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$\psi\psi$ PRODUCTION BY QUARKS, GLUONS AND B MESONS ")

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C O R R I G E N D U M

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FIG.2

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$\psi\psi$ PRODUCTION BY QUARKS, GLUONS AND B MESONS ^{")}

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A B S T R A C T

We determine, in the framework of perturbative QCD, the $O(\alpha_s^2)$ - $O(\alpha_s^4)$ parton cross-sections for $\psi\psi$ production by quarks, gluons and B mesons. The hadronic production characteristics are compared with recent data from π^-N scattering. At present energies, $\psi\psi$ production via B meson decay is negligibly small. At collider energies, B production by $(\bar{q}b)$ recombination constitutes, however, a considerable production mode.

^{")} This work was initiated and almost entirely carried out when one of the authors (P.M.) was a CERN Fellow.

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The production of $\psi\psi$ pairs in π^-N collisions has recently been investigated by the NA3 Collaboration at CERN¹⁾. The integrated cross-section is 18 ± 8 (30 ± 10) pb at 150 (280) GeV, with $6.5 < M_{\psi\psi} < 8.5$ GeV and $\langle p_t \rangle \sim 1$ GeV. One ψ is energetic and the other is slow.

Several production mechanisms can be accounted to describe these results. Heavy flavour excitation via recombination²⁾ and quark-antiquark annihilation³⁾ have been suggested.

In this letter we present the results of a systematic investigation of $\psi\psi$ production in the framework of perturbative QCD with graphs of order $O(\alpha_s^2) - O(\alpha_s^4)$ being considered. We first analyze "direct" $\psi\psi$ production by gluon fusion (Fig. 1a) and (low mass quark) $q\bar{q}$ annihilation (Fig. 1b) to produce a (charmed quark) $c\bar{c}$ pair which, via gluon exchange, creates another $c\bar{c}$ pair.

A significant fraction of the $\psi\psi$ events has been suggested⁴⁾ to originate from $B\bar{B}$ production with subsequent inclusive B ($\equiv b\bar{q}$) meson decay to a ψ . The size of the cross-section for $B\bar{B}$ production and the branching ratio $BR(B \rightarrow \psi + X)$ determine the rate. We have analyzed several mechanisms. The most simple B production is given by the lowest order graphs for $gg/q\bar{q} \rightarrow b\bar{b}$ (Fig. 1c); B meson formation follows from soft q quark association where the inclusive $b \rightarrow B$ transition probability is assumed to be unity. The production of an exclusive B meson via $b\bar{q}$ recombination, as suggested in Ref. 2), is shown in Fig. 1d. Subsequently, recombination between the b quark and the (fast) valence quark $\bar{q}$ of the projectile hadron takes place. The remaining $\bar{b}$ quark associates a soft q quark. The dynamical distributions of this model were shown to be compatible with the data²⁾; in particular a leading B meson and a slower $\bar{B}$ meson are expected. In order to expose the influence of $b\bar{q}$ recombination we also will analyze "double-recombination" meaning that the B and $\bar{B}$ formation, both exclusive, proceeds via the recombination mechanism (see Fig. 1e).

We now wonder about the cross-section size of exclusive B production other than recombination. We therefore reconsider, in close analogy to "direct" $\psi\psi$ production, the processes of Figs. 1a and 1b where $c\bar{c}$ and associated $q\bar{q}$ production is assumed. We are aware that the use of perturbative QCD is not well justified and that we have limited ourselves to a subclass of graphs. We nevertheless believe that such evaluations will give useful order of magnitude estimates⁵⁾.

The considered subprocesses are thus :

- (1),(2) : $gg/q\bar{q} \rightarrow \psi\psi$ ("direct" = g fusion/annihilation)
- (3),(4) : $gg/q\bar{q} \rightarrow b \bar{b} \rightarrow \psi\psi$ (lowest order)
- (5) : $\bar{q}g \rightarrow B_r \bar{b} \rightarrow \psi\psi$ (recombination)
- (6) : $\bar{q}q \rightarrow B_r \bar{B}_r \rightarrow \psi\psi$ (double recombination)
- (7),(8) : $gg/q\bar{q} \rightarrow B \bar{B} \rightarrow \psi\psi$ (as "direct")

where the B meson produced via recombination is given the index r .

