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LIMITATIONS OF THE BUNCH-CURRENT
BY TRANSVERSE MODE-COUPPLING

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Abstract

In large storage rings such as PETRA or PEP, the current per bunch has been found to be limited due to the transverse impedance of the RF cavities. This behaviour is usually explained by coupling of two transverse modes with adjacent mode numbers, e.g. $m=0$ and $m=1$. However, recent experiments at DCI established the existence of negative mode-numbers, which were already included theoretically in Ref.4, and the threshold for coupling of modes $m=0$ and $m=-1$ is usually much lower. For LEP, the current threshold is then estimated as only 0.4 mA/bunch (1.6 mA total) and several possibilities to increase this limit are discussed.

Introduction

During the running-in of the electron-positron storage ring PETRA it was found that the beam current was limited by a vertical single-bunch instability which became stronger as more RF cavities were installed. A similar effect in both transverse planes has been observed at PEP after the installation of the "mini-beta scheme" which had raised the betafunctions at the RF cavities.

In smaller storage rings, the single-bunch "head-tail" instability had been known for quite some time ¹⁾. However, the instability observed in the larger rings PETRA and PEP did not depend strongly on chromaticity and did not vanish for $\xi=0$ which rules out the head-tail instability. In analogy to longitudinal mode-coupling proposed a few years earlier, it was found that transverse mode-coupling could explain most of the experimental observations ²⁾. However, the relevant theory is still in evolution and recently experiments were performed at DCI (Orsay) ⁵⁾ to test some of our predictions. As a result of these measurements it became clear that negative mode-numbers do not only exist but could also play a decisive role in mode-coupling. This LEP Note describes the consequences of including them in the theory.

1. Mode-Coupling Theory

The basic theory of transverse mode-coupling has been discussed in LEP Notes 363 and 385. Here we give a short review of the most important points.

The frequency of the m -th (uncoupled) transverse "head-tail" mode can be written

$$\omega_m = \omega_3 + m\omega_s + M_{mm} \quad (1)$$

when higher radial modes are neglected. Here ω_3 is the betatron and ω_s the synchrotron frequency, while M_{mm} are the diagonal elements of the matrix M discussed below. Here the off-diagonal elements have been neglected and no mode-coupling occurs.

For larger machines, the off-diagonal elements become more important and we have to replace Eq(1) by the infinite determinant²⁾

$$\det [\omega - \omega_3 + m\omega_s + M_{mn}] = 0 \quad (2)$$

where ω are the coherent frequencies of oscillation. The matrix elements can be written

$$M_{mn} = j D F_m Z_{mn}^{\text{eff}} \quad (3)$$

$$\text{The factor } D = \frac{\omega_0 \beta I_b}{4\pi E/e} \quad (4)$$

is proportional to the current per bunch I_b and to the betafunction β at the location of the (biggest) transverse impedance Z (usually the RF cavities). For a Gaussian beam, the formfactor F_m is taken simply as $1/(m+1)$. The effective (transverse) impedance is defined by

$$Z_{mn}^{\text{eff}} = \sum_{p=-\infty}^{\infty} Z_T(\omega_p) h_{mn}(\omega_p - \omega_\xi) \quad (5)$$

$$\text{where } \omega_p = \omega_3 + m\omega_s + p\omega_0 \quad (6)$$

are the spectral frequencies, and

$$\omega_\xi = \xi \omega_3 / \alpha \quad (7)$$

is the "chromatic" frequency (ξ chromaticity, α momentum compaction). The normalized cross-power-density for a Gaussian beam is given by

$$h_{mn}(\omega) = j^{m-n} C_m C_n \cdot (\omega\tau)^{m+n} \exp(-\omega^2 \tau^2) \quad (8)$$

$$\text{where } C_m^2 = \frac{2}{3} \frac{\sqrt{2\pi}}{\Gamma(m+\frac{1}{2})} \quad (9)$$

is a normalization factor.

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Table 1

Parameters

	LEP		PETRA	DCI
	60°	90°		
R (m)	4243		366	15.1
E (GeV)	20		6	1
$\nu(-)$	66.2	94.2	25.2	1.7
$\nu_{so}(-)$	.096	.07	.063	.007
$\alpha(-)$	.004	.002	.027	.0788
$\beta(m)$	64	50	14.5	8
$\xi(-)$	0	0	0	0
$R_T (M\Omega/m)$	3.2*		1.6*	0.4
Q (-)	1		1	1
$f_r (GHz)$	1.3		2.0	1.3

*Obtained with $\langle b \rangle = 1.1$ b (K.Hübner, private communication).

In ref. 7, table 3, the transverse impedance of 2.2 M Ω /m has been obtained with $\langle b \rangle = 1.4$ b, which fits the cavity modes at low frequencies. For short bunches, the spectrum extends to higher frequencies and the smaller value of $\langle b \rangle$ should be used.

References

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2. Negative mode numbers

Most previous theories ¹⁻³⁾ have assumed that the mode-numbers m , which describe the azimuthal periodicity of the oscillation in longitudinal phase-space, and hence the bunch-shape, are either zero or positive. However, the expansion of the oscillation into a series with $\exp(jm\phi)$ permits both positive and negative values of m . The possible importance of that fact for longitudinal mode-coupling has been described more than a year ago³⁾, and more recently it has also been stressed for the transverse case⁴⁾. However, the decisive turn came with the measurement of frequency shifts in DCI⁵⁾ which could be identified with negative mode-numbers.

