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RELATION BETWEEN DIFFERENT AUXILIARY FIELD FORMULATIONS
OF $N = 1$ SUPERGRAVITY COUPLED TO MATTER

S. Ferrara, L. Girardello ⁺⁾ , T. Kugo ^{*)} and A. Van Proeyen ^{")}
CERN - Geneva

A B S T R A C T

We establish the connection between the old minimal set of auxiliary fields, the new minimal set and the Breitenlohner set. We show that the class of interactions which can be described by the latter two sets are particular cases of the interactions described in the old minimal set. The old and new minimal sets of auxiliary fields are equivalent for R symmetric Lagrangians. The precise correspondence of the bosonic part of the Lagrangians in the two minimal formulations is exhibited. In particular, this equivalence shows that one can have both a Fayet-Iliopoulos term and spontaneously broken supersymmetry with vanishing cosmological constant.

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- ⁺⁾ Permanent address : Istituto di Fisica dell'Università, Milano, INFN Sezione di Milano, Italy.
^{*)} On leave from Department of Physics, Kyoto University, Kyoto, Japan
^{")} Aangesteld navorser, N.F.W.O., Belgium.

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1. INTRODUCTION

Recently, general interactions of $N = 1$ supergravity coupled to an arbitrary $N = 1$ supersymmetric matter system have been constructed [1].

These interactions have raised great interest because of their potential application to particle physics. In fact, when local supersymmetry is spontaneously broken, one can exhibit the super BEH (Brout-Englert-Higgs) effect [2,3], the mass differences between ordinary particles and their superpartners, as well as the electroweak breaking scale in terms of gravitino mass $m_{3/2} \lesssim 0$ (1 TeV). The couplings of Yang-Mills theories to $N = 1$ supergravity have been obtained by the application of the tensor calculus applied to the supergravity multiplet $(2, \frac{3}{2})$ coupled to matter $(\frac{1}{2}, 0^+)$ and $(1, \frac{1}{2})$ multiplets.

However, in order to obtain these results one has to choose a definite description of the supergravity multiplet and in the current applications, the so-called minimal formulation of auxiliary fields has been used [4].

On the other hand, different auxiliary field formulations of $N = 1$ supergravity exist, namely the new minimal set [5] and the (non-minimal) Breitenlohner set [6] of auxiliary fields. It is therefore important to establish the connection among these different formulations and to understand whether different sets of auxiliary fields allow the construction of different interactions which may be relevant for physical applications.

The investigation carried out in this paper gives a complete answer to this problem.

We prove that all interactions constructed in terms of the so-called new minimal set as well as the Breitenlohner set of auxiliary fields are particular examples of the interactions that can be constructed with the original (old) minimal set.

In particular for R-symmetric interactions (to be defined below) the old minimal set and the new minimal set of auxiliary fields give completely equivalent formulations of the same theories at the classical level^{*)}. In particular we show

*) It has been shown [7] that these formulations are not equivalent at the quantum level. Auxiliary fields different from the old minimal ones generally lead to anomalies.

that in the new minimal set formulation of $N = 1$ supergravity coupled to the matter, the scalar potential is not positive definite. This in turn implies that, contrary to common belief, one can also in this case cancel the cosmological constant. Therefore in the new and old minimal set formulations, a Fayet-Iliopoulos term [8] is shown to be perfectly consistent with a vanishing cosmological term and spontaneous supersymmetry breaking. An explicit example of this mechanism has been provided in ref. [9].

The precise mathematical correspondence between these different auxiliary field formulations is reported here.

2. THE OLD MINIMAL SET OF AUXILIARY FIELDS

In the superconformal framework [10], the Lagrangian of the coupled $N = 1$ Matter-supergravity system of Cremmer et al. [1] is written as

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} [S_0 \bar{S}_0 \phi(S_i, \bar{S}^i (e^{2\tilde{J}V})_i{}^j)]_D \\ & + [g(S_i) S_0^3]_F - [f_{\alpha\beta}(S_i) W_a^\alpha \varepsilon^{ab} W_b^\beta]_F . \end{aligned} \quad (2.1)$$

Here S_i are a set of chiral matter multiplets whose component fields are denoted by (z_i, χ_{Li}, h_i) , $\bar{S}^i$ are the complex-conjugate multiplets and V is the gauge-multiplet, namely the connection superfield of the gauge group G . W_a^α is a spinor chiral superfield, the field-strength multiplet, $g(S)$ is a G -invariant chiral superfield, to be called superpotential and $f_{\alpha\beta}(S)$ is a chiral superfield transforming as the symmetric product of the adjoint representation of G . ϕ is a G invariant real superfield. The first components of these superfields are respectively denoted by

$$g(z_i) , \quad f_{\alpha\beta}(z_i) , \quad \phi(z_i, z^{*i}) . \quad (2.2)$$

The functions ϕ and g satisfy the G-invariance properties

$$\frac{\partial \phi}{\partial z_i} T_i^{\alpha j} z_j = z^{*i} T_i^{\alpha j} \frac{\partial \phi}{\partial z^{*j}} , \quad \frac{\partial g}{\partial z_i} T_i^{\alpha j} z_j = 0 . \quad (2.3)$$

$T_i^{\alpha j}$ are generator matrices of the group G in the representation of the chiral fields z_i . The matter fields S_i are assumed to have vanishing Weyl weight, while the compensating multiplet

$$S_0 = [z_0 , \chi_{L0} , h_0] \quad (2.4)$$

has Weyl weight +1.

The component form of the Lagrangian (2.1) has an easier derivation in the superconformal approach [11]. We will follow this approach from now on.

In the superconformal framework [10], the minimal auxiliary field formulations of locally supersymmetric interactions can be constructed with the help of the so-called supergravity conformal multiplet

$$(e_\mu^m , \psi_\mu , A_\mu , b_\mu) \quad (2.5)$$

with the additional compensating (chiral) multiplet given by eq. (2.4).

In eq. (2.5) the vector fields A_μ , b_μ are respectively the connection fields for local chiral transformations and dilatations (Weyl transformations).

After a suitable superconformal gauge choice the only non-vanishing components of S_0 are

$$S_0 = [1 , 0 , h_0 = \frac{1}{3}(S - iP)] . \quad (2.6)$$

Also b_μ can be completely eliminated by a gauge choice. In this superconformal gauge, the overall components of the Poincaré supergravity multiplet, including physical as well as auxiliary degrees of freedom, are therefore given by

$$(e_\mu^m , \psi_\mu , A_\mu , S , P) . \quad (2.7)$$

The "D" and "F" type density formulae which are needed to compute the component form of the Lagrangian density given by eq. (2.1) are respectively given [10, 4] by

$$\begin{aligned} [V_{(w=2)}]_D = e \left\{ D - \frac{1}{2} \bar{\psi} \cdot \gamma_i \gamma_5 \lambda - \bar{\psi} \cdot \gamma_i \gamma_5 \zeta + \frac{1}{3} C (R + e^{-1} \bar{\psi} \cdot R) \right. \\ \left. + \frac{i}{4} \epsilon^{mnkl} (\bar{\psi}_m \gamma_n \psi_k) (B_l - \frac{1}{2} \bar{\psi}_l \zeta - C A_l) \right\}, \end{aligned} \quad (2.8)$$

$$[\Sigma_{(w=3)}]_F = e (h + \bar{\psi}_R \cdot \gamma \chi_L + \bar{\psi}_{mR} \sigma^{mn} \psi_{nR} x) + h.c. , \quad (2.9)$$

with the convention $\bar{\psi} \equiv \psi^T \mathbf{C}$ ($\mathbf{C}$: charge conjugation matrix). Eqs. (2.8) and (2.9) define superconformal invariant Lagrangian densities for real vector multiplet

$V_{(w=2)} = [C, \zeta, H, K, B_m, \lambda, D]$ with Weyl weight 2 and for the chiral multiplet

$\Sigma_{(w=3)} = [z, \chi_L, h]$ with Weyl weight 3, respectively.

We now come to discuss R-invariance. In ref. [12] it was shown that the introduction of a Fayet-Iliopoulos term in eq. (2.1) requires an extra symmetry for the functions $\phi(z, z^*)$, $g(z)$. Suppose we assign charge $+n_i$ to the multiplet S_i and $-n_i$ to its complex conjugate multiplet $\bar{S}^i$. We also give charge -1 to S_0 (+1 to $\bar{S}_0$). Then the extra invariance mentioned above requires $\phi(S_i, \bar{S}^i)$ to be a function of zero charge and the superpotential $g(S)$ to have charge +3. With respect to the previous choices of charges we have

$$\phi(S_i, \bar{S}^i) = \phi(e^{-in_i q} S_i, e^{+in_i q} \bar{S}^i), \quad (2.10)$$

$$g(S_i) = e^{3iq} g(e^{-in_i q} S_i).$$

Here q is a real (constant) parameter.

The transformation defined by eq. (2.10) is called R-transformation, and any system with the previous invariance properties is called R-symmetric.