In our subsequent calculations, the $\psi(c\bar{c})$ and $B(b\bar{q})$ mesons are treated as non-relativistic bound states. This approximation has lead to sensible results in the mass spectrum⁶⁾, and it is often used for cross-section estimates⁵⁾. Each of the quarks carries the fraction $\xi_i \equiv (m_i/M)$ of the meson momentum P . The consequences of such assumptions have earlier been analyzed in Ref. 7). The set of gauge invariant graphs is indicated in Fig. 1. We have explicitly verified gluon gauge invariance. For the description of the non-relativistic bound state and the evaluation of parton cross-sections we refer to Ref. 8). The longitudinal component of the gluon polarization vector was eliminated by the transverse projector as shown in Ref. 9). The spin traces were carried out by the computer program SCHOONSHIP¹⁰⁾. We have averaged over all initial and summed over all final colour states.

The parton cross-sections for $\psi\psi$ production via gluon fusion (Fig. 1a) and $q\bar{q}$ annihilation (Fig. 1b) are given by

$$\frac{d\sigma}{d\hat{t}}(q\bar{q} \rightarrow \psi\psi) = \frac{3}{4\pi} \left(\frac{\pi \alpha_s}{3} \right)^4 \frac{|\psi_{(0)}|^2}{M_\psi} \frac{|\psi_{(0)}|^2}{M_\psi} \frac{\hat{s}^4 \sum_n y^n C_n(z)}{(\hat{t}-M_\psi^2)^4 (\hat{u}-M_\psi^2)^4}$$

$$\frac{d\sigma}{d\hat{t}}(g\bar{g} \rightarrow \psi\psi) = \frac{2}{3\pi} \left(\frac{\pi \alpha_s}{3} \right)^4 \frac{|\psi_{(0)}|^2}{M_\psi} \frac{|\psi_{(0)}|^2}{M_\psi} [3 + 16y^3 - 10z^2 - 24z^2y - 24z^4]$$

where $y \equiv (\hat{t}+\hat{u})/2\hat{s}$, $z \equiv (\hat{t}-\hat{u})/2\hat{s}$ and M is the ψ rest mass. The coefficients $C_n(z)$ read

$$\begin{aligned}
 C(0) &= (3981312Z^{12} + 258048Z^{10} - 1186048Z^8 + 35584Z^6 + 98144Z^4 - 13568Z^2 + 335) \cdot (2/3) \\
 C(1) &= (1935360Z^{10} - 596224Z^8 - 403840Z^6 + 168064Z^4 - 14288Z^2 + 137) \cdot (8/3) \\
 C(2) &= (1134336Z^8 - 1010944Z^6 + 239008Z^4 - 11472Z^2 - 1) \cdot (8/3) \\
 C(3) &= (-10176Z^6 + 688Z^4 + 828Z^2 + 5) \cdot (64/3) \\
 C(4) &= (16Z^4 + 8Z^2 + 1) \cdot 256
 \end{aligned}$$

Analogous evaluation of Figs. 1a and 1b for $B\bar{B}$ production leads to

$$\begin{aligned}
 \frac{d\sigma}{dt}(q\bar{q} \rightarrow B\bar{B}) &= \frac{1}{2\pi} \left(\frac{4\pi\alpha_s}{3} \right)^4 \frac{|\psi(u)|^2}{M_B} \frac{|\psi(\bar{u})|^2}{M_B} \left(\frac{1}{z_1 z_2} \right)^6 \frac{\hat{s}^4 \sum_{\ell=0}^4 \xi_1^{\ell} F^{(\ell)}}{(\hat{t} - M_B^2)^4 (\hat{u} - M_B^2)^4} \\
 \frac{d\sigma}{dt}(q\bar{q} \rightarrow B\bar{B}) &= \frac{1}{6\pi} \left(\frac{4\pi\alpha_s}{3\hat{s}^2} \right)^4 \frac{|\psi(u)|^2}{M_B} \frac{|\psi(\bar{u})|^2}{M_B} \left(\frac{1}{z_1 z_2} \right)^6 (\hat{u} - \hat{t})^2 (\hat{t} \hat{u} - M_B^4)
 \end{aligned}$$

