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PROSPECTS FOR SUPERSYMMETRIC $SO(10)$ MODELS
WITH AN INTERMEDIATE MASS SCALE

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A B S T R A C T

We investigate possible patterns of $SO(10)$ gauge symmetry breaking compatible with supersymmetry, limiting ourselves to the cases with one intermediate breaking scale. It is found that the one where a 54 representation breaks $SO(10)$ into a Pati-Salam group $SU(4)_C \times SU(2)_L \times SU(2)_R$ and the one where a 210 breaks it into $SU(3)_C \times U(1)_C \times SU(2)_L \times SU(2)_R$ are the most preferable patterns when supersymmetry is taken into account. Two models with the Pati-Salam intermediate symmetry are studied in more detail.

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1. - INTRODUCTION

An awkward feature of most grand unified theories (GUTs) is that the Higgs doublet with bare mass of the order of the electroweak unification scale, 10^2 GeV, receives undesirable mass "corrections" of the order of the grand unification scale, $\gtrsim 10^{15}$ GeV, when the higher order processes are taken into account. This is only evaded if the parameters of the model are retuned with an extraordinary accuracy in each order of perturbation theory. A partial improvement to this gauge hierarchy problem can be provided within the context of GUT models generalized with supersymmetry (SUSY), thanks to a suitable cancellation of diagrams generated by the boson/fermion correspondence of SUSY. If the bare parameters of the Lagrangian allow for different scales, SUSY protects this hierarchy from being spoiled by higher order corrections. Thus only fine tunings at the tree level need to be done.

The extension of the minimal $SU(5)$ GUT with global $N = 1$ supersymmetry has been widely studied ¹⁾. In the present work we will investigate the prospects of establishing a picture of the world based on the next simplest GUT model of that kind, the supersymmetric $SO(10)$. Several candidates for the $SO(10)$ SUSY GUT models have been proposed recently ²⁾⁻⁶⁾.

One main difference between the $SU(5)$ and $SO(10)$ GUTs lies in the question of neutrino masses. In the minimal version of $SU(5)$ non-zero neutrino masses are not allowed while in $SO(10)$ they appear and can be arranged in a way that agrees with low energy phenomenology. This is done by using a $\overline{126}$ tensor representation of Higgses in order to give a high Majorana mass to the right-handed neutrinos and keep the left-handed ones light ⁷⁾. An alternative mechanism was proposed by Witten ⁸⁾ who has shown that at the second order loop level neutrinos can acquire mass from a $\overline{16}$ spinor representation of Higgses, thus making the large representation $\overline{126}$ unnecessary. In the supersymmetric $SO(10)$, however, Witten's diagram is cancelled by other diagrams containing SUSY partners of the ordinary particles, and this mechanism will then be destroyed ⁴⁾. In fact, it does not work even when SUSY is broken, since the right-handed neutrinos will anyhow stay too light ⁹⁾. Therefore any realistic $SO(10)$ SUSY GUT model should contain a chiral superfield transforming as a $\overline{126}$.

Another important difference between the $SU(5)$ and $SO(10)$ GUTs is that in $SO(10)$ it is possible to have intermediate mass scales. $SO(10)$ thus forecasts more interesting physics beyond the electroweak scale, for example, it may allow for parity restoration down to low energies. Furthermore, because of

the intermediate scale(s), $SO(10)$ GUT is more flexible when the theory is confronted with experimental results (e.g., with the proton lifetime measurements). There is, however, a very great variety of possible breaking patterns (and possible intermediate stages) out of which the most suitable one should be selected. One might hope that SUSY could help to make the choice. It turns out, indeed, that when SUSY is imposed some breaking patterns are more natural, at least in the technical sense, than others. It should be mentioned at this point that in some $SO(10)$ SUSY GUT scenarios presented in the literature ^{4),10)} there are no intermediate scales at all but the gauge symmetry is broken directly down to $SU(3)_C \times SU(2)_L \times U(1)_Y$. In that kind of picture the attractiveness of $SO(10)$ over $SU(5)$ is, however, somewhat questionable.

It is clear that breaking of SUSY, either explicit, spontaneous or dynamical, has to occur in Nature, since no boson-fermion pairs with equal mass are present in the known particle spectra. Although there are a few phenomenological differences between existing models that break SUSY at very high energies (GUT scale or Planck mass) and others that break it at intermediate or low energies, they are equally viable as far as the solution to the hierarchy problem is concerned. We do not explore the question of SUSY breaking in any detail in our present work, but will allow it to occur spontaneously via either the Fayet-O'Raifeartaigh mechanism (F breaking) ¹¹⁾ or the Fayet-Iliopoulos mechanism (D breaking) ¹²⁾. In our analysis we will consider only those chiral Higgs superfields that are responsible for the gauge symmetry breaking, and assume that in this sector SUSY remains unbroken.

In Section 2 we investigate different breaking patterns of $SO(10)$ gauge symmetry in supersymmetric models, restricting ourselves to the cases with just one intermediate energy scale. The analysis covers the breaking of $SO(10)$ to intermediate symmetries using a spinor representation 16, an adjoint representation 45, a second order symmetric tensor representation 54, a fourth order antisymmetric tensor representation 210 and a fifth order antisymmetric tensor representation 126. In Section 3, two specific models with the Pati-Salam group $SU(4)_C \times SU(2)_L \times SU(2)_R = G_{PS}$ as an intermediate symmetry are considered in more detail. The breaking of $SO(10)$ down to G_{PS} is done with a 54, and both models involve a $\overline{126}$ for the neutrino mass reason referred to above. A number of parameter adjustments in the Lagrangian are found to be necessary so as to ensure the lightness of the Weinberg-Salam Higgs doublets and also to guarantee a consistent symmetry breaking pattern. It is pointed out that, because of the intermediate stage in breaking, these ad hoc adjustments may in general be complicated, especially as compared to the simple situation in the supersymmetric $SU(5)$ models.

However, an exception to this tendency is found in the model of Section 3.1, where a simple fine tuning is sufficient. We present our conclusions in Section 4.

2. - BREAKING PATTERNS OF SUPERSYMMETRIC SO(10)

Supersymmetry imposes non-trivial constraints on the breaking of the gauge symmetry. We investigate in this section several breaking patterns of the SO(10) SUSY GUT, displayed in Table I. We restrict ourselves to the cases where there is only one intermediate mass scale (M_I) between the electroweak unification scale (M_W) and the grand unification scale (M_U), and will in this section consider just the first stage of breaking, from SO(10) to the intermediate symmetry. The questions we want to answer are under which conditions the hierarchy of the mass scales, i.e., $M_W \ll M_I \ll M_U$, can be realized in the supersymmetric SO(10) and whether some of the breaking schemes are favoured or disfavoured over the others in this respect.

