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THE NON-PERTURBATIVE PARAMETER IN THE SVZ SUM RULES

CANNOT BE $\langle 0 | (\alpha_s / \pi) G_{a\mu\nu} G_a^{\mu\nu} | 0 \rangle$

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A B S T R A C T

New data on the T family and the SVZ sum rules are used to evaluate the parameter usually interpreted as $\langle 0 | (\alpha_s / \pi) G_{a\mu\nu} G_a^{\mu\nu} | 0 \rangle$. The result exceeds by almost two orders of magnitude that accepted for the ψ family. This excludes the now standard interpretation.

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The sum rules proposed by Shifman, Vainshtein and Zakharov ¹⁾ have been successfully applied to the spectroscopy of mesons ^{1),2)}, gluonia ³⁾ and baryons ⁴⁾. There has even been an attempt to predict from them the toponium mass ⁵⁾. As a byproduct the sum rules yield a non-perturbative parameter ¹⁾ interpreted as the vacuum expectation value of the energy density of the gluon condensate. Values obtained for this parameter from studies of the ψ family are of the order of 0.01 GeV^4 . Thus, e.g., the careful analysis from Ref. 6) gives

$$\langle 0 | \frac{\alpha_s}{\pi} G_{a\mu\nu} G_a^{\mu\nu} | 0 \rangle = (1.5^{+0.4}_{-0.2}) 10^{-2} \text{ GeV}^4 \quad (1)$$

In the following we denote this matrix element by G and its estimates as obtained by applying the SVZ sum rules to the ψ and T families by $\bar{G}(\psi)$ and $\bar{G}(T)$. According to the usual interpretation

$$G = \bar{G}(\psi) = \bar{G}(T). \quad (2)$$

The fact that $G \neq 0$ means that the physical vacuum is not perturbative. Some of the problems, where the actual numerical value of G is useful, are: testing lattice estimates of non-perturbative effects ⁷⁾, deriving confining $q\bar{q}$ potentials ⁸⁾, evaluating corrections to the energy levels of heavy quarkonia ⁹⁾, evaluating corrections to semi-leptonic decay widths of resonances ¹⁰⁾. Thus the question whether or not estimate (1) is reliable is important.

In spite of the phenomenological success of the SVZ sum rules, the estimate (1) has been questioned. Both potential models ¹¹⁾ and two-dimensional chromodynamics in the $N_c \rightarrow \infty$ approximation ¹²⁾ suggest that G exceeds $\bar{G}$ by at least a factor of two. Moreover, Nikolaev and Radyushkin ¹³⁾ have recently found that terms quartic in $G_{\mu\nu}$, which have been neglected in the SVZ analysis, have large perturbative coefficients and, at least for the ψ family are not negligible. When going from $\bar{G}(\psi)$ to $\bar{G}(T)$ the corrections of Nikolaev and Radyushkin are suppressed, as compared to the non-perturbative term included by SVZ by a factor $(m_b/m_c)^4 \sim 100$. Therefore, a careful evaluation of $\bar{G}(T)$ seems of great interest.

The SVZ sum rules have often been applied to the T family, but [cf., e.g., Ref. 6)] the precision was too low to make any strong statements about $\bar{G}(T)$. An indication that $\bar{G}(T)$ exceeds $\bar{G}(\psi)$ by perhaps as much as a factor of four has been reported in Ref. 5), but because of the large uncertainties involved the author did not stress it.

When applying the SVZ sum rules, experimental data for the resonances coupled to the e^+e^- channel are combined with theoretical prejudice for the corresponding continuum to evaluate the moments [cf., e.g., Ref. 6] :

$$M_n(V) = \frac{3}{4\pi\alpha^2 Q^2} \left[\sum_i \frac{\Gamma_{iV}}{M_{iV}^{2n+1}} + \frac{\alpha^2 Q^2}{3\pi} \left(1 + \frac{\alpha_{SV}}{\pi}\right) \frac{1}{n E_{TV}^{2n}} \right] \quad (3)$$

Here V denotes the family (ψ or T). Q is the charge of the corresponding quark in units of the electron charge. The index i labels the relevant resonances - six for the ψ family and four for the T family. M_{iV} is the resonance mass, while Γ is the product of the electronic width (Γ_e) and the hadronic width (Γ_h) divided by the total width (Γ_{tot}) of the resonance. Since $\Gamma_h \approx \Gamma_{tot}$, each Γ is in practice usually replaced by the corresponding electronic width. The coupling constants are usually chosen so that

$$\alpha_{s\psi} = 0.2 \quad , \quad \alpha_{sT} = 0.165 \quad (4)$$

but it has been checked ⁶⁾ that choosing $\alpha_{s\psi} = 0.3$, which corresponds to $\alpha_{sT} = 0.22$, reduces the parameter $\bar{G}(\psi)$ by a factor of about three compared to estimate (1). The energy E_{TV} is a threshold energy, where the continuum contribution to the reaction $e^+e^- \rightarrow$ hadrons becomes important. Its experimental estimate for $V = T$ is very uncertain and carries a large error. For instance, according to Ref. 6), $E_{TT} = (10.57 \pm 0.3)$ GeV.

