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SUPERSYMMETRY AND SUPERGRAVITY

B. Zumino
CERN - Geneva

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A review mostly of super-
gravity, the most exciting recent
development in supersymmetry. The
properties of simple and extended
supergravity theories are described
and the prospects for their physic-
al applications discussed.

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SUPERSYMMETRY AND SUPERGRAVITY

B. Zumino

CERN, Geneva, Switzerland

1. INTRODUCTION

Recent developments in supersymmetry have consisted mostly in the development of the theories of supergravity and extended supergravity. These are supersymmetric versions of Einstein's theory of gravitation, which involve, in addition to the spin 2 field of the graviton, one or more spin $\frac{3}{2}$ fields as well as spin 1, $\frac{1}{2}$ and 0 fields. These theories provide a route to a unification of the gravitational interaction with the other interactions -- weak, electromagnetic and strong -- known in physics. The unification is based on a new gauge principle, local supersymmetry; in extended supergravity, this principle permits one to give a geometric meaning not only to fields carrying spin 2 and 1, but also to fields of spin $\frac{3}{2}$, $\frac{1}{2}$ and 0. This geometric unification carries with it an additional advantage. Einstein's theory, when treated in perturbation theory as a gauge quantum field theory, is very divergent. It is known that pure gravity is one-loop renormalizable¹⁾, but very likely not renormalizable in the two-loop approximation. Furthermore, gravity coupled to other quantum systems (spin 1 or $\frac{1}{2}$ or 0) is not even one-loop renormalizable^{1, 2)}. This can be considered a shortcoming of the standard theory of gravitation. Supergravity is a supersymmetric version of gravitation theory and it has been suggested some time ago³⁾ that it will probably be less divergent, perhaps even renormalizable. [Supersymmetric field theories are known to have special convergence properties⁴⁾.] We shall see in the last section how far this hope has been realized.

Supergravity can be considered as the gauge theory of supersymmetry. Supersymmetry is a symmetry⁵⁾ which connects bosons (tensor fields) to fermions (spinor fields). The idea of gauging supersymmetry is not new and there are various approaches to this problem. We mention the work of Volkov and Soroka⁶⁾, of Wess and the author⁷⁾ and of Arnowitt and Nath⁸⁾, to which the author contributed at a given time. Although all these approaches share the same general philosophy, they differ in detail. That by Wess and the author is directly related and actually equivalent to what we shall call

supergravity in this lecture⁹⁾. The work of Volkov and Soroka makes essential use of a "Goldstone" spinor. That of Arnowitt and Nath connects with supergravity through a limiting process. Arnowitt and Nath and also Wess and the author use the concept of superspace (although in a somewhat different way). This is a space¹⁰⁾ having as coordinates, in addition to the usual four x^m , a certain number of additional anticommuting (fermionic) coordinates θ^α . A field in superspace or superfield is a function $V(x^m, \theta^\alpha)$. Expanded in a power series in θ^α a superfield is seen to be equivalent to a finite number of ordinary fields, since a power series in θ^α terminates after a finite number of terms (the square of each component θ^α vanishes). It turns out that a differential geometry of superspace can be worked out, in analogy with the ordinary Riemannian geometry of ordinary space-time, and a calculus of tensors and tensor densities exists. From this point of view the gauging of supersymmetry is equivalent to the requirement that the equations of motion be invariant with respect to general coordinate transformations in superspace. Because of lack of time we shall not describe in detail this approach here, although we believe that the differential geometry of superspace is the most natural technique for the study of supergravity.

2. SIMPLE SUPERSYMMETRY AND SUPERGRAVITY

To introduce supergravity^{11,12)} we start with a reminder of global supersymmetry. The algebra of global supersymmetry is an extension of the algebra of the Poincaré group. The generators of the Poincaré group are the total momentum P_a (translations) and the total angular momentum $M_{ab} = -M_{ba}$ (Lorentz transformations). They satisfy the well-known algebra

$$i[P_a, P_b] = 0 \quad i[M_{ab}, P_c] = \eta_{ac} P_b - \eta_{ab} P_c \quad (1)$$

$$i[M_{ab}, M_{cd}] = \eta_{ac} M_{bd} - \eta_{bc} M_{ad} + \eta_{bd} M_{ac} - \eta_{ad} M_{bc}.$$

