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NEUTRINOLESS DOUBLE ELECTRON CAPTURE AS A TOOL TO MEASURE
THE ELECTRON - NEUTRINO MASS

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A B S T R A C T

A nucleus (Z, A) may capture two atomic electrons to become $(Z-2, A)$. For Majorana neutrinos this may occur with no neutrino emission, the process is a virtual mixing of the parent atom and the daughter atom with two electron holes. The process becomes real as the daughter atom de-excites. In some cases where the daughter nucleus is excited, the neutrinoless decay may be enhanced by its proximity to a virtual resonance. We identify the $^{112}\text{Sn} \rightarrow ^{112}\text{Cd}$ transition as a good case. The no-neutrino lifetime for $m_\nu = 30 \text{ eV}$ ranges from 10^{22} to 10^{27} years as a function of the insufficiently well determined distance to resonance. The signatures of the two- or no-neutrino modes are very different.

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1. INTRODUCTION

The mass spectrum of today's "elementary" particles is presumably their most baffling mystery. For neutrinos, the problem is particularly acute. On the theoretical side, we have no compelling theory of neutrino masses; nor do we know of a convincing principle why neutrinos should be massless. On the experimental front most results are in the form of upper limits on neutrino masses, but for the tantalizing result of the ITEP group¹⁾ on the mass of the (anti)neutrino emitted in tritium β -decay:

$$14 \text{ eV} \leq m(\bar{\nu}_e) \leq 46 \text{ eV} ; \quad 99\% \text{ conf. level [Lubimov et al.}^{1)}] \quad (1.1)$$

To add to our ignorance, we do not know whether a possible neutrino mass term in the Lagrangian is lepton number conserving or not. In other words, we do not know whether neutrinos are "Dirac" particles (distinct from their antiparticles, like neutral kaons) or "Majorana" particles (identical to their antiparticles, like neutral pions). In this paper we investigate in some detail a process: double electron capture, that, like double β -decay, simultaneously addresses the issues of neutrino masses and lepton number conservation.

Only a year after the publication of Fermi's theory²⁾ of β -decay, Maria Goeppert-Mayer³⁾ used it to compute the lifetimes of atoms decaying by double β -emission:

$$(Z, A) \rightarrow (Z+2, A) + 2e^- + 2\nu \quad (1.2)$$

She estimated disappointingly long lifetimes, of the order of 10^{25} years. It was Wendal Furry⁴⁾ in 1939 who realized that Majorana neutrinos could be exchanged between the decaying neutrons, rather than emitted in the decay:

$$(Z, A) \rightarrow (Z+2, A) + 2e^- \quad (1.3)$$

Neutrino-less double β -decay has much less of a phase space suppression than the two neutrino processes of Eq. (1.2), possibly resulting in lifetimes many orders of magnitude shorter. As the knowledge on the chiral properties of weak interactions and the limits on neutrino masses improved, the theoretical expectation for the rate of neutrino-less double β -decay decreased. This is because the neutrino exchange amplitude vanishes for $m_\nu = 0$ and purely left-handed currents. The experimental situation on double β -decay is confusing and inconclusive: we do not attempt to summarize it here, and refer the reader to recent articles^{5,6)}.

In 1955, Rolf. G. Winter⁷⁾ estimated the lifetimes of otherwise stable isotopes that could slowly decay by a process that is the inverse of double β -decay:

$$\begin{aligned} (Z, A) &\rightarrow (Z-2, A)^{*HH'} + 2\nu_e \\ &\quad \downarrow \\ &\quad (Z-2, A) + \dots \end{aligned} \tag{1.4a}$$

$$\begin{aligned} (Z, A) &\rightarrow (Z-2, A)^{*HH'} \\ &\quad \downarrow \\ &\quad (Z-2, A) + \dots \end{aligned} \tag{1.4b}$$

The processes underlying the first rows in Eqs. (1.4a,b) are depicted in Figs. (1a,b), respectively. In both cases two atomic electrons are captured by two protons and turn into two neutrons. In the first case, two neutrinos are emitted. In the second, a Majorana neutrino is exchanged. The symbols H, H' label the vacant orbitals, or "holes", in the unstable daughter atom; the levels from which the electrons were captured. The stars in Eq. (1.4) refer to the possibility that the daughter nucleus be left in an excited state. The daughter atom $(Z-2)^{*HH'}$ de-excites by the emission of atomic X-rays (and/or Auger electrons) and possibly nuclear γ -rays (or other forms of nuclear debris). The dots in Eq. (1.4) refer to all of these radiations. It is the NONDEC (no-neutrino double electron capture) process of Eq. (1.4b) that is of interest to the question of massive Majorana neutrinos. The TWONDEC (two neutrino double electron capture) process of Eq. (1.4a) is of much less fundamental significance.

