ERRATUM AND ADDENDUM

to

LOOP CALCULATION IN CONFINED QCD
II: THE SELF ENERGY OF MASSIVE QUARKS

by

J. Baacke
CERN - Geneva

Y. Igarashi
Faculty of Education, Niigata University,
Niigata, 950-21 Japan

G. Kasperidus and H. Usler
Inst. für Physik, Univ. Dortmund, 4600 Dortmund 50
Federal Republic Germany

After sending out the preprint version of our paper we have realized - through a suggestion by J.D. Breit - that we have omitted a semicircular integral around the pole at $v=0$, arising from the $j=\frac{1}{2}^+$ intermediate state in the transversal part of the self energy. Explicitly this contribution is

$$E^{\text{pole}} = - \frac{\alpha}{R} 4n^2 \frac{p_0^2}{E_0^2} \iint r^2 dr r'^2 dr' \left\{ \frac{1}{3} \frac{Rr}{r'^2} + \frac{1}{6} \frac{rr'}{R^2} \right\} \times \\ \times j_0(p_0 r) j_1(p_0 r) j_0(p_0 r') j_1(p_0 r')$$

We had also to improve our routines generating the Bessel functions for large arguments, this leads to sizeable corrections of the boundary dependent contributions. A further numerical problem occurred in the calculation of the free contribution, but only at $m = 0$. The corrected numerical results and the pole contribution are given in Table 1.

We have also to correct misprints in some of the equations:

- There should be a - sign on the right hand side of eq. (2.3b).
- In eq. (2.5) j_0 should be replaced by j_0^2 .
- $(\not{p} - m)$ in eq. (4.2a) should have a factor $-\frac{\alpha}{4\pi} \cdot D$.
- d^3p in eq. (5.2) should be divided by $(2\pi)^3$.
- The integral in eq. (5.5) should have a factor $\alpha/4\pi$.
- g_3 in eq. (5.6) should be multiplied by m .
- In eq. (A.8) replace $-i\nu + E$ by $+i\nu - E$.

mR	$E_{\text{coul}}^{\text{bound}}$	$E_{\text{tr}}^{\text{pole}}$	$E_{\text{tr}}^{\text{bound}}$	$-E_{\text{tr}}^{\text{cut}}$	E_{free}	E_s
0	0.928	-.266	-.011	0.406	0.211	1.268
0.5	0.826	-.231	-.054	0.403	0.077	1.021
1.0	0.751	-.198	-.084	0.379	0.047	0.895
2.0	0.655	-.143	-.123	0.302	0.030	0.721
4.0	0.574	-.077	-.140	0.177	0.020	0.554
6.0	0.547	-.045	-.137	0.109	0.014	0.488
8.0	0.531	-.029	-.129	0.072	0.010	0.455
10.0	0.521	-.020	-.117	0.051	0.007	0.442

Table 1

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LOOP CALCULATION IN CONFINED QCD
II: THE SELF ENERGY OF MASSIVE QUARKS [†]

J. Baacke ^{*}
CERN - Geneva

Y. Igarashi
Faculty of Education, Niigata University,
Niigata, 950-21 Japan

G. Kasperidus and H. Usler
Inst. für Physik, Univ. Dortmund, 4600 Dortmund 50
Federal Republic Germany

ABSTRACT

We present an explicit numerical evaluation of the self energy of confined massive quarks for the ground state of the MIT bag model. The calculation is performed in the Coulomb gauge, using a technique developed in part I.

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^{*} On leave of absence from Inst. für Physik, Univ. Dortmund

1. Introduction

An efficient technique for calculating loop corrections within the framework of the MIT bag model [1] using confined QCD [2] has recently been set up by different authors [3,4,5]. The essential technical points are to use closed forms for the confined Green's functions instead of mode summation [6] and to avoid oscillating integrands by Wick rotation [3,5] or, equivalently, by using Euclidean propagators from the outset [4]. In our previous publication [4], referred to as I in the following, we have presented explicit expressions for the Euclidean confined Green's functions and the self energy of confined massive quarks. We consider here the numerical evaluation of the self energy, including the discussion of renormalization. We use the Coulomb gauge, as in Refs.[2,3], and dimensional regularization [7] for the free space part of the self energy. The boundary dependent contributions are found to be finite, in agreement with arguments given in Refs. [3,5].