In supersymmetry one adjoins spinorial generators Q_α , which transform as a (Majorana) spinor

$$i[M_{ab}, Q_\alpha] = \frac{i}{2} (\sigma_{ab})_\alpha{}^\beta Q_\beta \quad (2)$$

and satisfy

$$i[P_a, Q_\alpha] = 0 \quad \{Q_\alpha, \bar{Q}_\beta\} = -2(\gamma^a)_{\alpha\beta} P_a \quad . \quad (3)$$

Here $\bar{Q} = Q\gamma^0$. For convenience we use the Majorana representation, where all gamma matrices are real. The last equation (3) involves an anticommutator; therefore the supersymmetry algebra (1) to (3) is not an ordinary Lie algebra, but a graded Lie algebra. Just as the momentum and angular momentum can be obtained, in a local field theory, by integrating the conserved energy-momentum tensor T_{ab} , so the spinor charges Q_α can be obtained by integrating the time component of a conserved vector-spinor current J^a_α (a is a vector index, α a spinor index):

$$\frac{\partial}{\partial x^a} J^a_\alpha = 0 \quad Q_\alpha = \int J^0_\alpha d^3x \quad . \quad (4)$$

In a supersymmetric theory the Hamiltonian is the sum of the squares of the components of the spinor charge

$$H = \frac{1}{4} \sum_{\alpha=1}^4 (Q_\alpha)^2 \quad , \quad (5)$$

a relation which follows immediately from (3). The anticommutation relation (3) of the graded Lie algebra can be transformed into a commutation relation by using anticommuting (Majorana) spinor parameters α_1, α_2

$$[\bar{\alpha}_1 Q, \bar{\alpha}_2 Q] = -2 \bar{\alpha}_1 \gamma^a \alpha_2 P_a \quad . \quad (6)$$

Using anticommuting parameters the graded algebra can be exponentiated and gives rise to a (generalized) group with parameters belonging to a Grassmann algebra.

The representations of the graded Lie algebra (1) to (3) in terms of particle supermultiplets are easily obtained. For $m \neq 0$ one goes to the rest frame, where $P^a = (m, 0, 0, 0)$. The anticommutation relation (3) shows that the spinor charge is like a creation or destruction operator for spin, which changes the spin by $1/2$, according to (2). The supermultiplet consists of four one-particle states of spins $s - 1/2, s, s, s + 1/2$ (except for $s = 0$, in which case the spins are $0, 0, 1/2$). For $m = 0$ one goes to the

frame where $P_1 = P_2 = 0$ and one sees easily that the supermultiplet consists of two one-particle states of spins $s, s + \frac{1}{2}$. These supermultiplets can be realized in terms of fields and there exist local renormalizable interacting field theories admitting the global supersymmetry algebra (1) to (3). We give, as a simple example, the transformation laws of the spin $\frac{1}{2}, 1$ supermultiplet. It consists of a (Majorana) spinor field λ and a vector field A_a , which transform into each other as in

$$\begin{aligned}\delta A_a &\equiv [\bar{\alpha} Q, A_a] = i \bar{\alpha} \gamma_a \lambda \\ \delta \lambda &\equiv [\bar{\alpha} Q, \lambda] = -\frac{1}{2} F_{ab} \gamma^a \gamma^b \lambda,\end{aligned}\tag{7}$$

where

$$F_{ab} = \frac{\partial}{\partial x^a} A_b - \frac{\partial}{\partial x^b} A_a$$

and α is a constant spinor parameter.

The Lagrangian

$$L = -\frac{1}{4} F_{ab}^2 - \frac{i}{2} \bar{\lambda} \gamma \cdot \partial \lambda\tag{8}$$

changes by a total divergence under (7) and the action is invariant. The conserved spinor current for this system is given by

$$\begin{aligned}J^a &= F_{bc} \gamma^b \gamma^c \gamma^a \lambda \\ \frac{\partial}{\partial x^a} J^a &= 0.\end{aligned}\tag{9}$$

It is conserved as a consequence of the equations of motion. This very simple example can be generalized for instance by attributing A_a and λ to the adjoint representation of some internal symmetry group (supersymmetric Yang-Mills theory) and by introducing coupling to other supermultiplets.