In the present paper we improve the experimental determination of the moments (3) in two ways. Firstly, we use for the mass of the $T(9460)$ the new Novosibirsk result ¹⁴⁾, which carries an error sixteen times smaller than the previous world average from Ref. 15). Since the mass differences in the T family have been measured much better than the individual masses, the errors on the masses of the higher resonances also reduce, typically by a factor of three. Secondly, since we do not intend to make predictions concerning spectroscopy, we use the sum rules to fix the threshold energies E_T . This makes little difference for the ψ family, but the error on E_{TT} is reduced by a factor of almost four. Except for M_{1T} from Ref. 14), all the input data are taken from Ref. 15). The input not calculable from the tables in Ref. 15), i.e., Γ_{3T} and the relative errors of the widths Γ_i , has been taken from the card listings.

In the SVZ analysis the ratios of the experimental moments (3) are compared with the corresponding theoretical expressions

$$r_n(V) = \frac{M_n(V)}{M_{n-1}(V)} = \frac{\tau_n^{(0)}}{m_q^2} \left[1 + a_n \alpha_{sv} + b_n \frac{\bar{G}(V)}{m_q^4} \right] \quad (5)$$

Here m_q is the mass of the quark corresponding to family V . The coefficients $r_n^{(0)}$, a_n and b_n are known functions of n , explicitly given, e.g., in Refs. 1) and 6). In the theoretical expression various higher order terms have been omitted by SVZ. As stressed in Ref. 6), the correlation of errors in M_n and M_{n-1} is crucial for a correct estimate of the errors in r_n . When evaluating $\bar{G}(V)$ one has moreover errors on m_q and E_T , which are functions of the errors on the resonance masses and widths. In order to include all these correlations without getting involved with very long formulae, we proceeded as follows. Every mass or width is chosen at random from a Gaussian distribution centered at the input value and having a width equal to the input error. Then all the quantities of interest are calculated. The calculation is repeated a thousand times. The averages of the output values are given as our results and the corresponding dispersions are given as errors.

We calculated the quark masses and the threshold energies from the ratios r_2 and M_1^2/M_2 . The results for the coupling constants (4) are

$$m_c = (1.284 \pm 0.016) \text{ GeV}, \quad m_b = (4.185 \pm 0.032) \text{ GeV}$$

$$E_{T\psi} = (4.274 \pm 0.010) \text{ GeV}, \quad E_{T\tau} = (10.786 \pm 0.083) \text{ GeV} \quad (6)$$

consistently with previous estimates. The values of $\bar{G}(V)$ calculated separately for each r_n for $n = 2, 3, \dots, 10$ are collected in the Table.

The main result of this paper is that while the values for $\bar{G}(\psi)$ do not deviate significantly from previous results, the values of $\bar{G}(T)$ are larger by almost two orders of magnitude. The result can be restated as follows. The SVZ perturbative approximation to r_n contains a systematic error. According to the standard SVZ analysis this error is due to the non-zero value of G . This implies that for the T family it should be about hundred times smaller than for the ψ family. Experimentally, the errors for the two families are comparable, which is incompatible with the SVZ conjecture. For $\alpha_{s\psi} = 0.3$ and $\alpha_{sT} = 0.22$ the values of $\bar{G}(V)$ decrease, but the ratios $\bar{G}(T)/\bar{G}(\psi)$ somewhat increase. Thus increasing the coupling constant does not help.

The following comments may be of interest.