In section 2 we give a short resume of our previous work. The calculation of the different contributions to the self energy is described in sections 3,4 and 5. Our final results are discussed in section 6.

2. Description of the Method

The self energy of confined quarks can be calculated in second order as

$$E_s = E_{\text{Coul}} + E_{\text{tr}} \quad (2.1)$$

where

$$E_{\text{Coul}} = 4\pi\alpha \int d^3x \int d^3x' \int \frac{d\nu}{2\pi} G_{\text{Coul}}(\vec{x}, \vec{x}') \times \\ \times \bar{U}_0(\vec{x}) \gamma^0 \tilde{S}_E(\vec{x}, \vec{x}', \nu) \gamma^0 U_0(\vec{x}') \quad (2.2a)$$

and

$$E_{\text{tr}} = -4\pi\alpha \int d^3x \int d^3x' \int \frac{d\nu}{2\pi} \tilde{D}_{E,ij}^c(\vec{x}, \vec{x}', \nu) \times \\ \times \bar{U}_0(\vec{x}) \gamma^i \tilde{S}_E^c(\vec{x}, \vec{x}', -\nu - iE_0) \gamma^j U_0(\vec{x}') \quad (2.2b)$$

The colour Casimir factor is contained in $\alpha \equiv 4/3 \alpha_s$. The confined Euclidean Green's functions $\tilde{S}_E^c$ for quarks and $\tilde{D}_{E,ij}^c$ for gluons and the confined Coulomb propagator G_{Coul} have been given explicitly in I, separated into free space and boundary parts. ν is the Euclidean energy and the tilde indicates that the Fourier transform in Euclidean time has been taken. $U_0(x)$ is the MIT bag model wave function. We evaluate here the self energy for the $S_{1/2}$ ground state. After performing the angular integrations E_{Coul} and E_{tr} can be rewritten in the form

$$E_{Coul} = 2\pi\alpha \int r^2 dr \int r'^2 dr' (N_{11}(r, r') u(r) u(r') + 2N_{12}(r, r') u(r) v(r') + N_{22}(r, r') v(r) v(r')) \quad (2.3a)$$

$$E_{tr} = 4\pi\alpha \int r^2 dr \int r'^2 dr' (M_{11}(r, r') u(r) u(r') + 2M_{12}(r, r') u(r) v(r') + M_{22}(r, r') v(r) v(r')) \quad (2.3b)$$

$u(r)$ and $v(r)$ are the upper and lower component radial wave functions [8]

$$\begin{aligned} u(r) &= \sqrt{n} \sqrt{(E_0 + m)/E_0} j_0(p_0 r) \\ v(r) &= \sqrt{n} \sqrt{(E_0 - m)/E_0} j_1(p_0 r) \end{aligned} \quad (2.4)$$

with

$$n^{-1} = R^3 j_0(p_0 R) \frac{2E_0(E_0 - R^{-1}) + m/R}{E_0(E_0 - m)} \quad (2.5)$$

p_0 is defined by the boundary condition

$$u(R) = v(R) \quad (2.6)$$

Corresponding to the separation of the quark and gluon Green's functions into free (f) and boundary (b) parts one can decompose the kernels appearing in (2.3) as

$$N_{ij} = N_{ij}^{ff} + N_{ij}^{fb} + N_{ij}^{bf} + N_{ij}^{bb} \quad (2.7)$$

and correspondingly for the M_{ij} . The first superscript refers to the quark, the second one to the gluon propagators. We have to choose this separation since the kernels bf, fb and bb show a different (some of them singular) behaviour at $r = r'$ (see section 3), requiring different integration procedures, and since the free (ff) contribution has to be renormalized. This can be done in analogy to QED (see section 5).

Explicit expressions for the fb kernels have been given in I. Since the expressions for the kernels are quite lengthy and their number is large we give, in Appendix A, only the formulae for the 11 components, in order to display their general structure and to allow checking and comparison.