We note that gauge interactions can be introduced without any problem since the gauge potential V is inert under R-symmetry

$$V \rightarrow V. \quad (2.11)$$

This implies that the Lagrangian density (2.1) is globally R-symmetric provided eqs. (2.10) are fulfilled and, in addition, the "gauge metric" $f_{\alpha\beta}(S)$ is a chiral function of charge zero, i.e.

$$f_{\alpha\beta}(S_i) = f_{\alpha\beta}(e^{-in_i\theta} S_i) . \quad (2.12)$$

The Fayet-Iliopoulos term arises when the R-symmetry is "gauged" in the sense of supersymmetry [12, 13]; i.e., when the parameter of the R-transformation becomes a chiral multiplet. Let us introduce a gauge-potential multiplet V_R for this local $U_R(1)$ symmetry (Fayet-Iliopoulos gauge symmetry). The transformation laws are:

$$\begin{aligned} V_R &\rightarrow V_R + i(\Lambda - \bar{\Lambda})/g_R , \quad (\Lambda ; \text{chiral parameter}) \\ S_0 &\rightarrow e^{i\Lambda} S_0 , \quad \bar{S}_0 \rightarrow e^{-i\bar{\Lambda}} \bar{S}_0 , \\ S_i &\rightarrow e^{-in_i\Lambda} S_i , \quad \bar{S}^i \rightarrow e^{in_i\bar{\Lambda}} \bar{S}^i . \end{aligned} \quad (2.13)$$

Then the most general Lagrangian with local $U_R(1)$ symmetry is given by

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} \left[\bar{S}_0 e^{-g_R V_R} S_0 \phi(S_i, \bar{S}^i e^{n_i g_R V_R} e^{2\tilde{g}V}) \right]_D \\ & + \left[g(S_i) S_0^3 \right]_F - \left[f_{\alpha\beta}(S_i) W^\alpha W^\beta \right]_F - \left[f_R(S_i) W_R^2 \right]_F , \end{aligned} \quad (2.14)$$

where W_R is the field strength multiplet for V_R , $g_R = \kappa^2 \xi$ where ξ is a dimensional parameter and g_R ($\dim \xi = 2$) is the R-gauge coupling constant.

The Lagrangian density given by (2.14) is invariant with respect to $G \times U_R(1)$.

The component form of the Lagrangian (2.14) can still be obtained from refs. [1, 11]. Indeed, as observed in ref. 9, we can rescale the compensating chiral multiplet S_0 provided $g(S)$ is not vanishing:

$$S_0 \rightarrow S_0 g^{-1/3} \quad (2.15)$$

and then reduce the first two terms in eq. (2.14) to the form

$$-\frac{1}{2} \left[\bar{S}_0 S_0 \frac{\phi}{(g^* e^{3g_R V_R} g)^{1/3}} \right]_D + [S_0^3]_F \quad (2.16)$$

The last two terms of eq. (2.14) remain unchanged. The new S_0 is R-invariant while $e^{3g_R V}$ is exactly the R-covariantization due to non-invariance of $g(S)$.

We can now compute $\mathcal{L}$ in (2.16) as a function of $\mathcal{G}$ defined by

$$e^{\mathcal{G}/3} = \frac{1}{3} \frac{\phi(S_i, \bar{S}^i e^{n_i g_R V_R} e^{2\tilde{g}V})}{\{g^*(\bar{S}^i e^{n_i g_R V_R} e^{2\tilde{g}V}) g(S_i)\}^{1/3}} \quad (2.17)$$

Comparing with the Lagrangian in (2.1) we see that, in the case of local $U_R(1)$ symmetry, we simply have to replace $2\tilde{g}V \rightarrow 2\tilde{g}V + n_i g_R V_R$. Also the first component $\mathcal{G}(z_i, z^{*i})$ of the $\mathcal{G}$ supermultiplet as defined by (2.17) is the same as before (in the Wess-Zumino gauge). Therefore all the results of ref. 1, which were expressed in terms of the function $\mathcal{G}(z_i, z^{*i})$, remain unchanged provided the derivatives are now understood to be covariant with respect to $G \times U_R(1)$. Note in particular that the D-term of the R-gauge multiplet

$$D_R = g_R \mathcal{G}'^i n_i z_i \quad (2.18)$$

can contain a constant term because of the non-invariance of the superpotential $g(z_i)$. This is the usual Fayet-Iliopoulos term [8] which, in the flat limit, reduces to $(\xi V_R)_D$. In taking the flat limit one should also take into account that $g_R \rightarrow 0$, $n_i \rightarrow \infty$ with $n_i g_R$ kept finite.

3. CONNECTIONS BETWEEN DIFFERENT SETS OF AUXILIARY FIELDS

3.1 New minimal auxiliary field formulation [5]

In the framework of conformal supergravity, other sets of auxiliary fields are obtained by choosing different compensating multiplets [14]. In Section 2 a chiral multiplet was chosen and this brought us to the old minimal set of auxiliary fields. The new minimal set is obtained by choosing a linear (or tensor) multiplet. This consists of the components

$$L = [C, \zeta, V_\mu] \quad (3.1)$$

where V_μ is constrained so that it is the field strength of an antisymmetric tensor $a_{\mu\nu}$:

$$V^\mu = e^{-1} \epsilon^{\mu\nu\rho\sigma} (\partial_\nu a_{\rho\sigma} - \frac{i}{4} \bar{\Psi}_\nu \gamma_\rho \psi_\sigma C) - e^{\mu\kappa} \bar{\xi} \sigma_{\mu\kappa} \psi^n. \quad (3.2)$$

L can be viewed as a real vector multiplet with the constraint that the H and K components are zero. This implies more constraints, so that L written as a real multiplet is

$$L = [C, \zeta, 0, 0, V_m, -\not{D}\zeta, -\not{\square}C] . \quad (3.3)$$

It has Weyl weight 2. A conformal invariant action can be built

$$[L \ln(L/S_0 \bar{S}_0)]_D . \quad (3.4)$$

When S_0 has Weyl weight 1, then (3.4) is conformal invariant. Notice that S_0 introduces no new degrees of freedom since the Lagrangian (3.4) is invariant under the transformation

$$S_0 \rightarrow e^{i\Lambda} S_0, \quad \bar{S}_0 \rightarrow e^{-i\bar{\Lambda}} \bar{S}_0 \quad (3.5)$$

owing to $[L\Lambda]_D = 0$, Λ being a chiral multiplet. This is what we call additional gauge invariance of the new minimal set.

As possible gauge conditions we could take

$$\begin{aligned} b_\mu &= 0 && \text{for } \underline{K}\text{-invariance} \\ C &= 1 && \text{for dilatations} \\ \zeta &= 0 && \text{for } \underline{S}\text{-supersymmetry} \\ S_0 &= [1, 0, 0] && \text{for additional gauge.} \end{aligned} \quad (3.6)$$

Then (apart from constants) (3.4) becomes the supergravity Lagrangian for the new minimal set of auxiliary fields. Note that the conformal, chiral U(1) invariance has not been fixed by a gauge choice and the remaining fields from the conformal multiplet (2.5) and from L (3.1) are

$$(e_\mu^m, \psi_\mu, A_\mu, a_{\mu\nu} \text{ (or constrained } V_\mu)) . \quad (3.7)$$

In this formulation we can only use the multiplet L for Weyl-compensation. Therefore we can build any D-type action by multiplying with an appropriate power of L in such a way that the real multiplet has Weyl weight 2. However, chiral F-type actions cannot be compensated by L. Therefore we can only build actions of chiral multiplets which have Weyl weight 3. This requirement is the same as the requirement of R-invariance in the previous section.

A general form of the Lagrangian in this formulation would be

$$\begin{aligned} \mathcal{L} = & \left[L \left\{ e_m \left(\frac{f(L, \tilde{S}_i, \tilde{S}^i e^{m_i g_R V_R} 2 \tilde{g} V)}{S_0 \bar{S}_0} \right) + g_R V_R \right\} \right]_D \\ & + [g(\tilde{S}_i)]_F - [f_{\alpha\beta}(\tilde{S}_i) W^\alpha W^\beta]_F - [f_R(\tilde{S}_i) W_R^2]_F , \end{aligned} \quad (3.8)$$

where the term $g_R V_R$ is a possible FI term and f and g are arbitrary G-gauge invariant functions with Weyl weights $w = 2$ and 3 , respectively. $f_{\alpha\beta}$ and f_R are Weyl weight zero functions as before. This form is the most general one for the system of chiral matters $\tilde{S}_i$ and gauge vectors V, since we must retain the above-mentioned invariance under the additional gauge transformations: $S_0 \rightarrow e^{i\Lambda} S_0$ with L, $\tilde{S}_i$, V and V_R kept inert. For the purpose of discussing the connection to the old minimal system, we redefine the matter multiplets $\tilde{S}_i$ by

$$\tilde{S}_i = S_i (S_0)^{\tilde{n}_i}, \quad (\tilde{n}_i : \text{Weyl weight of } \tilde{S}_i), \quad (3.9)$$

so that the new S_i 's have zero Weyl weights as in the old minimal system, and then the additional gauge transformation becomes

$$\begin{aligned} S_0 &\rightarrow e^{i\Lambda} S_0, & \bar{S}_0 &\rightarrow e^{-i\bar{\Lambda}} \bar{S}_0, \\ S_i &\rightarrow e^{-i\tilde{n}_i \Lambda} S_i, & \bar{S}^i &\rightarrow e^{i\tilde{n}_i \bar{\Lambda}} \bar{S}^i, \\ V, V_R, L & ; \text{ inert} . \end{aligned} \quad (3.10)$$

