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BREAKDOWN OF LOCAL SUPERSYMMETRY THROUGH
GAUGE FERMION CONDENSATES

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ABSTRACT

It is shown that gauge fermion condensates can break local supersymmetry in the presence of non-minimal gauge kinetic terms. The resulting low energy effective theory is derived. It admits new ways for a gravitino mass induced breakdown of the weak interactions.

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The application of spontaneously broken local supersymmetry ^{1),2)} to models of high energy physics has recently attracted ³⁾⁻⁵⁾ much attention. It solves problems of globally supersymmetric models and it leads to a small mass scale in the theory : the gravitino mass $m_{\frac{3}{2}}$. The low energy effective potential contains terms proportional to $m_{\frac{3}{2}}$ which explicitly break supersymmetry and might induce a breakdown of the weak interactions ⁴⁾⁻⁷⁾.

The spontaneous breakdown of supergravity occurs in a hidden sector which couples to the ordinary particles only gravitationally. This hidden sector can either consist of singlet chiral superfields ¹⁾ or a new strong gauge interaction ³⁾. If one demands the absence of a cosmological constant at the absolute minimum, one is in the first case practically restricted to a superpotential $g = m^2(z + \beta)$ with $\beta = (2 - \sqrt{3})M$ where M is the Planck mass. The resulting gravitino mass is of order m^2/M and $m_{\frac{3}{2}} \sim \text{TeV}$ is obtained for $m \sim 10^{11}$ GeV. The second case is characterized by a scale μ where the new gauge interactions become strong. The induced gravitino mass is $m_{\frac{3}{2}} \leq \mu^3/M^2$ and a μ of the order of the grand unification scale could provide a gravitino mass of the required magnitude ⁸⁾. Up to now this case has only been discussed in the context of an effective Lagrangian approach. Meanwhile the most general coupling of non-Abelian gauge interactions to $N=1$ supergravity has been developed ²⁾ and these questions can be re-examined in detail. The results of this study are the content of this paper. We will show that the formation of gauge fermion condensates will in general break local supersymmetry and will derive the resulting low energy effective potential. This potential turns out to be more general than in the case where the hidden sector consists solely of chiral singlet superfields. As a result, the gravitino mass induced breakdown of the weak interactions might be easier to achieve.

Throughout our discussion we consider a pure supersymmetric Yang-Mills theory, i.e., a non-Abelian gauge theory with Majorana fermions λ in the adjoint representation of the gauge group. According to the general arguments ⁹⁾ we would expect a condensation $\langle \lambda\lambda \rangle$ of these gauge fermions in the limit of strong coupling. Global supersymmetry remains unbroken ¹⁰⁾ in the presence of these condensates, resulting in a degeneracy of vacua with $\langle \lambda\lambda \rangle$ equal to or different from zero. From our experience with non-supersymmetric theories we would expect that the $\langle \lambda\lambda \rangle \neq 0$ vacuum is dynamically preferred. This expectation is supported by the results of an effective Lagrangian approach where the $\langle \lambda\lambda \rangle = 0$ vacuum is shown to be non-sensible ¹¹⁾.

The central assumption of this paper is the persistence of this qualitative behaviour in the presence of gravity. As long as the scale μ of the gauge interactions is substantially smaller than M , we believe gravity to be a small perturbation that does not unbind the condensates, leaving the $\langle \lambda \lambda \rangle \neq 0$ state to remain the dynamically preferred vacuum state.

We start with a pure supersymmetric gauge theory (sugracolour) and n chiral superfields $S_i = (z_i, \chi_i, h_i)$ coupled to supergravity and follow the notation of Ref. 2)

$$S = \int d^4x d^4\theta E \left(\phi(S_i, S_i^*) + \text{Re} \frac{1}{R} g(S_i) + \text{Re} \frac{1}{R} f(S_i) W^\alpha W_\alpha \right) \quad (1)$$

The chiral fields are sugracolour singlets; they could participate in additional gauge interactions like $SU(3) \times SU(2) \times U(1)$ which we ignore for the moment. The chiral kinetic terms ϕ and the superpotential g combine to give a Kähler potential ¹⁾

$$g = 3 \log(-\phi/3) - \log |g|^2 \quad (2)$$

The second term in (1) corresponds to the gauge kinetic terms which are called non-minimal if $f(z_i)$ is not constant. Non-minimal gauge kinetic terms are present in all $N \geq 4$ extended supergravities and we take this as an argument to include them in the most general $N=1$ action ^{*)}. There could be some problems with non-trivial $f(z_i)$ such as wrong sign kinetic terms and negative contributions to the scalar potential of the form $\text{Re} f^{-1}(z_i) D^2$; we will, however, only discuss models where f is well behaved at the minimum of the potential. A shift in the fields can then be used to absorb f .

