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TWISTED EGUCHI-KAWAI MODELS: AN ANALYSIS OF THE SADDLE POINTS

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A B S T R A C T

For Eguchi-Kawai models with generalized periodic conditions, we perform an analysis of the gauge inequivalent maxima of the action, and we study the dependence of the solutions on arbitrary phases. The physically relevant cases are pointed out. We also establish the general relationship between twisted and Wilson model expectation values under the assumption that the $U(1)^d$ symmetry is not spontaneously broken.

Since the discovery of the Eguchi-Kawai (EK) model¹⁾, the interest in one-site lattice gauge theories has increased more and more. One may hope that such especially simple models will approximate the usual $SU(N)$ lattice gauge theories in the large N limit.

In this spirit, the twisted Eguchi-Kawai (TEK) model²⁾ was proposed to improve the behaviour of the EK model in the weak coupling region. The authors of Ref. 2) have performed numerical computations which confirm their arguments, but an analytic study of the model is also desirable.

In the weak coupling region, a saddle point method appears to be suitable for performing such an analysis; this method requires knowledge of all the configurations of link matrices which maximize the action [see Eq. (2b) below], and we are confronted with the problem of their classification. From a mathematical point of view, the relevant equations were shown to admit solutions in Refs. 3) and 4); a large class of solutions, including those of Ref. 2) was also given explicitly in Ref. 5). In this paper, we propose a straightforward classification of the link matrices which saturate the upper bound of the action in the TEK models; we have extensively used the gauge freedom in order to put the relevant matrices in a canonical position, and we have obtained families of gauge inequivalent solutions parametrized in general by several arbitrary phases.

As remarked in Refs. 6) and 7), the presence of such continuous degeneracies in the saddle point manifold is unwanted because it prevents the theory from being in agreement with the usual Wilson theory in the weak coupling region. Our classification is able to predict the number of arbitrary phases according to the twists; as a consequence, we can extract the particular values of twists for which the corresponding TEK model has a chance to reproduce the Wilson theory in the large N large β region. However, in order to make use of the twisted model results in the context of the large N Wilson theory, it is necessary to establish a formal connection between TEK and Wilson expectation values in the case of an arbitrary twist. This connection was established in Ref. 2) for the simplest twist $Z_{\mu\nu} = -1$, where the $\beta \rightarrow -\beta$ symmetry was exploited.

Starting from an analysis of the strong coupling (unbroken phase) behaviour of the general TEK model, we concluded that, for any $Z_{\mu\nu} = Z_{\nu\mu}^*$, and in the large N limit as long as the $U(1)^d$ symmetry is unbroken, the partition function (and the free energy) are the same as in the Wilson theory, and the dependence of the Wilson loop expectation values on the twist is parametrized by

$$W(C)_{\text{TEK}} = \prod_{\mu, \nu=1}^d (Z_{\mu\nu}^*)^{NP_{\mu\nu}} W(C)_W$$

where $NP_{\mu\nu}$ is the number of plaquettes belonging to a minimal surface of C and lying on the plane (μ, ν) .

CONSTRUCTION OF THE SADDLE POINTS

The main difference between the EK and TEK models lies in the method of identifying the link matrices $U_\mu(x)$ and $U_\mu(y)$ at different sites x, y of the lattice L . While the identification in the EK model is completely periodic [i.e., $U_\mu(x+v) = U_\mu(x)$], the identification in the TEK model is periodic up to a constant gauge transformation [i.e., $U_\mu(x+v) = \Omega_v^\dagger U_\mu(x) \Omega_v$]. The so-called twisting matrices³⁾ Ω_μ do not necessarily commute with each other but satisfy

$$\Omega_\mu \Omega_\nu = \Omega_\nu \Omega_\mu Z_{\mu\nu} \quad (1)$$

where $Z_{\mu\nu}$ are elements of the centre $Z(N)$ of the gauge group $SU(N)$. The reduction of the plaquettes to the unique site of the reduced theory then leads to the following partition function and action

$$Z(\beta) = \int \prod_{\rho=1}^4 dU_\rho \exp S_{\text{TEK}} \quad (2a)$$

$$S_{\text{TEK}} = N\beta \sum_{\mu \neq \nu} Z_{\mu\nu} \ln(U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger) \quad (2b)$$

where U_μ ($\mu = 1, 2, 3, 4$) are $SU(N)$ matrices and the $Z_{\mu\nu}$, defined above, are phases which can be parametrized in terms of integers $\eta_{\mu\nu}$ (defined modulo N) by

$$Z_{\mu\nu} = \exp \left(\frac{2i\pi}{N} \eta_{\mu\nu} \right), \quad \eta_{\mu\nu} = -\eta_{\nu\mu}. \quad (3)$$