Unlike in the old minimal case, the Fayet-Iliopoulos gauge transformation in this formulation becomes

$$\begin{aligned} V_R &\longrightarrow V_R + i(\Lambda' - \bar{\Lambda}')/g_R, \quad (\Lambda' : \text{chiral parameter}), \\ S_i &\longrightarrow e^{-im_i \Lambda'} S_i, \quad \bar{S}^i \longrightarrow e^{im_i \bar{\Lambda}'} \bar{S}^i \end{aligned} \quad (3.11)$$

However, we can define a combined gauge transformation of (3.11) with (3.10) as follows:

$$\begin{aligned} V_R &\longrightarrow V_R + i(\Lambda' - \bar{\Lambda}')/g_R, \\ S_0 &\longrightarrow e^{i\Lambda'} S_0, \quad \bar{S}_0 \longrightarrow e^{-i\bar{\Lambda}'} \bar{S}_0, \\ S_i &\longrightarrow e^{-in_i \Lambda'} S_i, \quad \bar{S}^i \longrightarrow e^{in_i \bar{\Lambda}'} \bar{S}^i, \\ V, L &; \text{ inert}, \quad n_i \equiv \tilde{n}_i + m_i \end{aligned} \quad (3.12)$$

which takes exactly the same form as the local $U_R(1)$ gauge transformation (2.13) defined in the old minimal system. So we also call the transformation (3.12) local R-transformation. In view of the invariance under (3.12), we can now rewrite the Lagrangian (3.8) simply into the form

$$\begin{aligned} \mathcal{L} &= \left[\mathcal{F}(L, S_I, \bar{S}^I e^{n_I g_R V_R} e^{2\tilde{g}V}) \right]_D \\ &\quad + \left[g(S_i) S_0^3 \right]_F - \left[(W^2\text{-terms}) \right]_F, \end{aligned} \quad (3.13)$$

$$(W^2\text{-terms}) \equiv f_{\alpha\beta}(S_i) W^\alpha W^\beta + f_R(S_i) W_R^2,$$

where we have introduced the capital index I to denote $S_I \equiv (S_0, S_i)$ and $n_I = (n_0 \equiv -1, n_i)$. Since S_0 is a G -singlet, the G -gauge vector matrix $V = (V_I^J)$ should be understood to be zero when either the index I or J is 0. We now show the equivalence of the new minimal system in the form (3.13) with the old minimal system (2.14) having a local $U_R(1)$ invariance.

To show the equivalence of linear multiplets to the chiral multiplets, one can perform a duality transformation [15]. In this way one easily shows that all actions with linear multiplets can be converted to actions in terms of chiral multiplets, and *vice versa* [16]. The analogous procedure will now be applied to

the linear compensating multiplet L to show the equivalence with a chiral multiplet, and thus with the old minimal set of auxiliary fields. But a novel feature here is that it is a chiral multiplet with zero Weyl weight that can be connected with L through duality transformation. So it is not the chiral compensating multiplet itself in the old minimal set which has Weyl weight 1.

3.2 From old to new minimal formulation

We start by showing that the old minimal system (2.14) can always be rewritten into the new minimal form (3.13). The converse will be shown later. The first step is to rewrite the chiral compensating multiplet, and the matter chiral multiplets as

$$\begin{aligned} S_0 &\Rightarrow S_0 e^{i\varphi} , & \bar{S}_0 &\Rightarrow \bar{S}_0 e^{-i\bar{\varphi}} , \\ S_i &\Rightarrow S_i e^{-in_i\varphi} , & \bar{S}^i &\Rightarrow \bar{S}^i e^{in_i\bar{\varphi}} . \end{aligned} \quad (3.14)$$

Here S_0 still has Weyl weight 1 [it will become the S_0 of (3.13)] and φ has zero Weyl weight and will be transformed to a linear multiplet. Note that by the decomposition (3.14) we introduced superfluous degrees of freedom which are compensated by the transformations

$$\begin{aligned} S_0 &\rightarrow S_0 e^{i\Lambda} , \\ S_i &\rightarrow S_i e^{-in_i\Lambda} , \\ \varphi &\rightarrow \varphi - \Lambda . \end{aligned} \quad (3.15)$$

Since we are assuming R-invariance properties (2.10) of functions ϕ and g , using (3.14) we can rewrite the interaction of chiral multiplets in (2.14) as

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} [S_0 \bar{S}_0 e^{-g_R V_R} e^{i(\varphi - \bar{\varphi})} \phi(e^{i(\varphi - \bar{\varphi})}; S_i, \bar{S}^i e^{n_i g_R V_R} e^{2\bar{g} V})]_D \\ & + [g(S_i) S_0^3]_F - [(W^2\text{-terms})]_F . \end{aligned} \quad (3.16)$$

An important point here is that φ does not appear in $[]_F$ terms and does in $[]_D$ term only in the form $i(\varphi - \bar{\varphi})$ by virtue of the R-symmetry. Eq. (3.16) can be rewritten further in the following form :

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} \left[S_0 \bar{S}_0 e^{-g_R V_R} e^U \phi(e^U; S_i, \bar{S}^i e^{n_i g_R V_R} e^{2\tilde{g}V}) \right]_D \\ & + \left[g(S_i) S_0^3 \right]_F - [(W^2\text{-terms})]_F + [UL]_D, \end{aligned} \quad (3.17)$$

where U is a real vector multiplet with Weyl weight $w = 0$ and L is a Lagrange multiplier which is a real linear multiplet (with $w = 2$) introduced above. The two Lagrangians (3.16) and (3.17) are completely equivalent; in fact by variation of (3.17) with respect to L one gets

$$\frac{\delta \mathcal{L}}{\delta L} = 0 \quad \Rightarrow \quad U = i(\varphi - \bar{\varphi}), \quad (\varphi: \text{chiral}) \quad (3.18)$$

which, inserted back into eq. (3.17), reproduces eq. (3.16). On the other hand, instead of eliminating U , we may also use the equation of motion for U

$$\frac{\delta \mathcal{L}}{\delta U} = 0, \quad \text{i.e.} \quad -\frac{\partial \Phi}{\partial U} + L = 0, \quad (3.19)$$

where

$$\begin{aligned} \Phi & \equiv \frac{1}{2} S_0 \bar{S}_0 e^{-g_R V_R} e^U \phi(e^U; S_i, \bar{S}^i e^{n_i g_R V_R} e^{2\tilde{g}V}) \\ & = \Phi(U, S_I, \bar{S}^I e^{n_I g_R V_R} e^{2\tilde{g}V}) \end{aligned}$$

with the notation $S_I = (S_0, S_i)$ introduced above. The (implicit) solution of eq. (3.19) is

$$U = U(L; S_I, \bar{S}^I e^{n_I g_R V_R} e^{2\tilde{g}V}). \quad (3.20)$$

Eq. (3.20) inserted back into eq. (3.17) yields the new minimal auxiliary field formulation of the same theory

$$\mathcal{L} = [\mathcal{F}(L, S_I, \bar{S}^I e^{n_I g_R V_R} e^{z \hat{g} V})]_D + [g(S_I) S_o^3]_F - [(W^2 \text{terms})]_F \quad (3.21)$$

with

$$\mathcal{F}(L, \dots) = [-\Phi(U, \dots) + UL]_{U=U(L; \dots)} \quad (3.22)$$

We call (3.22) a super Legendre transformation. In fact, for the first components of the multiplets

$$\begin{aligned} L &= [C, \dots] , \\ U &= [u, \dots] , \\ S_I &= [z_I, \dots] , \end{aligned} \quad (3.23)$$

Eq. (3.22) is nothing but a Legendre transformation between C and u:

$$\mathcal{F}(C, z, z^*) = -\Phi(u, z, z^*) + uC , \quad (3.24)$$

where, by (3.19),

$$C = \frac{\partial \Phi}{\partial u} . \quad (3.25)$$

The dual relation to (3.25) is given by

$$u = \frac{\partial \mathcal{F}}{\partial C} . \quad (3.26)$$

It is obvious that (3.25) and (3.26) can be regarded as supermultiplet relations

$$L = \frac{\partial \Phi}{\partial U} , \quad U = \frac{\partial \mathcal{F}}{\partial L} . \quad (3.27)$$

It is now obvious that the Lagrangian given by (3.17) has an additional gauge invariance due to the original R-symmetry (2.10) and the "superfluons invariance" under (3.15). In fact (3.17) is invariant under the following transformation

$$\begin{aligned}
 S_0 &\rightarrow e^{i\Lambda} S_0, & \bar{S}_0 &\rightarrow e^{-i\Lambda} \bar{S}_0, \\
 S_i &\rightarrow e^{-im_i\Lambda} S_i, & \bar{S}^i &\rightarrow e^{im_i\bar{\Lambda}} \bar{S}^i, \\
 U &\rightarrow U - i(\Lambda - \bar{\Lambda}).
 \end{aligned} \tag{3.28}$$

Therefore the new minimal set formulation of the same Lagrangian given by (3.21), (3.22) has an additional gauge invariance as it should, because $\mathcal{F}$ of (3.22) transforms as

$$\mathcal{F} \rightarrow \mathcal{F} - i(\Lambda - \bar{\Lambda})L \tag{3.29}$$

if

$$S_0 \rightarrow e^{i\Lambda} S_0, \quad S_i \rightarrow e^{-im_i\Lambda} S_i, \quad L \rightarrow L.$$

The transformation law (3.29) implies that $[\mathcal{F}]_D$ and so (3.21) is invariant. This ends the conversion of a general old minimal system with R-invariance to a new minimal one.