3. The Boundary Dependent Contributions

The evaluation of the boundary dependent kernels M_{ij}^λ and N_{ij}^λ with $\lambda = fb, bf$ and bb involves summation over ℓ and integration over v . The ℓ summation converges well except when r and r' are both near the boundary. In the latter case the summation extends typically to $\ell \approx 300$ if a relative accuracy of 10^{-4} is required. The convergence of the v integration is related to $|r-r'|$ for the Coulomb terms and to $R-r-r'$ for the transversal ones. The singular behaviour of some of the Coulomb kernels at $r = r'$ is caused by a divergent v integration (see e.g. Fig 1a in I). Specifically N_{11} and N_{22} have a logarithmic singularity and N_{12} has a pole at $r = r'$. which are both integrable. In these cases it is convenient for the numerical integration to separate

$$\begin{aligned} \int r'^2 dr' N_{11}^{fb}(r, r') u(r') &= \\ &= \int r'^2 dr' N_{11}^{fb}(r, r') (u(r') - u(r)) \\ &\quad + u(r) \int r'^2 dr' N_{11}^{fb}(r, r') \end{aligned} \quad (3.1)$$

The first integrand is non-singular, the second integral can be done analytically. For N_{12} and N_{22} one separates similarly, splitting $v(r') = (v(r') - r'v(r)/r) + r'v(r)/r$ to obtain a non-singular integral and one which can be done analytically. The other kernels have at most kinks or finite steps at $r = r'$ and the first radial integration (say over r') is unproblematic.

Next we have to perform the second radial integration (say over r). The

integrands, resulting from the r' -integration turn out to be singular at the boundary $r = R$. We have studied this singular behaviour numerically. For the fb contribution we find an integrable logarithmic singularity. For the bf and bb contributions we find a linear singularity ($-1/(R-r)$) in the 11 and 22 parts, this linear singularity cancels however in the sum if the bag boundary condition $u(R)=v(R)$ is satisfied. This agrees with the analysis given by Hansson and Jaffe [5] for the massless case, using the multiple reflection expansion.

After fitting the r -integrand with a logarithm of $(R-r)$ in the region above $r/R \sim .9$ we integrate then over r . We estimate the error introduced by this fitting procedure to be below 5%. The results for the boundary dependent contributions E_{Coul} and E_{tr} are given in table 1 and Fig. 2.

4. The free contribution

The calculation of the self energy with free space propagators for quarks and gluons is in principle straightforward and could be taken from textbooks on QED if we had used a covariant gauge. For the Coulomb gauge the problem of regularization and renormalization has been discussed only recently by Heckathorn [9] and Adkins [7] who obtain covariant counter terms using dimensional regularization. We have used the explicit expressions given by Adkins.

We have to consider

$$E_{\text{free}} = \int \frac{d^3 p}{(2\pi)^3} \bar{U}_0(\vec{p}) \Sigma(E_0, \vec{p}) U_0(\vec{p}) \quad (4.1)$$

where $U_0(\vec{p})$ is the Fourier transform of the bag model wave function (2.5) with energy E_0 and Σ is the self energy kernel given after dimensional regularization by

$$\begin{aligned} \Sigma(p) = & \frac{\alpha}{4\pi} m (3D+4) - \delta m - \frac{\alpha}{4\pi} (\not{p} - m) \\ & + \Sigma_f(p) \end{aligned} \quad (4.2a)$$

$$\Sigma_f(p) = -\frac{\alpha}{4\pi} (a^f(p)m + b^f(p)\not{p} + c^f(p)E\gamma^0) \quad (4.2b)$$

The functions a^f , b^f and c^f are given in Appendix B. They satisfy the condition

$$\left(a^f(p)m + b^f(p)m + c^f(p) \frac{E^2}{m} \right) \Big|_{p^2=m^2} = 0 \quad (4.3)$$

So with the choice $\delta m = (\alpha/4\pi)m(3D+4)$ the expectation value of $\Sigma(p)$ vanishes on shell. With this choice and using the fact [3,5] that the divergent part $(\alpha/4\pi)D(\not{p}-m)$ has vanishing expectation value not only on shell but also for a bag wave function (due to the bag boundary condition) we have finally

$$E_{\text{free}} = \int \frac{d^3 p}{(2\pi)^3} \bar{U}_0(\vec{p}) \Sigma^f(E_0, \vec{p}) \psi_0(\vec{p}) \quad (4.4)$$

The evaluation of (4.4) is straightforward. The angular integrals can be done explicitly, the kernel functions a^f , b^f , c^f are known analytically up to one Feynman parameter integration which has to be performed numerically. The integral over $|\vec{p}|$ converges rather slowly since the integrand oscillates and behaves in the average as $\ln|\vec{p}|/\vec{p}^2$. We have used averaging over the period of oscillation and numerical extrapolation to obtain a good accuracy ($\approx 1\%$). The procedure was checked by the simultaneous calculation of the norm $\langle \chi_0 \rangle = 1$ and of $\langle \not{p}-m \rangle = 0$.

