PHENOMENOLOGICAL SUPERGRAVITY *)

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ABSTRACT

The rich phenomenological consequences of simple supergravity theories are discussed in detail. Simple (N = 1) supergravity theories may provide the framework for consistently describing physics from energies close to (but below) the Planck scale (∼10^{19} GeV) down to low energies. It is shown that gravitational effects, as contained in supergravity theories, may play, for the first time, a rather fundamental role in particle physics at low as well as at superhigh energies. For example, gravitational induced SU(5) and SU(2) x U(1) breaking occurs naturally at vastly different energy scales, thus evading the cumbersome gauge hierarchy problem. In this extended, updated, written version of my lecture, I concentrate on the physical consequences of simple (N = 1) supergravity theories. After providing a few reasons which make SUPERGRAVITY theories relevant and maybe necessary to physics (Section 1), the physical structure of N = 1 supergravity is discussed at some length (Section 2), followed by a detailed discussion of the phenomenological consequences of supergravity theories, both at the grand unified (GUT) and electroweak energy scales (Section 3), and some final remarks (Section 4). Physics, not formalism, is our focus.

1. WHY SUPERGRAVITY?

Grand unified theories (GUTs), despite all their successes and aesthetical appeal, seem to be problematic\(^1\). Their main defect is the gauge hierarchy problem. It seems very difficult to create, and then keep intact to all orders in perturbation theory, the energy gap of thirteen decades that exists between the electroweak scale \(M_W(\sim 100 \text{ GeV})\) and the GUT scale \(M_X(\sim 10^{15} \text{ GeV})\):

\[
\frac{M_W}{M_X} \leq 0(10^{-13})
\]

Supersymmetry (SUSY), or Fermion-Boson symmetry\(^2\), solves at least the technical aspect of this problem automatically\(^3\). Given (1) at the tree level, SUSY theories are able to keep it intact to all orders in perturbation theory.

In supersymmetric theories, fermions and bosons are sharing the same multiplet (supermultiplet), and thus new relations between fermion and boson masses, as well as between gauge and scalar self-couplings, are enforced. This is analogous to GUTs where, by putting together quarks and leptons, we get relations between their masses, as well as relations between different gauge coupling constants\(^1\).

In the case of SUSY gauge theories, because of their high symmetry, a lot of "miraculous" cancellations take place and render the theory much more ultra-violet convergent than usual gauge theories. A quantitative expression of this behaviour is shown by the so-called non-renormalization theorems\(^4\). There is wave function renormalization for each chiral and gauge supermultiplet and gauge coupling renormalization
BUT no separate renormalization of masses, scalar and Yukawa couplings is needed. The non-renormalization theorems\textsuperscript{4} go beyond the fact that parameters related by SUSY are expected to be renormalized by the same constant. They say that some parameters are not renormalized at all. THE SET IT AND FORGET IT principle. It is exactly this property of SUSY theories that solves the technical aspect of the gauge hierarchy problem, as mentioned before\textsuperscript{3}. Also in SUSY theories, the technical aspect of the strong CP problem is solved automatically\textsuperscript{5}. The parameter $\theta_{\text{QCD}}$ which characterizes the magnitude of the effect ($\theta_{\text{QCD}} < 10^{-9}$ experimentally), if set equal to zero, will stay zero, if SUSY is good, because the fermion mass matrix is not renormalized\textsuperscript{5}. When SUSY is spontaneously broken, $\theta_{\text{QCD}}$ acquires a small, finite, acceptable value\textsuperscript{5}. Furthermore, if SUSY is exact, no vacuum renormalization is needed\textsuperscript{6}, a property that eventually may shed light on the absence of the cosmological constant.

Clearly, SUSY cannot be an exact symmetry in Nature, since then all members of a supermultiplet will have the same mass, which is experimentally excluded. For example, there is no scalar electron (selectron) with mass of 0.5 MeV! Actually, present experimental limits\textsuperscript{7} put the masses of any charged SUSY particle above $0(20 \text{ GeV})$. SUSY needs to be broken. We may consider spontaneous, or explicit but soft breaking, SB or ESB respectively. Arbitrary ESB is ad hoc and problematic (see below). SB of global SUSY is associated with the existence of a Goldstone fermion, the goldstino, $\psi^\prime$. If $g_{bf}$ denotes the coupling of the goldstino $\psi^\prime$ to a supermultiplet that contains a boson $b$ and a fermion $f$, then the boson–fermion mass splitting is given by a Goldberger-Treiman type relation:

$$
\begin{equation}
M_b^2 - M_f^2 = g_{bf} M_S^2
\end{equation}
$$

where $M_S$ denotes the SUSY breaking scale. A priori, $g_{bf}$ and $M_S$ are arbitrary parameters, but they have to satisfy certain physical constraints: (i) absence of anything "new" below 20 GeV\textsuperscript{7} imposes a lower bound, at least for the charged SUSY partners of the "observed" particles:

$$
\begin{equation}
g_{bf} M_S^2 \geq 0 \left( \frac{(20 \text{ GeV})^2}{\text{ }} \right)
\end{equation}
$$

(ii) absence of radiative corrections to the mass of the electroweak Higgs (Z-Higgs) larger than $O(M_W)$, i.e., solution of the gauge hierarchy problem (technical aspect), imposes an upper bound:

$$
\begin{equation}
g_{bf} M_S^2 \leq 0 \left( \frac{(M_W)^2}{\text{ }} \right)
\end{equation}
$$
Clearly, the most naïve way to satisfy (3) and (4) is to identify $g_{\tilde{f}}$ with some gauge coupling and $M_S$ with $M_W$:

$$g_{\tilde{f}} \sim g$$

$$M_S \sim O(M_W)$$

(5)

This approach has been tried for many years now by Fayet, with rather limited success. The Fayet-type models suffer from incurable diseases, such as broken colour and/or electromagnetism, and/or Adler-Bell-Jackiw anomalies, and/or dangerous flavour-changing neutral currents, etc. We need something better. The other way is to diminish $g_{\tilde{f}}$ with an approximate increase of $M_S$, such that (3) and (4) are still satisfied. This approach, instigated by the work of Barbieri, Ferrara and myself, seems to be more successful. The idea is very simple. One has to find a natural way, so that the effective goldstino coupling to "ordinary" matter (quarks, leptons, E-Higgs, $W^\pm$, Z, $\gamma$, g) and their SUSY partners is extremely weak. This tiny effective coupling can be arranged at the tree level, or if absent at the tree level, may be induced, after some gymnastics, at some number of loops by radiative corrections. In any case, in a large class of this type of models, one gets the following relations, as discussed in Ref. 10):

$$g_{\tilde{f}} \sim \frac{M_W}{M_{\ell \ell}}$$

$$M_S^2 \sim M_W M_{\ell \ell} \sim (10^{10} \text{ GeV})^2$$

(6)

which automatically satisfy the constraints (3) and (4). In other words, despite the fact that the whole theory is characterized by a SUSY breaking scale $M_S \sim 10^{10} \text{ GeV}$, the "low energy" world (⇔ "ordinary" matter plus SUSY partners) is feeling an effective SUSY breaking

$$\left(\frac{M_S}{M_{\ell \ell}}\right)_{\text{eff}} \sim O(M_W) \ll M_S$$

(7)

Thanks to the "magic" properties of SUSY theories, this decoupling has been shown to persist to all orders in perturbation theory.

Up to now, we have considered global SUSY gauge theories and have shown that the construction of "realistic" (?), phenomenologically accepted models, has enforced upon us Eq. (6). This fact is extremely important because the supergravitational (SUGAR) coupling between the goldstino and "ordinary" matter ($m \sim M_W$)
\[
(g_{BF})_{SUGAR} \sim \sqrt{G_N} \, \nu \sim \frac{M_W}{M_{PL}}
\]

(8)

is exactly of the same order of magnitude as the corresponding non-gravitational coupling, given by Eq. (6). In other words, (super) gravitational effects cannot be neglected anymore\(^{14},^{15},^{16}\).