We construct in each case the most general superspace potential (W) consistent with gauge symmetry and at most cubic in the fields. The necessary and sufficient condition for keeping the supersymmetry unbroken, namely that the vacuum expectation values (vevs) of auxiliary fields have to vanish, is then imposed. By this means it is found out whether the breaking pattern considered is acceptable or otherwise what modifications should be made in the superpotential to make it acceptable. What often turns out to be needed is an extra discrete symmetry among the chiral Higgs multiplets.

The auxiliary fields may be defined as follows ¹⁰⁾

$$\begin{aligned} F_{\phi_i}(\phi) &= \frac{\partial W}{\partial \phi_i} , \\ D^\alpha(\phi, \bar{\phi}) &= g \sum_{a,b} \bar{\phi}_a T^{\alpha a}{}_b \phi^b , \end{aligned} \quad (1)$$

where the ϕ_i 's are the scalar components of the chiral superfields and T^{α} 's are the generators of the gauge group. The conditions $F_{\phi_i}(\langle \phi \rangle) = 0$ for all i and $D^\alpha(\langle \phi \rangle, \langle \bar{\phi} \rangle) = 0$ for all α guarantee that the Higgs potential

$$V(\phi, \bar{\phi}) = \sum_i |F_{\phi_i}|^2 + \sum_\alpha |D^\alpha|^2 \quad (2)$$

obtains its minimum value $V = 0$ for the vacuum characterized by $\langle \phi \rangle$.

The only cases where one must pay attention to the D term are those in which the rank of the gauge group is lowered. This corresponds, in our case, to the breaking of $SO(10)$ by a spinor representation 16 or an antisymmetric fifth order tensor representation 126 . It turns out that in order to make the D term vanish in these cases, one should include also the $\overline{16}$ or $\overline{126}$ chiral superfields in the model ¹⁰.

The minimal set of chiral superfields (excluding matter supermultiplets which the ordinary fermions live in) we need in the case when the first step of the breaking is accomplished by a spinor will be $\phi(16)$, $\bar{\phi}(\overline{16})$, $\Sigma(45)$, $H(10)$ and $H'(10)$. These produce the following breaking pattern :

$$SO(10) \xrightarrow{\phi, \bar{\phi}} SU(5) \xrightarrow{\Sigma} SU(3)_c \times SU(2)_L \times U(1)_Y \xrightarrow{H, H'} SU(3)_c \times U(1)_Q. \quad (3)$$

Instead of the adjoint $\Sigma(45)$ a symmetric second order tensor representation $S(54)$ can be used to break the residual $SU(5)$. The reason why two 10 's of Higgses are introduced here is, as will be seen in the next section, to guarantee that the chosen vacuum in the electroweak breaking is supersymmetric.

Let us presume a hierarchy $\langle \phi \rangle, \langle \bar{\phi} \rangle \gg \langle \Sigma \rangle \gg \langle H \rangle, \langle H' \rangle$ among the vacuum expectation values of the scalar components of the superfields (we use the same symbol to denote the scalar component as the corresponding superfield itself). The part of the superpotential relevant for the first stage of breaking is then given by

$$W = f (\phi^2)_i H_i + g (\bar{\phi}^2)_i H_i + h (\phi \bar{\phi})_{ij} \Sigma_{ij} + m (\phi \bar{\phi}), \quad (4)$$

where the bilinear products of spinor representations are defined as $(\phi \bar{\phi})_{i\dots k} = \phi^T C_{\gamma_1 \dots \gamma_k} \bar{\phi}$, $i, j, \dots = 1, \dots, 10$ are $SO(10)$ tensor indices, γ 's are the generalized Dirac matrices and C is the generalized charge conjugation matrix. From this we obtain the following F terms :

$$\begin{aligned} \left\langle \frac{\partial W}{\partial H_i} \right\rangle &= f \langle (\phi^2)_i \rangle + g \langle (\bar{\phi}^2)_i \rangle, \\ \left\langle \frac{\partial W}{\partial \Sigma_{ij}} \right\rangle &= h \langle (\phi \bar{\phi})_{ij} \rangle, \end{aligned} \quad (5)$$

$$\left\langle \frac{\partial W}{\partial \phi_\alpha} \right\rangle = m \langle (\phi^\dagger C)_\alpha \rangle ,$$

$$\left\langle \frac{\partial W}{\partial \bar{\phi}_\alpha} \right\rangle = m \langle (\bar{\phi}^\dagger C)_\alpha \rangle$$

(5)
Contd.

[here $\alpha = 1, \dots, 16$ is an $SO(10)$ spinor index]. Breaking to $SU(5)$ can be achieved by giving a non-zero vev to the " ν_L^c " component of ϕ and to the " ν_R " component of $\bar{\phi}$. In order to guarantee the vanishing of the D term we must impose $|\langle \phi_{\nu_L^c} \rangle| = |\langle \phi_{\nu_R} \rangle|^{10}$. From Eq. (5) we can see that ensuring the vanishing of the F term requires $h = m = 0$, which can be arranged by imposing an additional discrete symmetry $\phi \rightarrow -\phi$ on the superpotential. Also a suitable tuning of the parameters f and g must be done.

Using the 54 instead of 45 does not change the situation essentially. Since the 54 is a symmetric tensor, there is no term corresponding to $\phi \bar{\phi} \Sigma$, but counterparts of the three other terms of Eq. (4) are still present. This breaking pattern is thus supersymmetric on the same conditions as the previous one.

When $SO(10)$ is broken by $\Gamma(126)$, and the residual $SU(5)$ with 45 or 54, the relevant superpotentials are, respectively,

$$W = f (\Gamma \bar{\Gamma})_{[ij]} \Sigma_{ji} + m \Gamma \bar{\Gamma} ,$$

$$W = f (\Gamma \Gamma)_{\{ij\}} S_{ji} + g (\bar{\Gamma} \bar{\Gamma})_{\{ij\}} S_{ji} + m \Gamma \bar{\Gamma} . \quad (6)$$

The discrete symmetry $\Gamma \rightarrow -\Gamma$ (or $\bar{\Gamma} \rightarrow -\bar{\Gamma}$) should be imposed, and in the latter case also a suitable choice of the parameters f and g is needed, to make these breaking patterns consistent with SUSY.

The adjoint representation $\Sigma(45)$ offers several patterns of $SO(10)$ breaking :

All of them are, however, strongly disfavoured in a supersymmetric theory : the relevant part of W , when a hierarchy among the different vevs is presumed, is given as

$$W = \frac{\mu}{2} \Sigma_{ij} \Sigma_{ji} \quad (8)$$

(the cubic term Σ^3 vanishes because Σ is antisymmetric) and leads to a non-vanishing F term, unless $\mu = 0$. In order to produce a realistic scenario one must thus go to a non-minimal model, e.g., one with two adjoint representations. Another possibility is to introduce a gauge singlet superfield N , with $\langle N \rangle = 0$, which enters the superpotential in the terms $\lambda \Sigma_{ij} \Sigma_{ji} N - m^2 N$ (see Witten¹⁾). This trick profits vanishing of $\langle F_\Sigma \rangle$, but costs, aside from employing the otherwise useless superfield N , a tuning of the parameters λ and m that is necessary to make $\langle F_N \rangle$ vanish. If one abandons the intermediate stage, a 45, in conspiracy with other representations (e.g., 54, 16 and $\overline{16}$), can, however, be used to break $SO(10)$ directly to $SU(3)_C \times SU(2)_L \times U(1)$ in a way consistent with SUSY^{4),10)}.