The results for $E_{\text{free}}(m)$ are given in Table 1 and Fig. 2. The contribution is relatively small except for $m \rightarrow 0$ where it shows a logarithmic cusp of the form $m \ln m$ (this type of dependence can be seen explicitly in the kernels).

5. The Contribution of the Cut

As discussed in I the Euclidean quark and gluon propagators have poles on the imaginary v axis, however the boundary and free parts taken separately have cuts along the imaginary v axis. We have evaluated the transversal part as

$$E_{tr} = -4\pi\alpha \int d^3x \int d^3x' \int \frac{dv}{2\pi} \tilde{D}_{E,ij}^c(\vec{x}, \vec{x}', v) \cdot \bar{U}_0(\vec{x}) \gamma^i \tilde{S}_E^c(\vec{x}, \vec{x}', -v - iE_0) \gamma^j U_0(\vec{x}') \quad (5.1)$$

where the v integration runs over the real v axis. For $E_0 < m$ we cross neither of the cuts of the separated free and boundary pieces of the propagators. However for $E_0 > m$ the cuts of the f and b parts of $\tilde{S}_E^c(x, x', -v - iE_0)$ move down through the real v axis. Giving a small imaginary part to E_0 , $E_0 \rightarrow E_0 + i\varepsilon$, the cuts are at $\text{Re } v = \varepsilon$, $m - E_0 < \text{Im } v < \infty$ and $-m - E_0 < \text{Im } v < -\infty$. The first one crosses the real axis for $E_0 > m$ to the right (left) of $v = 0$ for positive (negative) ε (see Fig.1). In our case $E_0 > m$, so we have to consider the complication arising from this cut.

Integrating just over the real v axis gives the correct answer if all contributions, ff , bf , fb and bb are taken together. In the actual calculation we have integrated only over real v for the boundary dependent parts. However the free (ff) part considered in section 4 contains also the contribution of the contour C in Fig. 1. This contribution has therefore to be subtracted.

Explicitly we find

$$E_{tr}^{cut} = -4\pi\alpha \int d^3p \bar{U}_0(\vec{p}) \sum_{tr}^{cut}(E_0, \vec{p}) U_0(\vec{p}) \quad (5.2)$$

where

$$\begin{aligned} \sum_{tr}^{cut}(E_0, \vec{p}) &= \int \frac{d^3p'}{(2\pi)^3} \gamma^i \frac{E' \gamma^0 - \vec{\gamma} \cdot \vec{p}' + m}{2E'} \gamma^j \delta_{ij}^\perp(\vec{q}) \cdot \\ &\quad \cdot \frac{1}{\vec{q}^2 - (E_0 - E' + i\varepsilon)^2} \Theta(E_0 - E') \end{aligned} \quad (5.3)$$

Here $\vec{q} = \vec{p} - \vec{p}'$ and $\delta_{ij}(q) = \delta_{ij} - q_i q_j / \vec{q}^2$. We have

$$\frac{1}{\vec{q}^2 - (E_0 - E' + i\varepsilon)^2} = P \frac{1}{\vec{q}^2 - (E_0 - E')^2} + i\pi \text{sign}(\varepsilon) \delta(\vec{q}^2 - (E_0 - E')^2) \quad (5.4)$$

The δ function in the imaginary part puts the gluon on shell. This

contribution gives the absorptive part of the free self energy. This part has already been left out in the free contribution by taking its real part (see Appendix B). However the principal value part in eq.(5.3) leads to a non-vanishing contribution which has to be subtracted.

We get

$$E_{tr}^{cut} = \frac{\alpha}{\pi} \int \frac{d^3 p}{(2\pi)^3} \bar{U}_0(\vec{p}) K(E_0, \vec{p}) U_0(\vec{p}) \quad (5.5)$$

where $K(E_0, p)$ is of the form

$$K(E_0, \vec{p}) = g_1 \gamma^0 + g_2 \vec{p} \vec{\gamma} + g_3 \quad (5.6)$$

The functions $g_i(E_0, p)$ are given in Appendix B.

The numerical evaluation presents no problems, the p' integration is over a finite range and the p integral converges well. The contribution $-E_{tr}$ is given in Table 1 and plotted in Fig. 2.