We proceed to discuss the effect of gauge fermion condensation in this model. Supersymmetry is broken if auxiliary fields receive a vacuum expectation value. The auxiliary field h_i of the superfields S_i is given by ²⁾

$$h_i = \exp \left(-\frac{g}{2} \right) g'_j (g''^j)^{-1} + \frac{1}{4} f'_k \lambda \lambda (g''^k)^{-1} \quad (3)$$

*) The sugracolour gauge interactions could as well be a sector of gauged extended supergravities.

where the omitted terms contain chiral fermions which are irrelevant for our purposes; $G_i^{\mu\kappa} = \partial^2 \mathcal{G} / \partial z_\kappa \partial z^{*\mu}$

The first term is the usual order parameter of local supersymmetry if there are only chiral superfields. The existence of the second term shows that $\lambda\lambda$ condensation can break supersymmetry in the presence of non-minimal gauge kinetic terms. The super-Higgs effect will occur as can be seen from the expression ²⁾

$$\frac{1}{8} f'^i \psi_R \gamma \cdot \chi_{Li} \langle \lambda\lambda \rangle \quad (4)$$

which appears in the action. The combination

$$\eta_L = \frac{1}{8} \sum_i f'^i \chi_{Li} \langle \lambda\lambda \rangle \quad (5)$$

denotes the Goldstino which will be absorbed by the gravitino. As a result, the gravitino will become massive and the magnitude of the $m_{3/2}$ will be discussed next. Suppose that $\langle \lambda\lambda \rangle = \mu^3$ and consider for example :

$$f = 1 + \sum_i \sigma_i \frac{z_i^i}{M} \quad (6)$$

where the vacuum expectation values of the z_i are assumed not to exceed M . The scale M_s^2 of supersymmetry breakdown is given by the vacuum expectation values of the auxiliary fields and can be estimated to be

$$M_s^2 = \langle h_i \rangle \sim \frac{\sigma_i \mu^3}{4M} \quad (7)$$

This results in a gravitino mass

$$m_{3/2} = \frac{\sigma_i \mu^3}{4\sqrt{3} M^2} \quad (8)$$

A scale $\mu = 10^{14}$ GeV of the strong gauge interaction and $\sigma_i \sim 10^{-1}$ would thus induce a gravitino mass of 1 TeV. Observe that in the global limit (μ fixed, $M \rightarrow \infty$) supersymmetry is restored in agreement with the no-go theorems for a dynamical breakdown of global supersymmetry ^{12),13)}.

The general form of the scalar potential is given by ²⁾

$$V = -3M^4 \exp(-G) + \left[M^2 \exp(-G/2) G'_j + \right. \\ \left. + \frac{1}{4M} f'_j(\lambda\lambda) \left(M^2 \exp(-G/2) G'_k + \frac{1}{4M} f'_k(\lambda\lambda) \right)^* (G'_k)^{-1} \right] \quad (9)$$

The second term is $|h_i|^2$ and thus a measure for supersymmetry breakdown. Note that the contribution of the scalar sector could cancel that of the condensate and restore supersymmetry. This does not happen in general as we will show later in special examples; it will, however, restrict the possible choice of $f(z)$. There we will also discuss the fine tuning of the cosmological constant.

First we want to derive the low energy effective potential in the flat limit ⁴⁾: $m_{3/2}$ fixed, $M \rightarrow \infty$. In order to render the formulae more transparent, we make some simplifying assumptions. We consider a hidden sector which (apart from sugracolour) consists only of one scalar field z and assume this field to be the only field that enters the function f of the non-minimal gauge kinetic terms. Additional fields L_i represent the light sector (quarks, leptons, Higgses) and are assumed to have only small vacuum expectation values. The function G is chosen to be

$$G = -\frac{zz^*}{M^2} - \frac{L_i L_i^*}{M^2} - \log |\tilde{g}(z, L_i)|^2 \quad (10)$$

where $\tilde{g}(z, L_i) = g(z) + h(L_i)$ ^{*)}. We impose the naturalness condition ⁶⁾ that the low energy superpotential $h(L_i)$ is scale invariant. With the abbreviation $C_z = \frac{1}{4} f'^z(\lambda\lambda)$ the potential then reads

$$V = -3M^4 \exp(-G) + |M^3 \exp(-G/2) G'^i|^2 \\ + |M^3 \exp(-G/2) G'^z + C_z|^2 \quad (11)$$

In the limit $M \rightarrow \infty$, $m_{3/2}$ fixed, this leads to ^{**)}

*) Superpotentials of the form $g(z)\exp(h(L_i))$ are equally well suitable ⁷⁾. In these cases the condensate can remove the degeneracy.

**) We only consider models with a vanishing cosmological constant.

$$V = |\tilde{h}_{1i}|^2 + A m_{3/2} (\tilde{h} + \tilde{h}^*) + B m_{3/2}^2 |L_i|^2 \quad (12)$$

where $\tilde{h} = \exp(z_0 z_0^*/2M^2)h$. The constant A has been present also in the case $C_z = 0$ ⁶⁾; here it is given by $A = -\sqrt{3} z_0/M$. In the absence of sugracolour B has been equal to one. With $C_z \neq 0$ it can differ from unity

$$B = -2 + M^2 |g'_z|^2 + \frac{1}{2} \left(g'_z \frac{C_z}{m_{3/2}} + h.c. \right) \quad (13)$$

In the case $C_z = 0$ one has the relation

$$B = 1 + \frac{\bar{E}_{vac}^4}{M^2 m_{3/2}^2} \quad (14)$$

and $B \neq 1$ is only possible if the vacuum energy differs from zero. For $C_z \neq 0$, however we can have $B \neq 1$ even in the absence of a cosmological constant.