3.3 From new to old minimal formulation

We now prove the converse; that is, any new minimal system can be converted to an old minimal system with R-invariance. This shows the complete equivalence of both formulations. The Lagrangian of a general new minimal system takes the form (3.13), which can be equivalently rewritten in the following form:

$$\begin{aligned}
 \mathcal{L} = & [\mathcal{F}(\tilde{U}, S_i, \bar{S}^i e^{m_i g_R V_R} e^{2\tilde{g}V})]_D - [i(\varphi - \bar{\varphi})\tilde{U}]_D \\
 & + [g(S_i) S_0^3]_F - [(W^2\text{-terms})]_F,
 \end{aligned} \tag{3.30}$$

where $\tilde{U}$ is a real vector multiplet and the Lagrange multiplier φ is a chiral multiplet. Indeed, the variation with respect to φ and $\bar{\varphi}$ yields the equation of motion

$$\frac{\delta \mathcal{L}}{\delta \varphi} = \frac{\delta \mathcal{L}}{\delta \bar{\varphi}} = 0 \quad \Rightarrow \quad \tilde{U} = L \text{ (linear multiplet)}, \tag{3.31}$$

which inserted back into eq. (3.30) reproduces eq. (3.12) since $[L\varphi]_D = [L\bar{\varphi}]_D = 0$.

On the other hand, the $\tilde{U}$ field equation implies that

$$\frac{\partial \mathcal{F}(\tilde{U}, \dots)}{\partial \tilde{U}} = i(\varphi - \bar{\varphi}), \quad (3.32)$$

which is solved implicitly by writing

$$\tilde{U} = \tilde{U}(i(\varphi - \bar{\varphi}); S_i, \bar{S}^i e^{n_i g_R V_R} e^{2\tilde{\mathcal{F}}V}). \quad (3.33)$$

Substituting this into (3.30) gives the old-minimal formulation form of the same theory

$$\mathcal{L} = -[\Phi(i(\varphi - \bar{\varphi}), S_i, \bar{S}^i e^{n_i g_R V_R} e^{2\tilde{\mathcal{F}}V})]_D + [g(S_i)S_0^3]_F - [W^2\text{-terms}]_F, \quad (3.34)$$

where

$$\Phi(i(\varphi - \bar{\varphi}), \dots) = [-\mathcal{F}(\tilde{U}, \dots) + i(\varphi - \bar{\varphi})\tilde{U}]_{\tilde{U} = \tilde{U}(i(\varphi - \bar{\varphi}); \dots)}. \quad (3.35)$$

The $[\]_D$ term of (3.34) can be cast into a more familiar form by using the additional gauge and local $U_R(1)$ invariances. In fact Φ defined by (3.35) is additionally gauge invariant if we assign to φ the transformation law $\varphi \rightarrow \varphi - \Lambda$ since $\mathcal{F}(\tilde{U}, \dots) \rightarrow \mathcal{F} - i(\Lambda - \bar{\Lambda})\tilde{U}$ by (3.29). Furthermore, Φ of (3.35) is invariant under the local $U_R(1)$ transformation (3.12) if φ is assumed inert, since $\mathcal{F}$ [and $\tilde{U}$ defined by (3.32) as well] are invariant under (3.12). We can now fix the redundant additional gauge by setting $\varphi = 0$ without affecting the local $U_R(1)$ invariance, which, as well as the fact that only S_0 carries non-zero Weyl and conformal $U(1)$ weights, imply that Φ in the $[\]_D$ term of (3.35) takes the form

$$\Phi|_{\varphi=0} = \frac{1}{2} S_0 \bar{S}_0 e^{-g_R V_R} \phi(S_i, \bar{S}^i e^{n_i g_R V_R} e^{2\tilde{\mathcal{F}}V}). \quad (3.36)$$

This is actually the old minimal system and concludes the proof.

3.4 An example

We have seen quite generally that the new minimal auxiliary field formulation is equivalent to the restricted class of old minimal formulation, namely the one with R-symmetry. This symmetry is a necessary and sufficient condition for the Fayet-Iliopoulos term to be introduced. It would, however, be interesting to see

this connection more explicitly for simple cases. Consider a special class of new-minimal system for which the $[]_D$ term in (3.8) takes the form

$$\left[L \left\{ \ln(L/S_0 \bar{S}_0) + g_R V_R - f(\tilde{S}_i, \bar{\tilde{S}}^i e^{2\tilde{J}V}) \right\} + h(\tilde{S}_i, \bar{\tilde{S}}^i e^{2\tilde{J}V}) \right]_D, \quad (3.37)$$

where f and h are, of course, assumed to be G -gauge invariant functions with Weyl weights $w = 0$ and 2 , respectively, and zero conformal chiral weights. After the redefinition (3.9), this corresponds to the function $\mathcal{F}(L, \dots)$

$$\begin{aligned} \mathcal{F} = L \left\{ \ln \left(\frac{L}{\bar{S}_0 e^{-g_R V_R} S_0} \right) - f(S_i, \bar{S}^i e^{n_i g_R V_R} e^{2\tilde{J}V}) \right\} \\ + \bar{S}_0 e^{-g_R V_R} S_0 h(S_i, \bar{S}^i e^{n_i g_R V_R} e^{2\tilde{J}V}) . \end{aligned} \quad (3.38)$$

In this case, the eq. (3.32) becomes

$$1 - f + \ln(\tilde{U}/\bar{S}_0 e^{-g_R V_R} S_0) = i(\varphi - \bar{\varphi}) , \quad (3.39)$$

which is solved explicitly as

$$\tilde{U} = S_0 \bar{S}_0 e^{i(\varphi - \bar{\varphi}) - g_R V_R + f - 1} . \quad (3.40)$$

This gives the function Φ of (3.35) as

$$\Phi = \bar{S}_0 e^{-g_R V_R} S_0 \left(e^{i(\varphi - \bar{\varphi}) + f - 1} - h \right) , \quad (3.41)$$

yielding the $[]_D$ term in the old minimal formulation^{*)}:

$$\begin{aligned} -[\Phi|_{\varphi=0}]_D = -[\bar{S}_0 e^{-g_R V_R} S_0 \{ \exp(-1 + f(S_i, \bar{S}^i e^{n_i g_R V_R} e^{2\tilde{J}V})) \\ - h(S_i, \bar{S}^i e^{n_i g_R V_R} e^{2\tilde{J}V}) \}]_D \end{aligned} \quad (3.42)$$

*) At first sight one might think that (3.42) already is the most general Lagrangian of the old minimal system. But it is special, of course, since the functions f and h introduced in (3.37) had Weyl weight $w = 0$ and 2 , respectively, and zero chiral weights. So they satisfy special relations:

$$\begin{aligned} f(e^{-in_i \Lambda} S_i, e^{in_i \bar{\Lambda}} \bar{S}^i) &= f(S_i, \bar{S}^i) , \\ h(e^{-in_i \Lambda} S_i, e^{in_i \bar{\Lambda}} \bar{S}^i) &= e^{-i(\Lambda - \bar{\Lambda})} h(S_i, \bar{S}^i) . \end{aligned}$$

This clearly explains why the simple Fayet-Iliopoulos term $\mathcal{G}_R^V$ in the new minimal case turns out to appear in the exponentiated form $e^{-\mathcal{G}_R^V}$ in the old-minimal case. Furthermore it is interesting to notice that it also explains the reason for the strange occurrence in the old minimal supergravity system: that is, a canonical form of kinetic terms of matter fields $\delta_{j\bar{m}}^i \partial_m z_i \partial_m^* z^{*j}$ was realized in the old minimal case only when the multiplets S_i were contained in the Lagrangian in an exponential form $e^{\delta_{j\bar{m}}^i S_i \bar{S}^{\bar{j}}}$. Indeed when $h = 0$, the choice $f(S_i, \bar{S}^{\bar{i}}) = \delta_{j\bar{m}}^i S_i \bar{S}^{\bar{j}}$ of the usual non-exponentiated form obviously corresponds to such a canonical kinetic term in the new-minimal case (3.37), while this f appears in the exponent in the resulting old minimal expression (3.42).

3.5 Non-minimal auxiliary field formulation of Breitenlohner [6]

We conclude this section by showing the relation between the old minimal set and the non-minimal Breitenlohner set. The latter exists with an arbitrary parameter n [17]. In the superconformal framework, this set is obtained by using a complex linear multiplet as a compensator. This multiplet can exist with an arbitrary Weyl weight w , which is related to the above-mentioned parameter n [14]:

$$w = \frac{2}{3n+1} \quad , \quad n \neq -\frac{1}{3} \quad , \quad 0 \quad (3.43)$$

Since we cannot change the Weyl weight of a linear multiplet by raising it to some power (a power of the linear multiplet is no longer a linear multiplet), we prove the equivalence with the old-minimal set for an arbitrary weight w . We will see that the interactions in the non-minimal set are again equivalent to a restricted class of interactions in the old-minimal set.

Consider first the old minimal formulation without gauge interactions (their inclusion is trivial as in the preceding discussion for the new-minimal formulation).

$$\mathcal{L} = -\frac{1}{2} [S_{\bar{0}} \bar{S}_{\bar{0}} \phi(S_i, \bar{S}^{\bar{i}})]_D + [g(S_i) S_{\bar{0}}^3]_F \quad (3.44)$$

Now suppose that $g(S)$ is a homogeneous function with respect to suitably assigned weights for S_i . Then we can show the equivalence to the Breitenlohner set. Indeed in this case we can redefine the multiplets

$$S_i \rightarrow \tilde{S}_i (S_0)^{-3n_i/m} \quad (3.45)$$

where m is the degree of homogeneity of $g(S)$ and n_i is the weight assigned to S_i .

