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SIMULATION OF QCD JETS INCLUDING SOFT GLUON INTERFERENCE

G. Marchesini

Istituto di Fisica dell'Università di Parma
and
INFN, Sezione di Milano, Italy

and

B.R. Webber

CERN -- Geneva

ABSTRACT

We present a new Monte Carlo simulation scheme for jet evolution in perturbative QCD which takes into account the results of recent analyses of soft gluon interference. Therefore, this scheme accounts correctly not only for the leading collinear singularities, as in previous schemes, but also for leading infra-red singularities. In this first paper we study the basic features of gluon jet evolution such as: i) the interference effects and the corresponding depletion of the parton distributions in the soft region; ii) the approach to asymptopia; iii) the efficiency of colour screening (preconfinement), which has been questioned recently by Bjorken.

^{*}) On leave from the Cavendish Laboratory, University of Cambridge and Emmanuel College, Cambridge, U.K.

1. - INTRODUCTION

Quantum chromodynamics (QCD) is considered a good candidate for a theory of strong interactions but the difficulty of reliable calculations makes the necessary tests and predictions rather scarce. Asymptotic freedom suggests a domain of phenomena (hard scattering processes) in which perturbation theory leads to reliable results. For instance, in e^+e^- collisions at high energy Q there are techniques which, in the perturbative expansion of inclusive distributions for off-shell partons (quarks and gluons), allow one to sum the contributions of all leading collinear singularities, i.e., all leading logarithmic terms of the type $[\alpha_s(Q^2) \ln Q^2/Q_0^2]^n$. Here Q_0 is the off-shell mass of emitted partons, which provides the cut-off for the collinear singularities and is such that the perturbative expansion is still justified, i.e.,

$$\alpha_s(Q_0^2)/\pi < 1 \quad . \quad (1.1)$$

With this technique the asymptotic parton inclusive distributions can be reliably computed, apart from the regions near the kinematical boundaries in x , the fraction of energy, i.e., for $x \approx 0$ and $x \approx 1$, where infra-red (IR) singularities arise.

An important feature of these calculations is the validity of a simple semiclassical picture, essentially that of the parton model¹⁾. This may be better understood^{2),3)} by working in a physical gauge, in which the leading collinear logarithms come from skeleton tree diagrams with no interference and the relevant phase space corresponds to the strong ordering of the virtual masses of partons in the tree. Such a picture suggested⁴⁾ a way to study the IR sensitive regions by taking the kinematical constraints into account properly. It was shown that this can be done systematically⁵⁾ by generalizing the jet calculus³⁾, which gives all asymptotic parton distributions in terms of a Markov process. Therefore, in hard processes the emission of off-shell partons can be studied by Monte Carlo simulations, such as those developed along these lines by Odorico and Mazzanti⁶⁾ and by Fox and Wolfram⁷⁾.

All these results are based on the leading collinear log approximation and they are accurate up to corrections of order $\alpha_s(Q^2)$. However, as mentioned above, for quantities such as the inclusive distributions near the kinematical boundaries $x \approx 0$ and 1 these corrections become important even at large Q^2 . In fact, the neglected contributions contain powers of $\ln x$ and $\ln(1-x)$ which are of IR origin since they arise when the energy of emitted gluons vanishes. Recently these soft gluon effects have been extensively studied⁸⁾⁻¹³⁾ and it has been shown that the branching process previously described requires important

modifications. In particular, in the region $x \approx 0$ interference effects arise⁸⁾⁻¹⁰⁾. It is, however, surprising that this typical quantum effect does not destroy the possibility of a semi-classical interpretation for the emission process, and so a (new) Monte Carlo simulation can still be defined.

In this paper we present a Monte Carlo simulation which takes into account this recent analysis of leading IR singularities. In order to describe the modifications with respect to the previous simulations, we recall the main new results obtained in the studies of soft gluon effects.

a. - The region $x \approx 0$

This region is relevant for the study of multiplicity. From the point of view of Monte Carlo simulations a correct treatment of this region is crucial since it provides the bulk of the emission. As mentioned above, it is here that interference effects due to soft gluons become significant. For instance, for the single gluon inclusive distribution in the soft region crossed Feynman diagrams are as important as the ladder graphs even when working in physical gauges.

The interference can be understood on the following basis: when no soft gluons are involved a strong ordering in the virtual masses in the ladder diagrams implies a strong ordering in the emission angles of successive gluons, which prevents interference. But when soft gluons are emitted an ordering in the masses does not necessarily imply ordering of the angles and thus interference can occur. It is actually in this phase space region where the angles are not ordered that the crossed diagrams are as important as the ladder ones. It is a surprising fact that in this non-ordered angular region the interference is completely destructive to leading order. The result is therefore that the distribution is still given to leading order by the ladder diagrams alone, but considered now only in the phase space region where the successive angles of emission are ordered; in the rest of phase space the distribution vanishes. Of course, when no soft gluons are involved one recovers the previous formulation, since in this case, mass and angular ordering are equivalent.

Some physical understanding of the completely destructive interference in the region where the angles are not ordered may be obtained by considering the limiting situation in which the observed soft gluon is emitted at a relatively large angle from a narrow jet of harder gluons. The probability for this non-ordered configuration is negligible for the following reason. To leading order the soft gluon, emitted from any of the harder gluons in the jet, cannot resolve the individual colour sources in the jet, which therefore act coherently as a single octet source. The lack of resolution of the gluons in the jet means that

all possible collinear singularities are cancelled in the coherent sum. This cancellation is of course effective only when the full class of diagrams is considered: for ladder diagrams alone the suppression of this configuration does not occur, since the sum is then incoherent. For the general proof of the cancellation of non-ordered regions we refer to Refs. 8)-10).

As is well known, this restriction of the allowed phase space introduces a drastic modification in the shape of the parton distribution in the soft region $x \approx 0$. In e^+e^- annihilation, for example, the two back-to-back jets produced are separated and the parton distribution should have no central plateau. It turns out in fact that the distribution is asymptotically peaked at the energy value $\omega \sim \sqrt{QQ_0}$. The depletion in the soft region also implies a reduction in the multiplicity.