where $\xi_1 = m_1/M_B$ takes care of the mass difference between the u and b quark. The first three of the functions $F^{(\ell)}$ have the structure

$$F^{(\ell)}(y, z) = \sum_{n=0}^4 y^n C_n^{(\ell)}(z) \quad (\ell = 0, 1, 2)$$

with the coefficients C_n given by

$$\begin{aligned}
 C0(0) &= (20736Z^{12} + 18432Z^{10} + 6688Z^8 - 2592Z^6 - 1003Z^4 + 88Z^2 + 32)/32 \\
 C0(1) &= (9792Z^{10} + 7952Z^8 - 20Z^6 - 1325Z^4 - 22Z^2 + 48)/4 \\
 C0(2) &= (4112Z^8 + 1816Z^6 - 847Z^4 - 174Z^2 + 52) \\
 C0(3) &= (208Z^6 - 8Z^4 - 35Z^2 + 6) \cdot 16 \\
 C0(4) &= (16Z^4 - 8Z^2 + 1) \cdot 64
 \end{aligned}$$

($\ell = 0$)

$$\begin{aligned}
 C1(0) &= (-94464Z^{10} - 63872Z^8 - 2240Z^6 + 11192Z^4 + 325Z^2 - 404)/32 \\
 C1(1) &= (-76032Z^{10} - 212864Z^8 - 66048Z^6 + 38328Z^4 + 6745Z^2 - 2144)/16 \\
 C1(2) &= (-32640Z^8 - 37936Z^6 + 6912Z^4 + 5253Z^2 - 1025)/2 \\
 C1(3) &= (-10128Z^6 - 2192Z^4 + 2857Z^2 - 419) \cdot 2 \\
 C1(4) &= (-528Z^4 + 256Z^2 - 31) \cdot 16
 \end{aligned}$$

($\ell = 1$)

$C2(0) = (377856 \times Z^{10} + 768768 \times Z^8 + 130048 \times Z^6 - 94720 \times Z^4 - 25732 \times Z^2 + 6981) / 128$
 $C2(1) = (152064 \times Z^{10} + 855040 \times Z^8 + 543360 \times Z^6 - 132080 \times Z^4 - 79546 \times Z^2 + 16227) / 32$
 $C2(2) = (883968 \times Z^8 + 1722880 \times Z^6 + 12256 \times Z^4 - 348512 \times Z^2 + 56181) / 32$
 $C2(3) = (188736 \times Z^6 + 96784 \times Z^4 - 78164 \times Z^2 + 10575) / 4$
 $C2(4) = (25872 \times Z^4 - 12280 \times Z^2 + 1457)$

($\ell = 2$)

The remaining two follow from the $\xi_1 \leftrightarrow \xi_2 \equiv 1 - \xi_1$ symmetry of the cross-section :
 $F^{(3)} = -(F^{(1)} + F^{(2)})$, $F^{(4)} = (F^{(1)} + F^{(2)})$.

The wave function at the origin $\psi(0)$ is for the ψ meson determined by the $\psi \rightarrow e^+e^-$ decay width

$$|\psi(0)|^2 = \frac{M_\psi^2 T_e}{16\pi \kappa^2 e_c^2} = 0.0387$$

For the B meson its value is different. Non-relativistic model calculations reveal $|\psi(0)|^2 \sim \mu^{3/(2+v)}$ where μ is the reduced mass of the two-quark system¹¹⁾, and $0.1 < v < 1$. Thus we may expect moderate variation in the mass range we consider leaving subsequent insights practically unchanged.