The simplest version of supergravity^{11,12)}, which we shall call simple supergravity, is the theory of the self-interacting massless spin $\frac{3}{2}$ -spin 2 supermultiplet. The fields which enter this theory are the vierbein field e_m^a , the connection ω_m^{ab} , and the Majorana Rarita-Schwinger field ψ_m^α . (m, n , etc., are world indices; a, b are vectorial tangent space indices; α, β

spinorial tangent space indices.) They can be considered as the gauge fields of the supersymmetry algebra (1) to (3) associated with the respective generators as in

$$e_m^a P_a + \frac{1}{2} \omega_m^{ab} M_{ab} + \psi_m^\alpha Q_\alpha, \quad (10)$$

which is a Lie-algebra valued vector field. The Lagrangian of simple supergravity is the sum of the Einstein Lagrangian and of the Rarita-Schwinger Lagrangian

$$L = L_E + L_{RS}, \quad (11)$$

where

$$\begin{aligned} L_E &= \frac{1}{8} R_{mn}{}^{ab} e_z^c e_s^d \varepsilon^{mnrs} \varepsilon_{abcd} = \\ &= \frac{1}{2} R e \end{aligned} \quad (12)$$

and

$$L_{RS} = -\frac{i}{2} \varepsilon^{mnrs} \bar{\psi}_m \gamma_5 \gamma_n D_r \psi_s. \quad (13)$$

In (12) we use $e = \det e_m^a$ and $R = e_a^m e_b^n R_{mn}{}^{ab}$ is the Riemann tensor totally contracted with the inverse vierbein matrix e_a^m ; the covariant derivative in (13) is given by

$$D_r = \frac{\partial}{\partial x^r} - \frac{1}{2} \omega_r^{ab} \sigma_{ab} \quad (14)$$

and satisfies the commutation relation

$$[D_m, D_n] = -\frac{1}{2} R_{mn}{}^{ab} \sigma_{ab}, \quad (15)$$

We have taken the gravitational constant $\kappa = 1$. Observe that, because of the antisymmetry of the ε symbol D_r enters in (13) in a curl-like structure. Therefore we do not need to use a more complicated form for the covariant

derivative (namely one operating also on the world index of ψ_s) and the Lagrangian is generally covariant as it stands. This is perfectly analogous to the treatment of the electromagnetic field in curved space and reflects the fact that the Rarita-Schwinger field is also a gauge field as indicated by the flat-space gauge invariance of the Rarita-Schwinger action under

$$\psi_m \rightarrow \psi_m + \frac{2}{2x^m} \chi, \quad (16)$$

where χ is a Majorana spin $1/2$ field. In (11) e_m^a , ω_m^{ab} and ψ_m^α are taken as independent fields and must be varied independently.

The two terms of the action corresponding to (11) are invariant by construction under general coordinate transformations and under local Lorentz transformations. The new fact is that the sum is also invariant under the local supersymmetry transformation

$$\begin{cases} \delta e_m^a = i \bar{\alpha} \gamma^a \psi_m \\ \delta \psi_m = 2 D_m \alpha \\ \delta \omega_m^{ab} = B_m^{ab} - \frac{1}{2} e_m^b B_c^{ac} + \frac{1}{2} e_m^a B_c^{bc} \end{cases} \quad (17)$$

$$B_a^{\ell m} = i \bar{\alpha} \gamma_5 \gamma_a D_n \psi_r \varepsilon^{\ell m n r},$$

where $\alpha(x)$ is an anticommuting Majorana spinor parameter having an arbitrary x dependence. Observe that (17) can be considered as a generalization of the flat-space gauge invariance (16). Local supersymmetry is a new kind of gauge invariance. According to (6) the commutator of two global supersymmetry transformations is a translation. If one computes the commutator of two local supersymmetry transformations (17) of parameters $\alpha_1(x)$ and $\alpha_2(x)$ one finds (using the equations of motion) a general coordinate transformation of parameter $2i\bar{\alpha}_1 \gamma^m \alpha_2$, accompanied by field-dependent local Lorentz and supersymmetry transformations^{13,14)}. Local supersymmetry is here the more basic invariance, general coordinate invariance in some sense a consequence of it [that the algebra closes only if one uses the equations of motion is due to our having used the minimum set of fields; using suitable auxiliary fields this can be remedied¹⁵⁾].