The analysis described above can be generalized to multiparton emission: the interference effects of soft gluons are taken into account to leading order by the tree diagrams with the phase space restricted to the region where the opening angles are ordered in successive branchings. Therefore, the interference does not destroy the possibility of a semi-classical picture.

b. - The region $x \approx 1$

The single parton distribution in this region approaches the exclusive limit and thus exposes the Sudakov form factor. It has been shown¹¹⁾⁻¹³⁾ that the leading IR singularities for $x \approx 1$ are still given by the skeleton tree diagrams, provided that for each branching vertex i) the energy phase space constraints are properly taken into account and ii) the scale for the strong coupling constant α_s is taken to be the relative transverse momentum q_t rather than the off-shell mass q , so that the argument of α_s vanishes as $x \rightarrow 1$. This prescription leads to the correct form of the Sudakov form factor. As we shall see, the probability of branching in the tree is directly given by a form factor of the Sudakov type. The correct treatment of these IR singularities is therefore essential for a Monte Carlo simulation of the branching process.

Before discussing the Monte Carlo simulation further, let us recall that there are two major QCD effects that are still not included. First, we have considered only the leading IR and collinear singularities: non-leading corrections to these are not correctly included. The other point not considered yet is hadronization: in this formalism only partons of virtual mass Q_0 are emitted. As far as the second point is concerned, our interest in the parton distributions is based on the hope that they can be converted after a smooth momentum convolution into physical distributions for hadrons. The property of preconfinement¹⁴⁾ in QCD jets is the quantitative basis of this hope. This property comes from

the fact that a cluster of emitted partons forming a colour singlet has a total mass distribution which has a damping typical of a form factor of the Sudakov type^{5),14)}.

Recently, Bjorken¹⁵⁾ has questioned the validity of the preconfinement mechanism, especially in the case in which interference is taken into account. The basis for such skepticism is the following: consider e^+e^- annihilation into two jets, which are the result of the radiation from two colour quanta emitted directly by the virtual photon. For physical emission the colour of the two jets has to be screened and according to preconfinement this should be achieved by the formation of a colour singlet cluster containing partons from both jets. In order for this central cluster to have a small mass its constituents must be in the soft region. But the above-mentioned depletion of soft radiation may cause difficulty with this requirement. We can study this question directly with our Monte Carlo simulation and we are able to show that the inclusion of interference does not in fact give rise to special difficulties of this type: the central cluster mass still turns out to be typically small, of order Q_0 . It is crucial for this result to have the correct form of the branching form factor, i.e., a correct treatment of the region $x \approx 1$ [see also Ref. 16)].

In this paper we are mainly interested in studying some basic features of the new Monte Carlo simulation, rather than making comparisons with experiment, which requires further developments. For this reason we limit ourselves to the pure gluonic cascade process and we consider the following points: i) comparison of the new Monte Carlo scheme with the conventional one which does not properly include soft gluon effects, particularly in the soft region where interference is relevant and causes a depletion of the parton density; ii) study of the stability of the asymptotic results and comparison with analytic estimates wherever available; iii) the approach to the asymptotic regime and the relevance of sub-asymptotic corrections; iv) the question of colour screening, which in the purely gluonic case involves the study of the mass spectrum of pairs of colour-connected gluons.

The paper is organized as follows. In Section 2 we discuss the QCD branching process which forms the basis of Monte Carlo simulations. We explain the way in which we formulate a "coherent" branching scheme with soft gluon interference and compare it with the "conventional" scheme used previously. Section 3 presents the Monte Carlo results on the topics listed above, and Section 4 contains some concluding remarks.

2. - QCD BRANCHING

As stated in the Introduction, we consider here for clarity the development of purely gluonic jets (QCD with zero flavours), represented by a branching process like that in Fig. 1. At the end of this section we mention briefly how quarks may be included without any conceptual difficulties. For purposes of comparison we first recall the conventional formulation of the QCD branching process on which previous Monte Carlo simulations have been based^{6),7),17),18)}.

2.1 Conventional branching

Consider a process in which two jets are emitted back-to-back in the centre-of-mass (c.m.) system with total energy Q . The conventional branching process is conveniently described in a light-cone gauge with the gauge vector η parallel to one of the two partons emitted from the source in the c.m. frame. Then to leading order only the parton opposite to η can radiate and start the cascade. The maximum value for the virtual mass of this parton initiating the branching process is given by Q .

In this conventional treatment the subsequent evolution of the jets is characterized by the off-shellness or virtuality of the partons. At each branching a gluon of virtuality q^2 gives two with lower values q_1^2, q_2^2 . The branching continues until some predetermined cut-off scale $q^2 \sim Q_0^2$ is reached. At this point the perturbative evolution is terminated and some phenomenological model for the conversion of partons into hadrons is applied. We shall not consider such models here but limit our attention to the parton distributions that arise from the perturbative phase of the process.

The Monte Carlo simulation is defined by giving for every vertex of the tree, such as that in Fig. 2, the probability distribution P for the off-shell gluon of virtuality $q^2 < q_{\max}^2$ to branch into gluons of momentum fractions z and $(1-z)$, e.g., the light-cone momentum fraction $z = q_1^+/q^+$ where $q^+ = \omega + q_z$. The distribution for the pure gluon case is given by

$$dP(q^2, z) = \frac{dq^2}{q^2} dz P(z) \alpha_s \frac{[z(1-z)q^2]}{2\pi} \cdot \frac{\Delta_s(q_{\max}^2)}{\Delta_s(q^2)} \quad (2.1)$$

where the kinematically allowed region is defined by requiring $q_1^2, q_2^2 \geq Q_0^2$, so that

$$4Q_0^2 < \frac{Q_0^2}{z(1-z)} < q^2 < q_{\max}^2 \quad (2.2)$$

$P(z)$ is the Altarelli-Parisi branching function for the $g \rightarrow gg$ vertex:

