GAUGED SEVEN-DIMENSIONAL SUPERGRAVITY

P.K. Townsend
Laboratoire de Physique Théorique
Ecole Normale Supérieure, Paris

and

P. van Nieuwenhuizen +)
CERN - Geneva

ABSTRACT

We construct Yang-Mills theory and simple supergravity in seven dimensions. We gauge the rigid SU(2) symmetry of the latter. The potential for the scalar field $\phi$ is of the form $\exp\phi$ and has no extremum. Possible improvement due to a "topological mass term" is discussed.

*) Laboratoire propre du Centre National de la Recherche Scientifique, associé à l'Ecole Normale Supérieure et à l'Université de Paris Sud. Postal address: 24, rue Lhomond, 75231 Paris Cedex 05, France.

+ On leave from the Institute of Theoretical Physics, SUNY, Stony Brook, NY 117940, U.S.A.

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1. - INTRODUCTION

Extended supergravity theories in four space-time dimensions \( d = 4 \) have a rigid chiral U\((N)\) invariance of which the O\((N)\) subgroup may be gauged by the vector fields in the supergravity supermultiplet. In \( d = 5 \) the rigid symmetry group is USp\((2N)\). For the (simple) \( N = 2 \) theory it is known that the U\((1)\) subgroup of USp\((2) \times \text{SU}(2)\) may be gauged \(^1\), but for \( N > 2 \) it is not known whether a gauged model exists. For dimension greater than 5 no gauged supergravity model is known. As gauging always requires the introduction of a cosmological constant, the question of whether gauged supergravity theories exist in higher dimensions is intimately related to the question of whether supersymmetry allows the existence of a cosmological constant. It is known that supersymmetry does not allow a cosmological constant in \( d = 11 \) \(^2\) and it is trivial to verify that this is also true of \( d = 10 \) and \( d = 9 \) simple supergravity because no gravitino mass term can be constructed. Interest in gauged supergravity models has recently been revived as a result of Kaluza-Klein ideas. It appears that the gauged \( N = 8 \) supergravity theory can be regarded as an effective four-dimensional theory arising from spontaneous compactification of 11-dimensional supergravity on the seven-sphere \(^3\). This raises the question of whether all gauged supergravity theories can be obtained by spontaneous compactification and possibly subsequent truncation. A test case for this conjecture is the \( SU(2) \times SU(2) \) gauged \( N = 4 \) supergravity of Freedman and Schwarz (FS) \(^4\). This is inequivalent to the standard gauged O\((4)\) model \(^5\) or its recent extension \(^6\) and cannot be obtained by truncation of a gauged \( N \geq 5 \) supergravity theory. One might suspect that the FS model could be obtained by spontaneous compactification of \( d = 10 \) supergravity and this was one of the motivations of a recent study of \( d = 10 \) supergravity \(^7\). It was found that \( d = 10 \) supergravity can spontaneously compactify to \( d = 7 \) on \( S^3 \), but not to \( d = 4 \) on \( S^3 \times S^3 \). Thus, although the higher dimensional origin of the FS model is left open a new question is raised. If \( d = 10 \) can spontaneously compactify to \( d = 7 \), does this imply the existence of a gauged simple \( (N = 2) \) supergravity theory in \( d = 7 \) ? The extended \( (N = 4) \) supergravity theory in \( d = 7 \) has been constructed \(^8\) and the question of whether it can be gauged was raised. We show in this article that the (simple) \( N = 2 \) supergravity theory in \( d = 7 \) can be gauged, the gauge group being \( \text{SU}(2); \) Whether the \( N = 4 \) theory can be gauged remains an open question.

Our results bear a striking resemblance to those of the \( d = 4 \) FS model, principally in that the scalar potential has no extremum, and if one fixes the scalar field arbitrarily to some constant then one finds a constant term in the spinor supersymmetry transformation law. We suspect that the "unorthodox"
potential of this model is related to the fact that in the solution describing spontaneous compactification of $d=10$ supergravity on $S^3$ all $d=7$ supersymmetries were broken.