We may go even further and assume that non-gravitational \(g_{BF}\)'s are altogether absent and only those induced by supergravity remain. This is a spectacular idea, because it is not only very exciting that gravity, for the first time, plays a major role in low energy physics, but also because the resulting models are the simplest of all the realistic models of broken SUSY.

If we follow this line of thought, then (8) and (4) set an upper limit\(^{15}\) on \(M_S\):

\[
M_S^2 \leq O \left( M_W M_{PL} \right)
\]

(9)

while (8) and (3) set a lower limit on \(M_S\):

\[
M_S^2 \geq O \left( \left( \frac{200 \text{ GeV}}{M_W} \right)^2 M_W M_{PL} \right)
\]

(10)

More or less in SUGAR-type models, \(M_S\) is determined uniquely from physics constraints to be

\[
M_S^2 \sim O \left( M_W M_{PL} \right)
\]

(11)

The coincidence between (6) and (8), (11) is rather amazing. Intuitively, one can easily understand the need to move from global to local SUSY or supergravity\(^{13}\). Consider the one-loop correction to the E-Higgs mass due to the graviton

\[
\sigma \sim \frac{n^4}{M_{PL}^6}
\]

(12)

This is catastrophic because the natural scale for the cut-off \(\Lambda\) is \(M_{PL}\), which, through (12), implies electroweak breaking at the Planck scale! We need either extremely accurate fine tuning or unavoidable cancellation of the graviton (boson) loop by a fermion loop, i.e., supersymmetric gravity, thus SUPERGRAVITY\(^{13}\). The needed spin 3/2 fermion sits on the same supermultiplet with the graviton (spin 2) and it is called gravitino. As the graviton may be considered the "gauge boson" of the Poincaré algebra, the gravitino is the
"gauge fermion" of local SUSY. After SB of local SUSY, the gravitino "eats" the goldstino, thus becoming massive\(^1\text{7}\):

\[
\mathcal{M}_{3/2} \sim \sqrt{\frac{\kappa_N^2}{\mathcal{M}_5^2}} \sim \frac{\mathcal{M}_5^2}{\mathcal{M}_{pl}^2}
\]  

(13)

which implies, by using \((11)\text{10}\):

\[
\mathcal{M}_{3/2} \sim O(M_W)
\]  

(14)

One may argue that, because of the anticommutation relation, which in standard notation\(^2\text{7}\) reads

\[
\{ Q_\alpha, \bar{Q}_\beta \} = -i \int (\delta_\mu)_{\alpha\beta} P_\mu
\]

global SUSY plus gravity implies automatically local SUSY or supergravity, so what's new? Well, that is correct, but it was thought for a long time that, as in the case of ordinary gravity, the effects of supergravity, at least for low energy physics, would be negligible. We have proved here that this is not necessarily the case. Only if the Fayet-type models\(^8\text{8}\) were correct, would \((8)\) indeed be much smaller than \((5)\) and SUGAR effects would be negligible. Nevertheless, Fayet-type models\(^8\text{8}\) do not work\(^9\text{9}\), thus supergravity enters the "low energy" physics world in full strength and glory.

2. PHYSICAL STRUCTURE OF SIMPLE \((N = 1)\) SUPERGRAVITY

We are then led to consider local SUSY gauge theories\(^1\text{8}\). The effective theory below the Planck scale must be \((19)\) \(N = 1\) supergravity. The restriction to \(N = 1\) follows from the apparent left-right asymmetry of the "known" gauge interactions. Since we are dealing with local SUSY, the breaking of SUSY must be spontaneous, not explicit, if Lorentz invariance or unitarity are not to be violated. It is remarkable that the effective theory below \(\mathcal{M}_{pl}\) has been uniquely determined\(^1\text{9}\) to be a spontaneously broken \(N = 1\) local SUSY gauge theory\(^1\text{8}\).

We start with a reminder of the structure of \(N = 1\) supergravity actions\(^1\text{8}\) containing gauge and matter fields (if not explicitly stated, we use natural units \(\kappa^2 \equiv 8\pi G_N = (8\pi/\mathcal{M}_{pl}^2) \equiv 1/M^2 = 1\))

\[
\mathcal{A} = \int d^4x \, d^4\theta \, E \left\{ \bar{\Phi}(\varphi, \bar{\Phi}) \, e^V + \mathcal{R} \left[ \mathcal{R}^{-1} g(\varphi) \right] + \mathcal{R} \left[ \mathcal{R}^{-1} f_{\alpha\beta}(\varphi) \, W_\alpha^\varphi \, e^{\alpha\beta} \, W_\beta^\varphi \right] \right\} 
\]  

(15)
where $E$ is the superspace determinant, $\Phi$ is an arbitrary real function of the chiral superfields $\phi$ and their complex conjugates $\phi^*$, $V$ is the gauge vector supermultiplet, $R$ is the chiral scalar curvature superfield, $g$ is the chiral superpotential, $f_{\alpha\beta}$ is another chiral function of the chiral superfields $\phi$, and $\omega^a$ is a gauge-covariant chiral superfield containing the gauge field strength. In addition to all the obvious general co-ordinate transformations, local supersymmetry and gauge invariance, the action (15) is also invariant under the transformations

$$
J \equiv 3 \ln \left(-\frac{1}{2} \Phi \right) \rightarrow J + K(\Phi) + K^*(\Phi)
$$

$$
G(\Phi) \rightarrow e^{K(\Phi)} G(\Phi)
$$

These transformations can be related to a description of the chiral superfields $\phi$ as co-ordinates on a Kähler manifold with Kähler potential $\Phi$, and the transformations (16) are known as Kähler gauge transformations. One particular manifestation of this Kähler gauge symmetry is in the effective scalar potential

$$
V = -e^{\Phi} \left( 3 + G_i G^i - \frac{1}{2} G^I G_I + (gauge \ terms) \right)
$$

where

$$
G \equiv J - 3 \ln \left( \frac{1}{4} |\Phi|^2 \right)
$$

which is clearly invariant under the transformations (16). In general, the action depends on a real function

$$
\Phi \equiv \sqrt{G(\Phi)}
$$

and on the chiral function $f_{\alpha\beta}(\phi)$. The most familiar forms of these functions are $J = -\phi \Phi/2$, giving canonical kinetic energy terms for the chiral superfields, $g(\phi)$ a cubic polynomial giving renormalizable matter interactions of dimension $\leq 4$, and $f_{\alpha\beta} = \delta_{\alpha\beta}$. We expect that more complicated functions will contain terms $O(\Phi/M_{pl})^n$ relative to these canonical leading terms.

Ellis, Tamvakis and myself have suggested interpreting Eq. (15) as an effective action suitable for describing particle interactions at energies $\ll M_{pl}$, just as chiral SU(N) x SU(N) Lagrangians were suitable for describing hadronic interactions at energies $\ll 1$ GeV. In much the same way as we know that physics gets complicated at $E = 1$ GeV, with many new hadronic degrees of freedom having masses of this order, we also expect many new "elementary particles" to exist with masses $O(M_{pl})$. It may well be that all the known light "elementary particles", as well as these
heavy ones, are actually composite, and that at energies $\gg M_{\phi}$
a simple preonic picture will emerge, analogously to the economical
description of high-energy hadronic interactions in terms of quarks
and gluons. It may even be that these preonic constituents are them-
selves ingredients in an extended supergravity theory. But let
us ignore these speculations for the moment and return to our pedes-
trial phenomenological interpretation of the action (15).

The well-known rules of phenomenological Lagrangians are that
one should write down all possible interactions consistent with the
conjectured symmetries [e.g., chiral $SU(2) \times SU(2)$], and only place
absolute belief in predictions which are independent of the general
form of the Lagrangian (e.g., $\pi\pi$ scattering lengths). These are
the reliable results which could also be obtained using current
algebra arguments. It does not make sense to calculate strong inter-
action radiative corrections (read: supergravity loop corrections)
to these unimpeachable predictions: these are ambiguous until we
know what happens at the 1 GeV scale (read: $M_{\phi}$), and our ignor-
ance can be subsumed in the general form of the phenomenological
Lagrangian, in which any and all possible terms are present a priori
(read: non-trivial $J$, non-polynomial $g$ and $f_{\phi\phi}$). On the other
hand, non-strong interaction radiative corrections can often be com-
puted meaningfully (e.g., the $\pi^+\pi^0$ mass difference, large numbers
of pseudo-Goldstone boson masses in extended technicolour theories).
Similarly, it makes sense to compute matter interaction (gauge,
Yukawa, Higgs) corrections to the tree-level predictions of the
effective action (15).