$$P(z) = N_c \left[\frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right] \quad (2.3)$$

where N_c is the number of colours. $\Delta_s(q^2)$ is the Sudakov form factor:

$$\Delta_s(q^2) = \exp \left\{ - \int_{4Q_0^2}^{q^2} \frac{dk^2}{k^2} \int_{\varepsilon(k)}^{1-\varepsilon(k)} dz P(z) \frac{\alpha_s[z(1-z)k^2]}{2\pi} \right\} \quad (2.4)$$

where on account of (2.2) $\varepsilon(q)$ is given by

$$\varepsilon(q) [1 - \varepsilon(q)] = Q_0^2 / q^2 \quad (2.5)$$

Note that this form of Δ_s ensures that the probability distribution is properly normalized in the phase space region (2.2). Finally, in (2.1) the argument of α_s has been taken to be $z(1-z)q^2$, which is approximately the relative transverse momentum at the vertex [see Eq. (2.7)]. As we recalled in the Introduction, this gives the correct form of the Sudakov form factor; it also gives the proper behaviour of anomalous dimensions as $N \rightarrow \infty$. Using $\alpha_s(Q^2) = (b \ln Q^2/\Lambda^2)^{-1}$, we have asymptotically

$$\ln \Delta_s(q^2) \sim -\frac{N_c}{\pi b} \left\{ \ln(q^2/\Lambda^2) \ln[\alpha_s(Q_0^2)/\alpha_s(q^2)] - \ln(q^2/Q_0^2) \right\} \quad (2.6)$$

As one proceeds along the tree, the probabilities for the branchings of gluons 1 and 2 are still given by (2.1) where the maximum values $q_{1\max}^2$ and $q_{2\max}^2$ are obtained by requiring that the exact relative transverse momentum of these gluons is real:

$$q_t^2 = z(1-z) \left(q^2 - \frac{q_1^2}{z} - \frac{q_2^2}{1-z} \right) \geq 0 \quad (2.7)$$

The fact that the values of $q_{1\max}$ and $q_{2\max}$ are not independent causes some difficulties for this Monte Carlo scheme, since it leads to ambiguities about how to share the available phase space between the two outgoing lines at each vertex [see Odorico, Ref. 6)].

Neglecting angular correlations due to gluon polarization [which are known¹⁹⁾ to be small for final states of up to four partons], the azimuthal direction of $\vec{q}_t$ is distributed uniformly. This completes the specification of all the probability distributions needed for the conventional simulation of gluon jets.

2.2 Coherent branching

As described in the Introduction, interference effects modify the branching process but they can be taken into account simply by ordering the opening angles of successive branchings. To be specific¹⁰⁾, we introduce for any vertex, such as that in Fig. 2, the angular variable

$$\xi \equiv q_1 \cdot q_2 / \omega_1 \omega_2 \quad (2.8)$$

which reduces to $(1 - \cos\theta_{12})$ when the virtualities of the produced gluons are small compared with their energies. The effect of interference of soft gluons is taken into account to leading order by requiring (see Fig. 3)

$$\xi_1, \xi_2 < \xi < \xi_{\max} \leq 1 \quad (2.9)$$

where ξ_1 and ξ_2 are the angular variables of the eventual branchings of gluons 1 and 2, while $\xi_{\max}$ is the angular variable of the vertex at which the incoming gluon was produced. The virtuality of this gluon is

$$q^2 = q_1^2 + q_2^2 + 2\omega_1\omega_2\xi \quad (2.10)$$

Therefore, as expected, ordering of ξ is equivalent to ordering of virtuality when all the energies ω_i are of the same order, but may differ when some gluon energies become relatively small.

An important difference from the previous scheme is clear in the frame in which the original gluon is moving forwards. In the coherent branching scheme the angular ordering (2.9) implies that the emission of partons is essentially forwards, while in the conventional branching the kinematical boundary (2.2) allows partons to be emitted backwards also.

The Monte Carlo scheme we use is defined as follows: the probability distribution $\mathcal{P}$ for a gluon of energy ω branching, as in Fig. 2, with angular variable ξ into two gluons of energy fractions $z = \omega_1/\omega$ and $(1-z) = \omega_2/\omega$ is given by

$$dP(\xi, z) = \frac{d\xi}{\xi} dz P(z) \frac{\alpha_s}{2\pi} \frac{\Delta(\xi_{\max})}{\Delta(\xi)} \theta(\xi_{\max} - \xi) \theta(\xi - \xi_{\min}) \quad (2.11)$$

where $\xi_{\min}$ is the lower kinematical boundary on ξ defined by the requirement $q_1^2, q_2^2 > Q_0^2$ (see later). The argument of α_s is given by $2z^2(1-z)^2\omega^2\xi$ [$\gamma z(1-z)q^2$, from Eq. (2.10)] and $P(z)$ is the usual Altarelli-Parisi function of Eq. (2.3).

Note that since in this case the gluons are emitted forwards the variable z in $P(z)$ can be taken to be either the fraction of energy, as used here, or the light-cone momentum fraction, as in the previous scheme. In fact, the two variables essentially coincide in the forward hemisphere. Note moreover that the usual gauge cut-off^{2),5)} in $P(z)$, present when one works in a general planar gauge, is not effective when the light-cone momentum fraction is restricted to the forward hemisphere. In the coherent branching formalism therefore one does not need to refer to a specific gauge vector. We choose to define z as the energy fraction because, as we shall see shortly, the exact kinematics of the coherent scheme are simplified by this choice.