As for $d=10$ supergravity the field content of $d=7$ supergravity includes an antisymmetric tensor gauge field, which may come in one of two dual forms. In $d=7$ we have a choice between an $A_{\mu \nu}$ and an $A_{\mu \nu \rho}$. In general, one expects that if two antisymmetric tensor fields occur, e.g., $A_{\mu \nu}$ and $A_{\mu}$, one will require modified field strengths of the type $(F_{\mu \nu \rho} + F_{\mu \nu \rho} A_{\mu})$. These occur on dimensional reduction $^9$. Even if they do not occur in a given theory it may be that a duality transformation will produce them $^{10}$. Conversely, if they do occur a duality transformation may remove them. In particular, this is the case in $d=7$. If one starts with an $A_{\mu \nu}$ field, one finds that a modified field strength is required, whereas it is not required in the dual theory. For this reason it is technically simpler to choose the $A_{\mu \nu \rho}$ form of the theory, and we shall present our results in terms of $A_{\mu \nu \rho}$. We mention that this trick can also be used to simplify results of the Maxwell-Einstein coupling in $d=10$ $^{11}$ or its extension to the Yang-Mills-Einstein coupling $^{12}$.

The ungauged model presented here is presumably equivalent to a duality transformed truncation of the model of Ref. 8), but it is most easily constructed directly by Noether coupling. We have streamlined a few of these techniques. The coupling to matter is presumably straightforward in principle, but complicated in practice by non-polynomial scalar interactions. We present here only the flat space super Yang-Mills theory in $d=7$.

In the $A_{\mu \nu \rho}$ form of gauged $d=7$ supergravity there is a possible modification that is not possible in the $A_{\mu \nu}$ form of the theory. One can add to the action a "topological mass term" $^{13}$

\[ \varepsilon_{\kappa \lambda \mu \nu} \left( \sigma - \tau \right) F_{\kappa \lambda \mu \nu} A_{\rho - \tau} \]

with some interesting results.

In Section 5 we finally discuss some Kaluza-Klein ideas in connection with our work.
Those of our conventions not mentioned explicitly are
\[ \{ \Gamma^\mu, \Gamma^\nu \} = 2 \delta^\mu_\nu, \quad \epsilon = \det e^m_\mu, \quad \epsilon_i^j \epsilon_j^k = \delta_i^k, \quad e^i_\mu = e_i^k e^k_\mu \]
\[ R = R^m_\mu e^{m\mu}, \quad R_{\mu\nu} = R^m_{\mu\nu\mu n} e^{m\nu} \]
\[ R^m_{\mu\nu\mu n} = \partial_\mu \omega_{\nu\mu n} + \omega_{\mu n} \partial_\nu \omega_{\nu\mu n} - \omega_{\mu n} \omega_{\nu\mu n} \]
\[ F_{\mu_1 \ldots \mu_n} = n \frac{1}{2} \delta_{\mu_1 \ldots \mu_n} \Gamma^{\mu_1 \ldots \mu_n} = \Gamma^\mu \ldots \Gamma^\mu \]

2. - **SEVEN-DIMENSIONAL SIMPLE SUPERGRAVITY**

The fields of the model are \( A^a_\mu \), the gravitino \( \psi^a_\mu \), a triplet of vectors \( A^{a}_{\mu \nu} \), a scalar \( \phi \), a spinor \( \lambda^a_\mu \), and a gauge antisymmetric tensor \( A_{\mu \nu \rho} \). The (rigid) symmetry group is \( \text{USp}(2) \cong \text{SU}(2) \) and the spinors are eight-component complex \( \text{SU}(2) \) Majorana spinors. One can choose the charge-conjugation matrix to be unity so, for example, using a northwest contraction convention,
\[ \lambda^a = \epsilon^{ab} \lambda^b \]
where the bar indicates the Dirac-conjugate spinor. We first give the result for the Lagrangian and then comment on its derivation.