6. Results and Conclusions

In this paper we have discussed the explicit numerical evaluation of the self energy of massive quarks confined to a spherical cavity, using a technical framework set up in I. We have calculated the boundary dependent parts and the free space contributions separately since the latter one has to be renormalized. The boundary dependent contributions E_{Coul} and E_{tr} whose evaluation was described in section 3 were found to be finite. The free contribution E^{free} and its renormalization were discussed in section 4. E^{free} contains a part arising from integrating around a cut (contour C in Fig.1) which is not present in the total self energy and has been left out ab initio in E_{tr} . This contribution E_{cut} , given in section 5 has therefore to be subtracted.

The total self energy is obtained as

$$E_s = E_{Coul}^{bound} + E_{tr}^{bound} + E^{free} + (-E_{tr}^{cut}) \quad (6.1)$$

The single terms on the right hand side and the total result E_s are given in Table 1 for those values of the quark mass for which we have performed the calculation. E^{free} , which has a cusp at $m = 0$ has also been calculated

at $m = .1, .01$ and $.001$. The curves displayed in Fig. 2 are interpolations by hand.

At large m the self energy approaches the non-relativistic limit [10] obtained as the expectation value of the non-relativistic potential [2,10]

$$V(\xi = \frac{r}{R}) = \frac{\alpha}{2R} \left(\frac{\xi^2}{1-\xi^2} - \ell_n(1-\xi^2) \right) \quad (6.2)$$

in the MIT bag model ground state.

For the self energy at $m = 0$ we obtain $E(m=0) = 1.5 \alpha/R$. Two different results have been published recently. Chin et al. [6], who use a covariant gauge and mode summation for the propagators, obtain, in our notation, $E(m=0) = .31 \alpha/R$. However they use a different convention for the boundary term of the $\ell=0$ mode. This difference in convention is irrelevant if self and interaction energies in a colour singlet state are added [2]. If we understand their notation correctly a term

$$\Delta E = \frac{\alpha}{R} \int d^3x \bar{U}_0(\vec{x}) \left(\frac{9}{10} - \frac{1}{2} \vec{x}^2 \right) U_0(\vec{x}) = .63 \frac{\alpha}{R} \quad (6.3)$$

should be added before comparing their result to ours. This increases their value to $E_s(m=0) = .94 \alpha/R$. Though this does not differ in sign and order of magnitude, the disagreement is sizeable. Breit [3], who uses essentially the same technique as us and also the same convention for the $\ell=0$ mode, obtains a still smaller value $E_s(m=0) = .76 \alpha/R$. We can certainly claim to have performed the calculation with higher numerical accuracy, less numerical integrations were involved and the ℓ summation has been extended to very high ℓ whenever necessary to obtain a good numerical accuracy. It is however clear that the whole calculation is sufficiently involved and contains many occasions for even trivial mistakes. It will simply take some time and correspondence between the different authors to resolve the disagreement between the different values obtained. An independent evaluation of the self energy, using a covariant gauge, announced by Hansson and Jaffe [5] will certainly be helpful for settling the question.

For phenomenological purposes the message that the self energy is $\approx .5 \alpha/R$ for large quark masses and between $.76 \alpha/R$ and $1.5 \alpha/R$ for small ones should at present be sufficiently accurate in view of the large uncertainties in the value of α to be used.

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Appendix A

We give here the explicit expressions for some of the kernels needed for the calculation of the boundary dependent parts of the self energy. The 11 components of the Coulomb kernels are given by

$$N_{11}^{bf}(r, r') = \frac{1}{2\pi R} \int_{-\infty}^{\infty} \frac{dv}{2\pi} \kappa^2 \sum_{l=0}^{\infty} \frac{r_{<}^l}{r_{>}^{l+1}} \frac{1}{2l+1} \{ (l+1) a_e^+ + l a_e^- \} i_e(\kappa r) i_e(\kappa r') \quad (A.1)$$

$$N_{11}^{bb}(r, r') = \frac{1}{2\pi R} \int_{-\infty}^{\infty} \frac{dv}{2\pi} \kappa^2 \sum_{l=1}^{\infty} \frac{l+1}{l} \left(\frac{rr'}{R^2} \right)^l \frac{1}{2l+1} \{ (l+1) a_e^+ + l a_e^- \} i_e(\kappa r) i_e(\kappa r') \quad (A.2)$$

with

$$a_e^{\pm}(\kappa R) = \frac{m\kappa (k_{e\pm 1} i_{e\pm 1} - k_e i_e) + \kappa^2 k_{e\pm 1} i_e + (v^2 - m^2) k_e i_{e\pm 1}}{\kappa^2 (i_{e\pm 1}^2 + i_e^2) + 2m\kappa i_{e\pm 1} i_e} \quad (A.3)$$

where the argument of the modified spherical Bessel functions $i_l(z) = \sqrt{\pi/2z} I_{l+1/2}(z)$ and $k_l(z) = \sqrt{2/\pi z} K_{l+1/2}(z)$ is κR and $\kappa = \sqrt{v^2 + \mu^2}$.