The energetics of the DEC process is depicted in Fig. 2, where the mass difference between ground state atoms $M(Z) - M(Z-2)$ is labelled Q. The quantity Q is the difference between Q_β and Q_{EC} , the corresponding mass differences in the β -decay and electron-capture decay of the "in-between" atom (Z-1). The NONDEC decay of Eq. (1.4b) can be visualized as a two step process, as depicted in Fig. 3b. "First" the neutrino exchange diagram of Fig. 1b induces a mixing between the atoms Z and $(Z-2)^{*HH'}$. "Subsequently" the unstable daughter state $(Z-2)^{*HH'}$ decays by electron or photon emission. The mixing amplitude is second order in the Fermi interaction, much as in the $K_0 - \bar{K}_0$ system, and can also be thought of as a non-diagonal mass term, ΔM , in a two-dimensional atomic mass matrix. Let $E^*, E_{H,H'}$ label the excitation energy of the daughter nuclear state and the (positive) ionization energies of the H, H' electronic vacancies. The total excitation energy and width Γ of the $(Z-2)^{*HH'}$ state are

$$E \approx E^* + E_H + E_H', \quad (1.5a)$$

$$\Gamma \approx \Gamma^* + \Gamma_H + \Gamma_{H'}, \quad (1.5b)$$

The intermediate daughter atom in Fig. 3b is off the energy shell by $Q-E$. The NONDEC decay rate is of the form

$$\frac{1}{\tau} \approx \frac{(\Delta M)^2}{(Q-E)^2 + \Gamma^2/4} \Gamma \quad (1.6)$$

where ΔM is the atom-mixing amplitude, to be explicitly computed below for a favourable isotope. In Eq. (1.6) we have used the width Γ as given by Eq. (1.5b), and thus neglected small corrections due to phase space (the intermediate state is not precisely on-shell) and to the interaction between the three sources of instability of the daughter atom. Clearly, the most interesting cases are those for which $Q-E$ is closest to zero and the lifetimes are correspondingly short. Since atomic mass differences are of the order of MeV's and Γ is of the order of eV's, to find a case of perfect degeneracy ($Q-E \lesssim \Gamma$) would be, as R.G. Winter⁷⁾ put it, "a monumental coincidence". There are, however, a handful of cases that are degenerate within present experimental errors, typically in the keV range. Their importance has recently been re-emphasized by Georgi et al.⁸⁾; and some interesting cases, perhaps not the best from the point of view of degeneracy, have been analyzed by Vergados⁹⁾.

The signature for DEC could be indirect (the abundance of $Z-2$ atoms in Z -ores) or direct [the X-rays and γ -rays emitted by $(Z-2)^{*HH'}$]. The TWONDEC process characteristically proceeds with a highest branching ratio into the nuclear ground state $(Z-2)^{HH'}$, yielding X-rays of tens of keV's but not γ -rays in the MeV range. As the hoped-for degeneracy that would speed up the NONDEC process becomes better, the TWONDEC process into $(Z-2)^{*HH'}$ is strongly suppressed by phase space. Observation of the characteristic γ -rays of the daughter nucleus ultimately becomes a signature for NONDEC. Since the total energy release in the process is Q , and one does not expect it to be precisely equal to $E = E^* + E_H + E_{H'}$, some of the radiation from $(Z-2)^{*HH'}$ must occur at the "wrong" energy, shifted from its "official" value by $Q-E$. It is stated in Refs. 8 and 9 that this mismatched energy should be the one of the energetic γ -ray. It is in fact the less readily observable atomic radiation that is emitted with the wrong energy. The reason is that, at least in the cases where data exist, $\Gamma_H \gg \Gamma^*$. The atomic orbitals de-excite much before the nucleus "realizes" it, too is unstable.