For the parton cross-sections of $g\bar{q}q \rightarrow b\bar{b}$ (Fig. 1c) we refer to Ref. 12). The parton cross-sections for Figs. 1d and 1e cannot be cast in a simple form¹³⁾.

In the subsequent numerical evaluation, we set $\alpha_s = 0.3$.

We compare the size of the parton cross-sections for $\psi\psi$ production at $\hat{s} = 200 \text{ GeV}^2$ assuming for the B branching ratio : $BR(B \rightarrow \psi + X) = 0.01$ ¹⁴⁾. The differential cross-section $d\sigma/d\hat{t}$ for $q\bar{q} \rightarrow \psi\psi$ rises (with falling $\hat{t}$) to a flat plateau of $d\sigma/d\hat{t} \sim 6.10^{-39} \text{ cm}^2/\text{GeV}^2$ and decreases again towards the wide angle phase space boundary. In $g\bar{g} \rightarrow \psi\psi$ we find a dip of a factor five in the intermediate $\hat{t}$ range ; a maximum point thus appears in the upper and lower $\hat{t}$ region. As the centre-of-mass energy grows, a strong rise in the forward and backward direction develops. The $q\bar{q}(g\bar{g}) \rightarrow b\bar{b} \rightarrow \psi\psi$ subprocesses lead to a relatively flat angular distribution around $d\sigma/d\hat{t} \sim 10^{-37} \text{ cm}^2/\text{GeV}^2$ with a valley of a factor 1.3 (4.5) in the intermediate $\hat{t}$ range. The $B(b\bar{q})$ recombination process $\bar{q}g \rightarrow B + \bar{b} \rightarrow \psi\psi$ decreases steadily with falling $\hat{t}$. As

$|\hat{t}| \rightarrow 0$ it reaches $\sim 10^{-36} \text{ cm}^2/\text{GeV}^2$; it does not diverge. The double-recombination process $q\bar{q} \rightarrow B_r \bar{B}_r \rightarrow \psi\psi$ leads to similar characteristics but is a factor ten (or more) below. The exclusive B formations $gg/q\bar{q} \rightarrow B\bar{B} \rightarrow \psi\psi$ are one and a half orders of magnitude below "direct" $\psi\psi$ production. The angular distribution of both processes vanishes at the phase space boundaries; in the intermediate $\hat{t}$ region the gluon-fusion (annihilation) remains flat (vanishes).

We consider the integrated parton cross-sections. After the pronounced threshold onset reaching $\sim 3 \cdot 10^{-35} \text{ cm}^2$, the "direct" processes $gg/q\bar{q} \rightarrow \psi\psi$ (1,2 in the earlier list) decrease rapidly. The threshold onset of all other processes is less pronounced or even extended (processes 6,8). With increasing $\hat{s}$ the cross-sections either remain almost constant (processes 3,4,5,6) or decrease rapidly (processes 8,9). At $\hat{s} = 200 \text{ GeV}$ typical cross-section values are

$$\begin{array}{llllll} \sigma(3) & \approx & \sigma(4) & \approx & \sigma(5) & \approx & 10^{-35} \text{ cm}^2 \\ \sigma(2) & \approx & 2 \cdot \sigma(4) & \approx & 5 \cdot \sigma(6) & \approx & 10^{-36} \text{ cm}^2 \\ \sigma(7) & \approx & 5 \cdot \sigma(8) & & & \approx & 3 \cdot 10^{-38} \text{ cm}^2 \end{array}$$

where the numbers in brackets refer to the earlier listed subprocesses. The exclusive and inclusive production modes are related by a reduction factor which is determined in $e^+e^- \rightarrow B\bar{B}$. The exclusive B production is described perturbatively (analogous to Fig. 1a)⁷⁾ whereas the inclusive B production is given by $e^+e^- \rightarrow b\bar{b}$. The "branching ratio" for the exclusive $b \rightarrow B$ transition

$$BR(b \rightarrow B) = \left[\sqrt{2} \frac{4\pi\alpha_s}{3} \frac{|V_{ub}|^2 M_B}{m_b^2 \hat{s}} \right] = 0.1 \cdot \alpha_s$$

is evaluated in the threshold region. Assuming $\alpha_s = 0.3$ we expect a factor 10^{-3} suppression if the cross-section size of processes (3),(4) and (7),(8) is compared. The recombination subprocess does not follow this pattern since the exchange of a soft gluon is involved which causes a strongly peaked angular distribution. Similar arguments apply to the double-recombination process (6). Its slow threshold onset, however, permits a comparison only at higher energies.