We can rewrite (3.44) as follows

$$\mathcal{L} = -\frac{1}{2} \left[S_0 \bar{S}_0 \phi(\tilde{S}_i (S_0)^{-\frac{3n_i}{m}}, \bar{\tilde{S}}^i (\bar{S}_0)^{-\frac{3n_i}{m}}) \right]_D + [g(\tilde{S}_i)]_F \quad (3.46)$$

The Lagrangian (3.46) is entirely equivalent to

$$\mathcal{L} = -[\Phi(\mathcal{V}, \bar{\mathcal{V}}, \tilde{S}_i, \bar{\tilde{S}}^i)]_D + [g(\tilde{S}_i)]_F + [\mathcal{V}^{2-w} \mathcal{L} + \bar{\mathcal{V}}^{2-w} \bar{\mathcal{L}}]_D, \quad (3.47)$$

where $\mathcal{V}$ is a general complex multiplet of Weyl weight 1 and conformal chiral weight 1, and $\mathcal{L}$ is a complex linear multiplet (globally $\bar{D}\bar{D}\mathcal{L} = 0$) with Weyl weight w and chiral weight $(w-2)$;

$$\Phi(\mathcal{V}, \bar{\mathcal{V}}, \tilde{S}_i, \bar{\tilde{S}}^i) \equiv \frac{1}{2} \mathcal{V} \bar{\mathcal{V}} \phi(\tilde{S}_i \mathcal{V}^{-\frac{3n_i}{m}}, \bar{\tilde{S}}^i \bar{\mathcal{V}}^{-\frac{3n_i}{m}}) \quad (3.48)$$

Variation with respect to $\mathcal{L}(\bar{\mathcal{L}})$ of (3.47) gives $\mathcal{V} = S_0$ (chiral), which inserted back into (3.47) reproduces (3.46). On the other hand the $\mathcal{V}$ -field equations derived from (3.47) is solved implicitly as

$$\mathcal{V} = \mathcal{V}(\mathcal{L}, \bar{\mathcal{L}}, \tilde{S}_i, \bar{\tilde{S}}^i), \quad (3.49)$$

which, substituted into (3.47), gives rise to Breitenlohner formulation with a complex linear compensating multiplet $\mathcal{L}$ having arbitrary Weyl weight w :

$$\mathcal{L} = [\Psi(\mathcal{L}, \bar{\mathcal{L}}, \tilde{S}_i, \bar{\tilde{S}}^i)]_D + [g(\tilde{S}_i)]_F \quad (3.50)$$

with

$$\Psi(\mathcal{L}, \bar{\mathcal{L}}, \dots) = [-\Phi(\mathcal{V}, \bar{\mathcal{V}}, \dots) + \mathcal{V}^{2-w} \mathcal{L} + \bar{\mathcal{V}}^{2-w} \bar{\mathcal{L}}]_{\mathcal{V}=\mathcal{V}(\mathcal{L}, \bar{\mathcal{L}}, \dots)} \quad (3.51)$$

Notice that the previous restriction of homogeneity for superpotential g is now understandable as the Weyl weight requirement for the F-term. Indeed, the chiral multiplets $\tilde{S}_i$ in this formulation have Weyl weights $w_i = (3/m) \times n_i$ because of (3.45), and hence the homogeneous function $g(\tilde{S}_i)$ of degree m with respect to the weight n_i has Weyl weight $(3/m) \times m = 3$ as is required. This requirement imposed a real restriction on the possible form of the superpotential in the Breitenlohner formulation, since the complex linear multiplet $\mathcal{L}$ cannot be used as a compensator in the F-type action [10].

Since the Lagrangian (3.50) clearly gives the most general form in the Breitenlohner's formulation, we have completed the proof of its equivalence to the restricted class of old minimal systems with "homogeneous" superpotential term. At the same time, one can also give the converse proof by starting from (3.50) and considering the following duality transformation with chiral multiplet multiplier S_0 :

$$\mathcal{L} = [\Psi(\mathcal{V}, \bar{\mathcal{V}}, \tilde{S}_i, \bar{\tilde{S}}^i) - \mathcal{V} S_0^{2-w} - \bar{\mathcal{V}} \bar{S}_0^{2-w}]_D + [g(\tilde{S}_i)]_F .$$

Summarizing the results obtained in this section, we can say: i) In the absence of the Fayet-Iliopoulos term, the old minimal framework covers the most general interactions of matter multiplets in $N = 1$ supergravity. The Breitenlohner's non-minimal framework covers only a part of them, namely those with "homogeneous" superpotential. The new-minimal framework is further restrictive and describes only the systems with R-symmetry. [Indeed, the systems with R-symmetry necessarily have "homogeneous" superpotential of degree 3 by (2.10).] ii) Whenever the Fayet-Iliopoulos term can be introduced, all these three formulations become equivalent; i.e., the ranges of the possible interaction forms reduce to the common domain which was already covered by the new minimal framework in the absence of the FI term.

4. EXPLICIT RESULTS IN THE NEW MINIMAL FORMULATION

The results of the previous section still leave us with a question. In ref.[18] Lagrangians were calculated in the new minimal set of auxiliary fields and a positive

definite potential was obtained. It seems that this is a general feature of the new minimal field representation. The argument is the following. First consider how a non-positive potential was obtained in the old minimal formulation. The potential is

$$V = \frac{1}{2} (\text{Re } f_{\alpha\beta}) D^\alpha D^\beta + (-\mathcal{G}_i^{\prime\prime j}) h^{*i} h_j - 3 h_0^* h_0 \quad , \quad (4.1)$$

where the auxiliary fields D^α , h_i and h_0 represent their values from the field equations. The bosonic kinetic terms are

$$\begin{aligned} e^{-1} \mathcal{L}_{kin.} = & -\frac{1}{4} (\text{Re } f_{\alpha\beta}) F_{mn}^\alpha F_{mn}^\beta + \frac{1}{4} (\text{Im } f_{\alpha\beta}) F_{mn}^\alpha \tilde{F}_{mn}^\beta \\ & - (-\mathcal{G}_i^{\prime\prime j}) D_m z^{*i} D_m z_j - \frac{1}{2} R \quad . \end{aligned} \quad (4.2)$$

In order to have the right sign for the kinetic terms, the corresponding auxiliary fields must give a positive contribution to the potential. But the compensating multiplet must occur with the opposite sign to give the right sign of the Einstein Lagrangian.

On the other hand, in the new-minimal field representation there are no auxiliary scalars in the compensating multiplet. From this fact one might be tempted to conclude that there are only positive contributions to the potential, then implying a non-vanishing cosmological term if spontaneous supersymmetry breaking occurs ($\langle D^\alpha \rangle$ or $\langle h^i \rangle \neq 0$). But this cannot be true. In fact, in the previous section we have just shown that the new minimal representation is equivalent to the old one for all R-invariant Lagrangians. Furthermore we know of examples [9], in the old minimal formulation, of R-invariant potentials giving supersymmetry breaking with vanishing cosmological constant.

To solve this apparent contradiction we explicitly compute all bosonic terms in the new minimal formulation.

We disregard the gauge multiplets $(1, \frac{1}{2})$ as their Lagrangian does not involve the compensating multiplet and therefore is the same as in the old minimal formulation. We will find that the potential is again given by

$$V = 2 \mathcal{F}_j^i h_i h^{*j} \quad . \quad (4.3)$$

However, the elimination of the auxiliary fields V_m and A_μ will add extra terms to the original kinetic terms, $-2\mathcal{F}_j^i \partial_m z_i \partial_m z^{*j}$ to yield

$$e^{-1} \mathcal{L}_{kin.} = -2 \left(\mathcal{F}_j^i - \frac{\mathcal{F}_c^i \mathcal{F}_{cj}}{\mathcal{F}_{cc}} + \frac{2}{3} \mathcal{F}^i \mathcal{F}_j \right) \partial_m z_i \partial_m z^{*j} . \quad (4.4)$$

(In order to obtain only $\partial z \partial z^*$ and no $\partial z \partial z$ or $\partial z^* \partial z^*$ terms we will choose particular gauges for the additional gauge invariances discussed in Section 3). From eqs. (4.3) and (4.4) it is now obvious how the previous puzzle is solved. Indeed only the combination in (4.4) has to be positive and $\mathcal{F}_j^i$ can thus be non-positive definite. In the particular examples given in Ref. [18], $\mathcal{F}_j^i$ was taken separately positive definite. At the end of this section we will show that after the Legendre transformation of Section 3, the expression in (4.4) indeed becomes $\mathcal{G}_j^i$, and the potential also reduces to the same form as in the old minimal set [1], namely $-e^{-\mathcal{G}} [3 + \mathcal{G}_i' (\mathcal{G}''^{-1})^i_j \mathcal{G}^{*j}]$.

4.1 Calculation of bosonic terms of the Lagrangian (3.13)

We compute the bosonic terms (i.e., all the terms containing no fermion fields) for the general new minimal system, whose Lagrangian is given by eq. (3.13).

