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SUPERSYMMETRY, HIGGS AND SUPERHIGGS EFFECTS

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We discuss non-linear realizations of extended supersymmetries with central charges. Their coupling to N-extended supergravity provides a model-independent discussion of the Higgs and super-Higgs effects in higher N locally supersymmetric theories.

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ABSTRACT

We discuss non-linear realizations of extended supersymmetries with central charges. Their coupling to N-extended supergravity provides a model-independent discussion of the Higgs and superHiggs effect in higher N locally supersymmetric theories.

Non-linear Lagrangians describing chiral dynamics have been widely considered in the past¹⁾ to describe low-energy spontaneously broken chiral symmetries and in particular the low-energy properties of pion physics. When applied to N = 1 supersymmetry, non-linear representations of supersymmetry summarize the low-energy properties of goldstino amplitudes and contain all the information of supercurrent algebra, i.e., the way in which the goldstino, the spin-1/2 goldstone fermion of spontaneously broken supersymmetry, couples to ordinary matter. The theoretical set-up for the construction of non-linear realizations of supersymmetry and their consequent invariant Lagrangian was given several years ago by Volkov and Akulov²⁾ and further developed in Ref. 3).

When spontaneously broken Lagrangians are coupled to supergravity, the goldstino is eaten up by the spin 3/2 gravitino, the gauge fermion of local supersymmetry, and one obtains model independent relations between the gravitino mass and the scale vacuum energy of a global system which results from the spontaneous breaking of supersymmetry. This relation, as shown by Deser and Zumino⁴⁾, is model-independent, in the sense that it is the same relation that one would obtain by coupling linear realizations to supergravity and then, breaking spontaneously supersymmetry through the minimization of a given supersymmetric scalar potential⁵⁾.

In the present contribution we extend this analysis to the case of extended supersymmetry with or without central charges. When central charges are absent, the extension of the Volkov-Akulov formalism is trivial. However, the more interesting situation of matter with non-trivial central charges⁶⁾, or with central charges spontaneously broken is not trivial, and is the main result reported here. When non-linear Lagrangians with spontaneously broken central charges are coupled to N-extended supergravity, the simultaneous occurrence of the Higgs and superHiggs effect takes place. As a consequence, one obtains model-independent mass relations between the gravitinos, the gauge bosons of the supergravity multiplets, in terms of the vacuum energy and the Goldstone boson-goldstino-goldstino couplings of the global system. These couplings appear, in a purely geometrical fashion, in the non-linear globally supersymmetric original Lagrangian which describes the interaction of the Goldstone boson of central charges with the N-goldstinos.

We recall that the most general form of the N-extended supersymmetry algebra is⁶⁾

$$\{ Q_\alpha^i, \bar{Q}_\beta^j \} = -2 (\gamma^\mu)_{\alpha\beta} P_\mu \delta^{ij} - 2 Z^{ij} \mathbb{1}_{\alpha\beta} - 2 (\gamma_5)_{\alpha\beta} Z'^{ij}$$

where Q_α^i are a set of N-translationally invariant spin-1/2 Hermitian operators (antisymmetric in the indices i,j) belonging to the centre of the superPoincaré algebra.

The group composition, in the infinitesimal, implies the following commutation relation

$$[\bar{\epsilon}_1^i Q_\alpha^i, \bar{\epsilon}_2^j Q_\alpha^j] = -2 \bar{\epsilon}_1^i \gamma^\mu \epsilon_2^i P_\mu - 2 \bar{\epsilon}_1^i \epsilon_2^j Z^{\mu\nu} - 2 \bar{\epsilon}_1^i \gamma_5 \epsilon_2^j Z'^{\mu\nu} \quad (2)$$

where ϵ_1, ϵ_2 are spin-1/2 anticommuting constant spinors, and are interpreted as the parameter of a supersymmetric transformation.

If we denote now by $S_N IO(3,1)$ the N-extended superPoincaré algebra, an element of the coset-space $S_N IO(3,1)/O(3,1)$ is given by

$$\exp \frac{1}{d} (\bar{\theta} Q - i C_{ij} Z^{ij} - i C'_{ij} Z'^{ij}) \exp (-i x \cdot P) \quad (3)$$

where d is a dimensionful constant, $\dim d = 2$.

In the spirit of non-linear realizations the parameters $\theta_i, C_{ij}, C'_{ij}$ must be considered as fields, functions of space-time co-ordinates.

According to Callan, Coleman, Wess, Zumino and Weinberg¹⁾ the non-linear realizations of N-extended supersymmetry as well as of the $N(N-1)$ central charges are obtained by the action of a group element characterized by parameters $\epsilon_i, \omega_{ij}, \omega'_{ij}$ on the coset-space given by Eq. (3).