All examples of candidates for DEC measurements have the structure shown in Fig. 2, and correspond to the Z and Z-2 nuclei having even-even, $J^P = 0^+$ ground states. In Table 1 we give a non-exhaustive list of candidates, selected to have a non-negligible fractional natural abundance and a small figure of merit: the distance to resonance, Q-E. Less favourable examples of parent nuclides are ^{36}Ar , ^{40}Ca , ^{50}Cr , ^{54}Fe , ^{58}Ni , ^{64}Zn , ^{84}Sr , ^{92}Mo , ^{96}Ru , ^{108}Cd , ^{120}Te , ^{126}Xe , ^{132}Ba , ^{136}Ce , ^{138}Ce , ^{144}Sm , ^{156}Dy , ^{158}Dy , ^{174}Hf , ^{184}Os and ^{190}Pt .

Table 1

DEC transitions, abundance, daughter nuclear excitation, atomic vacancies and figure of merit of some isotopes¹⁰⁾.

Transition $Z \rightarrow Z - 2$	Z-natural abundance in %	Nuclear excitation E^* (in MeV), J^P	Atomic vacancies H, H'	Figure of merit Q-E (in keV)
$^{74}_{34}\text{Se} \rightarrow ^{74}_{32}\text{Ge}$	0.87	1.204 (2^+)	2S(P), 2S(P)	2 ± 3
$^{78}_{36}\text{Kr} \rightarrow ^{78}_{34}\text{Se}$	0.36	2.839 (2^+) 2.864 (?)	1S, 1S	19 ± 10 -6
$^{102}_{46}\text{Pd} \rightarrow ^{102}_{44}\text{Ru}$	1	1.103 (2^+) 1.107 (4^+)	1S, 1S	29 ± 9 25
$^{106}_{48}\text{Cd} \rightarrow ^{106}_{46}\text{Pd}$	1.25	2.741 (?)	1S, 1S	-8 ± 10
$^{112}_{50}\text{Sn} \rightarrow ^{112}_{48}\text{Cd}$	1.01	1.871 (0^+)	1S, 1S	-3 ± 10
$^{130}_{56}\text{Ba} \rightarrow ^{130}_{54}\text{Xe}$	0.11	2.502 (?) 2.544 (?)	1S, 1S 1S, 2S(P)	8 ± 13 -6
$^{152}_{64}\text{Gd} \rightarrow ^{152}_{62}\text{Sm}$	0.20	0 (0^+)	1S, 2S	4 ± 4
$^{162}_{68}\text{Er} \rightarrow ^{162}_{66}\text{Dy}$	0.14	1.783 (2^+)	1S, 2S	1 ± 6
$^{164}_{68}\text{Er} \rightarrow ^{164}_{66}\text{Dy}$	1.56	0 (0^+)	2S, 2S	9 ± 5
$^{168}_{70}\text{Yb} \rightarrow ^{168}_{68}\text{Er}$	0.14	1.355 (1^-) 1.393 (?)	1S, 2S 2S, 2S	-1 ± 4 8
$^{180}_{74}\text{W} \rightarrow ^{180}_{72}\text{Hf}$	0.13	0 (0^+) 0.093 (2^+)	1S, 1S 1S, 3S	26 ± 17 -4
$^{196}_{80}\text{Hg} \rightarrow ^{186}_{78}\text{Pt}$	0.15	0.689 (2^+)	1S, 2S	25 ± 9

The parameters that govern both NONDEC and neutrino-less double β -decay are the Majorana neutrino mass m_ν and the interference amplitude between right- and left-handed electron-to-neutrino weak couplings, η_{LR} . The NONDEC rate is much more sensitive to m_ν than to η_{LR} , in comparison to the $\beta\beta$ process. This is because the η_{LR} amplitudes are linear in electron momenta. The momenta in $\beta\beta$ decay are of the order of magnitude of the corresponding Q value, while in NONDEC they are of the order of magnitude of the much smaller atomic momenta. We will thus parametrize the NONDEC rate only in terms of m_ν . As we shall explicitly see in the next chapter, the leading NONDEC mechanism involves the capture of $L = 0$ electrons and is governed by an effective interaction that is a Lorentz scalar both in leptonic and hadronic variables. The favoured processes are therefore $0^+ \rightarrow 0^+$ nuclear transitions. If to improve the signature we demand that the transition be to an excited nuclear state (with established 0^+ quantum numbers), Table 1 narrows down to a single candidate: $^{112}\text{Sn} \rightarrow ^{112}\text{Cd}$. This is the case we shall treat in detail. An obvious remark is that it would be useful to improve upon the Q value measurements in the electron capture process $^{112}\text{In} \rightarrow ^{112}\text{Cd}$ ($Q_{EC} \approx 2578 \pm 8$ keV)^{10,11)} and the β -decay process $^{112}\text{In} \rightarrow ^{112}\text{Sn}$ ($Q_\beta = 0.658 \pm 6$ keV)^{11,12)} from which $Q(\text{DEC})$ is inferred. A second candidate that may be of interest is $^{130}\text{Ba} \rightarrow ^{130}\text{Xe}$. Here it would be useful to know whether either of the 2.502 MeV or 2.544 MeV nuclear states of ^{130}Xe is a 0^+ . This is a candidate for "geological" experiments, for after all ^{130}Xe is also the $\beta\beta$ decay product of ^{130}Te , an isotope for which such experiments have been carried out¹²⁾. Unfortunately the ^{130}Ba abundance in natural Ba is only 1%, while the corresponding number for ^{130}Te is 34.5%. We do not know whether the Ba abundance in Ba ores competes favourably with the corresponding Te abundance.