Since the supergravity action is non-renormalizable, and since
both the $\Phi$ and $f_{\phi\phi}$ terms in the action (15) have a $\int d^4\theta$ form,
we expect general variants of them to be generated by loop correc-
tions. Presumably, radiative corrections maintain the essential
geometry of the Kähler manifold. Therefore, we expect loop cor-
trections to fall into the class of Kähler gauge transformations (16).
The only analogous transformation allowed in a conventional renor-
malizable theory is $K = $ constant, corresponding to a wave function
renormalization. In our case, more general gauge functions $K(\phi)$
might appear.

In $N=1$ SB local SUSY gauge models, called for abbreviation
supergravity or SUGAR models, one usually distinguishes two sectors:
(i) the "observable" sector containing quarks, leptons, Higgs and
gauge bosons of electroweak and GUT types, as well as their SUSY
partners; (ii) the "hidden" sector containing at least the goldstino
and its SUSY associate. The "observable" and "hidden" sectors both
couple to supergravity, but not directly to each other. In other
words, ignoring supergravity, physics in the "observable" sector
would be completely supersymmetric. A realization of this programme
occurs, for example, by splitting the superpotential of the theory
into the sum of two terms.
\[ \Phi_i = \lambda_i (Z_i) + f(x_i) \]  

(20)

where \( \chi_i \) are the "observable" fields and \( Z_i \) the "hidden" fields. The \( f \)-part of the superpotential has to do with the "observed" sector and contains most of the physics, while the \( h \)-part of the superpotential has to do with SB of local SUSY and the vanishing of the cosmological constant. The scalar fields of the hidden sector typically have vev's of \( M_{P} \), which cause SUSY to be SB at a scale \( M_{G} \) determined by the parameters in the \( h \)-part. We may choose these parameters to be such that (11), or equivalently (14), are satisfied, as well as cancelling the cosmological constant at the same time ("superHiggs effect")\(^{18} \). A celebrated example is the Polonyi potential\(^{23} \):

\[ \lambda_i (Z) = M_{G}^{2} (Z + B) \]  

(21)

which, in the absence of other fields, and for \( B = (2 + \sqrt{3})M \) and \( \langle Z \rangle = (\sqrt{3} - 1)M \), \( M \equiv (M_{P}^{2}/\sqrt{8\pi}) \approx 2.5 \cdot 10^{18} \) GeV being the appropriate supergravity scale (superPlanck mass), implies SB of local SUSY at \( M_{G} \) and vanishing cosmological constant. The communication between the two sectors is mediated by the auxiliary fields of the SUGAR multiplet and takes place at tree-level in a model-independent way. Exchange of gravitons or gravitinos plays no role at this level. The elimination of these auxiliary fields produces non-renormalizable interactions between the two sectors. These non-renormalizable interactions include the ones between the hidden fields \( Z_i \), usually taken to be neutral under all gauge symmetries, and gauge fields.

All these effects have been summed into an effective scalar potential\(^{18} \) [(17), but with canonical kinetic energy terms for the chiral superfields].

\[ V(\Phi) = e^{\lambda} \left( \frac{4}{M^{2}} \sum_{\Phi} \frac{\partial \Phi \bar{\Phi}}{\partial \Phi} \right) \left\{ \sum_{\Phi} \left[ \frac{\partial g(\Phi)}{\partial \Phi} + \frac{\bar{\Phi} g(\Phi)}{M^{2}} \right]^{2} - \frac{3}{M^{2}} \right\} + \frac{1}{\alpha} \sum_{\alpha} D_{\alpha}^{2} (\Phi, \bar{\Phi}) \]  

(22)
This mess involves the scalar fields $\phi_i$, the superpotential $g$ and the $D$-terms, which have their usual global SUSY form. Incidentally, by comparing (22) with the global SUSY potential

$$V(\phi) = \sum_i \left| \frac{\partial g(\phi)}{\partial \phi_i} \right|^2 + \frac{1}{2} \sum_a D_a q^2 (\phi, \bar{\phi})$$

we notice the richer structure of the SUGAR potential [(22)] and thus we anticipate that more physics will come out from it. When the scalar components of the hidden fields are replaced by their vev's, all superheavy fields are integrated out, and expanding in powers of $1/M$, the resulting effective theory, just below $M$, for the light observable fields, will contain both the usual SUSY terms and a soft SUSY breaking piece$^{15), 24)}$--26):

$$\mathcal{L}_{\text{soft}} = \sum_i w_i q^2 |\chi_i|^2 + \sum_{n} (A_n f_n + h.c.) + \sum_{\alpha} \left( \frac{1}{2} M_\alpha \lambda_\alpha \lambda_\alpha + h.c. \right)$$

(23)

where $\lambda_\alpha$ is a gauge fermion [gluino ($\alpha=3$), wino ($\alpha=2$) or bino ($\alpha=1$)], $f$ is any term in $f$ and $\chi_i$ is any scalar (Higgs, squark or slepton). The $A_n$'s are expected to be of order one, while $m_i$ and $M_\alpha$ should be in general of order $m_3/2$$^{15), 19), 27})$. Actually, when corrections$^{26)}$ from integrating out superheavy fields are ignored, the following relations hold$^{15)}$:

$$w_i = \frac{w_i}{\sqrt{\kappa}}$$

(24)

for every scalar field, and$^{24), 25)}$

$$A_n = A - 3 + d_n$$

(25)

where $A$ is a universal number$^{25)}$ (assumed real) and $d_n$ is the number of fields multiplied together in term $n$. If we imagine that the low energy theory is embedded in a GUT model at some GUT scale below the Planck mass, then all gaugino masses are equal at the GUT scale, so that only one single parameter $M_0$ is needed$^{[M_0 \text{ in principle may be of } O(m_3/2)]}$.}
\[ M_\alpha(M_X) = M_0 \quad \forall \alpha = 1, 2, 3 \] (26)

while at lower energies \( M_\alpha \) evolves in a manner identical for the gauge couplings:

\[ \frac{M_\alpha(M)}{M_0} = \frac{\tilde{\alpha}_\alpha(M)}{\alpha_G} \quad \forall \alpha = 1, 2, 3 \] (26')

with \( \tilde{\alpha}_\alpha, \alpha_G \) the usual SU(3), SU(2), U(1), GUT fine structure constants.

In the following, we shall assume that corrections\(^{26}\) from integrating out superheavy fields are not very important and proceed with the simplified soft SUSY breaking piece, just below \( M_X \):

\[ L_{\text{soft}} = \frac{\lambda_3}{2} \sum_i |\chi_i|^2 + \frac{\lambda_1}{2} \sum_\nu (A - 3 + d_\nu) f_\nu + h.c. \] (27)

\[ - \frac{1}{2} M_0 \sum_\alpha \lambda_\alpha \lambda_\alpha \quad + \text{h.c.} \]

The soft operators in (23) or (27) are determined by the couplings of the hidden fields and by radiative corrections at the Planck scale, including those due to gravity. In spite of this, the very interesting thing is that sometimes it is possible to give the form of these operators without making detailed assumptions about either the hidden sector or the effects of gravitational radiative corrections. For example, in (27), the form of the hidden sector enters only through the three parameters \( m_{3/2}, A \) and \( M_0 \). The non-renormalizable interactions of the hidden fields with gauge fields, discussed before, will generally lead to soft Majorana masses, \( M_0 \), for the gauginos \( \tilde{f}_{\alpha \beta} \neq \delta_{\alpha \beta} \) in (15), while the non-renormalizable interactions between the "observable" and "hidden" sectors, commented before, will produce the soft operators in the scalar potential [first two terms in (23) or (24)].