In the weak coupling region (large $N\beta$), it is reasonable to assume that the integrals over U_ρ in (2a) will be dominated by the configurations U_μ which maximize the action S_{TEK} . Writing S_{TEK} as

$$S_{\text{TEK}} = N\beta \left[\sum_{\mu \neq \nu} \frac{1}{2} (z_{\mu\nu} U_\mu U_\nu - U_\nu U_\mu) (U_\mu^\dagger U_\nu^\dagger - z_{\mu\nu}^* U_\nu^\dagger U_\mu^\dagger) + d(d-1)N \right] \quad (4)$$

shows that the maximum is equal to $\beta d(d-1)N^2$ and is attained for $U_\mu = U_\mu^0$, so that

$$U_\mu^0 U_\nu^0 = z_{\mu\nu}^* U_\nu^0 U_\mu^0 \quad \mu, \nu = 1, \dots, 4. \quad (5)$$

It has also been shown⁸⁾ that the system of equations (5) can be satisfied only if the integers $\eta_{\mu\nu}$ [see Eq. (3)] obey

$$\frac{1}{4} \varepsilon_{\mu\nu\rho\sigma} \eta_{\mu\nu} \eta_{\rho\sigma} = 0 \mod N \quad (6)$$

Thus, the problem consists in constructing all the gauge inequivalent configurations of matrices U_μ satisfying (5) and (6). In an early paper³⁾, it was pointed out by 't Hooft that the equation (6) is invariant under the invertible transformation

$$\eta_{\mu\nu} \longrightarrow \eta'_{\mu\nu} = a_{\mu\rho} a_{\nu\sigma} \eta_{\rho\sigma} \quad (7)$$

for a belonging to $SL[4, Z(N)]$ [the set of 4×4 matrices with determinant one and coefficients in $Z(N)$]. Using a transformation of Eq. (7), one can put $\eta_{\mu\nu}$ in a canonical position; according to 't Hooft, the best we can do is

$$\eta_{\mu\nu} = \begin{pmatrix} 0 & 0 & \eta_{13} & 0 \\ 0 & 0 & \eta_{23} & \eta_{24} \\ -\eta_{13} & -\eta_{23} & 0 & 0 \\ 0 & -\eta_{24} & 0 & 0 \end{pmatrix}, \quad \eta_{13} \eta_{24} = 0 \mod N \quad (8)$$

In the following, we will only classify solutions of Eqs. (4) and (8). The solutions corresponding to the other (non-canonical) values of $\eta_{\mu\nu}$ can be constructed by using the transformation law of the U_μ^0 's under (7)

$$U_\mu^{0'} = \pi_\nu U_\nu^0 a_{\mu\nu} \quad (9)$$

So far, we have to solve in U_μ (for simplicity we do not put the 0 index in the construction) the system

$$\begin{aligned} U_1 U_3 &= \exp(-2i\pi \frac{\eta_{13}}{N}) U_3 U_1, & U_1 U_2 &= U_2 U_1 \\ U_2 U_4 &= \exp(-2i\pi \frac{\eta_{24}}{N}) U_4 U_2, & U_1 U_4 &= U_4 U_1 \\ U_2 U_3 &= \exp(-2i\pi \frac{\eta_{23}}{N}) U_3 U_2, & U_3 U_4 &= U_4 U_3 \end{aligned} \quad (10)$$

It is convenient to define the integers a, b, c, k, m, r in such a way that

$$\frac{\eta_{13}}{N} = \frac{a}{k}, \quad \frac{\eta_{24}}{N} = \frac{b}{m}, \quad \frac{\eta_{32} k m}{N} = \frac{c}{k} \quad (11)$$

where a, k are mutually prime, etc.; and to define the matrices above. $[A, B$ are matrices of dimension d^2 ; $\oplus(A, B, C, \dots, L)$ denote the usual direct sum of the matrices inside the parenthesis.]