The equations of motion obtained from (11) are the following. Variation of ψ_m gives

$$R^m \equiv \varepsilon^{mnrs} \left(\gamma_5 \gamma_n D_r \psi_s - \frac{1}{4} \gamma_5 \gamma_a C_{nr}{}^a \psi_s \right) = 0, \quad (18)$$

variation of the connection gives

$$C_{mn}{}^a = \frac{i}{2} \bar{\psi}_m \gamma^a \psi_n, \quad (19)$$

and variation of the vierbein

$$G^{am} = \frac{i}{2} \varepsilon^{lmnr} \bar{\psi}_l \gamma_5 \gamma^a D_n \psi_r, \quad (20)$$

where

$$C_{mn}{}^a \equiv D_m e_n{}^a - D_n e_m{}^a \quad (21)$$

is the torsion tensor and

$$G^m{}_a \equiv R^m{}_a - \frac{1}{2} e_a{}^m R, \quad R_{ma} = R_{me},{}^e \quad (22)$$

is the (non-symmetric) Einstein tensor. Because of the local supersymmetry of the action, Eqs. (18) to (20) are not independent. This interdependence of the equations is an essential element in assuring their consistency and indeed supergravity is the first known case of a theory involving interacting higher spin fields where no inconsistencies arise. The consistency can be verified directly. The usual check consists in performing a covariant differentiation on the left-hand side R^m of the Rarita-Schwinger equation (18). In our case one finds identically

$$D_m R^m \equiv -\frac{1}{2} G^{mn} \gamma_m \psi_n + \frac{1}{4} \varepsilon^{mnrs} C_{mn}{}^a \gamma_5 \gamma_a D_r \psi_s. \quad (23)$$

Now, the right-hand side of (23) vanishes as a consequence of the other equations of motion (19) and (20), as one can verify by means of a simple Fierz rearrangement (the anticommuting nature of the spinors is essential). It should be mentioned that the coupled spin $3/2$ -graviton system was thought

to be inconsistent and indeed other forms of the theory are. In our form the torsion plays a basic role in making the constraint (23) vanish. The vanishing of the constraint has also the consequence that no anomalous characteristic surfaces occur in our theory and propagation is causal along the local light cones¹²⁾.

Above we have formulated simple supergravity in first-order form, using the connection as an independent field. As always in theories of this type there is an equivalent second-order form, obtained by solving Eq. (19) for ω_m^{ab} and substituting the solution in the Lagrangian. In second-order form the connection is not an independent field, but is expressed in terms of derivatives of the vierbein (metric connection). The Lagrangian is then not just the sum of the Einstein and the Rarita-Schwinger Lagrangians, but has an additional contact term, which in the case of supergravity is

$$L_4 = \frac{1}{32} \left[\bar{\psi}^b \gamma^a \psi^c (\bar{\psi}_b \gamma_a \psi_c + 2 \bar{\psi}_a \gamma_b \psi_c) - 4 (\bar{\psi}_a \gamma \cdot \psi)^2 \right]. \quad (24)$$

Without this contact term the second-order form would not be locally supersymmetric.

Simple supergravity can be put in interaction with supersymmetric matter in a locally supersymmetric way. Just as the energy-momentum tensor is the source for Einstein's equation, the vector-spinor current is the source for the Rarita-Schwinger equation. The vector-spinor current for supersymmetry multiplets is known [see the example (9)] and therefore one has no difficulty in writing the trilinear interaction terms. These, however, turn out not to be locally supersymmetric by themselves. To complete the Lagrangian one must add appropriate contact terms constructed so as to preserve local supersymmetry and one must also modify the transformation laws of the fields by adding terms which depend on the interaction. This procedure is in principle trivial, but in practice requires rather complicated and lengthy calculations involving high-precision gamma matrix and Fierz rearrangement algebra. Clearly, a better understanding of the geometry of supergravity would simplify the construction of locally supersymmetric actions (this will probably be based on the technique of superspace). Nevertheless, the coupling of simple supergravity to matter supermultiplets has been completely worked out with the above methods for the scalar¹⁶⁾ (spins 0, 0, $\frac{1}{2}$) and for the massless vector (spins $\frac{1}{2}$, 1) and Yang-Mills¹⁷⁾ supermultiplets.