\[ \mathcal{L} (\text{ungauged}) = -\frac{i}{2} e R - \frac{i}{2} \Gamma^\mu \Gamma^{\nu \rho} D_\mu \psi^\nu - \frac{i}{4} \bar{\psi} \sigma^{-1} \psi^2 \]
\[ - \frac{e}{48} \frac{1}{\sqrt{2}} F_{\mu \nu \rho} \Gamma^\mu \psi^\nu \psi^\rho - \frac{e}{8} \frac{1}{\sqrt{2}} \bar{\psi} \sigma^2 (F_{\mu \nu \rho \sigma})^2 \]
\[ - \frac{e}{8} \frac{1}{\sqrt{2}} \Gamma^\mu \psi^{\mu \nu \rho \sigma} (\psi^\nu \psi^\rho \psi^\sigma) - \frac{e}{2} (\bar{\psi} \sigma^2 \psi)^2 \]
\[ - \frac{ie}{\sqrt{2}} \Gamma^\mu \psi^{\mu \nu \rho \sigma} (\psi^\nu \psi^\rho \psi^\sigma) - \frac{ie}{\sqrt{2}} \bar{\psi} \sigma^2 \psi \]
\[ + \frac{e}{24 \sqrt{10}} \left( \Gamma^\mu \Gamma^\nu \Gamma^\rho \psi^\delta \psi^\mu \psi^\nu \psi^\rho \psi^\sigma \right) F_{\psi \psi \psi \psi} - \frac{ie}{24 \sqrt{10}} \left( \psi^\mu \psi^{\mu \nu \rho \sigma} \psi^\nu \psi^\rho \psi^\sigma \right) F_{\psi \psi \psi \psi} \]
\[ + \frac{1}{2} \frac{1}{\sqrt{2}} \bar{\psi} \sigma^2 \psi \]
\[ - \frac{3ie}{20 \sqrt{10}} \left( \Gamma^\mu \Gamma^\nu \psi^\mu \psi^\nu \psi^\sigma \psi^\rho \psi^\sigma \psi^\rho \psi^\delta \psi^\mu \psi^\nu \psi^\rho \psi^\sigma \right) F_{\psi \psi \psi \psi} + \frac{ie}{48 \sqrt{2}} \bar{\psi} \sigma^2 \psi \]
\[ + \frac{1}{160 \sqrt{2}} \frac{1}{\sqrt{2}} \bar{\psi} \sigma^2 \psi \]
\[ \left( \bar{\psi} \sigma^2 \psi \right)^2 + \frac{1}{160 \sqrt{2}} \frac{1}{\sqrt{2}} \bar{\psi} \sigma^2 \psi \]

(2)
The quantity $e$ is $\det e_{\mu}^a$, $R$ is the Ricci scalar, and the quantity $\sigma$ is the following function of $\phi$

$$\sigma = e^{\exp\left(-\frac{i}{\sqrt{3}} \, \phi\right)}$$

We have not determined the four-fermion terms. This is straightforward in principle but very tedious in practice. The transformation rules are (again, not including three-fermion terms),

$$\delta \phi = \frac{i}{2} \bar{\psi}^i \gamma^i A^i, \quad \delta e_{\mu}^a = \bar{\psi}^i \gamma_\mu e_i$$

$$\delta \lambda_i = \frac{i}{2} \bar{\psi}^i \gamma^i \lambda_i$$

$$\delta \psi_i = \nabla \psi_i^c + \frac{i}{4} \bar{\psi}^i \gamma^i \lambda_c \gamma_\mu e_{\mu i} + \frac{i}{4\sqrt{10}} \left( \gamma^{\nu} \gamma_\mu - 8 \gamma^\nu \gamma_\mu \right) F_{\nu\mu}^c \gamma^i \gamma^j \psi_{ij} e_i e_j$$

$$\delta A_{\mu}^c = \left[ \frac{2}{\sqrt{10}} \bar{\psi}^i \gamma_\mu \gamma^i \lambda_c e_i + \frac{1}{\sqrt{10}} \bar{\psi}^i \gamma^i \lambda_c \gamma_{\nu} e_{\nu i} \right] F_{\mu\nu}^c$$

$$= \left[ \frac{1}{\sqrt{10}} \left( \bar{\psi}^i \gamma_\mu \gamma^i \lambda_c e_i - \frac{1}{2} \bar{\psi}^i \gamma^i \lambda_c \gamma_{\nu} e_{\nu i} \right) - \frac{i}{\sqrt{10}} \left( \frac{i}{2} \bar{\psi}^i \gamma_\mu \gamma^i \lambda_c e_i - \frac{1}{2} \bar{\psi}^i \gamma^i \lambda_c \gamma_{\nu} e_{\nu i} \right) \right] F_{\mu\nu}^c$$

Notice that the Lagrangian has the non-compact rigid symmetry

$$\phi \rightarrow \phi + c, \quad c \text{ a constant}$$

$$A_{\mu} \rightarrow A_{\mu} \exp\left(\frac{c}{\sqrt{3}}\right)$$

$$A_{\mu\nu} \rightarrow A_{\mu\nu} \exp\left(\frac{-ic}{\sqrt{3}}\right)$$

which is analogous to that of $d = 10$ supergravity and which determines the non-polynomial interactions of $\phi$. In accord with the way scalars usually occur in supergravity theories one can consider $\phi$ to parametrize the "coset" $G/H$ where $H$ is trivial and $G$ is $\text{U}(1)$ [the non-compact covering group of $\text{U}(1)$].