As a consequence of the fact that the phase space in (2.11) differs from that in (2.1) the branching form factor in this case is given by

$$\Delta(\xi, \omega) = \exp \left\{ - \int_{\xi_{\min}}^{\xi} \frac{d\xi'}{\xi'} \int_{\varepsilon(\omega, \xi')}^{1-\varepsilon(\omega, \xi')} dz P(z) \frac{\alpha_s [2z^2(1-z)^2\omega^2\xi']}{2\pi} \right\} \quad (2.12)$$

where the integration region is the same as in (2.11) so the probability distribution is correctly normalized. The cut-off $\xi_{\min}$ and consequently the form of $\varepsilon(\omega, \xi)$ are fixed by requiring that the branching is terminated when a gluon would otherwise have virtual mass less than Q_0 . Setting $q_1^2, q_2^2 \geq Q_0^2$ in (2.10) we obtain

$$\omega_1 \omega_2 \xi \geq Q_0^2 \quad (2.13)$$

To ensure that this is the case we require $\omega_1 \sqrt{\xi}, \omega_2 \sqrt{\xi} \geq Q_0$, i.e., $\varepsilon(\omega, \xi) < z < 1 - \varepsilon(\omega, \xi)$ where

$$\varepsilon(\omega, \xi) = \frac{Q_0}{\omega \sqrt{\xi}} \quad (2.14)$$

For the parent gluon we have

$$\omega^2 = (\omega_1 + \omega_2)^2 \geq 4 \omega_1 \omega_2 \quad (2.15)$$

and hence Eqs. (2.9) and (2.13) give

$$\omega^2 \xi_{\max} \geq 4 \omega_1 \omega_2 \xi \geq 4 Q_0^2 \quad (2.16)$$

so

$$\xi_{\min} = 4 Q_0^2 / \omega^2 \quad (2.17)$$

In other words, $\xi_{\min}$ is just the value of ξ at which the energy phase space for the branching vanishes:

$$\varepsilon(\omega, \xi_{\min}) = \frac{1}{2} \quad (2.18)$$

With the cut-offs given by Eqs. (2.14) and (2.17), the form factor (2.12) becomes a function of $\omega^2 \xi$ only. Using the notation $q^2 = \omega^2 \xi$ and $k^2 = \omega^2 \xi'$ (so that q^2 and k^2 are asymptotically proportional to, but not equal to, virtualities), we find

$$\ln \Delta(q^2) = - \int_{4Q_0^2}^{q^2} \frac{dk^2}{k^2} \int_{Q_0/k}^{1-Q_0/k} dz P(z) \frac{\alpha_s [2z^2(1-z)^2 k^2]}{2\pi} \quad (2.19)$$

$$\sim -\frac{1}{2} \cdot \frac{N_c}{\pi G} \left\{ \ln(q^2/\Lambda^2) \cdot \ln[\alpha_s(Q_0^2)/\alpha_s(q^2)] - \ln(q^2/Q_0^2) \right\}$$

[cf. Eq. (2.6)]. As a result of the reduction of the available phase space, we have a branching form factor Δ which is less damped than the one Δ_s of the previous scheme which was just the Sudakov form factor. More precisely, one has

$$\Delta_s(q^2) \sim [\Delta(q^2)]^2 \quad (2.20)$$

This is consistent with the fact that this formalism describes the generation of a forward jet, while the Sudakov form factor relates to the production of both forward and backward jets.

In the angular ordering scheme the phase space regions available to the two outgoing lines at each vertex are uncorrelated: $\xi_1 < \xi$ and $\xi_2 < \xi$ in Fig. 3. This makes the Monte Carlo simulation simpler and more efficient than that for the conventional scheme, in which the kinematical boundaries are correlated via Eq. (2.7).

The coherent branching process starts with the energy E of the initial gluon and the angular variable $\xi \leq 1$ for the first branching. When the process has terminated the outgoing gluons are assigned virtual mass Q_0 . From the variables ω_i , ξ_i and a randomly-chosen azimuthal angle for each branching, the full four-momenta of all the gluons in the tree may be reconstructed. Furthermore, one may prove as follows that all these momenta are real, i.e., the branching process always evolves within the physical phase space.

To prove this it suffices to show that a real relative transverse momentum q_t may be reconstructed at every vertex. In the notation of Fig. 2, we have

$$q_t^2 = [\vec{q}_1^2 \vec{q}_2^2 - \omega_1^2 \omega_2^2 (1-\xi)^2] / \vec{q}_t^2. \quad (2.21)$$

(In this scheme, "transverse" means perpendicular to the three-momentum $\vec{q}$ of the parent gluon.) A sufficient condition for q_t to be real is that

$$q_i^2 / \omega_i^2 \leq \xi \leq 1 \quad (i=1,2). \quad (2.22)$$

The upper limit on ξ is ensured by our initial conditions and the ordering of ξ . We establish the lower limit by induction backwards from the end of the tree. From the definition of the cut-off (2.14), it holds for any vertex at which both outgoing gluons are external lines ($q_i^2 = Q_0^2$). Now we show that if it holds for a given vertex it holds also for the preceding one in the tree. In the notation of Figs. 2 and 3 we have to show that $q^2 / \omega^2 \leq \xi_{\max}$. From (2.10) and (2.22) .

$$\begin{aligned} q^2 &= q_1^2 + q_2^2 + 2\omega_1\omega_2\xi \leq \omega_1^2\xi + \omega_2^2\xi + 2\omega_1\omega_2\xi \\ &= \omega^2\xi \leq \omega^2\xi_{\max}. \end{aligned} \quad (2.23)$$

Hence (2.22) holds at every vertex and the proof is complete.

Notice that our choice of initial conditions such that $\xi \leq 1$ ($\theta_{12} \leq \pi/2$) for the first branching amounts simply to a choice of Lorentz frame in which the branching process is viewed. For the reasons discussed earlier, and to simplify the kinematics, we require the process to take place essentially only in the forward hemisphere. After reconstructing the four-momenta we may of course transform the event to any other frame; the invariant properties of the final state should be insensitive to the particular frame in which it was generated. This will in fact prove a useful consistency check on our procedure.

Although we have explained the coherent branching treatment for purely gluonic processes, there is no conceptual difficulty in applying it to processes involving both gluons and quarks. There will then be several types of vertices, each with its own well-known branching function analogous to (2.3), and the relative frequencies of the different types will be determined by the corresponding form factors. As far as the quark lines are concerned, ordering of opening angles and virtualities are equivalent, differing only in an asymptotically unimportant region of phase space, since the branching functions have no soft-quark singularities.