In order to determine the hadronic $\psi\psi$ production cross-sections we fold the parton cross-sections with the scale independent parton momentum distributions. The valence and sea distributions of the proton are those of Ref. 15), whereas those of the π^- meson are from Ref. 15b). The counting rule powers of the nucleon (meson) gluon distributions are $n_G = 5$ (3). Different parametrizations do not significantly change the subsequent insights.

In Fig. 2, we present the integrated cross-sections for $\psi\psi$ production in π^-p scattering. At lower energies ($p_{lab} < 400$ GeV) the subprocess $q\bar{q} \rightarrow \psi\psi$ constitutes the main production mode whereas at higher energies $gg \rightarrow \psi\psi$ will be dominant. This is understood from the size of the parton cross-sections around threshold and the behaviour of the momentum distributions, in particular the rise of the gluon distributions at low x values. The cross-sections for $\psi\psi$ production via B mesons are at present energies negligible. The smaller size of the parton cross-sections for reactions (4), (5) and (6), the higher $B\bar{B}$ production threshold and the small $BR(B \rightarrow \psi + X) = 0.01$, which enters quadratically in the cross-section formula, are the main reasons. The importance of the recombination process $q\bar{q} \rightarrow B_r + \bar{B} \rightarrow \psi\psi$ is understood from the flat valence and gluon momentum distributions of the π^- meson and the relatively large x values being accessible. The double recombination $q\bar{q} \rightarrow B_r \bar{B}_r \rightarrow \psi\psi$ is suppressed and the exclusive subprocesses (7) and (8) can be ignored. We realize that "direct" $\psi\psi$ production, even if it dominates, cannot explain the data ; a factor 3-5 is still missing. The higher order QCD corrections have been suggested as an explanation³⁾. The production and decay of the P wave states might also be considered. However, one should realize that single ψ production via the χ intermediate states is an important mechanism because P waves can be produced to order α_s^2 whereas the S waves need α_s^3 . Since in $\psi\psi$ production both subprocesses are of (lowest) order α_s^4 one has reasons to doubt the predominance of the P wave intermediate states. Their higher masses will also lead to a suppression factor.

We have extended our calculations to collider energies and find the subprocesses (1), (3), (5) to contribute each $\sim 4 \cdot 10^{-34}$ cm². We thus estimate the total $\psi\psi$ production cross-sections to be $\sigma_{\psi\psi} \gtrsim 10^{-33}$ cm².

The longitudinal momentum spectrum, shown in Fig. 3, reflects the influence of the two parton distributions and of the integrated cross-section. y_1 and y_2 are the rapidities of the two ψ 's. In order to explore the correlations we have varied y_2 and associated y_1 in different ways. The solid

(dashed/dotted) curves indicate $y_1 = 0$ ($y_1=y_2/y_1=-y_2$). The "direct" production by quarks and gluons dominates over the production via B mesons as anticipated from Fig. 2. The solid curves extend to large rapidity values where $q\bar{q}$ annihilation more and more dominates. If both ψ 's are produced with the same longitudinal momentum, the rapidity range is considerably reduced. If the ψ 's are produced in opposite direction, the decrease sets in fast with similar rates for the quark annihilation and gluon fusion. In $gg/q\bar{q} \rightarrow b\bar{b}$ production, we notice a rapid decrease for equally growing rapidities. Thus, different from "direct" $\psi\psi$ production, large opposite rapidities are here preferred. Similar characteristics are observed in the negative y_2 range.