Another interesting feature is the occurrence of the term

$$F_{\mu\nu\rho\sigma} \, F_{\kappa\lambda\iota\delta} \, A_{\tau\chi}^c \, e_{\mu\nu\rho\sigma - \kappa\lambda\iota\delta}$$

which is an analogue of the $\tilde{F}_{FA}$ term of $d = 11$ supergravity.
For the construction we have used standard Noether methods. We have found useful the following lemma which is valid for the construction of supergravity theories in any space-time dimension. For the normalization of gravitini and spinor kinetic terms as in (2), and for the normalization,

$$-\frac{1}{2n!} \left( F_{\mu_1 \cdots \mu_n} \right)^2, \quad F_{\mu_1 \cdots \mu_n} = n \partial_{[\mu_1} A_{\mu_2 \cdots \mu_n]}$$

of the kinetic term of a \((n-1)\)th rank antisymmetric tensor gauge field \(A_{\mu_1 \cdots \mu_{n-1}}\), the supersymmetry transformations of these fields into each other in \(d\) dimensions are

$$\delta \psi_\mu = a \left( \Gamma_{\mu \nu} \psi_{\nu} - \frac{n}{n-1} \delta \lambda \Gamma_{\mu} \Gamma_{\nu} \psi_{\nu} \right), \quad F_{\mu_1 \cdots \mu_n} \in$$

$$\delta A_{\mu_1 \cdots \mu_{n-1}} = (d-2) a n! \left( F_{\mu_1 \nu_2 \cdots \nu_{n-1}} \epsilon \right) + \epsilon \left( \Gamma^\mu_{\mu_1 \cdots \mu_{n-1}} \right)$$

$$\delta \lambda = \frac{p}{n!} \Gamma^\mu_{\mu_1 \cdots \mu_n} \epsilon F_{\mu_1 \cdots \mu_n}$$

One then deduces the Noether coupling terms to be

$$L^N_{(\text{Noether})} = \frac{a(\frac{2}{d-2})}{2} \left[ \frac{1}{n-1} \bar{\psi}_{\mu_1 \cdots \mu_n} \gamma^\nu \psi_{\nu} F_{\mu_1 \cdots \mu_n} + \right.$$

$$\left. + n \bar{\psi}_{\mu_1 \cdots \mu_n} \psi_{\nu} F_{\mu_1 \nu \mu_2 \cdots \mu_n} \right] + \frac{p}{n!} \left( \bar{\psi}_{\mu} \gamma^\cdash_{\mu_1 \cdots \mu_n} \psi \right) F_{\mu_1 \cdots \mu_n}$$

The constant \(a\) can be fixed from the supersymmetry algebra.

3. - GAUGED \(d = 7\) SUPERGRAVITY

To gauge the model of (2) we begin in the standard way by adding a cosmological constant, mass terms for \(\bar{\psi}_\mu\) and \(\lambda\), and new transformations for \(\psi_\mu\) and \(\lambda\) of the form \(\delta \psi_\mu = \gamma_\mu \epsilon, \delta \lambda = \epsilon\). When one comes to variations of the form \(\epsilon \phi F\) one discovers, usually, that they do not yet cancel, but that they can be cancelled by introducing gauge coupling; this yields an additional \(\epsilon \phi F\) term coming from the commutator of \((D_\mu B_\nu)\) in the variation of the gravitino action. Obviously this is only possible for vector fields and not for antisymmetric tensors. This is one reason why the presence of antisymmetric tensor fields can make impossible the addition of a cosmological constant and thereby
the construction of a gauged supergravity model. In our case, however, the \( \epsilon \psi F_\mu \), 
\( \epsilon A F_\mu \), and \( \epsilon A F_2 \) terms cancel by themselves. The cancellation of the \( \epsilon \psi F_\mu \) terms 
requires, as expected, the gauging of SU(2). The final result is that the action of (2) must be covariantized with respect to SU(2) as

\[
\mathcal{L}_\rho^\mu = \mathcal{L}_\rho + i \epsilon A \rho i j \chi^\mu j - \rho \rightarrow \rho
\]

where \( \alpha \) is the gauge coupling constant. In addition to covariantization one must add to the action the terms

\[
\mathcal{L}' = \frac{m^2}{2} \epsilon_{\mu
u} + \frac{e}{2} \epsilon_{\mu
u} \bar{\psi} \Gamma^\mu \psi \nu + \bar{\psi} \Gamma^\mu \lambda \psi \nu + \frac{e}{2} \epsilon_{\mu
u} \lambda
\]

and add to the transformation rules the terms

\[
\delta^' \psi_{\mu i} = m \xi_{\mu i} + \alpha A_{\mu i} \xi_{\mu j}
\]

\[
\delta^' \lambda_{\mu i} = -\sqrt{5} m \epsilon_{\mu i}
\]