In the rest of this paper we concentrate on ^{112}Sn decay. In Section 2 we compute the atomic mass-mixing parameter ΔM that governs the NONDEC process. In Section 3 we compute the rate of the TWONDEC process. Results and conclusions are given in Section 4. We find that the TWONDEC lifetime of ^{112}Sn is of the order of 10^{23} years, while the NONDEC lifetime varies in the range $\sim 3 \times 10^{21}$ to $\sim 10^{27}$ years as the degeneracy parameter $|Q-E|$ varies from zero to its maximum allowed value: ~ 10 keV within errors.

2. ATOM MIXING

The matrix element for the Majorana neutrino exchange diagram of Fig. 1b is

$$S_{fi} = -i \left(\frac{G_F \cos \theta_c}{\sqrt{2}} \right)^2 2m_\nu \frac{\bar{v}_e(p_1)(1-\gamma_5)u_e(p_2)}{2V\sqrt{E_1 E_2}} * \\ \frac{1}{(2\pi)^4} \int d^4q d^4x d^4x' \frac{\exp[i(q-p_1)x' - i(q+p_2)x]}{q^2 - m_\nu^2} \langle Z-2 | T(J_\lambda(x) J^\lambda(x')) | Z \rangle \quad (2.1)$$

where the momenta are labelled as in the figure, J_λ are hadronic weak currents and the rest of the notation is conventional. In writing the above expression we have used Fermi statistics to keep only the symmetric part ($\sim g^{\lambda\rho}$) of the leptonic matrix element of $\gamma^\lambda \gamma^0 (1-\gamma_5)$. Atomic electron momenta are negligible in the scale of inverse nuclear radii: the allowed approximation $\exp\{i(\vec{p}_1 \vec{x}' - \vec{p}_2 \vec{x})\} = 1$ is an excellent one. In this approximation, moreover

$$\frac{\bar{v}_e(p_1)(1-\gamma_5)u_e(p_2)}{2V\sqrt{E_1 E_2}} \simeq |\psi(0)|^2 \delta_{S0} \quad (2.2)$$

with the square of the electron wave functions $\psi(x)$ normalized to a unit d^3x integral, and $\delta_{S,0}$ a reminder that only electron pairs in a singlet spin state may be captured. We perform the $dq_0 dx_0 dx'_0$ integrals to obtain

$$S_{fi} = -2\pi i \delta(E_i - E_f) M_{fi} \\ M_{fi} = -G_F^2 \cos^2 \theta_c |\psi(0)|^2 \delta_{S0} m_\nu * \\ * \frac{1}{(2\pi)^3} \int d^3q d^3x d^3x' \exp\{i\vec{q}(\vec{x} - \vec{x}')\} \sum_n \frac{\langle Z-2 | J_\lambda(\vec{x}) | n \rangle \langle n | J^\lambda(\vec{x}') | Z \rangle}{|\vec{q}|(|\vec{q}| + E_n - M_Z - m_e)} \quad (2.3)$$

where we have introduced a complete set of intermediate states, and approximated neutrino energy by neutrino momentum. In the above expression the relevant neutrino momenta have a magnitude $|\vec{q}| \sim |\vec{x} - \vec{x}'|^{-1} \sim r^{-1}$, with r the nuclear radius. Relative to these values of $|\vec{q}|$, one may neglect typical nuclear excitation energies $E_n - M_Z - m_e \simeq 10$ MeV. Using this we may further simplify the matrix element:

$$m_{fi} = \frac{-G_F^2 \cos^2 \theta_c}{4\pi} m_\nu |\varphi(0)|^2 \delta_{S0} \int \frac{d^3x d^3x'}{|\vec{x} - \vec{x}'|} \langle Z-2 | J^\lambda(\vec{x}) J^\lambda(\vec{x}') | Z \rangle \quad (2.4)$$