3. EXTENDED SUPERSYMMETRY AND SUPERGRAVITY

The simplest way to combine supersymmetry and internal symmetry is to attribute all fields of a supermultiplet to the same representation of the internal symmetry group. In this case the spinor charges are singlets under the internal symmetry. This is the case, for instance, in the supersymmetric Yang-Mills theories^{3, 5)}. Another, and perhaps more interesting way, is what is called extended supersymmetry^{3, 18)}. The spinor charges are taken to belong to a representation of the internal symmetry group and carry, in addition to the spinor index (α or β), an internal index (i or j). In global extended supersymmetry one has the anticommutation relation, which generalizes (3)

$$\{Q_{\alpha}^i, \bar{Q}_{\beta}^j\} = -2 \delta^{ij} (\gamma^a)_{\alpha\beta} P_a \quad . \quad (25)$$

We consider the case where the internal symmetry is $O(N)$ and the charges belong to the N -dimensional vector representation. Since there are several spinor charges ($i, j = 1, \dots, N$), the multiplet structure is more complicated than that described in the preceding section. For instance, for zero mass, we can start with a state of helicity λ ; applying to it any of the spinor charges, we obtain N states of helicity $\lambda - 1/2$ and belonging to the vector representation of $O(N)$; applying again a spinor charge we obtain $N(N-1)/2 = N!/2!(N-2)!$ states of helicity $\lambda - 1$; in general, applying the spinor charges k times we obtain $N!/k!(N-k)!$ independent states of helicity $\lambda - (k/2)$. For $k = N$ we have one state of helicity $\lambda - (N/2)$; if this helicity equals $-\lambda$, we have a symmetric situation and the multiplet is complete; otherwise we must consider the states obtained by starting from helicity $-\lambda$ and applying the conjugate spinor charge to reach helicities $-\lambda + 1/2, \dots, -\lambda + (N/2)$. All these states together form an extended supermultiplet. There are interesting examples of renormalizable field theories^{19, 20)}, involving fields of maximum spin 1, and which admit a global extended supersymmetry for $N = 2, 3$ and 4 (for $N > 4$ one has necessarily spins higher than 1 and this leads to problems with renormalizability. Extended global supersymmetry has also been called hypersymmetry¹⁹⁾).

Corresponding to extended global supersymmetry one can construct extended supergravity theories. If one wants the graviton to be a singlet with respect to the internal symmetry, one must start with maximum helicity two.

One has then a number of different supergravity theories depending upon the value of N , which however can be at most equal to eight, since for $N > 8$ the supermultiplet will contain some spin > 2 and the graviton will not be a singlet (it is doubtful whether consistent interacting local theories with spins > 2 can be constructed; consistency seems to require a gauge principle, and fields of spin higher than two do not seem to allow an interpretation as gauge fields). The simplest extended supergravity theory is the $O(2)$ theory²¹⁾, for $N = 2$. There are massless particles of spins 2, $3/2$ and 1, with respective multiplicity 1, 2, 1. This theory admits a local supersymmetry with two Majorana anticommuting parameters (or one complex spinor). For $N = 3$ the spins are 2, $3/2$, 1, $1/2$, with multiplicity 1, 3, 3, 1, respectively, and the theory is invariant under a local supersymmetry with three Majorana parameters^{22, 23)}. For $N = 4$ the spins are 2, $3/2$, 1, $1/2$, 0, with respective multiplicities 1, 4, 6, 4, 2 (according to the above argument we need two scalars, corresponding to the two possible helicities of the other spins). The Lagrangian of this theory is also known^{24, 25)}. For $4 < N \leq 8$ it would be interesting to see if locally supersymmetric theories corresponding to the multiplet structure indicated by the algebra actually exist. It should be mentioned that arguments based on the spinor dual model²⁰⁾ imply the existence of some of these supergravity theories.