The quantity \( m \) is given in terms of \( \alpha \) and \( \phi \) as

\[
m = -\frac{\alpha}{\sqrt{5}} \exp \left( \frac{i}{\sqrt{5}} \phi \right)
\]

There is a de Sitter supergroup in \( d = 7 \), namely \( \text{OSp}(6,2|2) \) which would be a 
candidate for the symmetry group of the ground state. But it is not clear how one finds the 
ground state. Our results are rather similar to those of the FS model; the principal difference 
appears to be that a spinor mass term \( \lambda \lambda \) is required in our case.

For the gauged, as for the ungauged, model there is a choice between an 
\( A_{\mu
u} \) and an \( A_{\mu\nu\rho} \) field, the two being related by duality. However, the latter 
allows an additional term of the form given in (1) to be added along with the 
other mass terms in (10). Once such a term is added it is no longer possible to 
perform a duality transformation to exchange \( A_{\mu\nu\rho} \) for \( A_{\mu\nu} \). We have determined 
that a consistent modification of the results (10) and (11), up to terms in \( \delta \mathcal{L} \) which 
vanish for \( \phi = \text{constant} \), is
\[ L' = \text{with top. mass term} = 60 m^2 - 10 (m + \lambda \sigma^4) \]
\[ + \frac{5m}{2} \gamma^i \gamma^j \sigma \sigma' \gamma^i \gamma^j \sigma \sigma' \gamma^i \gamma^j \lambda_i \]
\[ + \left( \frac{3m}{2} + 6 \lambda \sigma^4 \right) \lambda_i \lambda_j - \frac{3}{36} \epsilon^{\mu \nu \rho \sigma} \tau \gamma_\mu \lambda_i \gamma_\nu \lambda_j \gamma_\rho \lambda_k \gamma_\sigma \lambda_l \]
\[ \epsilon^{\mu \nu \rho \sigma} A_{\mu \nu \rho \sigma} \]
\[ (13) \]

\[ \delta' \gamma_{\mu i} = m \gamma_{\mu i} e_i + \lambda A_{\mu i} \delta' e_i \]
\[ \delta' \lambda_i = - \frac{3}{36} (m + \lambda \sigma^4) e_i \]
\[ (14) \]

The \( \phi \) dependence of \( m \) is now determined to be

\[ m = - \frac{3}{5} i \lambda \sigma^4 - \frac{\lambda}{5 \sqrt{2}} \sigma^{-1} \]
\[ (15) \]

Supersymmetry requires that \( \lambda \) be a constant (and not a function of \( \phi \)) which is a useful check, as this is required also for gauge invariance. An interesting feature of (13) is that for \( \lambda \neq 0 \) the scalar potential has an extremum. We have not checked the cancellation of terms in \( \delta \bar{L} \) arising from this modification that contain \( \delta d \phi \) and it is not yet certain that (13) is a viable alternative to (10). We intend to return to this point in the future.

4. \( d = 7 \) Matter

There is only one "matter" theory in \( d = 7 \); that which reduces to the \( N = 4 \) super-Yang-Mills theory in \( d = 4 \). We expect that it can be coupled to the \( d = 7 \) simple supergravity model with results similar to those in \( d = 10 \) \((11), (12)\). We give here only the flat space result for \( d = 7 \) super-Yang-Mills theory. The fields are a triplet of scalars \( L_1^i \), a doublet of spinors \( \chi_i \), and a Yang-Mills gauge field \( B_{\mu} \), all in the adjoint representation of a gauge group \( G \). We denote the structure constants of \( G \) by \( f_{IJK} \). The Lagrangian is...
\[ \mathcal{L} = -\frac{1}{4} (G_{\mu\nu}^I)^2 - \frac{i}{2} \bar{\psi} \gamma^\mu \lambda_i^\Gamma A^\mu_i - \frac{i}{2} \left[ (D^\mu A_i^\Gamma) \right]^2 + \frac{i}{\sqrt{2}} \bar{\psi} \gamma^\mu \lambda_j^I \sigma_{jk}^I \epsilon_{kl} L_i^k L_j^l + \frac{1}{2} \bar{\psi} \gamma^\mu \lambda_i^I \epsilon_{jk} L_k^j L_l^l \]