Now that we have reduced the problem to the evaluation of a nuclear matrix element, it may be time to state what m_ν stands for if the electron neutrino is a superposition of "light" mass eigenstates ν_j :

$$\nu_e = \sum_j U_{ej} \nu_j \quad (2.5a)$$

$$m_\nu \equiv \sum_j (U_{ej}^*)^2 m(\nu_j) ; \quad m(\nu_j) \ll 1. \quad (2.5b)$$

We proceed to estimate the nuclear matrix element of Eq. (2.4) in the specific case of the $^{112}\text{Sn} \rightarrow ^{112}\text{Cd}$ transition. We employ the impulse approximation, in which the weak current matrix elements are sums over individual nucleons, and work in the non-relativistic limit for the motion of the nucleons. Only the time (space) components of vector (axial) currents survive, associated with Fermi (Gamow-Teller) matrix elements. Fermi transitions are suppressed unless the initial and final nuclei are in analog states with the same isospin -- not the case for $^{112}\text{Sn}, \text{Cd}$. We are thus led to consider the axial current

$$J^\lambda(\vec{x}) \rightarrow \vec{j}(\vec{x}) = g_A \sum_{n=1}^A \tau_n^- \vec{\sigma}_n \delta(\vec{x} - \vec{x}_n) \quad (2.6)$$

where g_A is the axial coupling of the nucleon and the rest of the notation is also conventional. Upon insertion of Eq. (2.6), the factor to the right of $\delta_{S,0}$ in Eq. (2.4) becomes

$$g_A^2 \text{M.E.} = g_A^2 \langle Z-2 | \sum_{n,m} \tau_n^- \tau_m^- \vec{\sigma}_n \vec{\sigma}_m \frac{1}{|\vec{x}_n - \vec{x}_m|} | Z \rangle \quad (2.7)$$

and is of order $\langle r^2 \rangle^{-1/2}$, with r the nuclear radius. To estimate M.E. for the case at hand, we have to descend to more nuclear detail.

The ground state nucleus of ^{112}Sn can be described as 50 protons filling magic closed shells up to the $1g_{7/2}$ configuration, and a 64 nucleon core with two holes in the $1g_{7/2}$ shell. Almost degenerate in mass with this state is the atom with two K-vacancies and a nucleus in the $^{112}_{48}\text{Cd}$ (1.871 MeV) excited state, see

Table 1. There is evidence that this excited state is a 0^+ three-phonon collectively excited level, with the same "intrinsic" nucleon configuration as the ground state: closed shells of neutrons up to the $1g_{7/2}$ level, and 50 protons in closed shells minus two proton holes in the $1g_{9/2}$ level. (An explicit description of the vibrational band structure of ^{112}Cd levels can be found in Ref. 14.) We thus approximate the NONDEC transition of interest as

$$(1g_{7/2}^2)^{-1}_n \rightarrow (1g_{9/2}^2)^{-1}_p \quad (2.8)$$

with two holes in a 0^+ state in an inert core¹⁵⁾ corresponding to $^{114}_{50}\text{Sn}$. The final state nucleus is in a quantized vibrational mode departing from the spherical ground state shape. This collective excitation is an adiabatic perturbation in the sense of being describable as the motion of nucleons in an average potential well, whose changes in shape and orientation are slow on the characteristic time scale of individual nucleon orbitals. As only the surface nucleons of Eq. (2.8) intervene in the NONDEC transition, we neglect in our estimate the perturbation induced by the surface motion on the nucleon wave functions. The adequacy of this approximation is confirmed by inspection¹⁰⁾ of the $\log(ft)$ values in the β^+ decays of $^{112}_{49}\text{In}$ into the different levels of the vibrational band of $^{112}_{48}\text{Cd}$: they are practically the same, independent of the different phonon structures.