In all the breaking patterns considered so far, SUSY puts non-trivial restrictions on the models. In the remaining two cases, where breaking is due to an $S(54)$ and to an antisymmetric fourth order tensor representation $T(210)$, the situation appears to be preferable. The critical difference is that these multiplets are present in both quadratic and cubic invariants in the superpotential.

The minimal model in the case of the 54 breaking^{*)} will contain the chiral superfields $S(54)$, $\phi(16)$, $\bar{\phi}(\overline{16})$, $H(10)$ and $H'(10)$. [The following analysis is still valid when $\phi(16)$ and $\bar{\phi}(\overline{16})$ are replaced by the pair $\phi(16)$ and $\bar{\Gamma}(\overline{126})$ or the pair $\Gamma(126)$ and $\bar{\Gamma}(\overline{126})$ which cases we will consider in the next section.] Again we assume $\langle \phi \rangle$, $\langle \bar{\phi} \rangle$, $\langle H \rangle$ and $\langle H' \rangle$ to be negligible as compared to $\langle S \rangle$. The relevant part of the superpotential is then

$$W = \frac{1}{2} \mu S_{ij} S_{ji} + \frac{1}{3} h S_{ij} S_{jk} S_{ki} \quad (9)$$

The possible breaking patterns consistent with SUSY are determined by setting the F terms $\langle F_{S_{ij}} \rangle$ equal to zero :

*) This breaking is also considered in Ref. 5).

$$\mu \langle S_{ij} \rangle + h \left(\langle S_{ik} \rangle \langle S_{kj} \rangle - \frac{1}{10} \delta_{ij} \text{Tr} \langle S^2 \rangle \right) = 0. \quad (10)$$

(Note that $S_{ij} = S_{ji}$ and $\text{Tr} S = 0$.) One can write the vev in a diagonal form $S_{ij} = s_i \delta_{ij}$, to obtain the equation

$$s_i^2 + \frac{\mu}{h} s_i = \frac{1}{10} \sum_{k=1}^{10} s_k^2. \quad (11)$$

From this it is seen that the s_i 's can have at most two different values, say u and v , the two roots of Eq. (11). Let the value u occupy the first n places of the diagonal and v the remaining ones. Since S is traceless, one must have $u = (1 - 10/n)v$. From Eq. (11), we then obtain the final solutions of Eq. (10) (no solution for $n = 5$)

$$\langle S_{ij} \rangle = \frac{10-n}{2n-10} \frac{\mu}{h} \text{diag}(\overbrace{1, \dots, 1}^{n \text{ places}}, x, \dots, x), \quad (12)$$

where $x = n/(n-10)$. This tells us that there are no objections from SUSY to break $SO(10)$ down to any of the intermediate symmetries $SO(9)$, $SO(8) \times U(1)$, $SO(7) \times SO(3)$ and $SU(4) \times SU(2) \times SU(2)$. We will not now try to find out how Nature makes the choice between these options and picks out the Pati-Salam symmetry $SU(4)_C \times SU(2)_L \times SU(2)_R$, which is the only acceptable one from the physical point of view. It is, however, possible that when SUSY is spontaneously broken, the degeneracy is lifted and G_{PS} becomes energetically preferred. This kind of mechanism is found to be possible in $SU(5)$ SUSY GUT models¹³⁾.

Let us finally consider the breaking with a fourth order antisymmetric tensor representation $T(210)$. In the minimal model $T(210)$ will be accompanied by the supermultiplets $\phi(16)$, $\bar{\phi}(\overline{16})$, $H(10)$ and $H'(10)$, but our analysis will stay unchanged if 16 and $\overline{16}$ are replaced by 126 and $\overline{126}$, as is the case in the model proposed in Ref. 3). The effective part of the superpotential will be

$$W = \frac{1}{2} \mu T_{ijkl} T_{ijkl} + \frac{1}{3} \lambda T_{ijkl} T_{klmn} T_{mnij}. \quad (13)$$

The necessary and sufficient conditions for preserving SUSY at the first stage of breaking are given by the following set of equations

$$\mu \langle T_{ijkl} \rangle + h \langle T_{ijmn} \rangle \langle T_{mnkl} \rangle = 0. \quad (14)$$

The classification of 210 under $SU(4)_C \times SU(2)_L \times SU(2)_R$ is given by

$$210 = (1,1,1) + (15,1,1) + (15,1,3) + (15,3,1) + (10,2,2) + (\bar{10},2,2) + (6,2,2). \quad (15)$$

Since $SU(2)_L$ should be preserved down to the electroweak unification scale, only the first three terms in this decomposition, i.e., $(1,1,1)$, $(15,1,1)$ and $(15,1,3)$, can acquire a large vev. They break $SO(10)$ down to the Pati-Salam group, to $SU(3)_C \times U(1)_C \times SU(2)_L \times SU(2)_R$ and to $SU(3)_C \times U(1)_C \times SU(2)_L \times U(1)_R$, respectively. There is still another possible breaking pattern, not readable from (15), namely that with an $SU(5) \times U(1)_Y$, intermediate symmetry.

We will now check which of these possible breaking patterns are consistent with SUSY. We fix our notation as in Ref. 14), where the generators of $SU(4)_C$ and $SU(2)_L \times SU(2)_R$ are denoted by Σ_{ij} , with $i,j = 1, \dots, 6$ and $i,j = 7, \dots, 10$, respectively. The generators Σ_{12} , Σ_{34} , Σ_{56} , Σ_{78} and $\Sigma_{910} \equiv \Sigma_{90}$ are diagonal, and are related to the diagonal generators of $SU(4)_C \times SU(2)_L \times SU(2)_R$ as follows :

$$\begin{aligned} T_3^C &= \frac{1}{4} (\Sigma_{34} + \Sigma_{56}), \\ T_8^C &= \frac{1}{4\sqrt{3}} (2\Sigma_{12} - \Sigma_{34} + \Sigma_{56}), \\ T_{15}^C &= \frac{1}{4}\sqrt{\frac{2}{3}} (\Sigma_{12} + \Sigma_{34} - \Sigma_{56}), \\ T_{3L} &= \frac{1}{4} (\Sigma_{78} + \Sigma_{90}), \\ T_{3R} &= \frac{1}{4} (\Sigma_{78} - \Sigma_{90}). \end{aligned} \quad (16)$$

It is useful to recall that 210 can be presented as a symmetrized product of two 45's, in the following way :

$$T_{ijkl} \sim \sum_{mn} \sum_{op} \varepsilon^{mnopqrstuv} \varepsilon_{qrstuvijkl}. \quad (17)$$

The singlet component $(1,1,1)$ of 210 is clearly T_{7890} . It is easy to check that a non-zero vev in this direction does not satisfy Eq. (14). Hence, although 210 can break $SO(10)$ to the Pati-Salam group, it cannot do it in a supersymmetric theory, at least if only the minimal set of chiral multiplets is included.

