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MEASUREMENTS OF BETATRON PHASE ADVANCE AND BETA FUNCTION IN THE ISR

by

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To check the optics of large storage rings measurements of the lattice functions are important. Some of these measurement methods have been tried out in the ISR. Information about the optics all round the ring is obtained by exciting a continuous betatron oscillation. The responses of different position monitors are compared with a network analyser. The square of the observed amplitude gives directly the relative value of the beta function. By measuring the relative phase of the responses the betatron phase advance between the locations of the monitors is obtained. Most of the error introduced by cable lengths is eliminated by observing both the upper and lower betatron side bands. The phase measurement does not depend on the monitor calibration. Its accuracy is therefore quite good as is confirmed by comparing the results with computations. Near the low-beta insertion, the beta functions are measured by observing the variation of the tune with changing quadrupole excitation.

1. Introduction

To understand the optics of a storage ring it is sometimes desirable to measure the lattice functions and to compare them with calculations. This can be particularly important for machines with a complicated optics containing low-beta insertions with elaborate chromatic corrections and for low-emittance lattices. In a very large machine like LEP access to the lattice elements is difficult and such measurements might be essential to locate sources of possible errors in the optical properties. With this goal in mind, a series of tests have been carried out in the ISR mainly to study the accuracy, technical problems and limitations of lattice measurements.

2. Betatron oscillations

We consider a storage ring with an equilibrium orbit of average radius R = circumference/ 2π , revolution time $T_0 = 2\pi/\omega_0$ and betatron tune Q . The azimuthal position around the ring is measured with the angle $\theta = s/R$ where s is the path length along the orbit. A continuous betatron oscillation is excited by a kicker at $\theta = 0$ and measured by a beam position sensitive monitor at a location θ ; fig. 1.

A beam with M equidistant bunches $m = 0, 1, 2 \dots M-1$ of equal intensity is considered. The current $I_m(t, \theta)$ due to bunch m measured at θ is

$$I_m(t, \theta) = \sum_{k=-\infty}^{+\infty} I_b(t - (k+m/M + \theta/2\pi)T_0)$$

Here $I_b(\tau)$ is the bunch form expressed as instantaneous current which we assume for convenience to be symmetric $I_b(-\tau) = I_b(\tau)$. Its Fourier transform is

$$\tilde{I}_b(\omega) = \int_{-\infty}^{+\infty} I_b(\tau) e^{-i\omega\tau} d\tau$$

A coupled bunch mode betatron oscillation is excited by the kicker. It is assumed that each bunch oscillates with the lowest bunch shape mode where all its particles follow closely the same betatron trajectory. The betatron oscillation of different bunches is related by the coupled mode number n , $n = 0, 1, 2 \dots M-1$, which determines the relative phase $\Delta\psi = n2\pi/M$ between the oscillations of adjacent bunches. The position sensitive monitors measure the dipole moment

of the beam and the obtained signal $U(t, \theta)$ is equal to the current times the displacement y times a gain factor g

$$U(t, \theta) = \frac{gMy_0}{T_0} \sqrt{\frac{\beta(\theta)}{\beta(0)}} \sum_{p=-\infty}^{\infty} \tilde{I}(\omega_p) \cos(\omega_p t + \phi(\theta) - \theta\omega_p/\omega_0) \quad (1)$$

with $\omega_p = \omega_0(pM+n+Q)$, y_0 = amplitude at $\theta = 0$, $\beta(\theta)$ = beta function and $\phi(\theta)$ = betatron phase advance

$$\phi(\theta) = \int_0^\theta \frac{R}{\beta(\theta)} d\theta$$

The sum in (1) contains positive and negative frequencies ω_p . For practical applications it is convenient to split (1) into two parts, each containing positive frequencies only

$$U(t, \theta) = \frac{gMy_0}{T_0} \sqrt{\frac{\beta(\theta)}{\beta(0)}} \left[\sum_{p_f} \tilde{I}(\omega_{p_f}) \cos(\omega_{p_f} t + \phi(\theta) - \theta\omega_{p_f}/\omega_0) + \sum_{p_s} \tilde{I}(\omega_{p_s}) \cos(\omega_{p_s} t - \phi(\theta) - \theta\omega_{p_s}/\omega_0) \right] \quad (2)$$

The first term is summed over all integer values of p_f for which $\omega_{p_f} = \omega_0(p_fM+n+Q)$ is positive; it represents the fast waves or upper side-bands. The second term is summed over all p_s for which $\omega_{p_s} = \omega_0(p_sM-n-Q)$ is positive and represents the slow waves or lower side-bands. The amplitude y_0 of the oscillation is only significantly different from zero if the frequency of the kicker is in the vicinity of one of the side-bands ω_{p_f} or ω_{p_s} ; in this case all the side-bands in (2) will appear in the monitor signal. For the case of a single bunch we set $M=1$ and $n=0$ in (1) or (2).