$$Q_m(A) = \oplus (\mathbb{1}_d, A, A^2, \dots, A^{m-1}) \quad (12)$$

$$P_m(B) = \begin{pmatrix} 0 & \mathbb{1}_d & 0 & 0 & \dots & 0 \\ 0 & 0 & \mathbb{1}_d & 0 & \dots & 0 \\ 0 & 0 & \dots & \dots & \dots & \mathbb{1}_d \\ B & 0 & 0 & \dots & \dots & 0 \end{pmatrix}, \quad m^2 \text{ blocks} \quad (13)$$

These matrices generalize the 't Hooft twisting matrices⁴⁾; they coincide with the latter if $A = \exp(2i\pi/m)$ and $B = 1$ in Eqs. (12) and (13).

CONSTRUCTION OF U_1 AND U_3

Using the gauge invariance of the theory:

$$U_\mu \longmapsto \Omega U_\mu \Omega^\dagger, \quad \Omega \in SU(N) \quad (14)$$

we can choose U_1 to be a diagonal matrix with equal eigenvalues close together, [i.e., $U_1 = \oplus(\lambda_1 \mathbb{I}_{n_1}, \lambda_2 \mathbb{I}_{n_2}, \dots, \lambda_I \mathbb{I}_{n_I})$, $\lambda_i \neq \lambda_j$]; then preliminary calculations lead to the following form for U_1 and U_3 . Given a partition of N/k by integers d_i [i.e., $N = k(d_1 + d_2 + \dots + d_I)$] we have, up to an overall permutation of the blocks,

$$U_1 = \bigoplus_{i=1}^I (Q_i(z) \otimes \varphi_{k,i} \mathbb{I}_{d_i}) \quad , \quad \varphi_{k,i} \text{ are different phases} \quad (15)$$

$$U_3 = \bigoplus_{i=1}^I M_i, \quad M_i = \begin{pmatrix} 0 & A_i & 0 & \dots & 0 \\ 0 & 0 & B_i & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & & J_i \\ K_i & 0 & \dots & & 0 \end{pmatrix} \quad (16)$$

where $A_i, B_i, \dots, K_i$ are k unitary matrices of dimensions d_i^2 . At this stage, we can use the little group of U_1 [i.e., $\bigotimes_{i=1}^I SU(d_i)^k$] in order to simplify M_i ; the best we can do is [see (13) for notation]

$$M_i = P_k(D_i) \quad (17)$$

where $D_i (i=1, \dots, I)$ are I unitary matrices of dimension d_i^2 . The canonical forms (15), (16) and (17) fix the remaining gauge invariance to be $\bigotimes_{i=1}^I SU(d_i)$.

CONSTRUCTION OF U_2 AND U_4

From their commutation relations with the two matrices constructed above, the general form for U_2 and U_4 is the following:

$$U_4 = \bigoplus_{i=1}^I \mathbb{I}_k \otimes R_i \quad (18)$$

$$U_2 = \bigoplus_{i=1}^I (Q_k(z_{23}) \otimes S_i) \quad (19)$$

R_i and S_i are like D_i . In order to fulfil completely the equations (10), these must obey the system above.

$$D_i R_i = R_i D_i \quad (20a)$$

$$D_i S_i = Z_{23}^k S_i D_i \quad (20b)$$

$$R_i S_i = Z_{24} S_i R_i \quad (20c)$$

Here again, the remaining gauge freedom $[\otimes_i SU(d_i)]$ allows us to put the matrices R_i, D_i, S_i in a canonical position; for instance, we can use it to put the R 's and D 's simultaneously diagonal, providing a trivial solution to Eq. (20a).