The 15th generator of $SU(4)_C$, T_C^{15} , determines the direction of breaking from $SO(10)$ down to $SU(3)_C \times U(1)_C \times SU(2)_L \times SU(2)_R^{*})$. The corresponding component in the $(15,1,1)$ part of 210 is given by

$$-T_{1234} + T_{1256} + T_{3456} \quad (18)$$

and hence the breaking takes place if we put

$$-\langle T_{1234} \rangle = \langle T_{1256} \rangle = \langle T_{3456} \rangle = v \neq 0 \quad (19)$$

and keep all other vevs zero. Equation (14) can now be satisfied, and SUSY preserved, provided the parameters are chosen so that

$$v = \frac{\mu}{h} \quad (20)$$

The breaking down to $SU(3)_C \times U(1)_C \times SU(2)_L \times U(1)_R$ is achieved when a non-zero vev is given to the neutral and colour singlet component of $(15,1,3)$. This is done by choosing

$$\begin{aligned} \langle T_{1278} \rangle = \langle T_{3478} \rangle = -\langle T_{5678} \rangle = -\langle T_{1290} \rangle = -\langle T_{3490} \rangle \\ = \langle T_{5690} \rangle \neq 0 \end{aligned} \quad (21)$$

with all other vevs vanishing. It can be checked from Eqs. (16) and (17) that the vev (21) transforms as T_{15}^C under $SU(4)_C$ and as T_{3R} under $SU(2)_R$. Equation (14) cannot, however, be satisfied with this choice and the breaking pattern is therefore not viable in the supersymmetric case.

As the last 210 breaking we now consider the pattern $SO(10) \rightarrow SU(5) \times U(1)_Y$. This breaking scheme is possible in the non-SUSY case since the decomposition of 210 under $SU(5) \times U(1)_Y$ contains a singlet $1(0)$. In order to see whether it is possible in the SUSY model, we have to write the singlet in terms of T_{ijkl} 's. To do this, one can proceed as follows. First decompose the product of a spinor 16 and a conjugation spinor $\overline{16}$:

$$16 \times \overline{16} = 1 + 45 + 210. \quad (22)$$

Each representation on the right-hand side contains a singlet $1(0)$ under $SU(5) \times U(1)_Y$. On the other hand, the spinor 16 decomposes as :

*) This breaking is also considered in Ref. 3).

$$16 = 1(-5) + \bar{5}(3) + 10(-1) \quad (23)$$

and therefore the three singlets in $16 \times \bar{16}$ come from the products 1×1 , $\bar{5} \times 5$ and $10 \times \bar{10}$:

$$\begin{aligned} 1(-5) \times 1(5) &\supset 1(0) \equiv S_1, \\ \bar{5}(3) \times 5(-3) &\supset 1(0) \equiv S_2, \\ 10(-1) \times \bar{10}(1) &\supset 1(0) \equiv S_3. \end{aligned} \quad (24)$$

The $1(0)$ components of 1 , 45 and 210 are associated with certain independent combinations of S_1 , S_2 and S_3 . The $SO(10)$ singlet 1 is obviously just the sum $S_1 + S_2 + S_3$. The $1(0)$ component of 45 is related to the generator Y' of $U(1)_Y$, and is $5S_1 - 3S_2 + S_3$. From orthogonality it then follows that the $1(0)$ of 210 is $5S_1 + S_2 - S_3$. It is a straightforward calculation to see that this corresponds to the following combination of T_{ijkl} 's :

$$\begin{aligned} &2T_{1234} - 2T_{1256} - 2T_{1278} + 2T_{1290} - 2T_{3456} \\ &- T_{3478} + 3T_{3490} + T_{5678} - 3T_{5690} - 2T_{7890}. \end{aligned} \quad (25)$$

The vev in this direction does not, however, satisfy Eq. (14) and consequently the breaking $SO(10) \rightarrow SU(5) \times U(1)_Y$, with 210 is not consistent with SUSY.

We summarize the results of this section as follows. From the breaking patterns considered, the one, where 54 breaks $SO(10)$ down to the Pati-Salam group $SU(4)_C \times SU(2)_L \times SU(2)_R$ and the one, where 210 breaks it down to $SU(3)_C \times U(1)_C \times SU(2)_L \times SU(2)_R$, are preferred over the remaining ones when SUSY is imposed to the theory. In the two specific models which we explore in the next section, breaking with 54 is used. The most unfavoured are the breakings with an adjoint representation 45 , whereas the 16 and 126 breakings may be acceptable, when suitable discrete symmetries and tunings of parameters are imposed on the superpotential.

3. - TWO SUPERSYMMETRIC SO(10) SCENARIOS

In this section we consider two supersymmetric SO(10) scenarios that have the Pati-Salam group $SU(4)_C \times SU(2)_L \times SU(2)_R$ as an intermediate symmetry. In order to create realistic neutrino masses, we shall use, as mentioned before, a $\overline{126}$ to break the intermediate group down to $SU(3)_C \times SU(2)_L \times U(1)_Y$. As is known¹⁰⁾ there are two possibilities to cancel the non-zero D term associated with the vev of this $\overline{126}$: we should include either 126 or 16 chiral multiplets in the model. These options form the two scenarios we will now study.

3.-. - The (126, $\overline{126}$) scenario

We take the following set of Higgs chiral superfields

$$S(54), \Gamma(126), \overline{\Gamma}(\overline{126}), H(10) \text{ and } H'(10),$$

and, as a generic representative of different fermion families, a matter chiral superfield

$$\psi(16).$$

All the Higgs scalar multiplets have non-zero vev, giving the following pattern of gauge symmetry breaking

$$\begin{aligned} SO(10) &\xrightarrow{S} SU(4)_C \times SU(2)_L \times SU(2)_R \xrightarrow{\Gamma, \overline{\Gamma}} SU(3)_C \times SU(2)_L \times U(1)_Y \\ &\xrightarrow{H, H'} SU(3)_C \times U(1)_Q. \end{aligned} \quad (26)$$

Scalar components of the matter superfield $\psi(16)$ do not acquire a vev different from zero. We consider the superspace potential (notation is schematical)

$$\begin{aligned} W = & g_0 \overline{\Gamma} \psi \psi + g_1 H \psi \psi + g'_1 H' \psi \psi + g_2 \overline{\Gamma} \overline{\Gamma} S + g_3 H H S \\ & + g'_3 H H' S + g''_3 H' H' S + \lambda H H + \lambda' H H' + \lambda'' H' H' \\ & + g_4 \Gamma \Gamma S + \frac{A}{2} S S + \frac{h}{3} S S S. \end{aligned} \quad (27)$$

This is the most general form of the superpotential for our set of superfields, except that it has an extra discrete symmetry under $\Gamma \rightarrow -\Gamma$, which has been introduced in order to remove the term $\Gamma \overline{\Gamma}$. This term is unwanted because it would lead into an F term vanishing only at $\langle \Gamma \rangle = \langle \overline{\Gamma} \rangle = 0$ thus forbidding this breaking pattern.