The $O(2)$ and $O(3)$ supergravity theories have an interesting invariance, which is a generalization of the duality invariance of Maxwell theory and of Maxwell-Einstein theory²⁶⁻²⁹⁾. That there exist theories with interacting vector and spinor fields having an exact duality invariance is a remarkable fact, which apparently had not been noticed earlier. We give here an extremely simple example of a theory of this type (this theory is not supersymmetric). Consider the Lagrangian (flat space)

$$\begin{aligned}
 L = & -\frac{i}{4} F_{ab}^2 - \frac{i}{2} \bar{\psi} \gamma \cdot \partial \psi - \frac{i}{2} \bar{\varphi} \gamma \cdot \partial \varphi \\
 & + \frac{if}{2} F^{ab} \bar{\psi} \sigma_{ab} \psi + g (\bar{\psi} \sigma_{ab} \varphi)^2,
 \end{aligned} \tag{26}$$

where, as usual,

$$F_{ab} = \frac{\partial}{\partial x^a} A_b - \frac{\partial}{\partial x^b} A_a, \tag{27}$$

and ψ and φ are two massless Majorana spin $1/2$ fields.

The equation of motion for F_{ab} is

$$\frac{\partial}{\partial x^a} \left(F^{ab} - if \bar{\psi} \sigma^{ab} \psi \right) = 0 , \quad (28)$$

while (27) can be replaced by

$$\frac{\partial}{\partial x^a} \tilde{F}^{ab} = 0 , \quad (29)$$

where $\tilde{F}^{ab} \equiv \frac{1}{2} \epsilon^{abcd} F_{cd}$ is the dual "electromagnetic" field. Equations (28) and (29) go into each other by the transformation

$$\begin{aligned} \delta F_{ab} &= \Lambda \left(\tilde{F}_{ab} - if \bar{\psi} \gamma_5 \sigma_{ab} \psi \right) \\ \delta \psi &= - \frac{\Lambda}{2} \gamma_5 \psi \quad \delta \bar{\psi} = - \frac{\Lambda}{2} \gamma_5 \bar{\psi} , \end{aligned} \quad (30)$$

where Λ is an infinitesimal constant parameter. This transformation is a generalization of the electromagnetic duality transformation. If one introduces the field

$$\hat{F}_{ab} = F_{ab} - \frac{if}{2} \bar{\psi} \sigma_{ab} \psi , \quad (31)$$

one finds that it transforms into its dual

$$\delta \hat{F}_{ab} = \Lambda \tilde{\hat{F}}_{ab} . \quad (32)$$

The equations of motion for the fields ψ and ϕ are also invariant under (30) provided $g = f^2/8$. This can be verified very simply if one observes that the particular combination (31) enters those equations. So, the theory given by (26) with $g = f^2/8$ has an exact duality invariance. This is true only with the particular value of the contact term and in the absence of minimal coupling. It is amusing that duality, which is a non-local transformation in terms of the Lagrangian variables, can fix the couplings of a local field theory.

Coming back to the $O(2)$ and $O(3)$ theories, they have just the right non-minimal (magnetic moment) couplings and contact terms required by duality invariance. The duality transformation of the $O(2)$ theory is²⁸⁾

$$\delta \hat{F}^{mn} = \Lambda e^{-1} \tilde{F}^{mn} \quad (33)$$

$$\delta \psi_m = -\frac{1}{2} \Lambda \gamma_5 \psi_m \quad \delta \varphi_m = -\frac{1}{2} \Lambda \gamma_5 \varphi_m ,$$

where ψ_m and ϕ_m are the two spin $3/2$ Majorana fields and $\hat{F}^{mn}$ is a suitably completed field [in a manner somewhat similar to (31)]. The theory is also invariant under a chiral transformation which, together with the $O(2)$ invariance, generates an $SU(2)$ symmetry

$$\begin{aligned} \delta_1 \psi_m &= \varphi_m & \delta_1 \varphi_m &= -\psi_m \\ \delta_2 \psi_m &= \gamma_5 \psi_m & \delta_2 \varphi_m &= -\gamma_5 \varphi_m \\ \delta_3 \psi_m &= \gamma_5 \varphi_m & \delta_3 \varphi_m &= \gamma_5 \psi_m \end{aligned} \quad (34)$$

[the vector field does not transform]. The duality (33) completes this to an $SU(2) \times U(1)$ symmetry of the S matrix.