The corresponding transversal parts are given by

$$M_{11}^{bf}(r, r') = \frac{1}{4\pi} \int_{-\infty}^{\infty} \frac{dv}{2\pi} \kappa \tilde{\kappa} (m - E_0 + i\nu) \sum_{l=1}^{\infty} \quad (A.4)$$

$$\begin{aligned} & [\{ (l+1) A_{e-\frac{1}{2}}^+ + l A_{e+\frac{1}{2}}^- \} i_e i_e i_e k_e \\ & + (l+1) A_{e-\frac{1}{2}}^- i_{e-1} i_{e-1} i_{e-1} k_{e-1} \\ & + l A_{e+\frac{1}{2}}^+ i_{e+1} i_{e+1} i_{e+1} k_{e+1}] \end{aligned}$$

and

$$\begin{aligned}
 M_{11}^{bb}(\tau, \tau') = & \frac{1}{4\pi} \int_{-\infty}^{\infty} \frac{d\nu}{2\pi} \kappa \tilde{\kappa} (m - E_0 + i\nu) \sum_{\ell=1}^{\infty} \left[\{ (\ell+1) A_{\ell-\frac{1}{2}}^+ + \ell A_{\ell+\frac{1}{2}}^- \} A_{\ell} i_{\ell} i_{\ell} i_{\ell} i_{\ell} \right. \\
 & + (\ell+1) A_{\ell-\frac{1}{2}}^- \kappa^2 B_{\ell} i_{\ell-1} i_{\ell-1} i_{\ell-1} i_{\ell-1} \\
 & + \ell A_{\ell+\frac{1}{2}}^+ \kappa^2 B_{\ell} i_{\ell+1} i_{\ell+1} i_{\ell+1} i_{\ell+1} \\
 & \left. - \frac{(\ell+1)(2\ell+1)}{\kappa^3} \frac{(\tau\tau')^{\ell-1}}{R^{2\ell+1}} A_{\ell-\frac{1}{2}}^- i_{\ell-1}(\tilde{\kappa}\tau) i_{\ell-1}(\tilde{\kappa}\tau') \right].
 \end{aligned} \tag{A.5}$$

Here $\kappa = |\nu|$ and $\tilde{\kappa} = \sqrt{(\nu+iE)^2 + m^2}$ and the arguments in the products of Bessel functions are consecutively $\tilde{\kappa}r$, $\tilde{\kappa}r'$, $\kappa r_{<}$ and $\kappa r_{>}$. From I we have the definitions

$$A_{\ell}(\kappa R) = - \frac{\frac{d}{dR}(R k_{\ell}(\kappa R))}{\frac{d}{dR}(R i_{\ell}(\kappa R))} \tag{A.6}$$

$$B_{\ell}(\kappa R) = \frac{1}{\kappa^2} \frac{k_{\ell}(\kappa R)}{i_{\ell}(\kappa R)} \tag{A.7}$$

$$A_j^{\pm}(\tilde{\kappa} R) = \frac{(m - i\nu + E) k_{j \pm \frac{1}{2}}(\tilde{\kappa} R) - \tilde{\kappa} k_{j \mp \frac{1}{2}}(\tilde{\kappa} R)}{(m - i\nu + E) i_{j \pm \frac{1}{2}}(\tilde{\kappa} R) + \tilde{\kappa} i_{j \mp \frac{1}{2}}(\tilde{\kappa} R)} \tag{A.8}$$