The F term is obtained from the following derivatives :

$$\begin{aligned}
\frac{\partial W}{\partial \psi} &= 2g_0 \bar{\Gamma} \psi + 2g_1 H \psi + 2g'_1 H' \psi, \\
\frac{\partial W}{\partial \bar{\Gamma}} &= g_0 \psi \psi + 2g_2 \bar{\Gamma} S, \\
\frac{\partial W}{\partial \Gamma} &= 2g_4 \Gamma S, \\
\frac{\partial W}{\partial H} &= g_1 \psi \psi + 2g_3 HS + g'_3 H' S + 2\lambda H + \lambda' H', \\
\frac{\partial W}{\partial H'} &= g'_1 \psi \psi + g'_3 HS + 2g''_3 H' S + \lambda' H + 2\lambda'' H', \\
\frac{\partial W}{\partial S} &= 2g_4 \Gamma S + \mu S + hSS + g_2 \bar{\Gamma} \Gamma + g_3 HH + g'_3 HH' + g''_3 H'H' + g_4 \Gamma \Gamma. \quad (28)
\end{aligned}$$

As was shown in Section 2, breaking of $SO(10)$ down to $SU(4)_C \times SU(2)_L \times SU(2)_R$ will be done by $S(54)$ with a vev

$$\langle S \rangle = \frac{2\mu}{h} \text{diag}(1, 1, 1, 1, 1, 1, -\frac{3}{2}, -\frac{3}{2}, -\frac{3}{2}, -\frac{3}{2}) \quad (29)$$

and keeping all other vevs vanishing. With this choice all the F terms are zero. The D terms vanish because all the diagonal generators annihilate the vacuum. The ratio μ/h in Eq. (29) determines the superlarge mass scale M_U ; we define $M_U = 2\mu/h$.

At this stage all components of $S(54)$, i.e., $S_{20} \sim (20, 1, 1)$, $S_6 \sim (6, 2, 2)$, $S_{13} \sim (1, 3, 3)$ and $S_{11} \sim (1, 1, 1)$, obtain mass squared $\sim M_U^2$. Here the three numbers in brackets denote the $SU(4)_C \times SU(2)_L \times SU(2)_R$ classification [branching rules for different $SO(10)$ representations under the Pati-Salam group and under $SU(3)_C \times SU(2)_L \times U(1)_Y$ are given in Table II]. The components $\Gamma_{15}^- \sim (15, 2, 2)$ and $\Gamma_6^- \sim (6, 1, 1)$ of $(\bar{\Gamma})^{(\overline{126})}$ obtain $(\text{mass})^2 \sim g_{4(2)}^2 M_U^2$. The fields in $H(10)$ and $H'(10)$ acquire masses as follows :

$$\begin{aligned}
m_{H_6}^2 &\sim 4(\lambda + g_3 M_U)^2 + (\lambda' + g'_3 M_U)^2, \\
m_{H'_6}^2 &\sim 4(\lambda'' + g''_3 M_U)^2 + (\lambda' + g'_3 M_U)^2, \\
m_{H_1}^2 &\sim 4(\lambda - \frac{3}{2} g_3 M_U)^2 + (\lambda' - \frac{3}{2} g'_3 M_U)^2, \\
m_{H'_1}^2 &\sim 4(\lambda'' - \frac{3}{2} g''_3 M_U)^2 + (\lambda' - \frac{3}{2} g'_3 M_U)^2, \quad (30)
\end{aligned}$$

where $H_6, H'_6 \sim (6,1,1)$ and $H_1, H'_1 \sim (1,2,2)$. In order to keep the $SU(2)_L$ Higgs doublets appearing in H_1 and H'_1 massless at this level, we should perform the following fine tunings :

$$\lambda = 3g_3 \frac{\mu}{h}, \quad \lambda' = 3g'_3 \frac{\mu}{h}, \quad \lambda'' = 3g''_3 \frac{\mu}{h}. \quad (31)$$

With these choices, the colour sextets H_6 and H'_6 become superheavy ($\sim M_U$), as is necessary because they mediate proton decay.

In order to investigate the next stage of the breaking, from G_{PS} to $SU(3)_C \times SU(2)_L \times U(1)_Y$, we must consider the superpotential W as expressed in terms of the $SU(4)_C \times SU(2)_L \times SU(2)_R$ decompositions of the superfields. We must also decouple the superheavy scalars from the low energy physics by dropping them from the Higgs potential V . Instead of writing down the lengthy expression of W , we will give only the derivatives that are relevant for our discussion (we disregard Clebsch-Gordan coefficients for simplicity) :

$$\begin{aligned} \frac{\partial W}{\partial H_1} &= g_1 \psi_4 \psi_{\bar{4}} + 2H_1(g_4 + g_3 S_{11}) + H'_1(g'_4 + g'_3 S_{11}), \\ \frac{\partial W}{\partial H'_1} &= g'_1 \psi_4 \psi_{\bar{4}} + H_1(g'_4 + g'_3 S_{11}) + 2H'_1(g''_4 + g''_3 S_{11}), \\ \frac{\partial W}{\partial \psi_{\bar{4}}} &= 2g_0 \bar{\Gamma}_{10} \psi_{\bar{4}} + g_1 H_1 \psi_{\bar{4}} + g'_1 H'_1 \psi_{\bar{4}}, \\ \frac{\partial W}{\partial S_{13}} &= g_2 \bar{\Gamma}_{10} \bar{\Gamma}_{10} + g_5 \Gamma_{10} \Gamma_{10} + g_3 H_1 H_1 + g'_3 H_1 H'_1 + g''_3 H'_1 H'_1, \\ \frac{\partial W}{\partial S_{20}} &= g_2 \bar{\Gamma}_{10} \bar{\Gamma}_{10} + g_2 \bar{\Gamma}_{10} \bar{\Gamma}_{10} + g_4 \Gamma_{10} \Gamma_{10} + g_4 \Gamma_{10} \Gamma_{10}, \end{aligned} \quad (32)$$

where $\psi_4 \sim (4,2,1)$, $\psi_{\bar{4}} \sim (\bar{4},1,2)$, $\bar{\Gamma}_{10} \sim (10,1,3)$, $\bar{\Gamma}_{10} \sim (\bar{10},3,1)$, $\Gamma_{10} \sim (10,3,1)$ and $\Gamma_{10} \sim (\bar{10},1,3)$.