The case of the $O(3)$ theory is even more interesting, since duality becomes part of a non-Abelian symmetry²⁸⁾. The duality transformation is now

$$\delta \hat{F}^{mn,i} = \Lambda^{ij} e^{-1} \tilde{F}^{mn,j} \quad (35)$$

$$\delta \psi_m^i = \left(\Lambda^{ij} - \frac{1}{2} \delta^{ij} \text{Tr} \Lambda \right) \gamma_5 \psi_m^j \quad \delta \lambda = -\frac{1}{2} \text{Tr} \Lambda \gamma_5 \lambda .$$

Here $\hat{F}^{mn,i}$, ψ_m^i are the three vector (suitably completed) and spin $3/2$ fields ($i, j = 1, 2, 3$), λ is the spin $1/2$ singlet and $\Lambda^{ij} = \Lambda^{ji}$ is a constant symmetric real matrix. The transformation (35) extends the original $SO(3)$ invariance of the theory to an $SU(3) \times U(1)$ invariance of the S -matrix, the matrix Λ^{ij} playing the role of the imaginary part of the anti-Hermitian 3×3 infinitesimal $U(3)$ matrix. In the Weyl representation for the gamma matrices, the upper components of the fields ψ_m^i transform by the three-dimensional representation 3 of $SU(3)$, the lower components by 3^* . Similarly $\hat{E}^i + i\hat{H}^i$ transform by the representation 3. Presumably, duality

transformations exist for all $O(N)$ theories, $N \leq 8$, and complete the invariance group to $U(N)$ [for $N = 4$ see Cremmer, Scherk and Ferrara^{25, 29}].

The $O(2)$ and $O(3)$ theories have the duality invariance because there is no minimal coupling between vectors and spinors. For instance, in the $O(3)$ theory, the trilinear coupling is given by

$$\frac{\kappa}{2} \varepsilon^{ijk} \bar{\psi}_m^i \left(F^{mn,k} - \gamma_5 \tilde{F}^{mn,k} \right) \psi_n^j \quad (36)$$

If one is willing to violate duality invariance one can introduce an additional minimal coupling, which in the $O(3)$ theory would be obtained by the replacement

$$D_m \psi_n^i \rightarrow D_m \psi_n^i + g \varepsilon^{ijk} A_m^j \psi_n^k \quad (37)$$

It turns out that local supersymmetry can be preserved³⁰⁻³³) by adding a mass term for the spin $3/2$ field

$$- \frac{g}{\kappa} e \bar{\psi}_m^i \sigma^{mn} \psi_n^i \quad (38)$$

and a cosmological term

$$\frac{3}{2} \frac{g^2}{\kappa^4} e \quad , \quad (39)$$

where, as always above, $e = \det e_m^a = \sqrt{-g}$. This type of theory is extremely interesting, because it provides examples of spin $3/2$ fields coupled by a charge-like minimal coupling to the vector field in a locally supersymmetric and therefore consistent way. Nevertheless, their physical interpretation is not clear, because the strength of the cosmological term is fixed by the other couplings and, as apparent from (39) they do not admit a perturbation expansion in the gravitational constant. Therefore, there is no flat space-time particle interpretation. A possible particle interpretation would require quantization about a background de Sitter space. Unfortunately, interesting values of the gauge constant g correspond to an undesirably large cosmological constant. A possible way out of this difficulty³⁴) is to couple supergravity to supersymmetric matter with spontaneously broken supersymmetry. One can show that spontaneous breaking of supersymmetry

gives rise to a cosmological term of strength dictated by the magnitude of the supersymmetry breaking, and having sign opposite to (39). In this way the total cosmological term can be adjusted to an arbitrarily small value, including zero.

It is interesting to observe³⁴⁾ that, even without coupling supergravity to supersymmetric matter, one obtains a classically consistent theory simply by dropping from the Lagrangian the cosmological term, such as (39). Naturally, this violates local supersymmetry, but this particular form of explicit symmetry-breaking is harmless enough; the resulting constraints are just those required for massive spin $3/2$ fields. In this way one obtains a theory of massive spin $3/2$ fields with charge-like coupling to the gauge vectors, free of the usual inconsistencies and acausal propagation. This theory can be quantized in flat space-time. Its consistency is dependent on the special relations between the minimal coupling constant g and the other couplings, such as the magnetic moment coupling and the gravitational constant, which are the remnants of the original supersymmetry.