3. - MONTE CARLO RESULTS

We have used the coherent branching scheme explained above as the basis of a Monte Carlo programme to generate gluon jets from a wide variety of initial conditions. Because it involves an infra-red cut-off Q_0 and a running coupling constant,

$$\alpha_s(Q^2) = \frac{4}{11 \ln(Q^2/\Lambda^2)} \quad (\text{for zero flavours}) \quad (3.1)$$

any such scheme has two dimensional parameters, Q_0 and Λ , whose dimensionless ratio determines the maximum value of α_s/π which can occur in the perturbative branching process. We have chosen $Q_0/\Lambda = 2$; since the minimum argument of α_s in our scheme is $\frac{1}{2}Q_0^2$ this means $(\alpha_s/\pi)_{\max} \approx 0.5$, which seems a reasonable value at which to terminate branching. In order that our energy scale should correspond roughly to GeV, we take

$$\Lambda = 0.2, \quad Q_0 = 0.4 \quad (3.2)$$

Our choice of Q_0 does not have a very strong motivation at this stage, since the value to be adopted depends primarily on the model to be used for hadronization, which we do not consider here. For a larger value of Q_0/Λ the

number of partons produced from given initial conditions will be reduced and the masses of colour-singlet combinations of partons will increase. Consequently, such a value would be appropriate for a hadronization scheme in which massive clusters decay into many hadrons, whereas our choice is appropriate for a scheme involving light clusters, each giving only a few hadrons. Any hadronization model must involve a parameter Q_0 defining the mass scale at which it takes over from the perturbative phase of jet evolution. Ideally, the final hadron distributions would be rather insensitive to Q_0 , even though the perturbative and hadronization schemes separately depend strongly on it, because it only defines the somewhat arbitrary scale at which our description of the evolution process changes.

3.1 A typical event

The gluon tree in Fig. 1 actually represents a fairly typical event generated by the programme with the following initial conditions: total energy $E = 300$, initial opening angle $= 90^\circ$ (i.e., $\xi = 1$ at the first vertex). Figure 4 shows the values generated for the energy ω of each line and the angular variable ξ at each vertex. A number of general features are apparent:

- i) the energy partition at a branching is usually very asymmetrical, because of the soft divergences of the branching function $P(z)$ at $z = 0$ and 1 ;
- ii) because of i) it is quite probable that one of the outgoing gluons from a given branching will have energy $\omega < 2Q_0/\sqrt{\xi}$, which means that it cannot branch further [see Eq. (2.17)] and becomes an external line, with mass Q_0 ;
- iii) there are also external lines with energies greater than $2Q_0/\sqrt{\xi}$, which had the opportunity of further branching but did not do so. We see by integration of the branching probability distribution (2.11) that the probability of this is given directly by the form factor $\Delta(\xi, \omega)$ in Eq. (2.12);
- iv) the cascade has an "inside-outside" structure, with the softer gluons tending to be produced earlier than the harder ones. In our scheme this may be seen rather directly from the phase space constraint (2.14): the range of available energies for gluons produced at a given vertex extends down to $Q_0/\sqrt{\xi}$, so softer gluons can come from vertices with higher ξ , i.e., those earlier in the tree.

From the ω and ξ values in Fig. 4 (and a random azimuthal angle for each vertex) we can reconstruct the four-momenta in the tree, according to the procedure outlined in the previous section. Figure 5 shows the virtual masses $\sqrt{q_1^2}$ obtained in this way. In this figure the external lines (all with mass $Q_0 = 0.4$) are labelled in order of emission, assuming that the time interval associated with each internal line is ω_1/q_1^2 . Note that the total invariant mass Q of the jet (not fixed by the initial conditions) turned out in this case to be 120.

Finally, in Figs. 6 and 7 we show the reconstructed three-momenta, projected onto the x - z plane, in two different reference frames. Figure 6 is in the frame in which the event was originally generated: the initial gluon travels along the $+z$ direction with energy $E = 300$ and the first branching defines the x - z plane. Thus any momenta out of the plane tend to be small (all $p_y < 2$ in this case). Remembering that the gluons are labelled in order of emission, we see the inside-outside structure very clearly. The 90° initial opening angle in this frame is reflected in the angle between the leading gluons, which are the last emitted from each of the two subjects formed in the initial branching.

Figure 7 shows the event in its own c.m. frame, with the z axis along the thrust axis of the final state, i.e., along the direction $\vec{n}$ which maximizes

$$T = \sum_i |\vec{p}_i \cdot \vec{n}| / \sum_i |\vec{p}_i| \quad (3.3)$$

The two subjects now appear back-to-back and the event is best described as two jet production from a static, vector, colour octet source of mass $Q = 120$ (the initial virtual gluon). In this frame all reference to the energy E at which the event was generated has disappeared; a useful check on the Lorentz invariance of our scheme, which we discuss later, is to verify that all distributions in this frame depend only on Q and not on E .

Two further interesting features of Fig. 7 are:

- v) although interference depletes the soft radiation, some soft gluons are still present; and
- vi) there are fluctuations in which gluons emitted earlier in the tree (such as no.1) appear at large angles or even in the wrong hemisphere. These observations will prove important in our discussion of the colour structure of final states.

3.2 Multiplicity distribution

The multiplicity of produced gluons is of great interest because it may be assumed to be roughly proportional to the hadron multiplicity and is known to be strongly affected by interference effects⁸⁾⁻¹⁰⁾. We have studied the gluon multiplicity from two points of view:

- i) as a function of jet energy E when the initial angular variable ξ is distributed according to Eq. (2.11) with $\xi_{\max} = 1$;
- ii) as a function of the jet invariant mass Q . In this case we find, as required for Lorentz invariance, that the multiplicity obtained has a negligible dependence on the initial conditions.