The breaking of G_{PS} is achieved by giving $\bar{\Gamma}_{10}$ and $\bar{\Gamma}_{10}$ a vev of the order of some intermediate mass scale M_I and keeping the other vevs zero. We can see from Eq. (32) that all the F terms will vanish [note that the dangerous-looking terms $\langle \bar{\Gamma}_{10} \Gamma_{10} \rangle$ and $\langle \bar{\Gamma}_{10} \bar{\Gamma}_{10} \rangle$ vanish at the vacuum because there is no $SU(3)_C$ singlet component in S_{20}]. The D term, on the other hand, is not trivial now, since the rank of the gauge group decreases. The broken

diagonal generator $\hat{Y}$ is that linear combination of the third generator of $SU(2)_R$, T_{3R} , and the uncoloured diagonal generator T_{15}^C of $SU(4)_C$, which is orthogonal to the electroweak hypercharge $Y = T_{3R} + \sqrt{2/3} T_{15}^C$, i.e.,

$$\hat{Y} = -\sqrt{\frac{2}{5}} T_{3R} + \sqrt{\frac{3}{5}} T_{15}^C. \quad (33)$$

The corresponding D terms for the two non-zero vevs are given by

$$\begin{aligned} D^{\hat{Y}}(\langle \bar{\Gamma}_{10} \rangle) &= -\sqrt{\frac{5}{2}} |\langle \bar{\Gamma}_{10} \rangle|^2, \\ D^{\hat{Y}}(\langle \Gamma_{\bar{10}} \rangle) &= \sqrt{\frac{5}{2}} |\langle \Gamma_{\bar{10}} \rangle|^2. \end{aligned} \quad (34)$$

Hence to preserve supersymmetry we must choose $|\langle \bar{\Gamma}_{10} \rangle| = |\langle \Gamma_{\bar{10}} \rangle|$.

We can see from Eq. (32) that in this scenario the $SU(2)_L$ Higgs doublets H_1 and H_1' do not obtain mass from the vevs that break G_{PS} and hence there is no need for any extra adjustments at the intermediate level.

From $\partial W / \partial S_{13}$ and $\partial W / \partial S_{20}$ in (32), it is easily seen that only the components

$$\Gamma_{10}(1, 3, -1), \quad \Gamma_{\bar{10}}(\bar{6}, 1, \frac{2}{3}), \quad \bar{\Gamma}_{10}(6, 1, -\frac{2}{3}), \quad \bar{\Gamma}_{\bar{10}}(1, \bar{3}, 1) \quad (35)$$

of $\Gamma(126)$ and $\bar{\Gamma}(\bar{126})$ acquire mass, $(\text{mass})^2 \sim g_2^2 M_I^2$, at the intermediate stage. [The numbers in brackets in Eq. (35) denote the $SU(3)_C \times SU(2)_L \times U(1)_Y$ classification.] The term $\partial W / \partial \psi_{\bar{4}}$ will develop a $(\text{mass})^2 \sim g_0^2 M_I^2$ for the scalar superpartner of the right-handed neutrino ν_R , the $\psi_{\bar{4}}(1, 1, 0)$ (the ν_R itself obtains a Majorana mass of the same size, of course).

The last stage of symmetry breaking, $SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_Q$, is achieved by giving non-zero vevs to the neutral components of the doublets $H_1(1, 2, \frac{1}{2}) \equiv H_{1+}$, $H_1(1, 2, -\frac{1}{2}) \equiv H_{1-}$, $H_1'(1, 2, \frac{1}{2}) \equiv H_{1+}'$ and $H_1'(1, 2, -\frac{1}{2}) \equiv H_{1-}'$. The rank of the gauge group is in this breaking decreased further by one unit but the D term associated with the broken generator $Q' = -T_{3L} + Y$ is zero as long as we choose

$$\langle H_{1+} \rangle = \langle H_{1-} \rangle = \langle H_{1+}' \rangle = \langle H_{1-}' \rangle \sim \delta(M_w). \quad (36)$$

Concerning the F term, relevant derivatives are

$$\begin{aligned}
 \frac{\partial W}{\partial S_{13}(1,3,0)} &= g_3 H_{1+} H_{1-} + \frac{1}{2} g_3' (H_{1+} H_{1-}' + H_{1-} H_{1+}') + g_3'' H_{1+}' H_{1-}' + \dots \\
 &= \frac{\partial W}{\partial S_4(1,1,0)} + \dots, \\
 \frac{\partial W}{\partial S_{13}(1,3,-1)} &= g_3 H_{1+} H_{1+} + g_3' H_{1+}' H_{1+}' + g_3'' H_{1+}' H_{1+}' + \dots \\
 &= \frac{\partial W}{\partial S_{13}(1,3,1)} + \dots,
 \end{aligned} \tag{37}$$

where the dots stand for terms vanishing automatically at the vacuum. In order to make the F term vanish one must adjust

$$g_3 + g_3' + g_3'' = 0. \tag{38}$$

If we had only one supermultiplet 10 in the model (i.e., $g_3' = g_3'' = 0$) we would not have been able to break the electroweak symmetry in consistency with SUSY. Equation (38) would in that case force g_3 to vanish and consequently the Higgs doublets would stay massless.

Along with the doublets H_1 and H_1' , all the remaining members of Γ_{10} , $\bar{\Gamma}_{10}$, $\bar{\Gamma}_{10}$ and $\bar{\Gamma}_{10}$, except the components

$$\begin{aligned}
 &\Gamma_{10}(1,1,0), \quad \Gamma_{10}(1,1,1), \quad \Gamma_{10}(1,1,2), \\
 &\bar{\Gamma}_{10}(1,1,0), \quad \bar{\Gamma}_{10}(1,1,1), \quad \bar{\Gamma}_{10}(1,1,2),
 \end{aligned} \tag{39}$$

acquire mass of the order of M_W . The fields enumerated in Eq. (39) stay massless at the tree level.

To summarize, the spectrum of physical states in the exact SUSY limit is predicted to be as follows : particles in the matter superfield $\psi(16)$ are light, (Yukawa coupling constant) $\times M_W$, except the right-handed neutrino and its scalar superpartner which acquires mass of the order of M_I . Four colourless doublets in H_1 and H_1' included in the two $SO(10)$ decuplets are at the scale M_W . The $SU(4)_C$ sextets H_6 and H_6' are superheavy. The supermultiplets $\Gamma(126)$ and $\bar{\Gamma}(126)$ split into four mass scales : the sextets and fifteenplets under $SU(4)_C$, Γ_6 , $\bar{\Gamma}_6$, Γ_{15} , $\bar{\Gamma}_{15}$, become superheavy ; those members of the decuplets and antidecuplets which are given in (35) acquire mass at the intermediate scale M_I ; the remaining fields, except the six states in Eq. (39), have mass of

the order of M_W ; the states in (39) are massless at the tree level. All fields in the supermultiplet $S(54)$ obtain superlarge mass. Of course, some of the scalars are eaten by the broken generators at the different stages of breaking. The fermionic partner of such a scalar and of the corresponding gauge boson will at the same time combine to form a Dirac fermion with mass equal to the mass of the gauge boson.