This action induces a motion on the coset-space which is easily seen, by use of Eq. (2), to be of the form

$$\delta x^\mu = \frac{i}{d} (\bar{\epsilon} \gamma^\mu \theta) = \xi^\mu \quad (4)$$

$$\delta \theta_i = d \epsilon_i + \xi^\nu \partial_\nu \theta_i \quad (5)$$

$$\delta C_{ij} = d \omega_{ij} + \frac{i}{4} (\bar{\epsilon}_i \theta_j - \bar{\epsilon}_j \theta_i) + \xi^\nu \partial_\nu C_{ij} \quad (6)$$

$$\delta C'_{ij} = d \omega'_{ij} + \frac{i}{4} (\bar{\epsilon}_i \gamma_5 \theta_j - \bar{\epsilon}_j \gamma_5 \theta_i) + \xi^\nu \partial_\nu C'_{ij} \quad (7)$$

where

$$\xi^\mu = \frac{i}{d} \bar{\epsilon}^i \gamma^\mu \theta_i$$

In order to construct Lagrangians which are invariant under the non-linear transformations (4)-(7), we determine the covariant derivatives of the fields introduced above.

Consider an element of the graded Lie algebra of the form

$$\begin{aligned} g^{-1} \delta g = & \delta x^\mu \left\{ -i V_\mu^a P_a + \frac{i}{d} C_{\mu ij} Z_{ij} \right. \\ & \left. + \frac{i}{d} C_{\mu ij}^1 Z_{ij}^1 + \frac{1}{d} \bar{g}_\mu^i Q^i \right\} \end{aligned} \quad (8)$$

where g is a group element. A simple explicit computation gives

$$V_\mu^a = \delta_\mu^a + \frac{i}{d^2} (\bar{\theta}_i \gamma^a \partial_\mu \theta_i) \quad (9)$$

$$C_{\mu ij} = \partial_\mu C_{ij} - \frac{i}{4d} [\bar{\theta}_i \partial_\mu \theta_j - \bar{\theta}_j \partial_\mu \theta_i] \quad (10)$$

$$C_{\mu ij}^1 = \partial_\mu C_{ij}^1 - \frac{i}{4d} [\bar{\theta}_i \gamma_5 \partial_\mu \theta_j - \bar{\theta}_j \gamma_5 \partial_\mu \theta_i] \quad (11)$$

$$g_{\mu i} = \partial_\mu \theta_i \quad (12)$$

By construction, the above quantities behave, under a supersymmetry transformation, as covariant vectors:

$$\delta V_\mu^a = \xi^\nu \partial_\nu (V_\mu^a) + (\partial_\mu \xi^\nu) V_\nu^a \quad (13)$$

The matrix $V_\mu^a(\theta)$ can be regarded as a covariant vierbein and its inverse $[V_\mu^a(\theta)]^{-1} = V_a^\mu(\theta)$ can be used to transform world-vectors into world-scalars

$$C_{a ij} = V_a^\mu C_{\mu ij} \quad (14)$$

$$\delta(C_{a ij}) = \xi^\nu \partial_\nu (C_{a ij}) \quad (15)$$

A general invariant action for the geometrical fields θ , C , C' has the form

$$A = \int d^4x (\det V) \mathcal{L}(C_a, C_a^1, g_a) \quad (16)$$

where $\mathcal{L}$ is a Poincaré invariant function. The action given by Eq. (6) describes the most general interactions of the N -Goldstone fermion with $N(N-1)$ Goldstone boson, associated with the spontaneous breaking of the corresponding central charges. A proper normalization of the Goldstone boson fields is

$$C_{ij} = \frac{1}{g_{ij}} Z_{ij} \quad , \quad C'_{ij} = \frac{1}{g'_{ij}} Z'_{ij} \quad (17)$$

The minimal Lagrangian is therefore

$$\mathcal{L}_{\text{min}} = -\frac{d^2}{2} \det V - \frac{1}{4} \det V \eta^{ab} \sum_{ij} [g_{ij}^2 C_{a ij} C_{b ij} + g_{ij}'^2 C'_{a ij} C'_{b ij}] \quad (18)$$

where

$$g_{ij} C_{\mu ij} = \partial_\mu Z_{ij} - \frac{i}{4} \frac{g_{ij}}{d} (\bar{\theta}_i \partial_\mu \theta_j - \bar{\theta}_j \partial_\mu \theta_i) \quad (19)$$

The coefficients have to be chosen so that θ_i , z_{ij} , z'_{ij} are property normalized fields.