In a nutshell, the SB of SUSY in SUSY gauge theories coupled to \( N = 1 \) SUGAR leads to an effective theory below the Planck scale in which global SUSY is explicitly broken by a constrained set of soft operators, at an effective scale:

\[ (M_\sigma)^\text{eff} \approx M_{3/2} \] (28)
As discussed before, physics constraints [see (7)] impose the condition \( m_{3/2} \sim 0(m_W) \) [see (14)], which creates a new hierarchy problem. Since we are dealing with gravitational phenomena, naively, the natural mass scale for the gravitino is \( m_{P_2} \) and not \( m_W \). I call this the supergravity hierarchy problem, or the SUGAR hierarchy problem. On the other hand, it should be emphasized that the automatic soft breaking of global SUSY that is provided in the SUGAR framework not only splits "low energy" supermultiplets in the right way [all scalars and gauginos getting masses \( O(m_{3/2}) \)], BUT it has the correct form to pass unscathed through all the traps set out by low energy phenomenology.

In the physics applications which follow, we shall make extensive use of two main characteristics of the general framework discussed above. First, since we are dealing with an effective theory (the \( N = 1 \) SUGAR action is non-renormalizable), the superpotential \( g \) is not anymore necessarily constrained by renormalizability to be at most cubic, but it may contain any higher powers, suitably scaled by inverse powers of \( m_{P_2} \), the natural cut-off of the theory\(^{10} \). Secondly, because of the non-renormalization theorems\(^{4} \) of SUSY (SET IT AND FORGET IT principle), we may set, as we wish, certain parameters equal to zero, even if no symmetry implies that - a very different situation from ordinary gauge theories. Here, no apologies are needed. As explained in detail before, most of the physics is contained in the "observable" sector superpotential \( f(\chi_i) \) [see (20)]. Here we shall assume that, in one way or another, the "hidden" sector has played its role, as discussed previously, and we shall concentrate on the form of \( f(\chi_i) \). We follow the natural (cosmic) evolution of things starting at energies below \( m_{P_2} \) and "coming down" to \( m_W \). So we distinguish physics around the GUT scale \( (M_X) \) and physics around the electroweak (E-W) scale \( (M_W) \).

All physics from \( m_{P_2} \) down to (and including) low energies should emerge from such a programme. We will show next that this is indeed possible.

3. PHYSICS WITH SIMPLE \( (N = 1) \) SUPERGRAVITY

A. Physics Around the GUT Scale \( (M_X) \)

The superhigh energy regime \( (\sim 10^{16} \text{ GeV}) \) is the theorists' paradise. There is a lot of freedom in building models, even though the constraints both from particle physics and cosmology become tighter and tighter. For definiteness, simplicity, and out of habit, we shall take as our prototype GUT an SU(5) type model\(^{11} \). All GUT physics information will be contained in \( f_{\text{GUT}} \), the GUT part of the "observable sector" superpotential. There is no consensus about the definite form of this superpotential, but it should unavoidably contain a piece \( (f_I) \) that breaks SU(5) down to SU(3) \( \times \) SU(2) \( \times \) U(1)
and if possible, a piece \( f_{II} \), providing some explanation about the tree-level gauge hierarchy problem, so we write:

\[
f_{\text{GUT}} = f_I + f_{II}
\]  

(29)

For example, we may take \(^{28}\)

\[
f_I = \frac{a_1}{M} \chi^4 + \frac{a_2}{M^2} \chi^2 T_\tau (\Sigma^3)
\]

(30)

and \(^{29}\)

\[
f_{II} = \overline{\Theta} H \left( \frac{\lambda_1}{M} \frac{\Sigma^2}{M} + \frac{\lambda_2}{M^2} \frac{\Sigma^3}{M^3} + \cdots \right) + \overline{H} \Theta \left( \frac{\lambda_1'}{M} \frac{\Sigma^2}{M^2} + \frac{\lambda_2'}{M^3} \frac{\Sigma^3}{M^3} + \cdots \right) + M_\Theta \overline{\Theta} \Theta
\]

(31)

where \( X = 1 \), \( \Sigma = 24 \), \((\Theta)^c = (3), \overline{(H)^c} = (5)^c\) are chiral superfields of SU(5). The Higgs fields \( H \) and \( \overline{H} \) couple to quark and lepton fields in the usual way. All components of \( \Theta \) and \( \overline{\Theta} \) have a bare mass \( M_\Theta \) (which is taken to be of order \( M_X \) or larger), and so remain heavy after SU(5) breaks to SU(3) \( \times \) SU(2) \( \times \) U(1). After minimizing the potential, obtained by plunging into (22) the sum of \( f_I \) and \( f_{II} \) as given by (30) and (31), we get zero vev's for \( H \) and \( \overline{H} \) but non-zero ones for

\[
<X> = \left( \frac{m_{3/2}}{M} \right)^{3/8} M
\]

\[
<\Sigma> = \left( \frac{m_{3/2}}{M} \right)^{1/4} M
\]

(32)

Furthermore, we find\(^{28}\) that the SU(3) \( \times \) SU(2) \( \times \) U(1) symmetric minimum is the lowest one for all values of \( a_1 \) and \( a_2 \), with a value\(^{28}\)

\[
V_{\text{eff}} \sim - \left( \frac{m_{3/2}}{M} \right)^{5/4} M^4
\]

(33)
What do these results mean? First, since the vev of $\Sigma$ sets the scale of $SU(5)$ breaking, we find that the GUT scale $M_X$ satisfies

$$M_X^4 \sim O\left(\frac{m_3}{M} M^3\right)$$

(34)

which is a highly successful relation. Using as an input the non-hierarchical and easy to explain ratio $(M_X/M) \sim 10^{-2}-10^{-4}$, we obtain that $m_{3/2} \sim O(100 \text{ GeV})$. More generally, relations of the form $M_X^{2p-2} \sim O(m_{3/2} M^2 p^{-3})$ with $p \geq 3$, are also possible by suitably modifying the exponents in (30). The supergravity hierarchy problem has been solved in a rather simple way.

Secondly, the $SU(3) \times SU(2) \times U(1)$ symmetric minimum is lower in energy density than the $SU(5)$ symmetric minimum $X = \Sigma = 0$ by an amount $(m_{3/2}/M)^{5/2} M^6$. Thirdly, the barrier between these two minima is never larger than $(m_{3/2}/M)^{5/2} M^6$, the same as the splitting between the states. Why this is so can be seen by noting that if we replace $X$ by its vev (32) in (30), the effective renormalizable self-coupling of $\Sigma$ is $10^{-12} \text{ tr}(\Sigma^3)$. Thus we have generated a small renormalizable coupling for $\Sigma$ from our starting point of only non-renormalizable interactions among $X$ and $\Sigma$. This small coupling suppresses the barrier between the $SU(5)$ and the $SU(3) \times SU(2) \times U(1)$ phases. The consequences of this suppression for supercosmology are difficult to overestimate. Simply, it now makes possible the transition from the $SU(5)$ to the $SU(3) \times SU(2) \times U(1)$ phase at temperatures $T \sim 10^{10}$ GeV, which was previously blocked, since the barrier between the two phases was of the order of $(M_X)^4$. Incidentally, in this picture, the number density of GUT monopoles is naturally suppressed below its present experimental upper bound.