We proceed to estimate the matrix element of Eq. (2.7) in the framework of the approximations just discussed. The spin part is computed with use of the j - j coupling basis for the two nucleon holes. The spatial wave functions of the individual nucleons are $\ell = 4$ spherical harmonics times radial $1g$ wave functions. The latter are approximated as harmonic oscillator solutions with a parameter $\alpha = M\omega/2$ to be empirically adjusted to the nuclear mean-square radius. We follow Hornyak¹⁶⁾ in parametrizing the oscillator frequency ω as

$$\omega \approx \frac{53}{A^{1/3} + 1.41} \text{ MeV} \quad (2.9)$$

to reproduce the empirical value $\langle r^2 \rangle^{1/2} \sim (0.82 A^{1/3} + 0.58) \text{ fm}$, valid for $A > 8$. In our case $A=112$, $\alpha^{1/2} = 63 \text{ MeV}$ and $\langle r^2 \rangle^{1/2} = 5.4 \text{ fm}$. After a lengthy but straightforward integration of radial and angular variables we obtain, for the matrix element of Eq. (2.7)

$$\text{M.E.} [^{112}_{48}\text{Cd}(0^+), ^{112}_{50}\text{Sn}(0^+)] = 5.1 \langle r^2 \rangle^{-1/2} \quad (2.10)$$

and for the atomic mass mixing parameter appearing in Eq. (1.6), the value

$$\Delta M = \left(\frac{G_F \cos \theta_c}{\sqrt{2}} \right)^2 |\varphi(0)|^2 m_\nu \frac{g_A^2}{2\pi} |M.E.| \quad (2.11)$$

In the Coulomb approximation, which is a fair one for $n = 1$ orbitals, $|\varphi(0)|^2 = (Z\alpha m_e)^3/\pi$ and the numerical value of Eq. (2.11) is

$$\Delta M \approx 1.9 \cdot 10^{-22} \text{ eV} \left(\frac{m_\nu}{30 \text{ eV}} \right) \quad (2.12)$$

This value is two orders of magnitude smaller than the rough estimate of Georgi et al.⁸⁾ and, for a "typical" value $m_\nu = 30 \text{ eV}$, it is some sixteen orders of magnitude smaller than the $K_L - K_S$ mass difference, reflecting the smallness of the electron mass and of the assumed neutrino mass on the scale of hadron masses and inverse radii.

Before we discuss the NONDEC signature and the rates corresponding to Eqs. (2.12) and (1.6) we devote a section to the competing TWONDEC process.

3. DOUBLE ELECTRON CAPTURE WITH EMISSION OF TWO NEUTRINOS

The S-matrix element corresponding to the TWONDEC diagram of Fig. 1a is

$$S_{fi} = \left(\frac{G_F \cos \theta_c}{\sqrt{2}} \right)^2 \frac{\bar{u}(p_2)(1-\gamma_5)u(p_1)}{2V\sqrt{p_2^0 p_1^0}} \frac{\bar{u}(k_1)\gamma^\lambda(1-\gamma_5)\gamma^\rho v(k_2)}{2V\sqrt{k_1^0 k_2^0}} * \quad (3.1)$$

$$* \int d^4x d^4x' \exp \{ i(k_1 - p_1)x_1 + i(k_2 - p_2)x_2 \} \langle Z-2 | T(J_\lambda(x_1) J_\rho(x_2)) | Z \rangle$$

We perform the time integrations and, as in the NONDEC case, we select the dominant symmetric piece of $\gamma^\lambda(1-\gamma_5)\gamma^\rho$ and use the "allowed" approximation $\exp \{ i(\vec{k}_1 - \vec{p}_1) \cdot \vec{x}_1 \} = 1$. This allows us to write the transition matrix element in the form:

$$M_{fi} = \left(\frac{G_F \cos \theta_c}{\sqrt{2}} \right)^2 |\varphi(0)|^2 \delta_{S0} \frac{\bar{u}(k_1)(1+\gamma_5)v(k_2)}{2V\sqrt{k_1^0 k_2^0}} \{M.E.\} \quad (3.2a)$$

$$M.E. \equiv \int d^3x_1 d^3x_2 \sum_n \frac{1}{2} \left\{ \frac{\langle Z-2 | J_\lambda(\vec{x}_1) | n \rangle \langle n | J^\lambda(\vec{x}_2) | Z \rangle}{k'_0 + E_n - M_Z - m_e - i\epsilon} + (x_1 \leftrightarrow x_2, k'_1 \leftrightarrow k'_2) \right\} \quad (3.2b)$$