To illustrate these points we present some simple examples. Throughout we consider $f(z) = 1 + (\sigma z/M)$ so that $f' = \sigma/M$ is a constant. The first example is based on a superpotential $g(z) = m^2 z$ where the parameter m is used to fine-tune the cosmological constant. The potential (11) was solved numerically and it was found to admit a solution where the absolute minimum has zero vacuum energy. This situation corresponds to $z_0 \simeq 0.766 M$ and $m^2 \simeq 0.717 \sigma \mu^3/M$, and results in a gravitino mass

$$m_{3/2} \simeq 0.737 \frac{\sigma \mu^3}{M^2} \quad (15)$$

A sufficiently small σ and μ of the order of the grand unification scale could thus give $m_{3/2} = 0(\text{TeV})$. The parameters in the low energy effective potential (12) are $A \simeq -1.33$ and $B \simeq 1.59$. Observe that B differs significantly from one. There are two properties of this example which have to be mentioned: i) $g'_z \neq 0$ at the minimum, i.e., the scalar sector contributes to the breakdown of supersymmetry as well as the condensate. This happens in most of the cases, sometimes these two contributions can even cancel each other; ii) consider the same model but with $C_z = 0$. In this case the minimum is found to be at $z_0 = 0$ and has a positive cosmological constant. The presence of the condensate lowers the vacuum energy and the new vacuum is energetically favoured. This, however, does not happen in general, and there one has to argue that the $C_z \neq 0$ vacuum is dynamically preferred, as we did earlier.

In a second example, we will show that B can even become negative. We choose the superpotential $g = m^2(z + 2M)$. Again there exists a solution where the absolute minimum has a vanishing cosmological constant: $z_0 = -M$ and $m^2 \approx -0.0875 \sigma \mu^3 / M^*$). This leads to $A = \sqrt{3}$ and $B = -2$, which shows that B can become negative. In this example G'_Z vanishes at the minimum and the breakdown of supersymmetry is induced only by the condensate. The gravitino mass is $m_{\frac{3}{2}} = (-\sigma \mu^3) / (4\sqrt{3} M^2)$ *).

It has been shown that potentials of type (12) can lead to a gravitino mass induced breakdown of the weak interactions. For models with $B = 1$ a necessary condition ⁶⁾ for this to happen was $A \geq 3$. In the general case for $B > 0$ this bound will be replaced by $A \geq 3\sqrt{B}$, thus A could be smaller. New possibilities arise for $B \leq 0$. A value of $B = -2$ as in the second example, however, has probably disastrous consequences as it will potentially lead to a breakdown of colour-SU(3) as well. The models of phenomenological interest seem to be those where B is close to zero ⁷⁾. Such models exist as we can already argue from our two examples. They both have a superpotential of the type $g = m^2(z + \beta)$ and for a range $0 \leq \beta \leq 2M$ B varies continuously from 1.59 to -2. Values of B close to zero are interesting since in such a case radiative corrections ^{3), 14), 15)} from the SU(3) x SU(2) x U(1) gaugino masses tend to increase the masses of the coloured scalars while reducing those of the Higgses. This could then lead to an induced breakdown of SU(2) x U(1) while preserving SU(3). These questions have to be studied in detail.

We have already mentioned the SU(3) x SU(2) x U(1) gaugino masses, which are in general present in models of the considered type. We had assumed the existence of non-minimal gauge kinetic terms and it is natural to assume the same for the low energy gauge interactions. The induced gaugino masses will then in general be given by $m_g^2 \approx M f' m_{\frac{3}{2}}^2$. For $f = 1 + \sigma z/M$ this will, e.g., lead to $m_g^2 \approx \sigma m_{\frac{3}{2}}^2$.

To conclude, we have shown that gauge fermion condensation can break local supersymmetry. The gravitino mass is given by μ^3/M^2 ; thus two large scales conspire to induce a small scale. Non-minimal gauge kinetic terms as well as the sugracolour gauge interaction could be motivated by gauged extended supergravities. Furthermore, in formulations à la Kaluza-Klein of $N=8$, condensation of fermionic bilinears is advocated as the only way

*) σ has to be chosen negative in this example.

to obtain spontaneous compactification with a zero cosmological term ¹⁶⁾. The derivation of the low energy potential has been given in the limit $M \rightarrow \infty$, $m_{\frac{3}{2}}$ fixed, and it turns out to be more general than in the previously known cases. Models with the parameters B close to zero might have interesting consequences with regard to a gravity induced breakdown of the weak interactions. We have not yet discussed the incorporation of grand unified models in this context. It might be that the scale μ of sugracolour coincides with the grand unification scale. The potential problems ¹⁷⁾ of these models should, however, be also present in this case.

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