It should be clear that the basic result — small renormalizable couplings arising from non-renormalizable ones suppressed only by inverse powers of $M$ — is quite general and does not depend on the detailed form of the superpotential (fT)28). The main characteristics of these types of models28),31) are that they provide relations of the type (34); they make possible "delayed" SU(5) to SU(3) × SU(2) × U(1) phase transitions at $T \sim 10^{10}$ GeV, and they contain more "light" particles than the ones in the minimal SUSY SU(3) × SU(2) × U(1) model. This last fact may sound dangerous when calculating $N_X$, $\sin^2 \theta_{\text{EW}}$, and $m_b/m_t$, since in general an arbitrary increase of "light" stuff gives an out-of-hand increase32),33), and thus experimentally unacceptable values for, the above-mentioned quantities. A more careful analysis34) of these cosmological acceptable models (CAM)34) shows that they make predictions as successful (for $\sin^2 \theta_{\text{EW}}, \ m_b/m_t, \ldots$) as at least the ones32),33) of the phenomenologically acceptable minimal type models (MIMO). For a detailed, thorough phenomenological analysis of CAMs, see Ref. 34).
Next, we discuss\textsuperscript{29} physics related to $f_{II}$ as given in (31). The vev of $\Sigma$ does not only break SU(5) to SU(3) $\times$ SU(2) $\times$ U(1) but also provides a mass term which mixes the colour triplets in $H$ and $\bar{H}$ with those in $\theta$ and $\bar{\theta}$. However, there is no weak doublet in the $\bar{\mathbf{50}}$, and so the weak doublets in $H$ and $\bar{H}$ remain massless. The colour triplets will have a mass matrix\textsuperscript{29}

$$
\begin{pmatrix}
0 & \sim \frac{M_X}{M} \\
\sim \frac{M_X}{M} & \frac{M_\theta}{M}
\end{pmatrix}
$$

(35)

where $M_\theta$ should be of order $M_X$ or larger ($\leq M$), to avoid having particles from $\theta$ and $\bar{\theta}$ influencing the renormalization group equations at scales below $M_X$ (or even $M$). The eigenvalues of this mass matrix are $O(M_\theta)$ and $O(M_X^2/M_\theta M^2)$; this latter eigenvalue is about $10^{10}$ GeV for $M_X \sim O(10^{16}$ GeV) and $M_\theta \sim O(M)$. In this case, the Higgs colour triplet can be used to generate\textsuperscript{28,30} the baryon number of the Universe after the SU(5) to SU(3) $\times$ SU(2) $\times$ U(1) transition which, as discussed earlier, occurs at temperatures $T \sim 10^{10}$ GeV in CMB\textsuperscript{34}. It is remarkable that $O(10^{10}$ GeV) is the lower bound\textsuperscript{35} allowed for colour triplet Higgs masses from present limits on proton decay ($T_\beta > 10^{31}$ years). If indeed there are $10^{10}$ GeV Higgs triplets, then protons should decay predominantly\textsuperscript{36} to $\nu_\mu K\pi$ with a lifetime $\sim O(10^{31}$ years).

The role of supergravity in this natural explanation\textsuperscript{29} of the Higgs triplet-doublet mass splitting (a tree-level gauge hierarchy problem) is fundamental, in several aspects. The same kind of explanation had been suggested before\textsuperscript{37} in the framework of renormalizable global SUSY GUTs, where $\Sigma^2$ in (31) was replaced by a $\mathbf{75}$ of SU(5) and higher than two powers of $\Sigma$ were absent\textsuperscript{37}. Unfortunately, the use of $\mathbf{75}$ drastically conflicts with cosmological scenarios\textsuperscript{28-31} based on SUSY GUTs. The barrier between the SU(5) and SU(3) $\times$ SU(2) $\times$ U(1) phases is impossible to overcome unless most of the $\mathbf{75}$ is very light ($\sim M_\theta$). But then, all hell breaks loose. A light $\mathbf{75}$ makes the gauge coupling in the SU(5) phase decrease at lower energies so there is no phase transition at all\textsuperscript{28,30}. Furthermore, the presence of these new light particles in the SU(3) $\times$ SU(2) $\times$ U(1) phase changes the renormalization group equations, and prevents perturbative unification. On the contrary, in SUGAR theories, since we may use non-renormalizable terms, we may replace the fundamental $\mathbf{75}$ by an "effective" $\mathbf{75}$ contained in $\Sigma^2$. Unlike a light $\mathbf{75}$, a light $\mathbf{24}$ neither makes the SU(5) gauge coupling decrease at energies below $M_X$, nor upset perturbative unification\textsuperscript{29}. The previously mentioned cosmological scenarios\textsuperscript{28-31} can proceed without modification. In addition, SUSY non-renormalization theorems\textsuperscript{4} ensure the stability of the triplet-doublet splitting.
to all orders in perturbation theory. Since the only modifications of the theory are at the GUT scale $M_X$, it seems that we have got a harmless and elegant solution of the tree-level, and for that matter, to all orders in perturbation theory, gauge hierarchy problem.

SUGAR models give good physics at the GUT scale - unique, cosmologically acceptable breaking of $SU(5)$ to $SU(3) \times SU(2) \times U(1)$, with an explanation of the smallness of the gravitino mass $^{28}$ [(34)-like relations], and a natural explanation $^{29}$ of the Higgs triplet-doublet splitting, cosmologically fitted and general enough. We believe that even if the very specific form of $f_T$ in (30) may change, then $f_{TT}$ as given by (31) (or its obvious generalization to other GUT models) will be always a useful part of the $f_{GUT}$.

After finding plausible explanations for the SUGAR hierarchy problem [gravitino mass $\propto 0(100 \text{ GeV})$], the tree-level and higher orders gauge hierarchy problem (triplet-doublet Higgs splitting), it is time to explain the last gauge hierarchy problem (1), i.e., why does $M_H/M_X \propto 0(10^{-13})$? This problem brings us naturally to our next subject.

B. Physics Around the Electroweak (E-W) Scale ($M_X$)

Although there is no consensus on the best way to incorporate grand unification in SUGAR models, a unique minimal low energy model has recently emerged. In this model, the physics of the TeV scale is described by an effective $SU(3) \times SU(2) \times U(1)$ gauge theory, in which the breaking of weak interaction gauge symmetry is induced by renormalization group scaling of the Higgs (mass) operators $^{19}$. Much of the attractiveness of this model stems from the fact that no gauge symmetries or fields beyond those required in any low energy SUSY theory are included. Sometimes, it may happen, as is the case of Cosmological Acceptable Models $^{34}$ (CAMs), that there are GUT relics which are light ($\propto M^2$), but they do not seem to play any fundamental role at low energies, so we may neglect them in our present discussion. Furthermore, adding random chiral superfields to the low energy theory may be problematic. For example, the presence of a gauge singlet superfield coupled to the Higgs doublets and added to trigger $SU(2) \times U(1)$ breaking $^{41}, 24, 27$, usually (but not always $^{42}$) destroys $^{43}$ any hope of understanding the gauge hierarchy problem; the reason being $^{43}$ that in a GUT theory, the gauge singlet does not only couple to the Higgs doublets but also to their associate, superheavy colour triplets. Then we have to try hard $^{42}$ to avoid $10^{15}$ GeV Higgs doublet masses, generated by $^{43}$ one-loop effects involving colour triplets. Something smells fishy.

We focus then on the standard low energy $SU(3) \times SU(2) \times U(1)$ gauge group, containing three generations of quarks and leptons, along with two Higgs doublets, as chiral superfields. The low energy effective superpotential ($f_{LES}$) of the model consists only of the
usual Yukawa couplings of quark and lepton superfields to the Higgs superfields, along, in general, with a mass term coupling the two Higgs doublets, $H_1$ and $H_2$. Explicitly, in a standard notation:

$$
\tilde{f}_{\text{LES}} = h_{ij} U_i^C Q_j H_2 + \tilde{h}_{ij} D_i^C Q_j H_1 \\
+ f_{ij} L_i E_j^C H_1 + \nu_4^C H_1 H_2
$$

where a summation over generation indices $(i,j)$ is understood and $Q(U^c)$ denote generically quark doublets (charge $-2/3$ antiquark singlet) superfields, while $L(E^c)$ refers to lepton doublets (charge $-1$ antilepton singlet) superfields. With the exceptions of the top quark Yukawa coupling and the mass parameter $m_t$, which in principle may be of order $O(m_t/2)$, all other parameters appearing in (36) contribute to the masses of the observed quarks and leptons and are known to be small. Neglecting these small couplings, the effective Low Energy Potential ($V_{\text{LEP}}$) can be written as [see (22), (23) and (27),]