From Eq. (19), it is obvious that when we set some of the $g_{ij} = 0$, the corresponding central charge is unbroken and the corresponding z_{ij} becomes a normal neutral matter field which transforms non-linearly under supersymmetry but which is neutral under central charge transformations.

Irrespective of the fact whether g_{ij} vanish or not, we can introduce complex scalar fields non-neutral under central charge symmetry, so that

$$[Z_{ij}, \phi] = i\sqrt{d} q_{ij} \phi \quad q_{ij} = -q_{ji} \quad (20)$$

Under supersymmetry ϕ transforms non-linearly

$$\delta \phi = \xi^\mu \partial_\mu \phi + i q_{ij} \xi_{ij} \phi \quad (21)$$

and

$$\xi_{ij} = \frac{-i}{2\sqrt{d}} (\bar{\epsilon}_i \theta_j - \bar{\epsilon}_j \theta_i)$$

We can also introduce the covariant derivative of ϕ

$$D_\mu \phi = \partial_\mu \phi - \frac{i}{d\sqrt{d}} q_{ij} (\bar{\theta}_i \partial_\mu \theta_j) \phi \quad (22)$$

so that

$$D_a \phi = V_a^\mu D_\mu \phi \quad (23)$$

transforms like ϕ .

As a consequence the action

$$A_M = \int d^4x \det V \mathcal{L}_{\text{MATTER}}(\phi, \phi^*, D_a \phi, D_a \phi^*) \quad (24)$$

is invariant if $\mathcal{L}_{\text{MATTER}}$ is Poincaré invariant as well as invariant under U(1)-central charge transformation.

Of course, if some g_{ij} are non-vanishing, then the corresponding central charges are spontaneously broken, and one can write arbitrarily couplings provided ϕ is coupled to the Goldstone boson field as follows:

$$\psi = \exp \left[\frac{2i}{\sqrt{d}} q_{ij} C_{ij} \right] \phi \quad (25)$$

so that ψ is inert under the action of Z_{ij} .

The coupling of non-linear realizations of supersymmetry and central charges to N-extended supergravity provides the simultaneous occurrence of the Higgs and superHiggs effect.

In order to work out the mass formulae which follow, it is sufficient to work out the full bilinear part of the coupled locally supersymmetric invariant Lagrangian as well as those trilinear couplings which are needed to make the bilinear couplings invariant. Of course, we are assuming that the full non-linear coupling exists. This may not be true for higher N theories. These results are derived in Ref. 7).

The cancellation of the cosmological constant implies the relation

$$m_{3/2}^2 = \frac{1}{6} k^2 d^2 \quad (26)$$

where k is the gravitational coupling constant.

The Higgs effect gives masses to the $N(N-1)/2$ vector bosons of the supergravity multiplet in terms of g_{ij} and $m_{3/2}$:

$$m_{ij}^2 = \frac{3}{16} g_{ij}^2 m_{3/2}^2 \quad (27)$$

The charge coupling of the vector bosons to the gravitino is

$$g_c = k m_{3/2} \quad (28)$$

and is universal.

Strictly speaking, these results, obtained by working out the linearized coupling, are only valid if $N < 4$. For $N \geq 4$ scalar fields appear in the supergravity multiplet and their v.e.v. can destroy the above relations. These relations remain valid only if extremes of the potential exist for which all scalar fields of the gravitational multiplet have vanishing v.e.v.s.

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REFERENCES

- 1) S. Weinberg, Phys. Rev. Lett. 18 (1967) 188;
S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177 (1969) 2239;
C. Callan, S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177 (1969) 2247.
- 2) V. Akulov and B. Volkov, Phys. Lett. 46B (1973) 109.
- 3) E. Ivanov and A. Kapustinkov, J. Phys. A11 (1978) 2375; J. Phys. G8 (1982) 167;
T. Uematsu and Z. Zachos, Nucl. Phys. B201 (1982) 250;
J. Wess, Karlsruhe preprint (1982).
- 4) S. Deser and B. Zumino, Phys. Rev. Lett. 38 (1977) 1433.
- 5) E. Cremmer et al., Phys. Lett. 79B (1978) 231; Nucl. Phys. B147 (1979) 105;
E. Cremmer, S. Ferrara, L. Girardello and A. van Proeyen, Phys. Lett. 116B (1982) 231; Nucl. Phys. B. B212 (1983) 413.
- 6) R. Haag, J.T. Lopuszanski and M. Sohnius, Nucl. Phys. B88 (1975) 257.
- 7) S. Ferrara, L. Maiani and P.C. West, CERN preprint TH.3513 (1983).

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