For a continuous unbunched beam of total current I_0 the signal $U(t, \theta)$ observed with a monitor is

$$U(t, \theta) = gY_0I_0 \sqrt{\frac{\beta(\theta)}{\beta(0)}} \cos(\omega_p t + \phi(\theta) - \theta\omega_p/\omega_0) \quad (3)$$

with $\omega_p = \omega_0(n+Q)$. Here the mode number n gives the number of transverse wiggles the beam contains for a fixed t ; n can take positive or negative values depending on the particle in front of a reference particle having a phase advance or phase lag. The expression (3) can contain a positive or a negative frequency. For practical applications it is better expressed either as a fast wave or upper side-band.

$$U_f(t, \theta) = gY_0I_0 \sqrt{\frac{\beta(\theta)}{\beta(0)}} \cos(\omega_{p_f} t + \phi(\theta) - \theta\omega_{p_f}/\omega_0) \quad (4a)$$

with $\omega_{p_f} = \omega_0(n_f+Q) > 0$, $n_f > -Q$ or in a slow-wave or lower side-band

$$U_s(t, \theta) = gY_0I_0 \sqrt{\frac{\beta(\theta)}{\beta(0)}} \cos(\omega_{p_s} t - \phi(\theta) - \theta\omega_{p_s}/\omega_0) \quad (4b)$$

with $\omega_{p_s} = \omega_0(n_s-Q) > 0$, $n_s > Q$.

In the case of the unbunched beam only the frequency given by the kicker is excited.

The betatron side-bands (4) have a certain width $\Delta\omega$ due to the dependence of the revolution frequency ω_0 and the betatron tune Q on momentum

$$\Delta\omega_{pf} = \omega_{oo} \left(-\eta(n_f + Q_o) + \frac{dQ}{dp/p} \right) \frac{\Delta p}{p} \quad (5)$$

$$\Delta\omega_{ps} = \omega_{oo} \left(-\eta(n_s - Q_o) - \frac{dQ}{dp/p} \right) \frac{\Delta p}{p}$$

Here ω_{oo} and Q_o are the revolution frequency and tune in the center of the momentum distribution in the beam, Δp is the momentum spread,

$\eta = \frac{-d\omega/\omega}{dp/p}$ and $\frac{dQ/Q}{dp/p}$ is the chromaticity.

3. Observation of amplitude and phase of the oscillation

A continuous betatron oscillation is excited at one of the frequencies ω_{pf} or ω_{ps} and observed in two position sensitive monitors located at θ_1 and θ_2 , the first one serving as reference, the second one as measurement (fig. 1). The amplitudes and phases of the two signals are compared. (In principle we could also compare one signal with the excitation of the kicker, however, the relation between the two is much more complicated and depends on the particle distribution¹). We can excite the beam with a swept frequency covering one or two side-bands and compare the signals with a network analyser. It is also possible to excite with band-width limited, uniform noise and compare the signals with a double channel FFT (Fast Fourier Transform).

The ratio between the signal amplitudes from the measurement and reference monitor is

$$\frac{\hat{U}(\theta_2)}{\hat{U}(\theta_1)} = \frac{g_2}{g_1} \frac{\sqrt{\beta(\theta_2)}}{\sqrt{\beta(\theta_1)}} \quad \text{or} \quad \frac{\beta(\theta_2)}{\beta(\theta_1)} = \left(\frac{g_1}{g_2} \frac{\hat{U}(\theta_2)}{\hat{U}(\theta_1)} \right)^2 \quad (6)$$

The gains g_1 and g_2 include monitor calibrations, cable attenuations and amplifier gains. They have to be known in order to obtain the relative beta functions with this measurement.^{2,3}

The relative phases $\Delta\xi$ of the signals from two monitors located at θ_1 , and θ_2 are according to (2) or (4)

$$\Delta\xi = \xi(\theta_2) - \xi(\theta_1)$$

$$\Delta\xi_f = \phi(\theta_2) - \phi(\theta_1) - (\theta_2 - \theta_1)\omega_{pf}/\omega_o - \omega_{pf} \left(\frac{L_2}{c_2} - \frac{L_1}{c_1} \right) \quad (7)$$

$$\Delta\xi_s = -(\phi(\theta_2) - \phi(\theta_1)) - (\theta_2 - \theta_1)\omega_{ps}/\omega_o - \omega_{ps} \left(\frac{L_2}{c_2} - \frac{L_1}{c_1} \right)$$

where L_2, L_1 are the lengths and c_1, c_2 the signal velocities in the cables between the monitors and the measurement station. From the measurement of the relative signal phase $\Delta\xi_f$ or $\Delta\xi_s$ the betatron phase advance between the locations θ_1 and θ_2 can be determined.