where we have introduced a complete set of intermediate states into the definition of M.E. To estimate M.E. we employ the usual trick of substituting the energy denominators by a mean excitation energy $\bar{\omega}$ and we retain only the dominant "Gamow-Teller" space components of the axial current:

$$M.E. \sim \frac{g_A^2}{\bar{\omega}} \langle Z-2 | \sum_{n,m} \tau_n^- \tau_m^- \vec{\sigma}_m \vec{\sigma}_n | Z \rangle \quad (3.3)$$

For the case we are concentrating upon, $^{112}_{50}\text{Sn} \rightarrow ^{112}_{48}\text{Cd}$, the active "nucleons" (two $lg_{7/2}$ neutron holes that become two $lg_{9/2}$ proton holes) have the same principal and orbital quantum numbers: Eq. (3.3) is non-vanishing and its calculation reduces to simple angular-momentum combinatorics. The result, in a j-j coupling scheme, is:

$$\left| \langle ^{112}_{48}\text{Cd}(0^+) | \sum_{m,n} \tau_m^- \tau_n^- \vec{\sigma}_m \vec{\sigma}_n | ^{112}_{50}\text{Sn}(0^+) \rangle \right| = \frac{240}{\sqrt{5}} W\left(\frac{7}{2} \frac{1}{2} \frac{9}{2} \frac{1}{2} \middle| 41\right) = \frac{80}{9\sqrt{5}} \quad (3.4)$$

where W is a Racah coefficient.

The TWONDEC ^{112}Sn is dominated, as is the case for NONDEC, by transitions into 0^+ daughter nuclear states of excitation energy E^* , with the two electrons captured from $L = 0$ orbitals. The corresponding TWONDEC partial decay rate is computed from Eqs. (3.2), (3.3), and (3.4) to be:

$$\Gamma_{2\nu} = \frac{1}{4\pi^3} |\psi(0)|^4 (G_F \cos\theta_c)^4 \int_0^{Q-E} k_0^2 (Q-E-k_0)^2 |M.E.|^2 dk_0 = \quad (3.5a)$$

$$= m_e \left(\frac{2}{3\pi}\right)^5 (G_F \cos\theta_c m_e^2)^4 \left(\frac{Q-E}{m_e}\right)^5 (Z\alpha)^6 \left(\frac{m_e}{\bar{\omega}}\right)^2 g_A^4 \quad (3.5b)$$

where E is given by Eq. (1.5a) and, in the last expression, we have specified $\Gamma_{2\nu}$ for the dominant capture of two 1s electrons. The spectrum of low-lying excited levels of ^{112}Cd is shown in Fig. 4. TWONDEC decay of ^{112}Sn is energetically allowed into all daughter nuclear states below (and perhaps including) the almost

degenerate state $0^+(1.871)$. Decay into states with $J^P \neq 0^+$ is strongly suppressed by angular momentum. Decay into 0^+ states other than the ground state is suppressed by the $(Q-E)^5$ phase space factor in Eq. (3.5); the TWONDEC branching ratio into $0^+(1.2242)$, for instance, is $\sim 0.5\%$. Thus the ^{112}Sn TWONDEC signatures are the two K-X-rays of the daughter Cd atom which, unlike for NONDEC decay, should have the "correct" energy and be unaccompanied by nuclear γ -rays. All this is very beautiful, if not because $\Gamma_{2\nu}^{-1} \sim 1.5 \times 10^{23}$ years, as given by Eq. (3.5) with $\bar{\omega} = 10$ MeV.

4. RESULTS AND CONCLUSIONS

Double electron capture with the emission of two neutrinos, TWONDEC, should occur whenever energetically allowed, with a lifetime given by Eq. (3.5a). For the specific case we have studied, ^{112}Sn , the TWONDEC capture lifetime is 0 (10^{25} years). Partially responsible for this very long lifetime is the small energy release in this particular decay. A TWONDEC decay may in principle be detected via the characteristic K-X rays of the daughter atom. These ought to appear in pairs, with their energies slightly shifted by satellization.