4. RENORMALIZATION

Pure gravity is one-loop renormalizable on the mass shell^{1,2)}. The possible divergent counter terms are of the form

$$a R^2 + b R_{mn} R^{mn} + c R_{mnrz} R^{mnrz} \quad . \quad (40)$$

In the background field method, one can go to the mass shell by using the classical equations of motion, which for pure gravity are

$$R_{mn} = 0 \quad (41)$$

Therefore the first two terms in (40) vanish. The third term can be transformed by using the Gauss-Bonnet formula

$$\int d^4x \sqrt{-g} \left(R_{mnrz} R^{mnrz} - 4 R_{mn} R^{mn} + R^2 \right) = 0 \quad (42)$$

for an asymptotically flat space. Therefore, in the action, it is equivalent to a linear combination of the first two and it vanishes on the mass shell. At the two-loop level pure gravity is probably not renormalizable.

Gravity in interaction with spin 0, 1, or $\frac{1}{2}$ fields is not even one-loop renormalizable. Equation (41) is no longer true and explicit calculations show that the divergent counter terms have non-vanishing coefficients^{1, 2)}.

For supergravity the situation is better. An argument based on (42), duality, and global supersymmetry shows that in simple supergravity and in the O(2) and O(3) extended supergravity theories (without minimal coupling!) all four particle amplitudes are one-loop finite²⁷⁾. Furthermore, certain leading divergences in the two-loop four-particle amplitudes vanish in simple supergravity³⁵⁾ [these divergences do not vanish in pure gravity]. Recently, arguments have been given to generalize the above results to more than four-particle amplitudes and to all two-loop divergences^{36, 37)}. Therefore, supergravity and the O(2) and O(3) theories are probably renormalizable up to two loops. (At the three-loop level, difficulties occur in pure supergravity. These difficulties may be absent in the extended theories).

These results are very remarkable if one compares them with the situation before supergravity was considered. Simple supergravity is a theory in which the graviton interacts with the spin $\frac{3}{2}$ field: its two-loop renormalizability should be contrasted with the lack of one-loop renormalizability of gravity interacting with spin 0, 1, or $\frac{1}{2}$. The O(2) theory contains as a subtheory the Maxwell-Einstein theory (which is not one-loop renormalizable). The O(3) theory contains as a subtheory the Einstein-Dirac theory, and also the theory of simple supergravity coupled to the $\frac{1}{2}$, 1 supermultiplet, both of which are one-loop non-renormalizable³⁸⁾. These examples indicate that the convergence properties are better when all fields belong to the same supermultiplet (have a common geometrical meaning) and a single coupling constant governs all interactions. At this time nothing is known of the renormalization properties of the extended supergravity theories with additional minimal couplings of the type (37).

5. CONCLUDING REMARKS

Supergravity and extended supergravity provide us with theories which are consistent at the classical level and are remarkably convergent when quantized. These properties are the consequence of the invariance under a new type of gauge transformation, local supersymmetry transformations. Fields with spin 0, $\frac{1}{2}$, 1, $\frac{3}{2}$ and 2 can be unified as constituents of a single (irreducible) geometric object.

In spite of these qualitative successes, the application of the ideas of supergravity to the physical world is still beset with difficulties. The simplest approach³⁹⁾ would be to require that the minimal group of non-Abelian gauge theories, which is $SU(2) \times U(1) \times SU(3)$ [weak, electromagnetic and colour], should be a subgroup of the internal symmetry group of supergravity. This is unfortunately impossible³⁹⁾, since the largest possible group of extended supergravity is $O(8)$, which is too small. The enlargement of $O(8)$ to $U(8)$ by means of duality transformations will not solve this problem, since it requires the absence of minimal, charge-like couplings to the vectors, so that the group $U(8)$ would be a global, not a gauge, invariance. An interesting approach which is being pursued at present^{40,41)} is to develop a supergravity theory based on the gauging of the extended superconformal algebra⁴²⁾, which has $U(N)$, rather than $O(N)$, as the group of internal symmetry. This would give directly a $U(N)$ gauge group. If, furthermore, the various fields arrange themselves according to acceptable representations, this approach could become very promising and one could go on to consider the difficult problem of symmetry-breaking. Alternatively, one may hope that some version of the approach followed by Arnowitt and Nath⁸⁾ will provide the desired unified theory.

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