$$
V_{\text{LEP}} = \sum_{i=1}^{3} \left[ m_{L_i}^2 |L_i|^2 + m_{E_i}^2 |E_i|^2 + m_{Q_i}^2 |Q_i|^2 + \\
+ m_{U_i}^2 |U_i|^2 + m_{D_i}^2 |D_i|^2 \right] \\
+ m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + A h_t m_{3/2} (U_3 C Q_3 H_2 + h.c.) \\
+ B m_{3/2} \tilde{m}_4 (H_2 H_2 + h.c.) + h_t \nu_4^C (H_2 H_1 C U_3 + h.c.) \\
+ h_4^2 (|Q_3|^2 |U_3|^2 + |Q_3|^2 |H_2|^2 + |U_3|^2 |H_2|^2) \\
+ "D-term"
$$
The effective parameters appearing in (37) take, at large scales \((\sim M_X)\), the values

\[
\begin{align*}
\lambda_1^{\varphi} (M_X) &= \lambda_2^{\varphi} (M_X) = \lambda_3^{\varphi} + \lambda_4^{\varphi} (M_X) \\
\lambda_{1_i}^{\varphi} (M_X) &= \lambda_{2_i}^{\varphi} (M_X) = \lambda_{3_i}^{\varphi} (M_X) = \lambda_{4_i}^{\varphi} (M_X) = \lambda_{5_i}^{\varphi} (M_X) = \lambda_{6_i}^{\varphi} (M_X) \\
A (M_X) &= A \\
B (M_X) &= A - 1
\end{align*}
\]  

(i = 1, 2, 3)

as dictated by (27). It should be stressed once more that the boundary conditions (38) are exact, if we only neglect corrections at the Planck scale, ignore the scaling of parameters from \(M\) to \(M_X\), and pay no attention to corrections at the GUT scale. All these effects are expected to be small and it is assumed that they do not seriously disturb (38) and the picture hereafter.

It is apparent from (37) that SUGAR models can easily succeed in giving weak interaction scale masses \((m_{3/2} \sim M_X)\) to squarks, sleptons and gauginos [see (27)]. Alas, SUGAR models also give large positive \((\text{mass})^2\) to the Higgs doublets, thus making the breaking of \(SU(2) \times U(1)\) difficult. One way to overcome this difficulty is the introduction \(^{41}\), \(^{24}\), \(^{27}\) of a gauge singlet coupled to \(H_1\) and \(H_2\), but, as mentioned above, with disastrous effects \(^{43}\) for the gauge hierarchy. A particularly simple solution to the \(SU(2) \times U(1)\) breaking relies upon the fact that the boundary conditions (38) need be satisfied only at \(M_X\) (or \(M\)), and that large renormalization group scaling effects can produce a negative value for \(m_{12}^2\) at low energies \(^{19}\). The full set of renormalization group equations for the parameters in \(V_{\text{Eff}}(37)\) has been written elsewhere \(^{44}\). Here we concentrate on the most interesting equation, the one for the mass-squared of the Higgs \((m^2)\), which gives mass to the top quark:

\[
\begin{align*}
\mu \frac{g_t^2}{8 \pi^2} \begin{bmatrix} m_{12}^2 \\ m_{23}^2 \\ m_{33}^2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = & \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} m_{12}^2 \\ m_{23}^2 \\ m_{33}^2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 3 \\ 3 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} a_t^2 m_{12}^2 \\ a_t^2 m_{23}^2 \\ a_t^2 m_{33}^2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}
\end{align*}
\]  

(39)
where we have neglected gauge couplings other than the "coloured" one, \( \alpha_3 = g_3^2/4\pi \), \( M_3 \) is the gluino mass [see (26) and (27)], and Yukawa couplings other than \( h_t \), for the top, have been dropped. The physics content of (39) is apparent. Since \( \mu \) is decreasing (we come from high energies down to low energies), the sign of the first two terms in (39) is such as to make all \( m_{2 \tilde{u} \nu \nu}, Q_{3 \nu} \) smaller at low energy, with the decrease of \( m_{2 \tilde{u} \nu \nu} \) becoming more pronounced because of the 3:2:1 weighting. On the other hand, the sign of the last two terms in (39) is such as to make \( m_{\tilde{d}} \) and \( m_{\tilde{f}} \) (the squark masses) larger at low energy, but have no direct effect on \( m_{2 \tilde{u} \nu \nu} \) [notice the "zeros" in the corresponding matrices in (39)]. Indirectly though, the net effect on \( m_{2 \tilde{u} \nu \nu} \) of the last two terms in (39) is to enhance further its decrease at low energies, by increasing \( |	ilde{H}_u| \) and \( m_{\tilde{q}} \), which then drive down \( m_{2 \tilde{u} \nu \nu} \) via the first two terms of (39). This is exactly what we are after! We want large \( (\sim M_3) \) and positive squarks and sleptons (masses)\(^2\), BUT negative Higgs (mass)\(^2\) to trigger SU(2) \( \times \) U(1) breaking. The ways of obtaining negative Higgs (mass)\(^2\) now become clear [see (39)]. We have to use either a large top Yukawa coupling \( (h_t) \), or large \( A \), or large \( m_{\tilde{t}} \), or a fourth generation to provide large Yukawa couplings, or some suitable, physically plausible combination of the above possibilities. There are pros and cons for every one of the above situations. In the case of large \( h_t \), a lower bound on the mass of the top quark is set\(^{19,38,40}\):

\[
M_t > 0 \quad (60 \text{ GeV})
\]  

(40)

which some people may find uncomfortable. We may avoid a large \( h_t \) by moving it into the large \( A(>3) \) regime\(^{38,45}\). The price, though, is high\(^{38,45}\). The phenomenologically acceptable vacuum becomes unstable against tunnelling into a vacuum in which all gauge symmetries, including colour and electromagnetism, are broken. We must\(^{38,45}\) then arrange things in such a way that the lifetime for this vacuum decay process is greater than the age of the Universe. Some people, not without reason, may find this possibility dreadful. We may avoid large \( h_t \) and/or large \( A \) by using\(^{34}\) non-vanishing \( m_{\tilde{t}} (\sim m_{3/2}) \) where a rather satisfactory picture then emerges\(^{34}\). Some people may object here to the basic assumption of large \( m_{\tilde{t}} (\sim m_{3/2}) \), since in the case of natural triplet-doublet Higgs splitting-type models\(^{29}\) [see (31)], \( m_{\tilde{t}} \) has a tendency to be small, if not zero, even though other sources of \( m_{\tilde{t}} \) may be available.

Finally, we come to the possibility of a fourth generation which, suitably weighed, may help us to avoid large \( h_t, A, \) or \( m_{\tilde{t}} \). The problem here is that low energy phenomenology (evolution of coupling constants, \( m_{\tilde{t}}/m_{\tau, \, \ldots} \))\(^{32-34}\) as well as firm cosmological results like nucleosynthesis (especially \( ^4\text{He} \) abundance\(^{46}\)), may suffer almost unacceptable modifications. Furthermore, one has to watch out for the mass of the fourth generation charged lepton, since it
is going to behave like $m^2$ in (39), and thus $m_{\text{Higgs}}^2$ may easily go negative, breaking electromagnetic gauge invariance.