Double electron capture with no neutrino emission, NONDEC, may occur if the electron neutrino has a non-vanishing Majorana mass. This process is resonance-enhanced if the daughter atom with two electron holes is degenerate in energy with the parent atom. The best case we have found, which may well not be the best existing case, is the "virtual" decay or atom mixing $^{112}\text{Sn} \rightarrow ^{112}\text{Cd} [0^+(1.871)]$, with the daughter state "leaking" via the de-excitation of its two 1s electron holes and unstable nucleus. The NONDEC signature are the daughter-nucleus γ -rays, with energies and branching ratios as given in Fig. 4, accompanied by Cd K-X rays, one of which may have its energy shifted by the mismatch in energy between the parent atom and the almost degenerate virtual daughter state. The lifetime for the overall process is given by Eq. (1.6). The degeneracy parameter $Q-E$ is in this case -3 ± 10 keV. The mixing mass ΔM is estimated in Eq. (2.12) and the daughter state width of Eq. (1.56) is of order $\Gamma \sim 20$ eV. The computed NONDEC lifetime, as a function of $|Q-E|$, is plotted in Fig. 5, for $m_\nu = 30$ eV. Even for "perfect degeneracy" $|Q-E| \lesssim \Gamma$, the predicted NONDEC lifetime is long: 0 (10^{22} years). For $Q-E > 1$ keV the NONDEC lifetime becomes desperately long, 10^{26} years or more. The shorter lifetimes unfortunately demand very good degeneracy. For $|Q-E| > 200$ eV, $\tau(\text{NONDEC}) > 10^{24}$ years. Though our predictions for ^{112}Sn NONDEC lifetimes may be as much as 14(10) orders of magnitude shorter than the ones computed by Vergados⁹⁾ for ^{58}Ni , ^{92}Mo (^{96}Ru) decay, this considerable improvement may not be enough to make the experiments easily feasible.

Optimistically assume that $Q-E \sim 100$ eV. In this case, as can be read off Fig. 5, the NONDEC lifetime is comparable to the TWONDEC lifetime ~ 0 (10^{23} years). Fifty decays of each type, with clearly distinguishable signatures, would take place per year and per kilogram of ^{112}Sn (~ 100 kg of natural tin). An experiment capable of detecting ^{112}Sn TWONDEC is quite thinkable, whereas the NONDEC process may become unobservable for $m_\nu < 30$ eV or $|Q-E| > 100$ eV, the size of the necessary source increasing quadratically with $(Q-E)/m_\nu$. An improved measurement of the β -decay and electron capture Q values in ^{112}In decay (a determination of the figure of merit $Q-E$) is a prerequisite to the decision of whether the NONDEC decay of ^{112}Sn may shed light on the fundamental question of the nature and mass of electron neutrinos.

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- 15) The numerical adequacy of this description is difficult to assert. The spherical equilibrium shape of the two active nucleon holes, relatively tightly bound in a O^+ pair, is an argument against the presence of an important configuration mixing, which would invalidate the "active nucleons" approximation. We have evaluated in the same approximation the β -decay and electron capture rates of ^{112}In into ^{112}Sn and ^{112}Cd . The theoretical rates are overestimates by factors of 2 and 7, respectively. Here, the parent In nucleus is in an odd-odd 1^+ state and the "active nucleon" approximation may be worse than for the $\text{Sn} \rightarrow \text{Cd}$ case. We do not expect the rate calculation of the DEC processes to be in error by more than an order of magnitude.
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Figure captions

- Fig. 1 : Diagrams describing double electron capture. a) with emission of two neutrinos. b) with no neutrino emission.
- Fig. 2 : Energetics and transitions in double electron capture (DEC). The atom denoted Z is stable but for DEC into the ground state of $Z-2$ with two electron holes (of energy $E_H + E_{H'}$); or to a state whose nucleus is also excited to a level of energy E^* .
- Fig. 3 : Shorthand diagrams for a) TWONDEC, b) NONDEC. The latter decay proceeds through a virtual state $(Z-2)^{*HH'}$.
- Fig. 4 : Low-lying energy levels¹⁰⁾ and some γ -ray branching ratios¹⁷⁾ of the ^{112}Cd nucleus.
- Fig. 5 : DEC lifetimes of ^{112}Sn . The no neutrino mode is shown as a function of the degeneracy parameter $Q-E$.

(a) TWONDEC

(b) NONDEC

Fig. 1

Fig. 2

(a)

(b)

Fig. 3

Fig. 4

Fig. 5