.
- IC/82/118 PARTHA CHOSE - Scalar loops and the Higgs mass in the Salam-Weinberg-Glashow model.
- IC/82/119 A. QADIR - The question of an upper bound on entropy.
- IC/82/122 C.W. LUNG and L.Y. XIONG - The dislocation distribution function in the plastic zone at a crack tip.
- IC/82/124 BAYANI I. RAMIREZ - A view of bond formation in terms of electron momentum distributions.
- IC/82/127 N.N. COHAN and M. WEISMANN - Phonons and amplitudons in one dimensional incommensurate systems.
- IC/82/128 M. TOMAK - The electron ionized donor recombination in semiconductors.
- IC/82/129 S.P. TEWARI - High temperature superconducting of a Chevrel phase ternary compound.
- IC/82/130 LI XINZ HOU, WANG KELIN and ZHANG JIAMZU - Light spinor monopole.
- IC/82/131 C.A. MAJID - Thermal analysis of chalcogenides glasses of the system $(As_2Se_3)_{1-x}(Te_2Se)_x$.
- IC/82/132 K.M. KHANNA and S. CHAUBA SINGH - Radial distribution function and second virial coefficient for interacting bosons.
- IC/82/133 A. QADIR - Massive neutrinos in astrophysics.
- IC/82/134 H.B. GHASSIB and S. CHATTERJEE - On back flow in two and three dimensions.
- IC/82/137 M.Y.M. HASSAN, A. RABIE and E.H. ISMAIL - Binding energy calculations using the molecular orbital wave function.
- IC/82/138 A. BREZINI - Eigenfunctions in disordered systems near the mobility edge.
- IC/82/140 Y. FUJIMOTO, K. SHIGEMOTO and ZHAO ZHIYONG - No domain wall problem in SU(N) grand unified theory.
- IC/82/142 G.A. CHRISTOS - Trivial solution to the domain wall problem.
- IC/82/143 S. CHAKRABARTI and A.H. NAYYAR - On stability of soliton solution in NLS-type general field model.
- IC/82/144 S. CHAKRABARTI - The stability analysis of non-topological solitons in gauge theory and in electrodynamics.
- IC/82/145 S.N. RAM and C.P. SINGH - Hadronic couplings of open beauty states.
- IC/82/146 BAYANI I. RAMIREZ - Electron momentum distributions of the first-row homonuclear diatomic molecules, A_2 .
- IC/82/147 A.K. MAJUMDAR - Correlation between magnetoresistance and magnetization in Ag Mn and Au Mn spin glasses.
- IC/82/148 E.A. SAAD, S.A. EL WAKIL, M.H. HAGGAG and H.M. MACHALI - Pade approximant for Chandrasekhar H function.
- IC/82/149 S.A. EL WAKIL, M.T. ATIA, E.A. SAAD and A. HENDI - Particle transfer in multiregion.