As can easily be checked, the only bosons that can mediate proton decay, are in this model the $SU(4)_C$ sextets H_6 and H'_6 . As we have given them superheavy mass by making suitable fine tunings, this model is safe from the point of view of proton decay.

3.2. - The $(16, \overline{126})$ scenario

We will consider now the following set of Higgs chiral superfields

$$S(54), \bar{F}(\overline{126}), \phi(16), H(10) \text{ and } H'(10),$$

and one matter chiral superfield

$$\psi(16).$$

It turns out that in the simplest version of this model we consider that one meets difficulties when trying to produce a realistic fermion mass spectra. Especially, the right-handed neutrino will not receive high Majorana mass although there is a $\overline{126}$ Higgs multiplet present; SUSY prohibits indirectly the relevant coupling, as we will see immediately.

The most general superpotential in this scenario would be

$$\begin{aligned} W = & f_{11} H \psi \psi + f_{12} H \psi \phi + f_{13} H \phi \phi + f'_{11} H' \psi \psi + f'_{12} H' \psi \phi \\ & + f'_{13} H' \phi \phi + f_2 \bar{F} \bar{F} S + f_3 H H S + f'_3 H H' S + f''_3 H' H' S \\ & + g_1 \bar{F} \phi \phi + g_2 \bar{F} \phi \psi + g_3 \bar{F} \psi \psi + \lambda H H + \lambda' H H' \\ & + \lambda'' H' H' + \frac{\mu}{2} S S + \frac{h}{3} S S S. \end{aligned} \quad (40)$$

It is directly clear that an extra discrete symmetry $\bar{\Gamma} \rightarrow -\bar{\Gamma}$ should be imposed on W in order to ensure conservation of SUSY. Otherwise, namely the term $\bar{\Gamma}\phi\phi$ in (40) would give a non-vanishing F term, $\langle F_{\bar{\Gamma}\nu_L^C\nu_L^C} \rangle \sim \langle \phi_{\nu_L} \rangle \langle \phi_{\nu_L^C} \rangle$, at the intermediate stage of breaking. On the other hand, it is the corresponding term for the matter spinor, i.e., $\bar{\Gamma}\psi\psi$, that could offer the source of a Majorana mass for the right-handed neutrino. Since the extra symmetry will prevent also this coupling, the model predicts a light right-handed neutrino and might therefore be phenomenologically unacceptable.

The same considerations as in the previous model can now be repeated. We will omit here the details and just refer to main points.

In the breaking from $SO(10)$ to $SU(4)_C \times SU(2)_L \times SU(2)_R$, carried out with $S(54)$, superheavy masses are generated for $\bar{\Gamma}_{15}$ and $\bar{\Gamma}_6$ and for the whole $S(54)$. As in the previous case, we can perform fine tunings among the parameters so that the colour sextets H_6 and H'_6 of the two decuplets become superheavy whereas the weak doublets within H_1 and H'_1 remain massless. However, these fine tunings are not enough to keep the doublets light this time. Because of the terms $H\phi\phi$ and $H'\phi\phi$ in W , H_1 and H'_1 will now, in contrast with the first scenario, acquire mass also at the intermediate level. In order to avoid this, complicated additional adjustments involving several parameters must be done.

The intermediate breaking is performed with the " ν_L^C " component of $\phi_4 \sim (\bar{4}, 1, 2)$ and the " $\nu_L^C \nu_L^C$ " component of $\bar{\Gamma}_{10} \sim (10, 1, 3)$. In order to ensure the vanishing of the D term, the vevs should be related as $|\langle \phi_4 \rangle| = \sqrt{2} |\langle \bar{\Gamma}_{10} \rangle|^{10}$.

As in the $(126, \bar{126})$ scenario, three components of $\bar{\Gamma}(\bar{126})$, i.e., $\bar{\Gamma}_{10}(1, 1, 0)$, $\bar{\Gamma}_{10}(1, 1, 1)$ and $\bar{\Gamma}_{10}(1, 1, 2)$ remain massless at the tree level.

Finally, the proton decay occurs here in the same manner as in the previous model. So this scenario is also safe in this respect.

4. - CONCLUSIONS

We have investigated the breaking patterns of gauge symmetry in the supersymmetric $SO(10)$ grand unified theory, assuming that there is only one intermediate mass scale between the GUT scale and the electroweak scale.

Breaking of $SO(10)$ down to different intermediate symmetries done with several different low dimensional Higgs multiplets (16, 45, 54, 126 and 210) was first studied (see Table I). It was found that the intermediate groups $SU(4)_C \times SU(2)_L \times SU(2)_R$ (the Pati-Salam group) obtained with the second order symmetric tensor representation 54 and $SU(3)_C \times U(1)_C \times SU(2)_L \times SU(2)_R$ obtained with the fourth order antisymmetric tensor representation 210 are the most preferable ones when the requirements of SUSY are considered.

We have investigated more thoroughly two models with the Pati-Salam intermediate symmetry. In one of them, the intermediate breaking was generated by 126 and $\overline{126}$ and in the other one by 16 and $\overline{126}$. It was shown that in these two cases the whole breaking chain can be carried through consistently with the conservation of SUSY. Because of the intermediate stage, the adjustments required for this are generally more complicated and numerous than, for example, in the $SU(5)$ model. The (126, $\overline{126}$) scenario is, however, a favourable exception, because there the Higgs doublets responsible for the electroweak breaking do not acquire any mass contributions at the intermediate level.

In supersymmetric models the mechanisms for producing fermion masses radiatively are invalid. Therefore, in order to obtain realistic neutrino masses, the $SO(10)$ SUSY GUT model should necessarily contain a $\overline{126}$ chiral supermultiplet. This alone will not, however, guarantee that the right-handed neutrino obtains a high Majorana mass as desired. In fact, in the (16, $\overline{126}$) scenario SUSY prevents $\overline{126}$ from coupling to the ordinary fermion multiplet, which makes that model much less realistic than the (126, $\overline{126}$) model in this respect.

The $SO(10)$ SUSY GUT models we have presented in this work are incomplete since the breaking of supersymmetry is not considered. We will leave this question, along with a detailed investigation of the phenomenological implications of the models, for a future study. According to the observations we have made in the present paper there are two questions that will require special attention in this connection : will the SUSY breaking effects give large enough masses to the undesired massless particles that appeared, and can the quite large number of relatively light, $O(M_W)$, particles (mainly from 126 and $\overline{126}$) be made compatible with the low energy phenomenology ?