Hereafter, we denote by $\mathcal{F}$ the (real) function $\mathcal{F}(C, z_I, z^{*I})$ with arguments of the first components of multiplets and use notations such as

$$\mathcal{F}_c = \frac{\partial \mathcal{F}}{\partial C} , \quad \mathcal{F}^I = \frac{\partial \mathcal{F}}{\partial z_I} , \quad \mathcal{F}_J^* = \frac{\partial^2 \mathcal{F}}{\partial z_I \partial z^{*J}} \quad \text{etc.}$$

The invariance properties of the function $\mathcal{F}$, written for the 1st components, are

$$(Weyl) \quad \left(2C \frac{\partial}{\partial C} + z_0 \frac{\partial}{\partial z_0} + z_0^* \frac{\partial}{\partial z_0^*} \right) \mathcal{F} = 2 \mathcal{F} , \quad (4.5a)$$

$$(chiral) \quad \left(z_0 \frac{\partial}{\partial z_0} - z_0^* \frac{\partial}{\partial z_0^*} \right) \mathcal{F} = 0 , \quad (4.5b)$$

$$(add. gauge) \quad \tilde{n}_I z_I \frac{\partial}{\partial z_I} \mathcal{F} = \tilde{n}_I z^{*I} \frac{\partial}{\partial z^{*I}} \mathcal{F} = C \cdot \tilde{n}_I = (-1, \tilde{m}_i) \quad (4.5c)$$

The identities (4.5a) and (4.5b) show that $\mathcal{F}$ carries Weyl (conformal) weight 2 and (conformal) chiral weight 0, respectively, and eq. (4.5c) is the first component version of the additional gauge invariance property (3.29).

In the density formula (2.8) we need C and D components of the multiplet $\mathcal{F}(L, S_I, S^{*I})$ in order to calculate the boson terms of $[\mathcal{F}]_D$. For a general function $\Phi(V_i)$ of (a set of) vector multiplets $V_i = [C_i, \dots, D_i]$, we have the following formula giving its C and D components:

$$\begin{aligned} C(\Phi) &= \Phi(C_i) \ , \\ D(\Phi) &= \Phi^i D_i + \frac{1}{2} \Phi^{ij} (H_i H_j + K_i K_j - B_i^m B_j^m - D_m^c C_i D_m^c C_j) \\ &\quad + (f.t.), \end{aligned} \quad (4.6)$$

where D_m^c is the conformal covariant derivative and (f.t.) denotes the other terms containing fermion fields. We can apply this formula to the present function $\mathcal{F}(L, S_I, S^{*I})$ by noticing that the linear multiplet L and the chiral multiplets S_I , regarded as vector multiplets have the following components [vector embedding]:

$$\begin{aligned} L &= [C, \xi, 0, 0, V_m, -\mathcal{D}^c \xi, -\square^c C] \\ S_I &= [z_I, -i\chi_{LI}, -h_I, ih_I, iD_m^c z_I, 0, 0] \end{aligned} \quad (4.7)$$

Thus we find by (4.6) the components of the $\mathcal{F}$ -multiplet:

$$\begin{aligned} C(\mathcal{F}) &= \mathcal{F} \ , \\ D(\mathcal{F}) &= -\mathcal{F}_c \square^c C + \frac{1}{2} \mathcal{F}_{cc} (-V_m^2 - D_m^c C D_m^c C) \\ &\quad + \{ \mathcal{F}_c^i (-iV_m D_m^c z_I - D_m^c C D_m^c z_I) + h.c. \} \\ &\quad + 2 \mathcal{F}_J^i (h_I h^{*J} - D_m^c z_I D_m^c z^{*J}) + (f.t.) \ . \end{aligned} \quad (4.8)$$

When substituting these into the action formula (2.8), we perform a partial integration of the first d'Alembertian term in (4.8):

$$\int d^4x (-e \mathcal{F}_2 \square^c C) = \int d^4x e \left\{ \partial_m \mathcal{F}_2 \cdot D_m^c C - \frac{1}{3} \mathcal{F}_2 C R + (f.t.) \right\}, \quad (4.9)$$

since the conformal covariant d'Alembertian is given by

$$\square^c C = e^{-1} \partial_\nu (g^{\mu\nu} e D_\mu^c C) + \frac{1}{3} C R + (f.t.). \quad (4.10)$$

Then (2.8) yields

$$\begin{aligned} e^{-1} [\mathcal{F}(L, S_I, \bar{S}^I)]_D \\ = \frac{1}{3} R (\mathcal{F} - C \mathcal{F}_c) + \frac{1}{2} \mathcal{F}_{cc} ((\partial_m C)^2 - V_m^2) \\ - i (\mathcal{F}_c^I D_m^c \bar{z}_I - \mathcal{F}_{cI} D_m^c \bar{z}^{*I}) V_m + 2 \mathcal{F}_J^I (h_I h^{*J} - D_m^c \bar{z}_I D_m^c \bar{z}^{*J}) + (f.t.). \end{aligned} \quad (4.11)$$

Here we have already used the fact that D_m^c on C is simply an ordinary derivative ∂_m aside from fermion terms^{*)}. This is true also for z_I except for z_0 , for which

$$D_m^c z_0 = \partial_m z_0 - \frac{i}{2} A_m z_0 + (f.t.). \quad (4.12)$$

The residual superpotential term $[g(S) S_0^3]_F$ in the Lagrangian (3.13) is easily given by

$$e^{-1} [g(S_i) S_0^3]_F = (3 z_0^2 h_0 g + g^i h_i z_0^3 + h.c.) + (f.t.), \quad (4.13)$$

where $g = g(z_i)$. This follows from the F-type action formula (2.9) and the component formula for the general function $\varphi(\Sigma_i)$ of (a set of) chiral multiplet Σ_i :

$$\varphi(\Sigma_i) = [\varphi(\bar{z}_i) \equiv \varphi, \quad \varphi^i \chi_{Li}, \quad \varphi^i h_i - \varphi^{ij} \bar{\chi}_{Li} \chi_{Lj}] . \quad (4.14)$$

By (4.11) and (4.13), we have completed the first step of the calculation of the bosonic terms for the general new minimal Lagrangian (3.13).

*) This is the case when we take $b_\mu = 0$ as $\bar{K}$ -gauge fixing.

4.2 Elimination of auxiliary fields V_m and A_m

At this stage we still have many "redundant" gauge invariances to be fixed; that is, $\underline{D}$ (Weyl), $\underline{A}$ (chiral), and $\underline{S}$ (conformal supersymmetry) gauges of superconformal and $\underline{Q}$, $\underline{X}$ and $\underline{f}$ gauges of the additional gauge invariance with a chiral parameter $\Lambda = [\underline{a}, \underline{x}, \underline{f}]$. (In the present context of calculating only boson terms, $\underline{S}$ and $\underline{X}$ gauge fixing is irrelevant and the $\underline{K}$ -gauge was already fixed by $b_\mu = 0$.) These gauge-fixing freedoms are convenient in the following calculations for re-writing the Lagrangian (4.11) and (4.13).

First we fix the $\underline{D}$ -gauge by the condition

$$\mathcal{F} - C \mathcal{F}_c = -3/2 \quad (4.15)$$

so that the Einstein term in $\mathcal{L}$ takes the canonical form $-(\frac{1}{2})R^{(*)}$. Further we fix the $\underline{Im} \underline{Q}$ -gauge of additional gauge invariance by

$$\mathcal{F}_c = 0 \quad (4.16)$$

[Notice that $\mathcal{F}$ transforms under the $\underline{Q}$ gauge transformation as $\mathcal{F} \rightarrow \mathcal{F} + 2 \underline{Im} \underline{Q} \cdot C$ by (3.29). The reason for this gauge choice will be made clear below.] That is, we have set the conditions

$$\mathcal{F} = -3/2 \quad , \quad \mathcal{F}_c = 0 \quad , \quad (4.17)$$

by which the components C and z_0 no longer become independent variables but functions of the other matter variables z_i and z^{*i} ; $C = C(z_i, z^{*i})$, $z_0 = z_0(z_i, z^{*i})$.

Now we proceed to the task of eliminating the auxiliary fields A_m and V_m . For this purpose, we have to make the component V_m free from the linear multiplet constraint:

$$0 = D_m^\epsilon V_m = e^{-1} \partial_\mu (e V^\mu) + (f.t.) \quad (4.18)$$

*) It is at this stage that the superconformal tensor calculus framework is more flexible than the Poincaré one (the new minimal framework in this case). Indeed, in the latter tensor calculus, the component C is already fixed as $C = 1$ and hence one has to perform tedious Weyl rescaling in order to obtain the same result as given by (4.15).

**) The phase of z_0 is undetermined by the (real) condition (4.17) alone. However, since this is a consequence of conformal chiral invariance, the final result we will obtain is independent on the choice of phase for z_0 .

So we add the Lagrange multiplier term

$$\varphi \partial_\mu (e V^\mu) = -e (V_m \partial_m \varphi) + \text{total derivatives} . \quad (4.19)$$

However, this term can be immediately eliminated by using the $\text{Re } \underline{Q}$ -gauge freedom: indeed, the transformation law (3.29) of the $\mathcal{F}$ -multiplet, $\mathcal{F} \rightarrow \mathcal{F} - i(\Lambda - \bar{\Lambda})L$, implies that

$$[\mathcal{F}]_D \rightarrow [\mathcal{F}]_D + e 2 V_m \partial_m (\text{Re } Q) + \text{total derivatives} \quad (4.20)$$

under $\underline{Q}$ -gauge transformation, which demands that the multiplier φ should transform as

$$\varphi \rightarrow \varphi + 2 \text{Re } Q \quad (4.21)$$

So we can set $\varphi = 0$ as a $\text{Re } \underline{Q}$ gauge condition.