Then Eq. (20c) is similar to Eq. (10) in U_1, U_3 for which we gave the most general solutions (15), (16) and (17). Using a decomposition such as (15) for R_i , we have shown that the system (20) is compatible only if

$$d_i = m r \quad i = 1, \dots, I \quad (21)$$

In fact, the d_i 's can be multiples of mr , but these cases are included in (21) up to an overall permutation matrix. In the appendix, we show that the condition (21) is always compatible with the definition of the d_i 's. Then the solution of Eq. (20) is

$$D_i = \varphi_{3,i} (Q_m(Z_{32}^k) \otimes Q_n(Z_{32}^{km})) \quad (22a)$$

$$S_i = P_m(P_n(\varphi_{2,i})) \quad (22b)$$

$$R_i = \varphi_{4,i} (Q_m(Z_{42}) \otimes 1_n) \quad (22c)$$

where $\varphi_{\nu,i}$ are arbitrary phases. We have chosen the convention that the P_m and Q_m matrices are, without argument, the 't Hooft matrices.

SUMMARY AND COUNTING THE PHASES

As seen above, the four matrices U_μ ($\mu=1,\dots,4$) can be written as direct sums of $J(\equiv N/kmr)$ block matrices of dimension $(kmr)^2$, each block depending on an arbitrary phase. In the gauge we used, one can write the saddle points U_μ^0 as

$$U_1^0 = \bigoplus_{i=1}^J \left\{ \varphi_{1,i} (Q_k(Z_{13}) \otimes \mathbb{1}_{mr}) \right\} \quad (23a)$$

$$U_3^0 = \bigoplus_{i=1}^J \left\{ P_k(\varphi_{3,i} Q_m(Z_{32}^k) \otimes Q_r(Z_{32}^{km})) \right\} \quad (23b)$$

$$U_4^0 = \bigoplus_{i=1}^J \left\{ \mathbb{1}_k \otimes \varphi_{4,i} (Q_m(Z_{42}) \otimes \mathbb{1}_r) \right\} \quad (23c)$$

$$U_2^0 = \bigoplus_{i=1}^J \left\{ Q_k(Z_{23}) \otimes P_m(P_r(\varphi_{2,i})) \right\} \quad (23d)$$

So far, the solutions depend on $4J-4$ arbitrary phases, (four phases being fixed by the conditions of the unit determinant); if $J > 1$, some $SU(N)$ generators commute with all the U_μ^0 's; these generators are those of the form $G_J \otimes \mathbb{1}_{kmr}$ [with G_J a diagonal generator of $SU(J)$.]

This fact is rather undesirable because, as discussed extensively in Ref. 7), the presence of generators commuting with all the U_μ 's (zero modes) prevents the partition function $Z(\beta)$ from agreeing with the Wilson one in the large N , large β region. One way to escape the zero modes is to choose the numbers $n_{\mu\nu}$ in such a way that $J = 1$ (i.e., $N = kmr$). In this case, the matrices U_μ are irreducible in $SU(N)$ and unique [up to a gauge transformation and multiplication by an element of the discrete centre $Z(N)$ of $SU(N)$]; moreover, we have verified that the quadratic form of the fluctuations around the saddle point is not degenerate.

A consequence of this result is that all the quadratic integrations can be performed in the saddle point expansion; as an example, the first few terms of the free energy $F(= \ln Z)$ are

$$F_{\text{TEK}}(\beta) = A(d-1)N^2\beta + (d-1)(N^2-1)\frac{\ln\beta}{2} + O\left(\frac{1}{\beta}\right) \quad (24)$$

and agree with their analogue in the Wilson theory in the large N limit. A classification of the acceptable twists is given in the appendix. The above argument pleads in favour of the non-symmetry breaking of the famous Z_N^d symmetry of the TEK action (2b), which, according to Eguchi and Kawai¹⁾, must not be broken in order for the model to be valid.

Despite the fact that we do not have a direct proof that this symmetry is not spontaneously broken in the weak coupling region, we do have a second argument which strengthens the one given above. It is based on the high cyclicity of the matrices U_μ^0 in (23) when $J = 1$ and when k and m remain unbounded in the large N limit (e.g., $N = \eta_{13} \cdot \eta_{24}$ with η_{13}, η_{24} two arbitrary large primes). Indeed, as pointed out in Ref. 1), the necessary condition for the reduced model to be valid is that for any open loop on the original (non-reduced) lattice; the expectation value of the corresponding loop in the reduced model has to be zero. In the weak coupling region where the saddle point dominates the measure, we expect

$$\langle \text{tr} U_\mu U_\nu U_\rho^* \dots U_\sigma \rangle \doteq \text{tr} (U_\mu^0)^{a_1} \dots (U_\sigma^0)^{a_d} \quad (25)$$

where at least one of the a_i 's is different from zero in order for the loop to be open.