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Table I - Breaking patterns of $SO(10)$ with one intermediate symmetry.

SO(10)	$SU(4)_C \times SU(2)_L \times SU(2)_R$
S(54)	$(20,1,1) + (6,2,2) + (1,3,3) + (1,1,1)$ $\equiv S_{20} + S_6 + S_{13} + S_{11}$
$\Gamma(126)$	$(15,2,2) + (10,3,1) + (\overline{10},1,3) + (6,1,1)$ $\equiv \Gamma_{15} + \Gamma_{10} + \Gamma_{\overline{10}} + \Gamma_6$
$\overline{\Gamma}(\overline{126})$	$(15,2,2) + (\overline{10},3,1) + (10,1,3) + (6,1,1)$ $\equiv \overline{\Gamma}_{15} + \overline{\Gamma}_{\overline{10}} + \overline{\Gamma}_{10} + \overline{\Gamma}_6$
H(10)	$(6,1,1) + (1,2,2) \equiv H_6 + H_1$
H'(10)	$H'_6 + H'_1$
$\psi(16)$	$(4,2,1) + (\overline{4},1,2) \equiv \psi_4 + \psi_4^-$
$\overline{\psi}(\overline{16})$	$(\overline{4},2,1) + (4,1,2) \equiv \overline{\psi}_4 + \overline{\psi}_4$

Table IIa - Branching rules for the SO(10) chiral multiplets under the Pati-Salam group.

$SU(4)_C \times SU(2)_L \times SU(2)_R$	$SU(3)_C \times SU(2)_L \times U(1)_Y$
S_{20}	$(\bar{6}, 1, \frac{2}{3}) + (6, 1, -\frac{2}{3}) + (8, 1, 0)$ $\equiv S_{20}(\bar{6}, 1, \frac{2}{3}) + S_{20}(6, 1, -\frac{2}{3}) + S_{20}(8, 1, 0)$
S_6	$S_6(3, 2, \frac{1}{6}) + S_6(3, 2, -\frac{5}{6}) + S_6(\bar{3}, 2, \frac{5}{6}) + S_6(\bar{3}, 2, -\frac{1}{6})$
S_{13}	$S_{13}(1, 3, 1) + S_{13}(1, 3, 0) + S_{13}(1, 3, -1)$
S_{11}	$S_{11}(1, 1, 1)$
Γ_{15}	$\Gamma_{15}(1, 2, \frac{1}{2}) + \Gamma_{15}(1, 2, -\frac{1}{2}) + \Gamma_{15}(3, 2, \frac{7}{6}) + \Gamma_{15}(3, 2, \frac{1}{6})$ $+ \Gamma_{15}(\bar{3}, 2, -\frac{1}{6}) + \Gamma_{15}(\bar{3}, 2, -\frac{7}{6}) + \Gamma_{15}(8, 2, \frac{1}{2})$ $+ \Gamma_{15}(8, 2, -\frac{1}{2})$
Γ_{10}	$\Gamma_{10}(1, 3, -1) + \Gamma_{10}(3, 3, -\frac{1}{3}) + \Gamma_{10}(6, 3, \frac{1}{3})$
$\Gamma_{\overline{10}}$	$\Gamma_{\overline{10}}(1, 1, \frac{1}{2}) + \Gamma_{\overline{10}}(1, 1, -\frac{1}{2}) + \Gamma_{\overline{10}}(1, 1, -\frac{3}{2})$ $+ \Gamma_{\overline{10}}(\bar{3}, 1, \frac{2}{3}) + \Gamma_{\overline{10}}(\bar{3}, 1, \frac{1}{3}) + \Gamma_{\overline{10}}(\bar{3}, 1, -\frac{2}{3})$ $+ \Gamma_{\overline{10}}(6, 1, \frac{2}{3}) + \Gamma_{\overline{10}}(6, 1, -\frac{2}{3}) + \Gamma_{\overline{10}}(6, 1, -\frac{4}{3})$
Γ_6	$\Gamma_6(3, 1, -\frac{1}{3}) + \Gamma_6(\bar{3}, 1, \frac{1}{3})$
$\bar{\Gamma}_{15}$	as Γ_{15}
$\bar{\Gamma}_{10}$	$\bar{\Gamma}_{10}(1, 1, -2) + \bar{\Gamma}_{10}(1, 1, 0) + \bar{\Gamma}_{10}(1, 1, -1)$ $+ \bar{\Gamma}_{10}(3, 1, \frac{2}{3}) + \bar{\Gamma}_{10}(3, 1, -\frac{1}{3}) + \bar{\Gamma}_{10}(3, 1, -\frac{4}{3})$ $+ \bar{\Gamma}_{10}(6, 1, \frac{4}{3}) + \bar{\Gamma}_{10}(6, 1, \frac{1}{3}) + \bar{\Gamma}_{10}(6, 1, -\frac{2}{3})$
$\bar{\Gamma}_{\overline{10}}$	$\bar{\Gamma}_{\overline{10}}(1, 3, 1) + \bar{\Gamma}_{\overline{10}}(\bar{3}, 3, \frac{1}{3}) + \bar{\Gamma}_{\overline{10}}(\bar{6}, 3, -\frac{1}{3})$
$\bar{\Gamma}_6$	as Γ_6
	.../

/...

$SU(4)_C \times SU(2)_L \times SU(2)_R$	$SU(3)_C \times SU(2)_L \times U(1)_Y$
H_6	$H_6(3, 1, -\frac{1}{3}) + H_6(\bar{3}, 1, \frac{1}{3})$
H_1	$H_1(1, 2, \frac{1}{2}) + H_1(1, 2, -\frac{1}{2})$
ψ_4	$\psi_4(1, 2, -\frac{1}{2}) + \psi_4(3, 2, \frac{1}{6})$
ψ_4^-	$\psi_4^-(1, 1, 0) + \psi_4^-(1, 1, 1) + \psi_4^-(\bar{3}, 1, \frac{1}{3}) + \psi_4^-(\bar{3}, 1, -\frac{2}{3})$
$\bar{\psi}_4$	$\bar{\psi}_4(1, 1, 0) + \bar{\psi}_4(1, 1, -1) + \bar{\psi}_4(3, 1, \frac{2}{3}) + \bar{\psi}_4(3, 1, -\frac{1}{3})$
$\bar{\psi}_4^-$	$\bar{\psi}_4^-(1, 2, \frac{1}{2}) + \bar{\psi}_4^-(\bar{3}, 2, -\frac{1}{6})$

Table IIb - Branching rules of the Pati-Salam representations
under $SU(3)_C \times SU(2)_L \times U(1)_Y$.

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