- i) The dependence of $\bar{G}(T)$ on n is not stronger than that for $\bar{G}(\psi)$. Thus phenomenologically the SVZ sum rules may be as good for the T family as they are for the ψ family, provided that a much larger value of the parameter $\bar{G}$ is chosen.
- ii) The quark masses have been calculated [cf., e.g., 1) and 6)] under the assumption that $\bar{G}(V)$ contributes little to r_2 . The results given in the Table suggests that $\bar{G}(V)$ increases with decreasing n until it is forced down by assumption. By a slight change in the quark masses one can extend the increase of $\bar{G}(V)$ down to $n = 2$. The corresponding values of $\bar{G}(V)$ are equally plausible a priori and, as seen from the Table, are larger by over fifty per cent than the usual ones.
- iii) Most of the n dependence of the difference between r_n and the perturbative part of its SVZ approximation is taken into account by the coefficient $r_n^{(0)} b_n$. Nevertheless the calculated $\bar{G}(V)$ exhibits a systematic n dependence. In the framework of the SVZ approach the calculations leading to formula (5) were clearly unreliable for n exceeding 6, 7 or 8 ^{1),6)}. Since the derivation seems doubtful for all n , perhaps some n dependent parametrization of $\bar{G}(V)$ could extend the applicability range of the SVZ sum rules.

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N	100 $\bar{G}(\psi)$ GeV ⁴		100 $\bar{G}(T)$ GeV ⁴	
	(a)	(b)	(a)	(b)
2	0	2.1 ± 0.1	0	97 ± 3
3	0.9 ± 0.3	2.0 ± 0.2	50 ± 5	112 ± 2
4	1.2 ± 0.3	1.7 ± 0.2	58 ± 7	99 ± 4
5	1.1 ± 0.2	1.4 ± 0.2	54 ± 7	83 ± 5
6	1.0 ± 0.2	1.2 ± 0.2	48 ± 7	68 ± 5
7	0.9 ± 0.1	1.1 ± 0.1	42 ± 7	57 ± 5
8	0.8 ± 0.1	0.9 ± 0.1	37 ± 6	48 ± 5
9	0.7 ± 0.1	0.8 ± 0.1	32 ± 5	41 ± 4
10	0.6 ± 0.1	0.7 ± 0.1	29 ± 5	36 ± 4

TABLE

Parameters $\bar{G}(V)$ calculated from Eq. (5). For case (a), the quark masses and the thresholds are given in Eq. (6). For case (b), $m_c = (1.271 \pm 0.013)$ GeV, $m_b = (4.158 \pm 0.027)$ GeV, $E_{T\psi} = (4.218 \pm 0.092)$ GeV, $E_{TT} = (10.717 \pm 0.081)$ GeV.

REFERENCES

- 1) M.A. Shifman, A.I. Vainshtein and V.I. Zakharov - Nuclear Phys. B147 (1979) 385, 448.
- 2) L.J. Reinders, H.R. Rubinstein and S. Yazaki - Nuclear Phys. B186 (1981) 109, B196 (1982) 125.
- 3) M.A. Shifman - Z.Phys. C9 (1981) 347.
- 4) B.L. Ioffe - Nuclear Phys. B188 (1981) 317 ;
A.V. Smilga - Yadernaya Fizika 35 (1982) 473 ;
L.J. Reinders, H.R. Rubinstein and S. Yazaki - CERN Preprint TH. 3405 (1982).
- 5) M. Tanimoto - Z.Phys. C9 (1981) 59.
- 6) B. Guberina, R. Meckbach, R.D. Peccei and R. Rückl - Nuclear Phys. B184 (1981) 476.
- 7) A. Di Giacomo and G.P. Paffuti - Phys.Letters 108B (1982) 327 ;
J. Kripfganz - Phys.Letters 101B (1981) 169 ;
T. Banks et al. - Nuclear Phys. B190 (1982) 129.
- 8) C.A. Flory - Phys.Letters 113B (1982) 262 ;
A. Soni and M.D. Tran - Phys.Letters 109B (1982) 393.
- 9) M.B. Voloshin - Nuclear Phys. B154 (1979) 365 ;
H. Leutwyler - Phys.Letters 98B (1981) 447 ;
D. Gromes - Phys.Letters 115B (1982) 482.
- 10) K. Shinzuya - Phys.Letters 105B (1981) 406.
- 11) J.S. Bell and R.A. Bertlmann - Nuclear Phys. B177 (1981) 218 ; B187 (1981) 285 ;
L. Durand, B. Durand and J.B. Whitenton - "The SVZ Method: Why it Fails and Ways to Improve it", Fermilab Preprint 82/88-THY (1982).
- 12) A. Bradley, C.S. Langensiepen and G. Shaw - Phys.Letters 102B (1981) 359 ;
P. Distas and G. Shaw - Phys.Letters 116B (1982) 188.
- 13) N.N. Nikolaev and A.V. Radyushkin - Dubna Preprint (1982).
- 14) A.S. Artamonov et al. - Phys.Letters 118B (1982) 225.
- 15) Particle Data Group - Phys.Letters 111B (1982) 1.

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