The purpose of the first study is primarily for comparison with asymptotic analytic predictions, which can be deduced from differential equations for the multiplicity moments³⁾⁻⁵⁾ under these initial conditions⁹⁾. For the average multiplicity we expect asymptotically^{*)}

$$N(E) \equiv \langle n \rangle_{\text{fixed } E} \propto \left(\ln \frac{E}{\Lambda} \right)^{-\frac{1}{4}} \exp \left(\frac{48}{11} \ln \frac{E}{\Lambda} \right)^{\frac{1}{2}}. \quad (3.4)$$

As shown in Fig. 8a, this form agrees with the Monte Carlo results quite well over the entire energy range that we studied ($1 < E < 10^4$), leaving little room for subasymptotic corrections to Eq. (3.4). On the other hand, for the ratio of multiplicity moments $R_2(E) \equiv \langle n(n-1) \rangle / \langle n \rangle^2$ the results in the same energy range (Fig. 8b) are far from the asymptotic prediction, $R_2 = 4/3$. We have computed the first subasymptotic correction to this prediction for our scheme and find that it is indeed large:

$$R_2(E) = \frac{4}{3} - \frac{10}{9} \left(\frac{48}{11} \ln \frac{E}{\Lambda} \right)^{-\frac{1}{2}} + O \left(\ln \frac{E}{\Lambda} \right)^{-1}. \quad (3.5)$$

This is much more satisfactory (dashed curve), but further corrections are clearly important and we conclude that for R_2 (and for higher multiplicity moments) the leading asymptotic predictions are of little relevance at any attainable energy.

The multiplicity at fixed jet mass Q seems more physical by virtue of its Lorentz invariance, but detailed analytical predictions are more difficult to derive. The leading asymptotic behaviour may be seen from Eq. (2.10) to correspond to two uncorrelated jets, each with initial conditions as in the discussion above and with energy of order Q , as might be expected from our examination of a typical event in the c.m. frame in the previous subsection. Consequently we expect, at sufficiently high Q ,

$$\bar{N}(Q) \equiv \langle n \rangle_{\text{fixed } Q} \sim 2 \langle n \rangle_{\text{fixed } E=Q} = 2 N(Q) \quad (3.6)$$

^{*)} The power of the logarithm in the pre-exponential factor of Eq. (3.4) is sensitive to non-leading QCD corrections [see Ref. 20)]. The value of $-1/4$ is the one expected from the leading terms as incorporated in our branching scheme.

and

$$\bar{R}_2(Q) \equiv \left[\frac{\langle n(n-1) \rangle}{\langle n \rangle^2} \right]_{\text{fixed } Q} \sim \frac{1}{2} [1 + R_2(Q)] \quad (3.7)$$

As shown in Fig. 9, these two asymptotic predictions in fact agree surprisingly well with the Monte Carlo results for the entire range of masses that we studied ($Q \lesssim 10^4$).

A second surprise is that (either at fixed E or at fixed Q) the ratio of the mean to the dispersion for the multiplicity distribution is almost energy-independent, corresponding to the empirical formula

$$\bar{R}_2(Q) \approx \text{constant} - \frac{1}{\bar{N}(Q)} \quad (3.8)$$

shown by the dotted curve in Fig. 9. A related observation is that KNO scaling²¹⁾ holds very well (Fig. 10). In view of the importance^{*} of the subasymptotic terms in $R_2(E)$, however, this must be a transient phenomenon unrelated to the truly asymptotic KNO scaling predictions of Refs. 3)-5) and 9).

The effect of soft gluon interference is seen very clearly in the exponent of the average multiplicity: the 48 in Eq. (3.4) would be 96 in the conventional branching scheme, leading to a much greater asymptotic rate of increase. However, other authors have found^{6),7)}, and we confirm, that in the conventional approach the asymptotic exponential behaviour is not completely exposed until extremely high energies, and this leads to a rate of increase that is not much greater than that of Eq. (3.4) at the energies we have studied.

3.3 Inclusive distributions

For the presentation of results on momentum distributions one has to decide on a definite frame of reference and for this purpose we have chosen the second frame discussed in subsection 3.1, i.e., the c.m. frame with the z axis along the thrust axis. The distribution of the c.m. energy fraction $x = 2\omega'/Q$ (ω' denoting the c.m. energy) is shown in Fig. 11 for $Q = 10^2$ and 10^3 . The figure also illustrates the Lorentz invariance of the results: to a good approximation they depend only on Q and not on the energy E at which the event was originally generated.

*) -----
The second term in Eq. (3.5) still represents a correction of 0.1 when $E = 10^{12}$.

The reduction of the exponent of the average multiplicity comes from the cancellation of soft gluon production, as explained in the Introduction. This is seen very clearly when Fig. 11 is compared with the form of the x distribution obtained from the conventional Monte Carlo procedure (Fig. 12). In the latter case the distribution peaks very near its kinematical lower limit, $x = 2Q_0/Q$, whereas the coherent distribution peaks at much higher $x \sim \sqrt{Q_0/Q}$. The depletion of the soft region is illustrated further in Fig. 13a, which shows the integral of the distribution for $Q_0 \leq \omega' \leq 2$ ($= 5Q_0$) together with the value at the peak, $\omega' \sim \sqrt{QQ_0}$. The peak value rises nearly as rapidly as the average multiplicity: asymptotically⁸⁾⁻¹⁰⁾

$$\left(\frac{x}{\sigma} \frac{d\sigma}{dx} \right)_{\text{peak}} \propto \left(\ln \frac{Q}{\Lambda} \right)^{-\frac{3}{4}} \bar{N}(Q) \quad (3.9)$$

(dotted curve), whereas the soft region $\omega' \leq 2$, although contributing a slowly increasing absolute multiplicity, represents a rapidly vanishing fraction of the total. One would expect that at sufficiently high Q^2 the soft gluon depletion would give rise to a corresponding depletion of the hadronic x distribution at small x , but the details of the observability of such an effect clearly depend on the model of hadronization and are therefore outside the scope of the present study.