The terms containing V_m and A_m in $\mathcal{L}$ of (4.11) are collected as follows: (notice that A_m appears only in $D_m^C z_0$)

$$\begin{aligned} e^{-1} \mathcal{L}^{V_m, A_m} = & -\frac{1}{2} \mathcal{F}_{cc} V_m^2 - i (\mathcal{F}_c^I \partial_m z_I - h.c.) V_m \\ & - \frac{1}{2} (\mathcal{F}_c^0 z_0 + h.c.) A_m V_m + i (\mathcal{F}_I^0 z_0 \partial_m z^{*I} - h.c.) A_m \\ & - \frac{1}{2} \mathcal{F}_0^0 |z_0|^2 A_m^2 . \end{aligned} \quad (4.22)$$

Eliminating A_m and V_m we obtain

$$e^{-1} \mathcal{L}^{V_m, A_m} = -\frac{1}{2 \mathcal{F}_{cc}} (\mathcal{F}_c^I \partial_m z_I - h.c.)^2 + \frac{1}{3} (\mathcal{F}^I \partial_m z_I - h.c.)^2 . \quad (4.23)$$

Here we have used the gauge conditions $\mathcal{F} = -3/2$ and $\mathcal{F}_c = 0$ [(4.17)] as well as equalities

$$\begin{aligned} z_0 \mathcal{F}_c^0 &= z_0^* \mathcal{F}_{c0} = -C \mathcal{F}_{cc} , \\ z_0 \mathcal{F}_I^0 &= \mathcal{F}_I - C \mathcal{F}_{cI} , \\ |z_0|^2 \mathcal{F}_0^0 &= \mathcal{F} - C \mathcal{F}_c + C^2 \mathcal{F}_{cc} , \end{aligned} \quad (4.24)$$

all of which follow from identities (4.5a) and (4.5b) with suitable differentiations.

We next rewrite the term $\mathcal{F}_{cc}(\partial_m C)^2/2$ in (4.11) as

$$\frac{1}{2} \mathcal{F}_{cc} (\partial_m C)^2 = (2 \mathcal{F}_{cc})^{-1} (\mathcal{F}_c^I \partial_m \bar{z}_I + h.c.)^2, \quad (4.25)$$

since the gauge condition $\mathcal{F}_c = 0$ of (4.17) leads to

$$\mathcal{F}_{cc} \partial_m C + (\mathcal{F}_c^I \partial_m \bar{z}_I + h.c.) = 0. \quad (4.26)$$

If we add a term

$$-\frac{1}{3} (\mathcal{F}^I \partial_m \bar{z}_I + h.c.)^2 \quad (4.27)$$

which actually vanishes because of (4.17), the sum of (4.23) and (4.25) takes the simple form:

$$\begin{aligned} e^{-1} \mathcal{L}^{V_m, A_m} + \frac{1}{2} \mathcal{F}_{cc} (\partial_m C)^2 - \frac{1}{3} (\mathcal{F}^I \partial_m \bar{z}_I + h.c.)^2 \\ = \left\{ 2 \frac{\mathcal{F}_c^I \mathcal{F}_{cJ}}{\mathcal{F}_{cc}} - \frac{4}{3} \mathcal{F}^I \mathcal{F}_J \right\} \partial_m \bar{z}_I \partial_m \bar{z}^{*J}. \end{aligned} \quad (4.28)$$

Notice that with a gauge choice different from (4.16) we would have obtained bizarre kinetic terms such as $\partial_m z^I \partial_m z_J$ and $\partial_m z^{*I} \partial_m z^{*J}$ after elimination of V_m and A_m .

With the result (4.28), the Lagrangian $\mathcal{L}$ given by (4.11) plus (4.13) now becomes

$$\begin{aligned} \mathcal{L} &= -\frac{e}{2} R + \mathcal{L}_{kin.} + \mathcal{L}_{aux.}, \\ e^{-1} \mathcal{L}_{kin.} &= -2 \left(\mathcal{F}_J^I - \frac{\mathcal{F}_c^I \mathcal{F}_{cJ}}{\mathcal{F}_{cc}} + \frac{2}{3} \mathcal{F}^I \mathcal{F}_J \right) \partial_m \bar{z}_I \partial_m \bar{z}^{*J}, \\ e^{-1} \mathcal{L}_{aux.} &= 2 \mathcal{F}_J^I h_I h^{*J} + (3 \bar{z}_0^2 h_0 g + g^i h_i \bar{z}_0^3 + h.c.). \end{aligned} \quad (4.29)$$

This gives the desired component expression for the bosonic sector of the general matter-coupled new minimal supergravity system, corresponding to Cremmer et al's [1] general result for the old minimal case. We now show that the expression (4.29) in fact coincides with Cremmer et al.'s result. As a general argument we have already shown the equivalence of the new minimal system to the old minimal one

with an R-symmetry. But here we can see a term-by-term correspondence by making an explicit connection between the present function $\mathcal{F}$ and the function $\mathcal{G}$ of Refs. [1, 3]. Before considering this correspondence we make an important observation on our result (4.29), in connection with the non-positivity property of the potential.

As anticipated at the beginning of this section we notice, from (4.29), that the coefficients of the quadratic forms $-\partial_m z_I \partial_m z^{*J}$ and $h_I h^{*J}$ are different. The difference results from the elimination of the auxiliary fields V_m and A_m which give an additional contribution

$$- \frac{\mathcal{F}_c^I \mathcal{F}_{cJ}}{\mathcal{F}_{cc}} + \frac{2}{3} \mathcal{F}^I \mathcal{F}_J \quad (4.30)$$

to the matrix-coefficient of the kinetic term $-2\partial_m z_I \partial_m z^{*J}$. As explained before this is the solution of the apparent paradox related to the possibility of having non-positive definite potentials in the new minimal set formulation.

4.3 The matter kinetic term $\mathcal{L}_{\text{kin}}$.

We now show that the matter kinetic term in (4.29) actually coincides with the old minimal result of Ref. [1]:

$$-2 \left(\mathcal{F}_J^I - \frac{\mathcal{F}_c^I \mathcal{F}_{cJ}}{\mathcal{F}_{cc}} + \frac{2}{3} \mathcal{F}^I \mathcal{F}_J \right) \partial_m z_I \partial_m z^{*J} = \mathcal{G}_J^i \partial_m z_i \partial_m z^{*j} \quad , \quad (4.31)$$

where the function $\mathcal{G}$ is a Kahler invariant [19] combination of the kinetic function $\phi(z_i, z^{*j})$ and the superpotential $g(z_i)$ appearing in the old minimal system (2.1):

$$\mathcal{G} = 3 \ln(\phi/3) - \ln |g|^2 \quad . \quad (4.32)$$

The relation between the present function $\mathcal{F}$ and the old minimal one Φ

$$\Phi(u, z_i, z^{*i}) \equiv \frac{1}{2} |z_0|^2 e^u \phi(z_i, z^{*i}) \quad , \quad (4.33)$$

is given by the Legendre transformation (3.24) with (3.25) and (3.26). Since $u = \mathcal{F}_c$ vanishes by the $\text{Im}\mathcal{Q}$ -gauge condition (4.16), the relation (3.24) says

$$\mathcal{F} = -\Phi = -\frac{1}{2} |\bar{z}_0|^2 \phi(\bar{z}_i, \bar{z}^{*i}) \quad \left(= -\frac{3}{2} \right) . \quad (4.34)$$

Furthermore, we have the following equations by differentiating (3.24):

$$-\mathcal{F}_I = \Phi_I = \begin{cases} \Phi/\bar{z}_0^* & \text{for } I=0 \\ \frac{3}{2} \frac{\phi_i}{\phi} & \text{for } I=i \end{cases} , \quad (4.35a)$$

$$(4.35b)$$

$$\begin{aligned} \Phi_J^I &= -\mathcal{F}_J^I - \left[\left(\frac{\partial}{\partial \bar{z}^{*J}} \right)_c \frac{\partial \mathcal{F}}{\partial C} \right] \left(\frac{\partial C}{\partial \bar{z}_I} \right)_u \\ &= - \left(\mathcal{F}_J^I - \mathcal{F}_c^I \mathcal{F}_{cJ} / \mathcal{F}_{cc} \right) . \end{aligned} \quad (4.36)$$

Here in (4.36) the u -fixed derivative $(\partial C / \partial \bar{z}_I)_u$ has been evaluated by using (3.26):

$$\mathcal{F}_c = u \quad \Rightarrow \quad \mathcal{F}_c^I + \mathcal{F}_{cc} \left(\frac{\partial C}{\partial \bar{z}_I} \right)_u = 0 . \quad (4.37)$$

The eqs. (4.35) and (4.36) lead to

$$\begin{aligned} & -2 \left(\mathcal{F}_J^I - \frac{\mathcal{F}_c^I \mathcal{F}_{cJ}}{\mathcal{F}_{cc}} + \frac{2}{3} \mathcal{F}^I \mathcal{F}_J \right) \partial_m \bar{z}_I \partial_m \bar{z}^{*J} \\ &= 2 \left(\Phi_J^I - \frac{2}{3} \Phi^I \Phi_J \right) \partial_m \bar{z}_I \partial_m \bar{z}^{*J} \\ &= 2 \left(\Phi_J^i - \frac{2}{3} \Phi^i \Phi_J \right) \partial_m \bar{z}_i \partial_m \bar{z}^{*j} \quad (\because \bar{z}_0 \Phi^0 = \Phi = 3/2) \\ &= 3 \left(\frac{\phi_j^i}{\phi} - \frac{\phi^i \phi_j}{\phi^2} \right) \partial_m \bar{z}_i \partial_m \bar{z}^{*j} \quad (\because \text{from (4.34)}) \end{aligned} \quad (4.38)$$

The last quantity is nothing but the desired one $g_{j \partial_m \bar{z}_i \partial_m \bar{z}^{*j}}^i$.