In many cases we can greatly reduce the errors due to the uncertainty of the cable length by observing both an upper and a lower side-band which are very close in frequency $\omega_{pf} \sim \omega_{ps}$.⁴ We split the betatron tune Q into a fractional and an integer or half integer part $Q = Q_{int} + \Delta Q$ if Q is close to an integer, $Q = Q_{int} + 1/2 + \Delta Q$ if Q is close to a half-integer.

The frequencies of the two side-bands closest by are

$$\omega_{pf}/\omega_o - \Delta Q = \omega_{ps}/\omega_o + \Delta Q$$

By taking the difference of the phases of the fast and slow waves (7) we get

$$\Delta\xi_f - \Delta\xi_s = 2 \left[\phi(\theta_2) - \phi(\theta_1) - \Delta Q \cdot (\theta_2 - \theta_1) - \omega_o \Delta Q \left(\frac{L_2}{c_2} - \frac{L_1}{c_1} \right) \right] \quad (8)$$

The cable delay is now multiplied with a rather small frequency $\Delta Q\omega_o$ and an inaccuracy in cable length will produce only a small error in the betatron phase measurement.

4. Experiments and Results

Most of the experiments were carried out with unbunched beams for which the instrumentation of the ISR is better adapted. A stack of protons with a total current of 2A and modest density was typically used.

The experimental set-up was basically the one shown in fig. 1. A frequency sweeping through one or two betatron side bands was used and the monitor signals were compared with a network analyser. An example of such a measurement of the phase difference and the amplitude ratio is shown in fig. 2.

The monitors used to measure the betatron oscillations were the standard pick-up stations distributed around the ISR for closed-orbit measurements which can be connected to the control room with relays⁵.

The relative beta function and the betatron phase advance were measured for 3 different configurations of the ISR and compared with AGS calculations.^{6,7,8} From comparison between theoretical and experimental betatron tunes⁹ and orbit bumps¹⁰ it is estimated that the latter represents the real machine with an error of less than $\approx 6^\circ$ in betatron phase and less than $\approx 5\%$ in beta function. The measurements presented here contain reading errors and uncertainties in amplifier gain and cable length giving estimated errors of $\approx 2^\circ$ in phase and 10% in beta function. However, some monitors gave occasionally signals leading to grossly wrong results, probably due to bad contact in the switching system. They were ignored in the data analysis. The amplitude measurement only gives information on the relative values of the beta function around the machine. The unknown scaling factor was adjusted to give the expected average value of the beta function.

The measurement of the relatively complicated double low-beta configuration¹¹ of the ISR gave an rms deviation from the calculation of 3° in betatron phase and 11% in beta function. The theoretical vertical beta function $\beta(\theta)$ and variation of the betatron phase $\phi(\theta) - Q\theta$ around the smooth phase advance is shown in fig. 3 together with the measurements for the quadrant of the ISR which contains the two low beta sections. Since several focusing elements are contained between adjacent monitors the measurement by itself cannot determine the lattice function everywhere but only check it at certain locations.

The method can also be used to measure the derivative of the lattice functions with respect to Δp . These functions $\frac{d\beta(\theta)/\beta(\theta)}{dp/p}$ and $\frac{d\phi(\theta)}{dp/p}$ represent the local chromatic effects and are important for chromaticity corrections.^{12,13} They were measured by observing the dependence of the amplitude ratio (6) and the relative phase (8) of the signals on momentum Δp or via (5) on ω within the same unbunched beam. The results agree with the general behaviour predicted by AGS calculations but showed variations with θ much larger than expected.

5. Measuring the beta function locally by a small change of a quadrupole excitation

The classical method to measure the beta function locally consists of measuring the change δQ of the betatron tune from the original value Q_0 due to a small change ΔK of the focusing strength in a single quadrupole. Within a short magnet approximation we have the relation¹⁴

$$\beta = \frac{4\pi}{\Delta K} \left(1 + \pi \delta Q \cot(2\pi Q_0) \right) \delta Q$$

Usually hysteresis effects limit the accuracy of such measurements. This can be improved by making several steps ΔK and using corrections based on the known magnetisation curve or by measuring the change in magnetic field directly.

This measurement method applied to the innermost quadrupoles of a low-beta insertion also gives information about the value of the beta function in the interaction point. Such measurements were carried out in the ISR¹⁴ and gave an agreement with AGS calculation of better than 3%. Since the betatron phase advance across a low-beta insertion is usually close to 180° a phase measurement would only give limited information.

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Fig. 1 Set-up for phase and amplitude measurement

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Fig. 2

Measurement of the phase difference $\Delta \xi$ and the amplitude ratio A_1/A_2 of the fast and slow wave of a vertical betatron oscillation with $Q = 8.887$ and $\Delta Q = -0.113$ observed with two monitors at different locations θ_1 and θ_2 .

Fig. 3 Vertical betatron phase advance $\phi(\theta)$ relative to Q_0 and beta function $\beta(\theta)$ of approximately one quadrant of the ISR. The lines give the AGS calculations and the points are the measurements.