Note that in both the conventional and coherent schemes the x distribution at $x > 0.1$ exhibits a slow decrease with increasing Q^2 , which is the usual QCD scaling violation. We have checked that the Q^2 dependences of the x^N moments ($N \geq 1$) of the distribution in the coherent scheme are consistent with the expected anomalous dimensions.

Figure 14 shows the gluon rapidity distributions for the same values of Q and E as in Fig. 12. Corresponding to the depletion at small x , there is a dip in the central rapidity region and a peak around $|y| \approx 0.7$ whose position is fairly insensitive to Q and to the cut-off parameter Q_0 . In Fig. 13b we show the values of the distribution in the dip and at the peak as functions of Q . In contrast to the low- x region, the rapidity distribution around $y = 0$ continues to increase as rapidly as the average multiplicity: as shown by the dotted curve, our results are consistent with an asymptotic behaviour

$$\left(\frac{1}{\sigma} \frac{d\sigma}{dy} \right)_{\text{dip}} \propto \bar{N}(Q) \quad (3.10)$$

To understand this we must remember that the central rapidity region is populated not only by soft gluons but also by wide-angle gluons with non-zero transverse momentum. For example, in the typical event of Fig. 7, together with the soft gluons 2, 3, 5 there is one (no. 1) that is not so soft but has low rapidity by virtue of its large angle of emission. It managed to reach such a large angle because it came from a massive subjet, corresponding to an opening angle greater than 90° in the c.m. frame. Although these large mass fluctuations, characteristic of the QCD branching process, eventually provide the bulk of the population in the low rapidity region, we see that they do not suffice to mask the small y dip completely.

The gluon transverse momentum distribution is shown in Fig. 15. Again we see the satisfactory degree of Lorentz invariance of the results (i.e., independence of energy E at fixed mass Q). It is interesting that this depends crucially on the fact that p_T is defined relative to a physical axis of the final state (in this case, the thrust axis) and not to some axis intrinsic to the branching process (e.g., the axis of the first branching in the c.m. frame). This is analogous to Odorico's observation⁶⁾ that one must use a physical axis rather than an axis related to the parton initiating the branching in the conventional scheme.

From the p_T distribution we can compute the average gluon transverse momentum as a function of Q . As shown in Fig. 16, it is in good agreement with the asymptotic prediction

$$\langle p_T \rangle \propto \alpha_s(Q^2) Q / \bar{N}(Q) \quad , \quad (3.11)$$

with little sign of important subasymptotic terms.

3.4 Colour structure

One of our main aims in this Monte Carlo study is to investigate the colour structure of the multigluon states produced. In particular, we wish to see whether the preconfinement of colour, i.e., the formation of low-mass colour singlet clusters in the perturbative phase of jet evolution, is consistent with the coherent branching scheme that we are advocating. A possible inconsistency, raised by Bjorken¹⁵⁾ and mentioned in the Introduction, seems to arise when we consider the neutralization of the colour charge of the two subjets produced in a given branching, e.g., at the first vertex of Fig. 1. The contents of the two subjets are approximately confined by angular ordering to two cones with opening angles determined by the branchings that follow the one in question. In the c.m. frame these two cones will be back-to-back and each subjet will be a colour non-singlet (an octet in the purely gluonic case). Therefore, at least one

colour singlet cluster must contain partons from both cones. Since the soft component of the parton distribution is depleted by interference, one might suspect that it is difficult to make such a cluster with low mass or even with a mass that does not increase in proportion to the c.m. energy Q . We shall refer to this cluster as the "central" cluster for the branching in question.

To discuss this problem we need some model for the formation of colour singlets from our multigluon "final states". As long as this model does not disrupt too much the connection between colour and momentum flow, its details do not matter because we are concerned only with qualitative features of the results. We have therefore used the simplest possible model, along the general lines proposed in Refs. 17) and 18), in which each gluon is split into a quark and an antiquark, each carrying one of the gluon's colour indices and half of its four-momentum. A colour singlet cluster is then formed from each $q\bar{q}$ pair sharing a colour index; the mass of the cluster is just one half of the invariant mass of the colour-connected gluon pair sharing that index.

This process is illustrated in Fig. 17 for the typical event discussed in subsection 3.1. The arrangement of the external lines has already been chosen such that the colour structure is planar, so adjacent external lines form colour singlets $b\bar{b}$, cc , ..., $k\bar{k}$, whose masses are as shown in the figure. The central cluster, involving one gluon from each subjet, is $e\bar{e}$; we see that its mass is the largest in the event but still quite small.

Carrying out this procedure on a large sample of Monte Carlo events at various values of Q , we show in Fig. 18 the results on the masses of various categories of colour singlet clusters. To avoid giving undue emphasis to the tail of the mass distribution, we show the mean value of the log of the mass for each category. There is a slight non-covariance in the masses obtained, especially for the central cluster: the widths of the bands in the figure indicate the kind of variation obtained when the energy at which the event was generated varies from $E \sim Q$ to $E \sim 10Q$. The effect is of the order of ± 0.1 in $\ln m$, i.e., $\pm 10\%$ in the cluster mass, which we do not regard as particularly worrying, although we would like to understand it better.

We see from Fig. 18 that, at least for our choice of Q_0 and Λ , the typical cluster masses are small, of the order of unity, as in the event in Fig. 17. The central cluster masses do tend to be higher than those of other clusters but they certainly do not increase in proportion to Q . All categories of cluster masses seem to be approaching reasonably small constant values at high energies. We conclude that there is no difficulty with the preconfinement of colour in the coherent branching scheme at any attainable energy.

The consistently higher average mass of the central cluster may be interpreted in the following way. We see from Fig. 17 that every cluster is a "central" cluster with respect to some vertex of the tree. Since the branching is a Markov process the portion of the tree starting at a given vertex is identical to a whole tree at lower Q^2 . Therefore, if the typical cluster mass approaches its asymptotic limit from below as Q^2 increases it is inevitable that the central cluster for the first vertex (the highest Q^2) will tend to have the highest mass.