Generalizing the theory to include negative mode-numbers, one finds that m has to be replaced by $|m|$ everywhere except in Eqs (2) and (6) where it occurs in the frequency shift $\Delta\omega_s$. Evaluating Eq (2) with the matrix elements of a (broad-band) resonator, one finds usually that the slope of the $m=0$ mode is negative and larger than that of all other modes. Hence if mode coupling occurs, it will be preferentially between modes 0 and -1. For longer bunches, it shifts to (-1,-2) or even (-2,-3).

In order to find the eigenvalues of Eq.(2), the infinite matrix was truncated to such a size that at least one uncoupled mode was found on either side of the coupled ones (from 5×5 to 9×9). For the parameters of LEP with 60° phase-advance/cell (see table I) mode-coupling occurs between modes (0,-1) or (-1,-2) depending on bunch lengths (see Fig.1). The threshold current as a function of bunch-length is shown in Fig.2. For the estimated RMS bunch-length $\sigma = 1.8$ cm and a broad-band resonator impedance $R/Q_T = 3.2 \Omega/\text{m}$ at 1.3 GHz, the threshold is only about 0.40 mA/bunch, (i.e. 1.6 mA beam-current). For a 90° lattice the threshold is even slightly lower (0.38 mA, see Fig.3).

The estimate for the threshold of LEP is much lower in ref.⁴⁾ (0.14 mA), but appears to be based on a too high value for the transverse impedance. The effect of higher radial modes has also been investigated in that reference and found to be of no consequence.

3. Thresholds in other machines

Considering the importance of such a severe limitation for LEP, we have attempted to verify our theory with two existing machines: PETRA and DCI.

For PETRA we assume a broad band resonator with a somewhat higher resonant frequency (2GHz) because of the smaller 500MHz cavities and a slightly lower vacuum-chamber. The results for the parameters shown in table I are given in Fig.4. The threshold increases from 4 mA for short bunches ($\sigma \approx 1$ cm) to over 20 mA for long ones ($\sigma \approx 6$ cm), in quite good agreement with observation. The frequency shift of the $m=0$ mode, however, appears to have a maximum while experimental points were fitted with a monotonous curve⁶⁾.

Since DCI is a small machine, bunch-lengthening is so large that transverse mode-coupling does not occur up to the maximum current that can be stored (300 mA). This experimental result⁵⁾ is also obtained from our theory, but again the slopes of the individual modes do not agree completely with observation (Fig.5).

This may be caused by the inherent limitations of the broad-band model, and we shall attempt to improve this rather rough estimate for the impedance of a set of cavities.

6. Conclusions and Recommendations

The inclusion of negative mode-numbers in the theory of transverse mode-coupling has lowered the expected threshold current by a factor of almost two. In order to still obtain the design luminosity, means have to be found to increase the current.

The most obvious method to increase the threshold is an increase of RF-voltage. This increases the synchrotron tune and reduces the bunch-length. Both effects increase the threshold if the original working-point is near a minimum (point A, Fig.2, 0.4 mA, increases to point B, 0.55 mA). There is enough voltage available at injection energy to increase the threshold even by a much larger factor, but at top energy this will require additional RF power if the synchrotron tune has to be kept constant during acceleration (to avoid synchro-betatron resonances). If that requirement can be relaxed, this method is the easiest one to raise the threshold, but experiments on existing machines are needed to verify the predictions.

Bunch-lengthening appears to be another possible cure, but quite large factors are required which may rule out simple reduction of the longitudinal damping partition number. Active devices like a dipole/octupole wiggler or a higher-harmonic RF system may be needed.

A reduction of the (vertical) beta function at the RF cavities is hardly possible with the present period length in the RF section. The transverse impedance could be reduced somewhat by installing fewer cavities which have to be driven harder, but that would increase the required power-level. Both methods are somewhat limited by the influence of the unaffected vacuum-chamber impedance (estimated to be about 1/4 of the cavity impedance⁶⁾).

Attempts to increase the current thresholds with feedback in PEP and PETRA were not successful. However, if one uses a reactive rather than the conventional resistive feedback - which has worked successfully in the ISR⁸⁾ - it may be possible to reduce the slope of the $m=0$ mode alone and thereby increase the threshold for coupling. Experiments on existing large storage rings would be very useful.

Fig 1: LEP-20 GeV (60° lattice)

Fig 2: LEP-60° lattice
20 GeV

Fig. 2 Threshold-current per bunch versus RMS bunch-length for LEP at 20 GeV (60° lattice).

Fig 3: LEP-90° lattice
20 GeV

Fig. 3 Threshold-current per bunch versus RMS bunch-length for LEP at 20 GeV (90° lattice).

Fig 4: PETRA 6 GeV

Fig. 4 Threshold-current per bunch versus RMS bunch-length for PETRA at 6 GeV.

Fig. 5 Tune-shift versus current in DCI at 800 MeV.