Whatever mechanism (if any) turns out to be correct, it is rather remarkable that in SUGAR-type models, there is a simple explanation of the breaking of $SU(2) \times U(1)$ and of the non-breaking of $SU(3) \times U(1)$ at $M_W$. Furthermore, for the first time, we have a simple explanation of why $M_W \ll M_X$ (or $M$), i.e., a simple solution of the cumbersome gauge hierarchy problem. Starting with a positive Higgs (mass)$^2$, of order $m_\text{Higgs}^2$ at $M_X$, and noticing that the evolution with $\mu^2$ of the Higgs (mass)$^2$ is very slow (logarithmic), it is not surprising that we have to come down a long way in the energy scale, before the Higgs (mass)$^2$ turns negative and is thus able to trigger $SU(2) \times U(1)$ breaking. For example, in a class of models characterized by "small" gravitino masses ($\ll O(M)$) and by a Coleman-Weinberg-type radiative $SU(2) \times U(1)$ breaking, occurring naturally, we get$^{48}$, by dimensional transmutation

$$M_W \simeq \Lambda_{\alpha_{\text{GUT}}} \frac{g_2(M_W) \alpha_X}{24} \rho \left( \frac{\eta}{\alpha_C} F \left( \frac{h_\text{X}(M_X)}{\alpha_C} ; A \right) \right) \quad (41)$$

where $g_2$ denotes the $SU(2)$ gauge coupling constant, $\alpha_C$ is the GUT fine-structure constant and $F$ is a rather involved function of its indicated variables. Using standard values for $g_2(M_Z)$ ($\simeq 0.67$), $\alpha_C$ ($\simeq 1/25$) and reasonable values for $h_\text{X}(M_X)$ ($0.2-0.3$), $A(M_X)$ ($2-3$), (41) gives$^{48}$

$$M_W \simeq (300-600) \Lambda_{\alpha_{\text{GUT}}} \quad (42)$$

a rather remarkable equation from many points of view. It does not only give $M_W \ll M_X$ as it was required by the gauge hierarchy, but it also provides a new and successful relation between the scale of electroweak unification $M_W$ ($\sim 80$ GeV) and the fundamental scale of strong interactions, $\Lambda_{\alpha_{\text{GUT}}}$ ($\sim 0.15-0.3$ GeV) in terms of dimensionless parameters. Clearly, this occurs because the rapid final stages of evolution of $m_\text{Higgs}^2$ are driven by the increases in t-quark Yukawa coupling, and more importantly, in the squark masses which occur when $g_3^2/4\pi$ becomes large. It is only in the SUSY Coleman-Weinberg scenario$^{38,34}$ that the weak interaction scale is related to that of strong interactions. This contrasts with what usually happens in weak gauge symmetry breaking in SUGAR models$^{39-40}$ where $M_W$
is connected to \( m_{3/2} \), but it is not directly related to the strong interaction scale.

Another very amazing fact is that the values of the parameters of the low energy world seem to co-operate with us. Since quarks are feeling strong interactions, (39) tells us that quarks may enjoy large masses (Yukawa couplings) without making squark (masses)\(^2\) negative, because of the last term (\( \sim \alpha_s \)), which easily balances off large Yukawa couplings, without any sweat. On the other hand, since leptons are not feeling strong interactions, the balance-off between the weak gauge couplings and large Yukawa coupling becomes extremely delicate and could be problematic. How nice that for all three generations, leptons and down quarks weigh less than 5 GeV and especially for the third generation that the top quark \((t)\) is heavier than the bottom quark \((b)\). An inverse situation would be disastrous, because in any reasonable GUT, a very heavy \( b \) quark would mean a very heavy \( \tau \) lepton, thus making electromagnetic gauge invariance tremble in such SUGAR-type schemes. I will not go any further into the esoterics of this type of \( SU(2) \times U(1) \) breaking models, since a rather thorough and detailed expose of these types of theories and of their phenomenological consequences is now available\(^34\). It should be stressed that things are now very constrained, as we see from the Table, taken from Ref. 34), where the whole low energy spectrum is worked out, in terms of very few parameters, \( m_{3/2} \), \( A(M_X) \), \( \xi \equiv M_0/m_{3/2} \) [see (26)] and \( m_q(M_X) \). Eventually, with more theoretical insight, we hope to determine even these very few parameters, thus predicting uniquely the low energy spectrum. For example, we have already discussed ways of determining \( m_{3/2} \) [see (34)], while some people may favour \( A(M_X) = 3 \) as a natural solution\(^49\) to the absence of the cosmological constant problem, etc. Among other interesting things contained in the Table, the existence of a very light \( (\sim 3-6 \, \text{GeV}) \) neutral Higgs, with the usual Yukawa couplings to matter, should not escape our attention. Since such a particle is a common feature of a large class of models\(^36\),34), a search in the \( Y + H^0 + \gamma \) channel, which is expected to be a few per cent of the \( Y + \mu^+\mu^- \) decay, may turn out to be very fruitful.

There are other, very interesting features concerning low energy phenomenology stemming out from the general form of \( V_{\text{LEP}} \) [(37) and (38)] in SUGAR models. Very tight constraints coming from natural suppression of flavour changing neutral currents\(^20\) (FCNC), absence of large corrections to \((g-2)\)\(^51\) and \( \phi \)\(^39,52\) \((\equiv (M_H/M_\chi \cos \theta)^2\) as well as to \( 8_{\text{QCD}} \), which have been the nemesis\(^50\)-52\(^,3\) of SUSY models with arbitrary and explicit soft SUSY breaking, are satisfied in SUGAR models. The highly-constrained set of soft SUSY operators (27) in SUGAR models fits the bill\(^19,39\). Concerning FCNC, (38) guarantees the super-GIM mechanism, since the mass matrices for the quarks and leptons are diagonalized by the same transformation that renders the mass matrices for their scalar partners and gluino couplings generation diagonal. Despite the fact that this property does
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<td>HZ, HZ-inos</td>
<td>78</td>
<td>82</td>
<td>79</td>
<td>79</td>
<td>82</td>
<td>75</td>
</tr>
<tr>
<td>Axino</td>
<td>26</td>
<td>23</td>
<td>24</td>
<td>24</td>
<td>17</td>
<td>9</td>
</tr>
</tbody>
</table>

Physical mass spectrum of the cosmologically acceptable model (CAM) and the minimal model (MIM) corresponding to the same gravitino mass $m_{_{3/2}} \simeq 15$ GeV for top quark masses equal to 25 GeV, 35 GeV and 50 GeV respectively. $\zeta$ denotes the ratio of the gaugino to the gravitino mass at $M_X$. All masses are in GeV units. The light neutral Higgs gets its mass via radiative corrections.
not survive, in general, after renormalization, it has been shown\(^{53},^{54}\) that these effects are controllable. Furthermore, the Buras stringent upper bound\(^{55}\) on the top quark mass \([< 0(40 \text{ GeV})]\), coming from kaon phenomenology \((K_L-K_S \text{ and } K_L + \mu^+\mu^- \text{ systems})\), is avoided\(^{53}\) in SUGAR models. There are a lot of cancellations between ordinary and SUSY contributions in \(K\) processes\(^{56}\), such that the top quark mass may be stretched up to 100 GeV without problems\(^{33}\). That sounds very satisfactory, especially for SUGAR models\(^{38}-^{40}\) that do need a large top quark mass for \(SU(2) \times U(1)\) breaking. It looks like a self-service situation. Similar comments apply in the case of \((g-2)_\mu\) or \(\rho\), where it has been shown that SUGAR model contributions are acceptable\(^{34},^{39},^{52}\). Typical values for SUGAR contributions are\(^{34}\) \(\Delta(g-2)_\mu \leq (3 \times 10^{-9})\) and\(^{39},^{52}\) \(\Delta \rho \leq 0.01\), which compare favourably with the present experimental upper bounds of \((4 \times 10^{-9})\) and \((0.03)\) respectively, but are large enough to be interesting. Better experimental bounds, especially on \(\Delta \rho\), could be revealing.