Appendix B

The kernels a^f , b^f and c^f are functions of the external energy E_0 and the integration variable $p=|\vec{p}|$. We use also $E = \sqrt{p^2 + m^2}$ and $p_0 = \sqrt{E_0^2 - m^2}$. Explicit expressions for these kernels have been obtained by Adkins [7] :

$$a^f = f_1 + 2 f_3 \quad (B.1a)$$

$$b^f = \frac{19}{6} - f_1 + f_2 + 2 f_4 - f_5 \quad (B.1b)$$

$$c^f = -\frac{8}{3} + f_1 - f_2 - 2 f_3 + f_5 \quad (B.1c)$$

where the functions f_i are given by

$$f_1 = 2 \frac{E}{p} \ln \frac{E+p}{E-p} - 4 \quad (B.2a)$$

$$f_2 = \frac{2}{3} \left(\frac{E}{p} \right)^3 \left[\ln \frac{E+p}{E-p} - 2 \frac{p}{E} - \frac{2}{3} \left(\frac{p}{E} \right)^3 \right] \quad (B.2b)$$

$$f_3 = \frac{p_0^2 - p^2}{E_0^2 - p^2} \ln |\sigma| - 1 \quad (B.2c)$$

$$f_4 = \frac{(p_0^2 - p^2)^2}{(E_0^2 - p^2)^2} \frac{1}{2} \ln |\sigma| - \frac{1}{2} \frac{p_0^2 - p^2}{E_0^2 - p^2} - \frac{1}{4} \quad (B.2d)$$

$$f_5 = \int_0^1 dx x^{1/2} (g (\ln g - 1) + \sigma (\ln |\sigma| - 1)) / (g + \sigma) - 2 \frac{p^2}{m^2} \int_0^1 dx x^{3/2} (g + \sigma (1 + \ln |\frac{g}{\sigma}|)) / (g + \sigma)^2 \quad (B.2e)$$

Here p and σ are defined as

$$g = 1 + \frac{p^2}{m^2} (1-x) \quad \sigma = \frac{p_0^2 - p^2}{m^2} \quad (\text{B.3})$$

Taking the absolute value of σ in the logarithms is equivalent to omitting the absorptive part of the free self energy. This absorptive part, which is not present in the confined self energy, is also left out in the cut contribution.

The kernels occuring in the evaluation of the cut contribution (see section 5) are explicitly given by

$$g_1 = 4 \int_0^{p_0} p'^2 dp' \frac{1}{2pp'} \ln \left| \frac{E_-^2 - p_-^2}{E_-^2 - p_+^2} \right| \quad (\text{B.4a})$$

$$g_2 = - \frac{1}{p^2} \int_0^{p_0} \frac{p'^2 dp'}{E'} \left\{ 2 - \frac{E_-^4 - p_-^2 p_+^2}{2pp'E_-^2} \ln \left| \frac{E_-^2 - p_-^2}{E_-^2 - p_+^2} \right| \right. \\ \left. + \frac{p_+^2 p_-^2}{2pp'E_-^2} \ln \frac{p_+^2}{p_-^2} \right\} \quad (\text{B.4b})$$

$$g_3 = - 4 \int_0^{p_0} \frac{p'^2 dp'}{E'} \frac{1}{2pp'} \ln \left| \frac{E_-^2 - p_-^2}{E_-^2 - p_+^2} \right| \quad (\text{B.4c})$$

Here we have used the definitions $E_- = E_0 - E'$ and $p_{\pm} = p \pm p'$.

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Table Captions

Table 1 : The self energy E_s of confined massive quarks in the ground state of the MIT bag model for different values of the quark mass. The different contributions are explained in the text. The energies are given in units α/R with $\alpha = 4/3 \alpha_s$.

Figure Captions

Figure 1 : Cuts in the complex v plane for the free contribution (see section 5). cuts of the gluon propagator, cuts of the quark propagator, integration contour. The cuts are displayed for $\epsilon > 0$ and $E_0 > m$.

Figure 2 : The self energy E_s of confined massive quarks in the ground state of the MIT bag model. The single contributions are labelled according to the definitions in the text.

mR	$E_{\text{Coul}}^{\text{bound}}$	$E_{\text{tr}}^{\text{bound}}$	E^{free}	$-E_{\text{tr}}^{\text{cut}}$	E_s
0.	.673	.133	.288	.406	1.500
0.5	.650	.066	.077	.403	1.196
1.	.632	.006	.047	.379	1.064
2.	.613	-.065	.030	.302	.880
4.	.587	-.120	.020	.177	.664
6.	.567	-.125	.014	.109	.565
8.	.555	-.120	.010	.072	.517
10.	.536	-.110	.007	.051	.484

Table 1

FIG. 1

FIG. 2