4.4 The potential term

Next consider the auxiliary field term $\mathcal{L}_{aux.}$ of (4.29):

$$\mathcal{L}_{aux.} = 2 \mathcal{F}_J^I h_I h^{*J} + (3 z_0^2 h_0 g + g^i h_i z_0^3 + h.c.) . \quad (4.39)$$

This produces (minus) the potential after elimination of the auxiliary fields h_I . But we have to fix the $\underline{f}$ -gauge freedom before that, because $\mathcal{F}_J^I$ is not invertible, owing to the presence of this gauge invariance; indeed, the $\underline{f}$ -transformation of the additional gauge (3.10), $\delta(\underline{f})h_I = -i\tilde{n}_I z_I$ (not summed) is just the zero eigenvalue vector of the matrix $\mathcal{F}_J^I$, since the identity (4.5c) leads to

$$\mathcal{F}_J^I \tilde{n}_I z_I = 0 . \quad (4.40)$$

The simplest way to fix the $\underline{f}$ -gauge would be to set

$$h_0 = 0 . \quad (4.41)$$

With this gauge condition adopted, one can immediately eliminate the other auxiliary fields h_i in $\mathcal{L}_{aux.}$ and obtain the potential V :

$$V = -\mathcal{L}_{aux.} = 2^{-1} (\mathcal{F}^{-1})_j^i g_i g^{*j} |z_0|^6 \quad (4.42)$$

But this form of potential looks different from that of the old minimal case [1]:

$$V^{old} = -e^{-g} [g^i (g^{-1})_i^j g_j + 3] . \quad (4.43)$$

Of course one could show the direct equivalence of (4.42) to (4.43) by a clever use of the additional gauge invariance property of $\mathcal{F}$, (4.5c) or (4.40). Instead of trying to do so, we take a short cut. We change the $\underline{f}$ -gauge condition and adopt the following particular condition in place of (4.41):

$$(\mathcal{F}^I + e^{i\theta} \sqrt{\Phi/\mathcal{F}_{cc}} \mathcal{F}_c^I) h_I = -3(z_0^*)^3 g^*/2 . \quad (4.44)$$

Here θ is a quite arbitrary phase for our purpose, which assures that we can always adopt this form of gauge condition; indeed, the left-hand side is variant under $\underline{f}$ -gauge transformation unless the ($\underline{f}$ -gauge invariant) coefficient of h_I in (4.44) is zero as a vector with I-index. But if it were zero for a value of θ , we could take another value for which it is non-zero.

Now we rewrite the quadratic part of (4.39) by using (4.44):

$$\begin{aligned}
 2 \mathcal{F}_J^I h_I h^{*J} &= 2 (\mathcal{F}_J^I - \mathcal{F}_c^I \mathcal{F}_{cJ} / \mathcal{F}_{cc}) h_I h^{*J} + 2 |\mathcal{F}_c^I h_I|^2 / \mathcal{F}_{cc} \\
 &= -2 \Phi_J^I h_I h^{*J} + \frac{2}{\Phi} |\Phi^I h_I - \frac{3}{2} (z_o^*)^3 g^*|^2 \quad \left(\begin{array}{l} \because (4.35), (4.36) \\ (4.44) \end{array} \right) \\
 &= -2 (\Phi_J^I - \Phi^I \Phi_J / \Phi) h_I h^{*J} + \frac{9}{2\Phi} |g z_o^3|^2 \\
 &\quad - \frac{3}{\Phi} (\Phi^I h_I g z_o^3 + h.c.) .
 \end{aligned} \tag{4.45}$$

Noticing that the $I = 0$ part of the last term is just cancelled by the contribution $3z_o^2 h_0 g$ from the superpotential term, and using (4.38) and $\Phi = -\mathcal{F} = 3/2$, we find that (4.39) becomes

$$\mathcal{L}_{aux.} = -\mathcal{G}_J^I h_I h^{*J} + [h_i (g^i - 2\Phi^i g) z_o^3 + h.c.] + 3|g z_o^3|^2 . \tag{4.46}$$

Since (4.32) and (4.35b) show that

$$g^i - 2\Phi^i g = -g \left(3 \frac{\Phi^i}{\Phi} - \frac{g^i}{g} \right) = -g \mathcal{G}^i ,$$

the elimination of h_i leads to

$$V = -\mathcal{L}_{aux.} = -|g z_o^3|^2 [\mathcal{G}_i (\mathcal{G}^{-1})^i_j \mathcal{G}^j + 3] . \tag{4.47}$$

This agrees exactly with the old minimal result (4.43) since

$$e^{-\mathcal{G}} = \left(\frac{3}{\Phi} \right)^3 |g|^2 = |g z_o^3|^2 \tag{4.49}$$

by (4.32) and (4.34).

5. CONCLUSIONS

By using conformal tensor calculus we covered all auxiliary field formulations of $N = 1$ supergravity coupled to matter systems. We concentrated mainly on the relation between old and new minimal sets of auxiliary fields. We showed that by a duality transformation, all matter couplings in the new minimal formulation are equivalent to all R-invariant matter interactions in the old minimal framework.

One of the outcomes is a complete understanding of the non-positiveness of the potential in the new minimal formulation.

The result of this investigation is that the new minimal as well as the non-minimal auxiliary field formulations do not lead to physical theories which are not already contained in the old minimal auxiliary field framework.

Finally we note that equivalence proofs between the old and new minimal formulations were given, for the pure supergravity sector, in ref. [20] using the superfield formalism and in ref. [21] in terms of a new tensor calculus. They did not include interactions of matter, i.e. chiral and gauge multiplets.

Therefore they did not clarify to what extent such an equivalence holds in general.

The proof of the equivalence between supergravity-matter coupled systems in different auxiliary field formulations was in fact the main motivation of the present investigation.

As far as the Fayet-Iliopoulos term is concerned it is also important to mention that the steps performed in section 2 to derive eq. (2.17) cannot be done for a vanishing superpotential function $g(S_i) = 0$.

In this case, using simple superspace arguments, it is easily seen that the final Lagrangian given by the sum of eqs. (4.17), (4.19) and (4.20) of ref. 1 is obtained by the following substitution rules:

- i) $e^{-g}, e^{-g/2} \Rightarrow 0$
- ii) $g^i_{\nabla_m} z_i \Rightarrow J^i_{\nabla_m} z_i + \frac{3}{2} i g_R^R \left(g^i_{\nabla_m} z^{*i} \Rightarrow J^i_{\nabla_m} z^{*i} - \frac{3}{2} i g_R^R \right)$
- iii) $\tilde{g} g^i_{T_i}{}^{\alpha j} z_j \Rightarrow \tilde{g} J^i_{T_i}{}^{\alpha j} z_j - \frac{3}{2} g_R^{\alpha} (J = 3 \log \frac{\phi}{3}, J^i = \frac{\partial J}{\partial z_i})$
- iv) for other derivatives of g not in the form given by ii) and iii), replace $g \Rightarrow J$.

The same substitution rule applies to the field transformation laws (4.24) of ref. 1. Note that the substitution rule ii) produces new minimal gauge couplings

of the R-gauge field to the gravitino, gauginos and chiral fermions (but not to the scalars) which could be eliminated through chiral redefinitions if $g(z)$ would be non-zero.

Some models discussed in the literature fall in this class [22]. However the most interesting situation with broken R-symmetry and supersymmetry and without cosmological constant requires a non-vanishing superpotential.

As a final comment we mention that our general Lagrangian (2.1) or (3.8) seem to imply that only gauge vector multiplets are treated. However, also vector multiplets without a gauge invariance, i.e. massive matter vector multiplets are actually included. Such vector multiplets are "gauge equivalent" [10] to a vector gauge multiplet with an Abelian gauge invariance plus one chiral multiplet transforming under this symmetry. E.g. in the old minimal formulation, an arbitrary function $[S_0 \bar{S}_0 \chi(V)]_D$ as Lagrangian is gauge equivalent to $[S_0 \bar{S}_0 \chi(\frac{1}{2} \ln \bar{S} e^{2V} S)]_D$ in our formulation. This is evident if one chooses the gauges $S = 1$ for all Λ -transformations. As S is the only chiral multiplet transforming under the symmetry, $\bar{S} e^{2V} S$ is the only invariant combination. Having already chosen Wess-Zumino gauges in our final expressions, we just impose there $z = ie^C$ to fix the remaining Abelian symmetry. The Lagrangian is then equal to the result obtained in [23]. One obtains massive vector multiplets with the right sign of kinetic and mass terms if

$$\mathcal{H}'' = \frac{3}{2} \left(\frac{\chi''}{\chi} - \left(\frac{\chi'}{\chi} \right)^2 \right) < 0$$

In particular, this is satisfied by a term which looks as a Fayet-Iliopoulos term $[\chi(V) = V]$, but which has no gauge invariance for the vector.

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