However, from the structure of the irreducible matrices Ω_μ (23), with $J = 1$ and k, m unbounded, we know that the trace in the right-hand side of (25) is zero if at least one of the exponents is finite. The only non-trivial loops for which the trace (25) is zero are loops which run a large number of times along a given direction; these loops are clearly closed due to the periodicity of the original lattice so that, in the large N limit, all the local but open loops have a zero expectation value.

THE STRONG COUPLING BEHAVIOUR OF THE TWISTED REDUCED MODELS

We know from Eguchi-Kawai's analysis that in the large N limit there is a one-to-one correspondence between the leading terms in the strong coupling expansion of the EK partition function and those appearing in the expansion of Wilson's partition function

$$Z_W = \int \prod_{\text{links}} dU_\ell \exp \beta N \sum_{\text{plaq}} \text{tr} (U_p + U_p^\dagger) \quad (26)$$

The same relationship holds whenever we compute a gauge invariant expectation value:

$$\frac{1}{N} \langle \text{tr} \prod_{\text{loop}} U_\ell \rangle = W(\text{loop}) \quad (27)$$

Let us now examine the detailed structure of the leading terms in any expression of the form

$$\int \prod_{\text{links}} dU_\ell \left(\exp \beta N \sum_{\text{plaq}} \text{tr} (U_p + U_p^\dagger) \right) \frac{1}{N} \text{tr} \prod_{\text{loop}} U_\ell \quad (28)$$

The large N contributions come from terms in the expansion containing as many powers of U_ℓ as of $U_\ell^\dagger$ for each link. The general structure of such terms is

$$\int dU_\ell \frac{1}{N} \text{tr} \prod_{\text{loop}} U_\ell \prod_{\text{plaq}} (\text{tr} U_p)^{\alpha_p} (\text{tr} U_p^\dagger)^{\beta_p} (\beta N)^{\sum_p \alpha_p + \beta_p} \quad (29)$$

with the condition that the content in U_p and $U_p^\dagger$ is such as is needed in order to form a minimal surface of the loop plus a closed surface.

As an immediate consequence, the leading β behaviour of a Wilson loop expectation value is $W(\text{loop}) \sim \beta^{NP}$, where NP is the number of plaquettes in a minimal surface.

The corresponding terms in the reduced models have the structure

$$\int dU_1 \frac{1}{N} \epsilon (U_\mu^\alpha U_\nu^\beta \dots U_\rho^\gamma U_\sigma^\delta) \prod_{\mu\nu} (\epsilon Z_{\mu\nu} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger)^{\alpha_{\mu\nu}} (\beta N)^{\sum_{\mu\nu} \alpha_{\mu\nu}} \quad (30)$$

where each U_ℓ is replaced by U_j oriented in the same direction and each U_p is replaced by $Z_{\mu\nu} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger$, (μ, ν) defining the (oriented) plane on which U_p lies.

Let us now parametrize the minimal surface in terms of the number of plaquettes belonging to it, which lie on each oriented plane $NP_{\mu\nu}$

$$NP = \sum_{\substack{\mu, \nu=1 \\ \mu \neq \nu}}^d NP_{\mu\nu} \quad (31)$$

Therefore, the dependence of each leading term in the strong coupling large N expansion on the twisted coefficients $Z_{\mu\nu}$ is of the form

$$\prod_{\mu\nu} (Z_{\mu\nu}^*)^{NP_{\mu\nu}} \prod_{\mu\nu} (Z_{\mu\nu})^{\alpha'_{\mu\nu}} \quad (32)$$

where $\alpha'_{\mu\nu}$ are such that they correspond to the contribution of a closed surface. This is accomplished by the condition $\alpha_{\mu\nu} = \alpha_{\nu\mu}$. We must now remember a fundamental property of the coefficients $Z_{\mu\nu}$, i.e., the fact that

$$Z_{\nu\mu} = Z_{\mu\nu}^* \quad (33)$$

As a consequence, the contribution of each leading term has the universal $Z_{\mu\nu}$ dependence

$$\prod_{\mu, \nu} (Z_{\mu\nu}^*)^{NP_{\mu\nu}} \quad (34)$$

In particular, any closed surface has no $Z_{\mu\nu}$ dependence.