In Fig. 4a, we expose the distribution in the invariant mass of the $\psi\psi$ system. After the sharp threshold rise, $d\sigma/dM_{\psi\psi}$ falls roughly exponential in the lower $M_{\psi\psi}$ range. No difference in the dynamical behaviour of the $gg/q\bar{q} \rightarrow \psi\psi$ subprocesses can be observed. We have also indicated the $b\bar{b} \rightarrow \psi\psi$ threshold onset which, however, may not be taken as a representative for the $\psi\psi$ mass distribution of this production mechanism. We consider these curves rather as a guide for order of magnitude estimates. Indeed a large amount of $\psi\psi$ pairs decaying from $B\bar{B}$ will be produced with an invariant mass lower than the $B\bar{B}$ threshold which, had the rates been sufficient, could have explained the large $M_{\psi\psi}$ events.

The p_t spectrum of $gg/q\bar{q} \rightarrow \psi\psi$ shows a broad maximum around $p_t \sim 1$ GeV/c which does not significantly change as the centre-of-mass energy grows. The ψ 's, produced via $b\bar{b}$, emerge with a factor two larger p_t value. As the centre-of-mass energy increases $\langle p_t \rangle_{b\bar{b}} \sim 2 \rightarrow 3$ GeV/c increases significantly. 75% (25%) of the over-all transverse momentum $\langle p_t \rangle \sim 1.5$ GeV/c result from the $q\bar{q}(gg) \rightarrow \psi\psi$ subprocess.

Our study of $\psi\psi$ production in π^-p scattering in the framework of perturbative QCD leads to the following results.

- (i) $\psi\psi$ production via $B\bar{B}$ states is at present energies small as compared to "direct" $\psi\psi$ production. At collider energies they, however, can contribute a significant fraction.
- (ii) The recombination process $\bar{q}g \rightarrow B_P + \bar{b}$ constitutes the main $B\bar{B}$ production mode²⁾.
- (iii) The "direct" $\psi\psi$ production mechanisms give a good order of magnitude estimate, but do not fully reach the size of the experimental values.

- (iv) The $q\bar{q} \rightarrow \psi\psi$ as compared to the $gg \rightarrow \psi\psi$ production mechanism reveals no significant difference in the kinematical distributions.
- (v) The $M_{\psi\psi}$ distribution with its peak value close to threshold decreases exponentially, and, if the present data are confirmed, it suggests the existence of another production mechanism with threshold around 8 GeV.
- (vi) The $\langle p_t \rangle_{\psi} \sim 1$ GeV of "direct" $\psi\psi$ production varies little with increasing centre-of-mass energy. The B meson intermediate states would lead to $\langle p_t \rangle_{B\bar{B}} \sim 2$ GeV with a substantial increase as the centre-of-mass energy grows.

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FIGURE CAPTIONS

- Figure 1 a) Order α_s^4 QCD graphs for $gg \rightarrow \psi\psi$. There are altogether 36 graphs, some of them vanishing due to colour conservation and others are related by crossing symmetry.
- b) Order α_s^4 QCD graphs for $q\bar{q} \rightarrow \psi\psi$ (two graphs).
- c) Lowest order graphs for $gg/q\bar{q} \rightarrow b\bar{b} \rightarrow \psi\psi$.
- d) Order α_s^3 QCD graphs for $\bar{q}g \rightarrow B_r + \bar{b}$. B production by recombination (three graphs).
- e) Order α_s^4 QCD graphs for $q\bar{q} \rightarrow B_r \bar{B}_r \rightarrow \psi\psi$. B production by double recombination (seven graphs).
- Figure 2 Integrated parton cross-sections of the investigated production mechanisms for $\pi^- p \rightarrow \psi\psi + X$.
- Figure 3 Rapidity distributions of the investigated production mechanisms for $\pi^- p \rightarrow \psi\psi + X$.
- Figure 4 Mass distribution of the $\psi\psi$ system for the investigated production mechanisms of $\pi^- p \rightarrow \psi\psi + X$.

FIG. 1a

FIG. 1b

FIG. 1c

FIG. 1d

FIG. 1e

FIG.2

FIG. 3

FIG. 4