Concerning \(\theta_{\text{QCD}}\), it has been shown\(^{19},^{57}\) that in SUGAR-type models, we not only understand the non-renormalization\(^{5}\) of \(\theta\), but we also understand\(^{19},^{57}\) why \(\theta\) is zero or small \(< 10^{-9}\) to start with. This fact is related with our freedom, discussed before, to use non-minimal \((\neq \delta_{\alpha\beta})\) \(f_{\alpha\beta}\) in \((15)\). It has already been observed\(^{18}\) that the gauge kinetic term in \((15)\), as well as giving rise to the canonical

\[
- \frac{1}{4} F^\alpha_{\mu
u} F^\beta_{\mu
u} (\not\omega \; f_{\alpha\beta})
\]

(43)

could also yield the CP-violating \(\theta\) vacuum term

\[
\mathcal{L}^{\mu\nu\rho\sigma} F^\alpha_{\mu\nu} F^\beta_{\rho\sigma} (\not\omega i \; f_{\alpha\beta})
\]

(44)

We know that \(\theta_{\text{QCD}}\) is \(< 0(10^{-9})\) experimentally, and that \(\theta\) is not renormalized in a supersymmetric theory\(^{5}\). It is finitely renormalized when supersymmetry is broken, but\(^{5}\) this is plausibly only by an amount \(\delta \theta = 0(10^{-16})\) in the popular Kobayashi-Maskawa model\(^{58}\). Thus we see that \(\theta\) should be less than \(0(10^{-9})\) in a supersymmetric GUT and may be very small. An attractive hypothesis is that \(f_{\alpha\beta}\) is a function with only real coefficients as found in extended supergravities\(^{59}\). In this case, \(\text{Im} f_{\alpha\beta} = 0\) when \(\langle 0 | \phi | 0 \rangle = 0\), and the theory is CP-invariant in the gauge sector. If some of the \(\phi\) then acquire complex vacuum expectation values, they will induce a non-zero value of \(\text{Im} f_{\alpha\beta}\) and hence violate CP spontaneously in the gauge sector, which is a new twist on an old proposal\(^{60}\). If the moduli of
some of these complex $<0|\phi|0>$ were $O(\Lambda_{\phi})$, then the effective $\Theta$
parameter would be $O(1)$ which is phenomenologically unacceptable.
However, it is easy to imagine scenarios where $\Theta$ is much smaller.
For example, if $\xi_{ab} = \delta_{ab} + O(\phi^2/m_{\phi}^2)$ and the culprit $<0|\phi|0> = 0(m_{\phi})$, then

$$\Theta = 0 \left( \frac{M_X}{M_W} \right)^2 < \text{small angle} \lesssim O(10^{-7})$$

(45)

and the phenomenological constraint on $\Theta_{QCD}$ could easily be res-
pected. If the only complex $<0|\phi|0>$ were $O(m_W)$, or if all the
$<0|\phi|0>$ were real as in all supersymmetric GUTs proposed to date,
then the bare $\Theta = 0$. Hence supergravity offers the other half of
an answer to the $\Theta$ vacuum problem. It should be stressed that low
energy supergravity models have new sources\(^{39,57,61,62}\) of CP
violation beyond the standard model. Unfortunately, they do not shed
more light on the smallness of the observed CP violation in the $K$
system\(^{61}\). On the other hand, potential large contributions\(^{61,57}\)
to $\xi^\prime$, $\Theta_{QCD}$ and to the Dipole Electric Moment Of the Neutron
(DEMON) put, in general, severe constraints\(^{57,61}\) on these new pos-
sible CP-violating phases. Actually, it seems almost unavoi-
dable\(^{57,39,57,62}\) that in SUGAR models the DEMON should be near, but
not above, the present experimental upper bound\(^{63}\) of $6 \times 10^{-25}$
e.cm, a rather drastic and experimentally testable prediction, in sharp
contrast with the standard model prediction $O(10^{-30}$ e.cm). We may
know soon.

Turning now to baryon decay, an interaction of the form\(^{19}\)

$$f \not\equiv \frac{2}{M} \tilde{F} T T T$$

(46)

where $\tilde{F}$ is a $\tilde{5}$ of matter (quark + lepton) chiral superfields in
$SU(5)$, $T$ is a $10$ of matter superfields and $\lambda$ is some generic
Yukawa coupling. Could replace the Higgs exchange in the Weinberg-
Sakai-Yanagida\(^{64}\) loop diagram for baryon decay. The magnitude of
the diagram with (46) relative to the conventional Higgs diagram is:

$$\left( \frac{g}{\sqrt{M}} \right) / ( \frac{2}{M_{H3}} ) \approx 0$$

(47)

The ratio (47) could easily be $>1$, making a non-renormalizable
superpotential interaction the dominant contribution to proton decay.
A careful analysis of SUGAR-induced baryon decay shows\(^{65}\),
surprisingly enough, that the expected hierarchy of decay modes is similar to that\textsuperscript{(32)} coming from conventional minimal SUSY GUTs. One might have wrongly expected that no hard and fast predictions could be made about gravitationally-induced baryon decay modes. Anyway, this mechanism could give observable baryon decay even if the GUT mass $M_X \approx M$.

Incidentally, similar terms like (46) have been considered\textsuperscript{(66)} in efforts to explain the "lightness" of the first two generations of quarks and leptons. One replaces\textsuperscript{(66)} direct Yukawa couplings for the first two generations with (very schematically):

$$\begin{align*}
\mathcal{L} &\supset \frac{\tilde{H}}{M} \Sigma T_2 \bar{f}_2 + \frac{\tilde{H}'}{M} H T_2 T_2 \Sigma \\
&\quad + \frac{\tilde{f}_1}{M^2} \Sigma^2 T_2 \bar{f}_1 + \frac{\tilde{f}_1'}{M^2} H T_2 T_2 \Sigma^2 + \ldots
\end{align*}$$

(48)

which not only repairs\textsuperscript{(19),(66)} wrong relations like $m_d(M_X) \approx m_e(M_X)$, very difficult to correct\textsuperscript{(67)} in conventional SUSY GUTs, but also provides reasonable masses for the first two generations. Indeed, it follows from (48) that the second generation is getting masses $(M_X/M)_Y \sim (0.1-1 \text{ GeV})$, while the first generation masses are $(M_X/M)_Y \sim (1-10 \text{ MeV})$, exactly what was ordered\textsuperscript{(66)}. It is amazing that in SUGAR models, by increasing $M_X(\sim 10^{14} \text{ GeV})$, relative to its ordinary GUT value $(N_X^{14} \text{ GeV})$, and by decreasing $M_Y$, what is relevant is the superPlanck scale $M(\sim 10^{18} \text{ GeV})$, the highly-desired ratio $(M_X/M) \sim 10^{-2}$, appears naturally. It seems now, for the first time, that gravitational interactions may be responsible for the masses of at least the first two generations. Once more, non-renormalizable interactions contained in SUGAR models provide a simple solution\textsuperscript{(66)} to another hierarchy problem, the fermion mass hierarchy problem.

4. **FINAL REMARKS**

We have shown that gravitational effects, as contained in SUGAR theories, cannot be neglected anymore in the regime of particle physics. On the contrary, it may be that supergravitational effects are really responsible: for the SU(5) breaking at $M_X$ with an automatic triplet-doublet Higgs splitting, for the SU(2) $\times$ U(1) breaking \[ (\text{and SU}(3) \times U(1)_{\Xi - \Xi} \text{ non-breaking}) \] at $M_Y$, naturally exquisitely smaller than $M_X$, for the "constrained" soft SUSY breaking at $M_3/2$, hierarchically smaller than $M$ in a natural way, for definite, at present experimentally acceptable departures from the
"standard" low energy phenomenology [like the DEMON, or $\Delta \rho$, with values below, but not far from, their present experimental upper bounds, or the existence of very light (< 0(10 GeV)) neutral Higgs bosons], as well as a rather well-defined low-energy SUSY spectrum, for observable baryon decay even if $M_X = M$, and for the light fermion masses of the first two generations. Furthermore, supergravity theories may provide, for the first time, a problem-free cosmological scenario, from primordial inflation through GUT phase transitions to baryon and nucleosynthesis, ostracizing troublesome particles such as GUT monopoles, gravitinos, Polonyi fields or other SUSY relics.

Putting the whole thing together, it becomes apparent that spontaneously broken $N = 1$ local SUSY gauge theories, with their prosperous and appropriate structure, may well serve as an effective theory describing all physics from $M_{Pl}$ down to (and including) low energies, with well-defined and rich experimental consequences. What's next, then? Well, we really have to understand where this highly successful theory comes from. There are reasons to believe that $N = 8$ extended SUPERGRAVITY may provide the fundamental theory. But this next move asks for a deep understanding of physics at Planck energies, which is as exciting as it is difficult, taking into account that even QUANTUM MECHANICS may need modification, if quantum gravitational effects have to be considered seriously.

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