The influence of these results on the strong coupling behaviour of the twisted models in the $N \rightarrow \infty$ limit is twofold:

- i) the partition function and the free energy, being sums over closed surfaces, do not depend on $Z_{\mu\nu}$

$$Z_{TEK}(\beta) = Z_W(\beta)$$

- ii) the dependence of the Wilson loop expectation values is simply parametrized by

$$W(loop)_{TEK} = \prod_{\substack{\mu, \nu=1 \\ \mu \neq \nu}}^d (Z_{\mu\nu}^*)^{N P_{\mu\nu}} W(loop)_W$$

If there is no spontaneous breakdown of the $U(1)^d$ symmetry in the reduced model, the two relationships that we have found must also be preserved for large β , and therefore provide the necessary connection between the twisted model expectations and the Wilson model results in the physically interesting region.

Incidentally, it is trivial to verify that the second of our equations is a consistent solution of the twisted Makeenko-Migdal equations connecting the Wilson loop expectation values in the TEK model in the phases where the symmetry is unbroken.

Note Added

After completion of this work, we were informed by G. Bhanot that general twisted models have also been studied by A. Gonzalez-Arroyo and M. Okawa⁹⁾, whose results agree with some of our conclusions.

APPENDIX

With the definition (11), we will show that

$$\frac{N}{k m r} \geq 1 \quad (A1)$$

We will use the usual θ function defined by

$$\begin{aligned} \theta(x) &= 0 & \text{if } x \leq 0 \\ \theta(x) &= 1 & \text{if } x > 0 \end{aligned} \quad (A2)$$

Then, decomposing N and $\eta_{\mu\nu}$ into products of prime

$$N = \prod_i p_i^{\alpha_i} \quad (A3)$$

$$\eta_{13} = x \prod_i p_i^{\beta_i}, \quad \eta_{24} = y \prod_i p_i^{\gamma_i}, \quad \eta_{32} = z \prod_i p_i^{\delta_i} \quad (A4)$$

$$\beta_i + \gamma_i \geq \alpha_i, \quad \forall i \quad (A5)$$

and considering an arbitrary factor, which we shall call p , we have respectively

$$\begin{aligned} k &= p^{\bar{\alpha}} & \bar{\alpha} &= (\alpha - \beta) \theta(\alpha - \beta) \\ m &= p^{\bar{\gamma}} & \bar{\gamma} &= (\alpha - \gamma) \theta(\alpha - \gamma) \\ r &= p^{\bar{\delta}} & \bar{\delta} &= (\alpha - \delta - \bar{\beta} - \bar{\gamma}) \theta(\alpha - \delta - \bar{\beta} - \bar{\gamma}) \end{aligned} \quad (A6)$$

Knowing (A5), it is straightforward to check the inequalities above which guarantee (A1) to be true.

$$\begin{aligned} \alpha - \bar{\beta} - \bar{\gamma} &\geq 0 \\ \alpha - \bar{\beta} - \bar{\gamma} - \bar{\delta} &\geq 0 \end{aligned} \quad (A7)$$

The inequality $N = k m r$ holds iff $\alpha = \bar{\beta} + \bar{\gamma} + \bar{\delta}$ and leads to the following classification of the cases "without zero modes":

<u>Cases</u>		<u>Conditions</u>
$\alpha - \bar{\beta} - \bar{\gamma} > \delta$		$\delta = 0$
$\alpha - \bar{\beta} - \bar{\gamma} \leq \delta$	$\beta, \gamma < \alpha$ $\gamma < \alpha \leq \beta$ $\beta < \alpha \leq \gamma$ $\alpha \leq \beta, \gamma$	$\alpha = \beta + \gamma$ $\gamma = 0$ $\beta = 0$ $\alpha = 0$

Table 1

It is clear that each of the p_i in (A3) has to obey one of the conditions of Table 1. It can also be noticed that the number $J = N/kmr$ agrees with the results of Ref. 6) (Abelian case; U_μ are arbitrary diagonal matrices, thus $k = m = r = 1$) and of Ref. 7).

This number is, however, invariant under the transformation (9) and consequently is independent on the canonical position.

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