(16)

and the transformation laws are

\[ \delta B^I_{\mu} = -\frac{1}{2} \epsilon^I_{\mu} \Gamma^\lambda_i \lambda_i^I \]
\[ \delta \lambda_i^I = \frac{i}{4} \Gamma^\nu_{\mu\nu} G_{\mu\nu}^I \epsilon_i^I + \frac{i}{\sqrt{2}} \left( \bar{\psi} L_i^j \gamma^\mu \right) \epsilon_j^I + \frac{1}{2} \epsilon_{\mu} L_i^j \gamma^\mu \epsilon_i^I \]
\[ \delta B^I_{\mu} = -\frac{1}{2} \epsilon^I_{\mu} \Gamma^\lambda_i \lambda_i^I \]

(17)

The coupling constant \( g \) occurs with the following normalization in \( G_{\mu\nu}^I \):

\[ G_{\mu\nu}^I = \partial_\mu B^I_{\nu} - \partial_\nu B^I_{\mu} + \frac{1}{2} \Gamma^I_{\mu\nu} B^I_{\rho} \gamma_{\rho} \]

(18)

An interesting question that has not yet been investigated is whether super-Yang-Mills theories can be coupled to gauged supergravity for arbitrary group \( G \) and arbitrary coupling constant, and whether the gauge symmetry is or is not spontaneously broken for non-zero cosmological constant. Compactification of \( d = 10 \) supergravity to \( d = 7 \) yields \( d = 7 \) supergravity coupled to three matter multiplets. Compactification on \( S^3 \) should yield an \( SU(2) \times SU(2) \) gauge theory. One \( SU(2) \) may be identified with the gauge group of gauged \( d = 7 \) supergravity, the other with a \( SU(2) \) matter gauge theory. The Kaluza-Klein results of Ref. 7) would suggest that only one linear combination of these \( SU(2)'s \) can remain unbroken.
5. COMMENTS

It is instructive to compare our results for the ungauged \( d = 7 \) theory with those for \( d = 10 \). To do this we set \( A_{\mu}^{j} = 0 \), which is consistent with its field equation; we perform a duality transformation to convert \( F_{\mu \nu \rho \sigma} \) into \( F_{\mu \nu \rho} \); and we make a Weyl rescaling such that the \( \phi \) kinetic term is cancelled. The bosonic part of the Lagrangian then takes the form

\[
\mathcal{L} = -\frac{1}{2} \epsilon \chi^{\alpha} \mathcal{R} - \frac{1}{12} \epsilon \chi^{-\alpha} (F_{\mu \nu \rho})^{2}
\]

(19)

with \( \chi \) some power of \( \exp(\phi) \), and \( \alpha \) a constant not equal to one. The bosonic action for \( d = 10 \) supergravity is of the same form but with \( \alpha = 1 \). This difference in \( \alpha \) is crucial to the question of whether the field equations allow spontaneous compactification on \( S^{3} \). The \( A_{\mu \nu} \) equation can be solved by

\[
\chi^{-\alpha}(\gamma) F_{\alpha \beta \gamma \delta \epsilon \zeta \eta} = \epsilon_{\epsilon \zeta \eta} ; \quad \alpha, \beta, \gamma = 5, 6, 7
\]

\[
F_{\mu \nu \rho} = 0 \quad \forall \mu, \nu, \rho = 1, 2, 3, 4
\]

(20)

which might be expected to lead to compactification. However the \( \chi \) equation and the trace of the Einstein equation yield

\[
-\Box \chi \sim \chi^{-\alpha}
\]

(21)

which has obvious solutions only for \( \alpha = 1 \). It therefore seems that the solution of the field equations found in Ref. 7) for \( d = 10 \) is special to \( d = 10 \).

We mention finally that in looking for solutions of \( d = 7 \) supergravity or of \( d = 10 \) coupled to matter it may be worthwhile to consider ansätze for which the quantity \( \epsilon_{\mu \nu \rho \sigma \tau} \cdot \text{tr}(F_{\mu \nu \rho \sigma}) \) is non-zero, i.e., "instanton-like" configurations.
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