Such an effect is also seen when we compare the masses of the clusters of highest and lowest rapidity in the final state, which are also shown in Fig. 18. Low-rapidity clusters tend to be produced early in the tree and have higher masses, while fast clusters come from the ends and are less massive. All these facts are quite well illustrated by the typical event of subsection 3.1 and Fig. 17.

We should mention that the association between higher cluster masses and earlier vertices of the tree is not a feature peculiar to the coherent branching process: we find that qualitatively identical results to Fig. 18 are obtained in the conventional scheme without angular ordering. The only quantitative difference is a slight reduction in the overall scale of cluster masses (about 20% for our values Q_0 and Λ).

In conclusion, we see from these results that although the soft radiation is depleted by interference effects, there are still enough soft partons to permit local screening of the colour charge between the two back-to-back subjects in the c.m. frame. As we saw in Fig. 13a, the mean number of gluons below a fixed energy is actually slowly increasing with Q even though this region provides an asymptotically vanishing fraction of the total gluon multiplicity. Furthermore, we recall from Fig. 13b that there is a more rapidly increasing number of gluons which are not quite so soft but have large emission angles in the c.m. frame (i.e., low rapidities); these are also available to form central clusters of moderate mass. Of course, in any branching scheme there will always be a few events (e.g., those with total parton multiplicity equal to two) with high-mass clusters, giving the mass distribution a long tail. Any hadronization model will therefore have to be capable of dealing with the fragmentation of very massive clusters, but the general features of the hadronic final states should not depend much on how this is done because such objects are extremely rare.

4. - CONCLUSIONS

We have presented a Monte Carlo scheme for gluon jet simulation which incorporates recent insights into interference effects by ordering successive opening angles (ξ variables) in the QCD branching process. The procedure is in fact somewhat simpler than the earlier one ignoring interference, which

suffered from ambiguities and inefficiencies in the implementation of phase space constraints. The new scheme should provide a good basis for efficient, stable and fast^{*)} simulation of the perturbative phase of jet development.

From the Monte Carlo results we found:

- i) although not manifestly covariant the scheme, in fact, produces results with a satisfactory degree of Lorentz invariance;
- ii) the jets generated have an "inside-outside" structure with softer partons tending to be emitted earlier and harder ones later;
- iii) where asymptotic analytic predictions exist, they are usually in fairly good agreement with the Monte Carlo results for accessible energies;
- iv) the higher moments of the multiplicity distribution are an exception to iii). In this case there are large, slowly-varying subasymptotic corrections and the predicted asymptotic form of the multiplicity distribution is not reached at any attainable energy;
- v) the expected depletions of the low- x and low-rapidity regions are observed but the mean multiplicity of gluons in these regions continues to grow with energy;
- vi) as a result of v) the scheme encounters no special difficulties with the preconfinement of colour, even in the central region.

In view of these encouraging results, the obvious next steps are the inclusion of quarks and a model for hadron production from the colour singlet clusters discussed in Section 3.4. It should then be possible to confront this theoretical approach with the wide variety of data on jets that are now being accumulated in all kinds of colliding-beam experiments.

We acknowledge with great pleasure many long and fruitful discussions with D. Amati and G. Veneziano.

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Our present program takes about 0.3ms on the CERN CDC 7600 computer to generate all the colour-singlet four-momenta in a jet of multiplicity n .

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FIGURE CAPTIONS

- Fig. 1 : Gluon branching process.
- Fig. 2 : Notation for a gluon branching.
- Fig. 3 : Ordering of angular variables according to Eq. (2.9).
- Fig. 4 : Energies ω and angular variables ξ for the typical event with the tree structure given in Fig. 1. The numbers (300, 25, 275, ...) next to the lines are their energies and those (1.0, 0.30, 0.27, ...) after the vertices are the corresponding values of ξ .
- Fig. 5 : Virtual masses of internal lines in Fig. 4, reconstructed according to Eq. (2.10). External lines are labelled according to their sequence of emission.
- Fig. 6 : Gluon momenta in the typical event, projected onto the x - z plane. Gluons are labelled in order of emission, as in Fig. 5. Note that the p_x and p_z scales are different.
- Fig. 7 : Same as Fig. 6, but in the c.m. frame with the z axis along the direction of maximum thrust.
- Fig. 8 : Multiplicity moments versus jet energy E , with initial conditions as explained in the text.
- Fig. 9 : Multiplicity moments versus jet mass Q .
- Fig. 10 : The KNO-scaled multiplicity distribution $\psi(v) = \langle n \rangle^{-1} P(n = v\langle n \rangle)$ at two different values of the jet mass: $Q = 63$ ($\langle n \rangle = 12.5$) and $Q = 1000$ ($\langle n \rangle = 37.5$). Discrepancies are consistent with statistical fluctuations (about 2000 events at each value of Q).
- Fig. 11 : Single gluon inclusive distribution $(x/\sigma) (d\sigma/dx)$ where x is the c.m. energy fraction. Monte Carlo results for the coherent branching scheme at various values of the jet mass Q and energy E as indicated. The curves are only to guide the eye.
- Fig. 12 : As Fig. 11, but for the conventional branching scheme, without interference effects.

- Fig. 13 : Dependence on jet mass (c.m. energy) Q of various features of the x and rapidity distributions in the coherent branching scheme.
- Fig. 14 : Rapidity distribution (symmetrical, and therefore shown as a function of $|y|$ only).
- Fig. 15 : Distribution of gluon momentum transverse to the thrust axis.
- Fig. 16 : Mean gluon transverse momentum versus c.m. energy Q .
- Fig. 17 : Colour index structure of the tree diagram for the typical event shown in Figs. 4-7. The numbers are the masses of the colour singlet clusters associated with the corresponding colour index lines as explained in the text. (The mass of the non-singlet $a\bar{1}$ is 0.8.)
- Fig. 18 : Mean values of the log of the mass for various categories of colour singlet clusters. The bands show the approximate regions in which the Monte Carlo results lie for various jet energy values at a given value of the jet mass Q .

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